



Středoevropský technologický institut
BRNO | ČESKÁ REPUBLIKA

Electron microscopy

InnoCore project

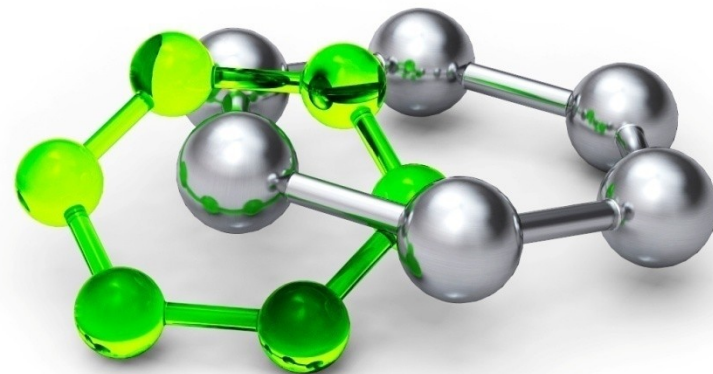
Jiri Novacek



EVROPSKÁ UNIE
EVROPSKÝ FOND PRO REGIONÁLNÍ ROZVOJ
INVESTICE DO VAŠÍ BUDOUCNOSTI



OP Výzkum a vývoj
pro inovace



Syllabus

- **Lecture 1: Applications of electron microscopy in life-science research**
- **Lecture 2: Transmission electron microscope, cryo-electron microscopy, principles of image formation**
- **Lecture 3: Fourier transform, techniques for 3D model determination in cryo-EM**

Syllabus

- Lecture 1: Applications of electron microscopy in life-science research
- Lecture 2: Transmission electron microscope, cryo-electron microscopy, principles of image formation
- **Lecture 3: Fourier transform, techniques for 3D model determination in cryo-EM**

Content

- **Fourier transform**
- **Contrast transfer function**
- **Single particle analysis**
- **Cryo-electron tomography**

Fourier transform

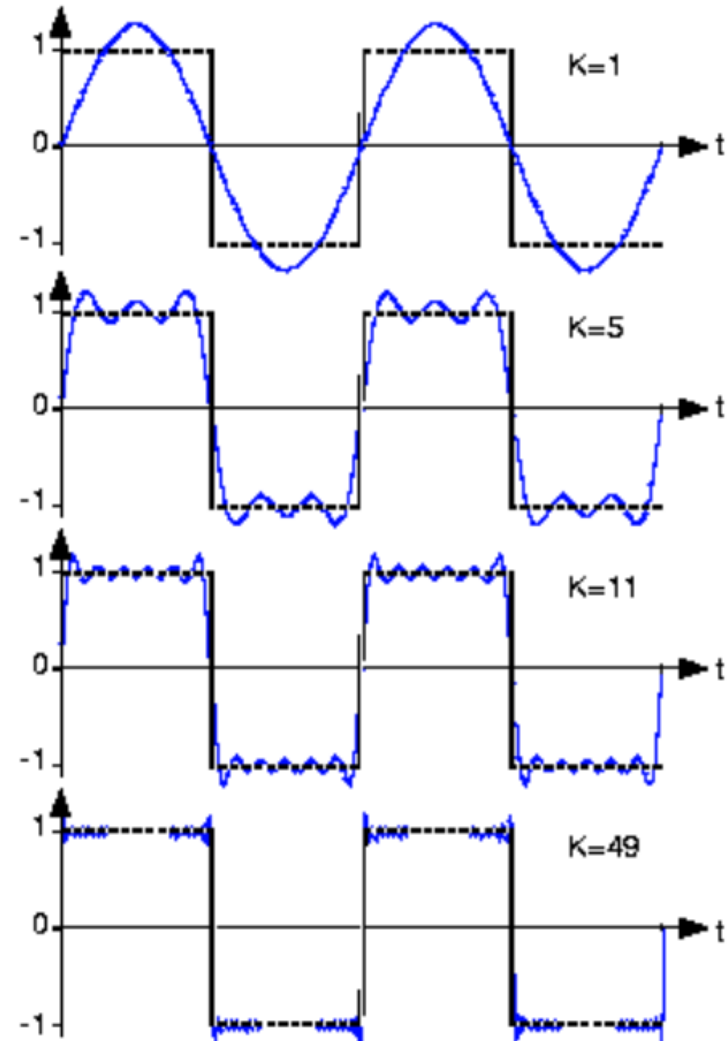
Fourier series is an expansion of a function $f(x)$ in terms of an infinite sum of sines and cosines

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

The higher the spatial frequencies (i.e. higher resolution) are included, the more faithful the representation of the original signal will be.



<http://cnx.org>

Fourier transform

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx$$

f : function which we are transforming (1D)

x : axis coordinate

i : $\sqrt{-1}$

k : spatial frequency


$F(k)$: Fourier coefficient at frequency k

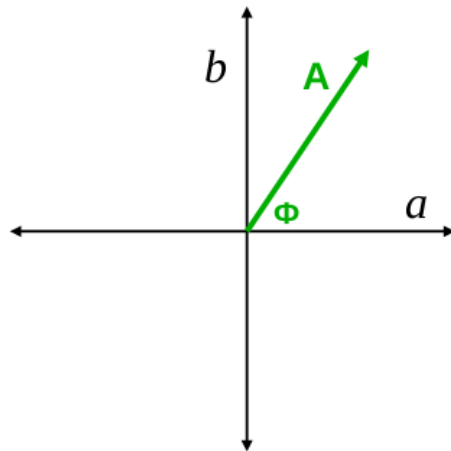
Euler's Formula: $e^{i\phi} = \cos \phi + i \sin \phi$

$$F(k) = \underbrace{\int_{-\infty}^{\infty} f(x) \cos(-2\pi kx) dx}_a + i \underbrace{\int_{-\infty}^{\infty} f(x) \sin(-2\pi kx) dx}_b$$

Fourier transform

$$F(k) = \int_{-\infty}^{\infty} f(x) \cos(-2\pi kx) dx + i \int_{-\infty}^{\infty} f(x) \sin(-2\pi kx) dx$$

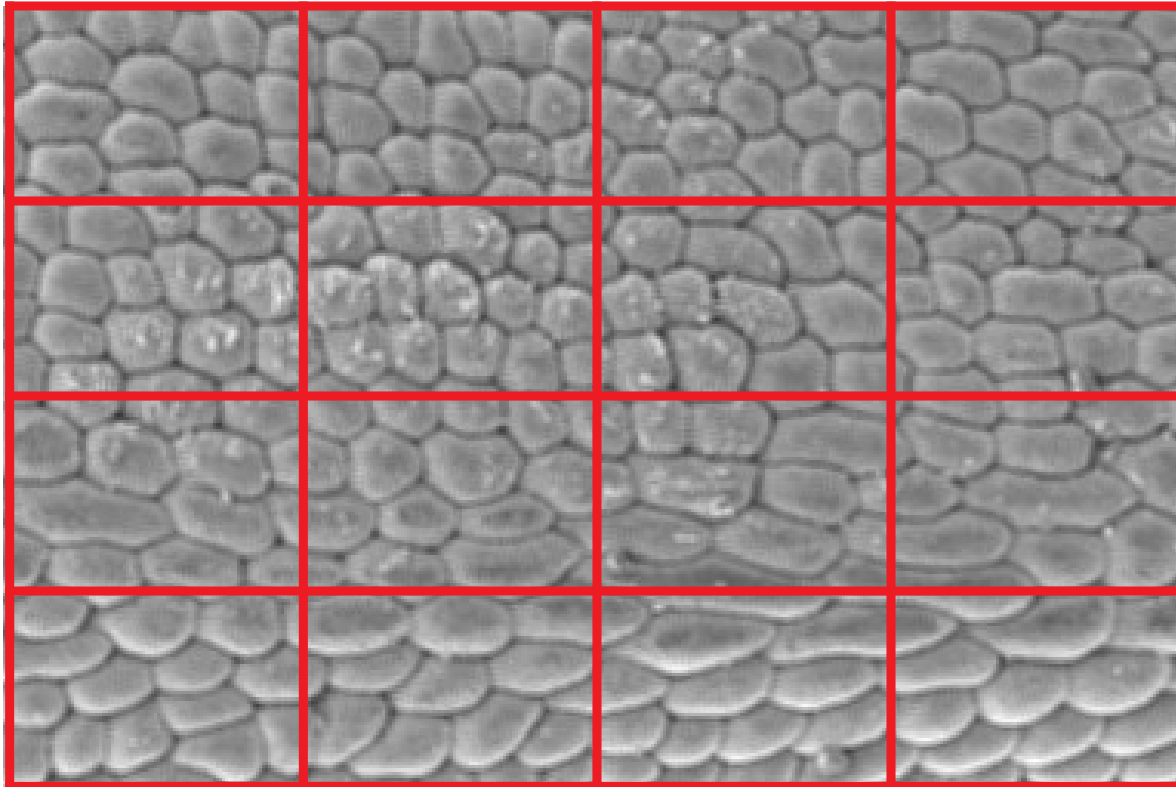




Amplitude, A: $\sqrt{a^2 + b^2}$

Phase, Φ : $\arctan \frac{b}{a}$

Digitization



Discrete Fourier transform

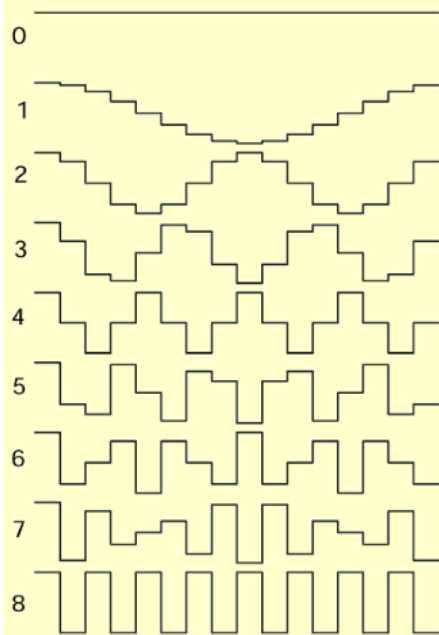
- 1D discrete Fourier transform of function $f(x)$

$$\Phi(\omega_x) = \sum_{x=0}^{N-1} f(x) e^{-i\left(\frac{2\pi}{N}\omega_x x\right)}$$

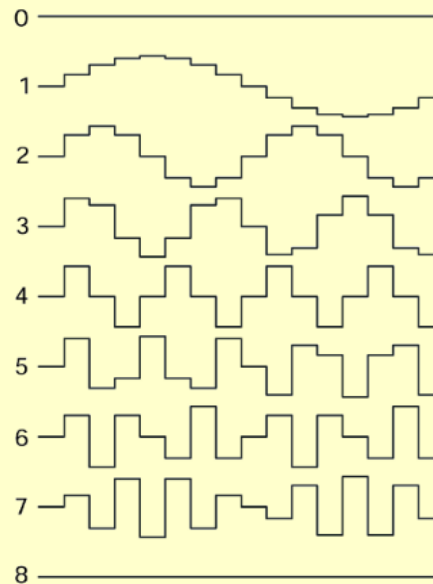
- 1D inverse discrete Fourier transform of function $\Phi(\omega_x)$

$$f(x) = \frac{1}{N} \sum_{\omega_x=0}^{N-1} \Phi(\omega_x) e^{i\left(\frac{2\pi}{N}\omega_x x\right)}$$

Cosine base functions for N=16

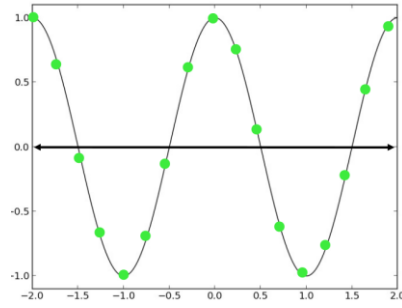


Sine base functions for N=16

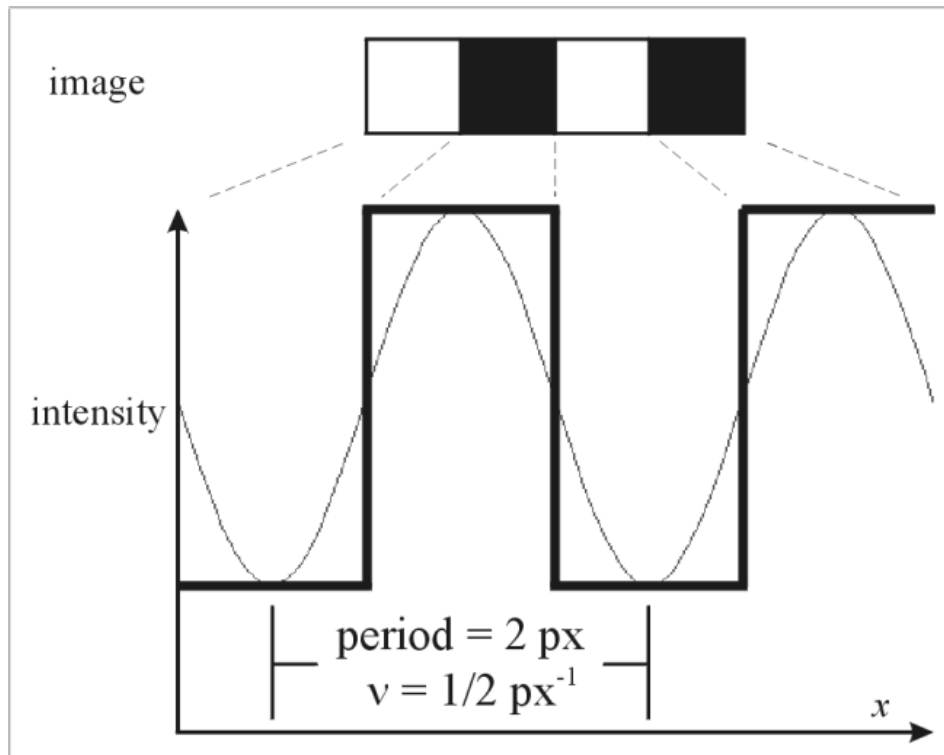
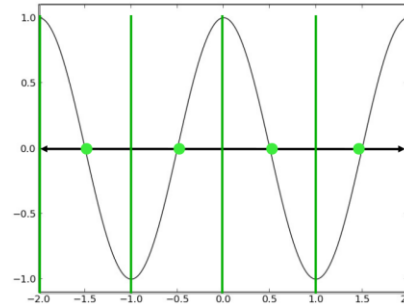


Discrete Fourier transform - sampling

Sufficient sampling



Insufficient sampling



- the period of the finest oscillation is 2 pixels
- the spatial frequency of this oscillation is 0.5 px^{-1}
- the finest detectable oscillation is what is known as “Nyquist frequency”
- the edge of the Fourier transform corresponds to Nyquist frequency

Nyquist frequency

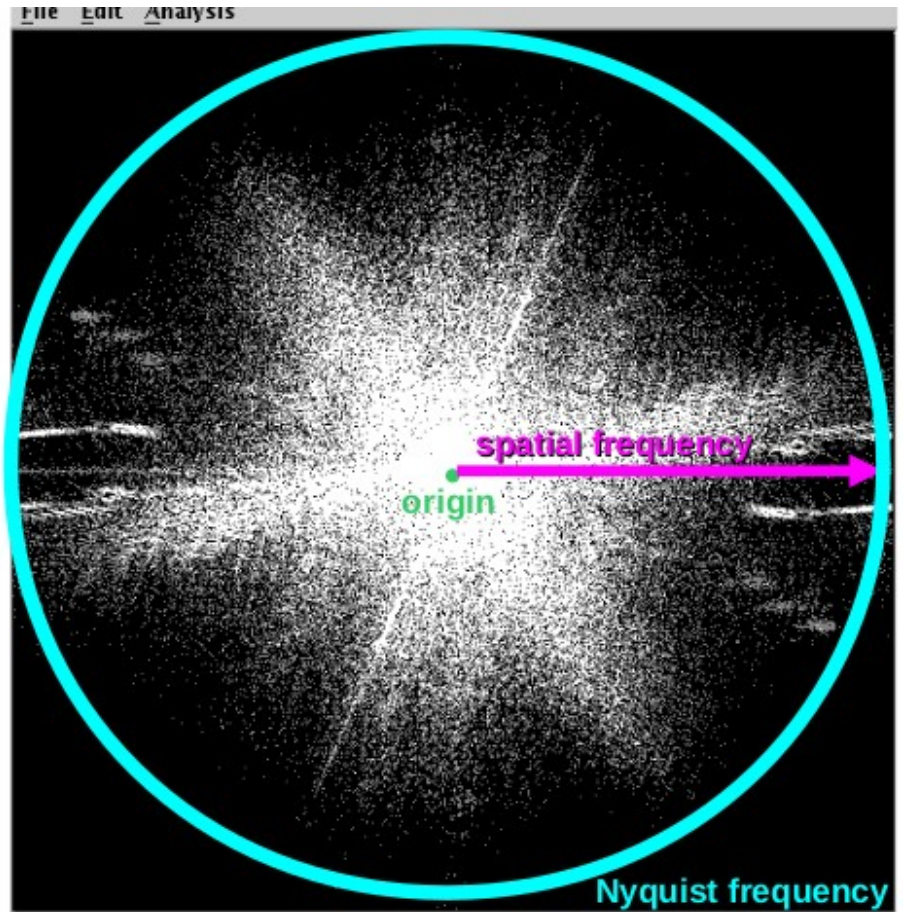


Image formation

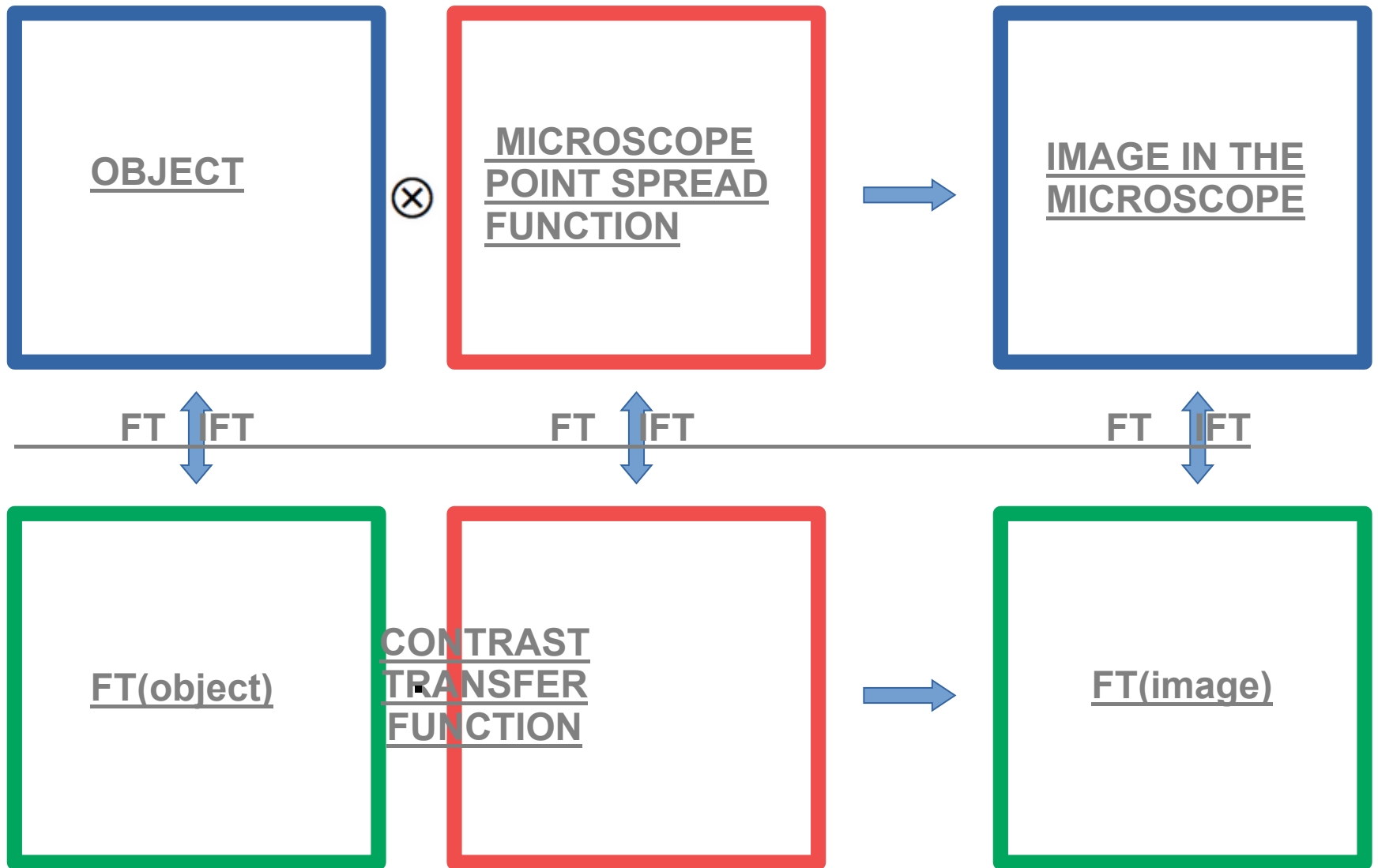
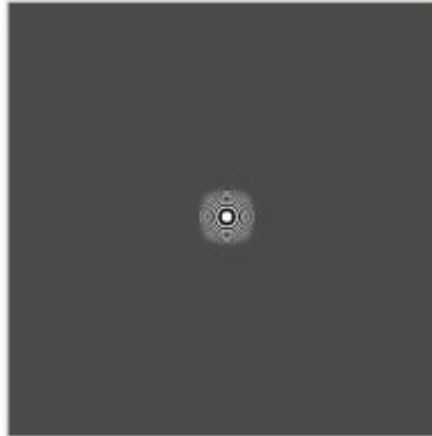


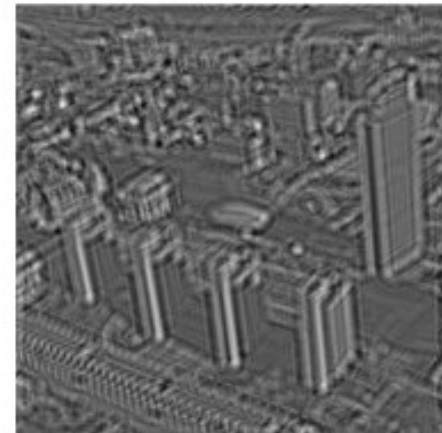
Image formation



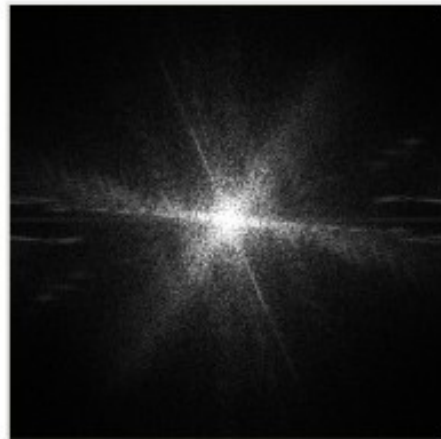
$f(x)$



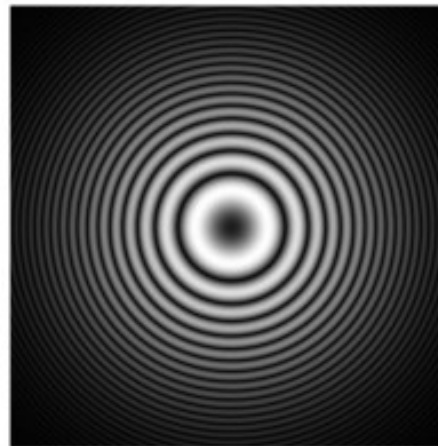
$g(x)$



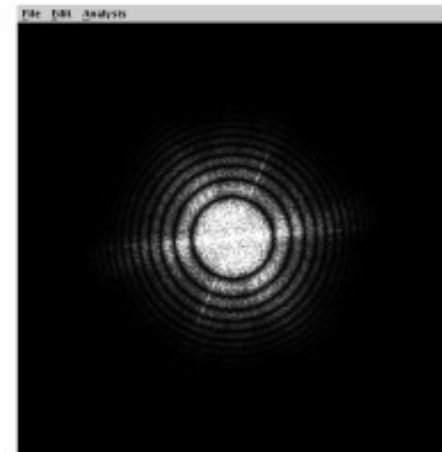
$f(x) \cdot g(x)$



$F(X)$



$G(X)$



$F(X) \cdot G(X)$

Contrast transfer function

$$\text{CTF}(\vec{s}) = -\sqrt{1 - A^2} \cdot \sin(\gamma(\vec{s})) - A \cdot \cos(\gamma(\vec{s}))$$

$$\gamma(\vec{s}) = \gamma(s, \theta) = -\frac{\pi}{2} C_s \lambda^3 s^4 + \pi \lambda z(\theta) s^2$$

A – amplitude contrast

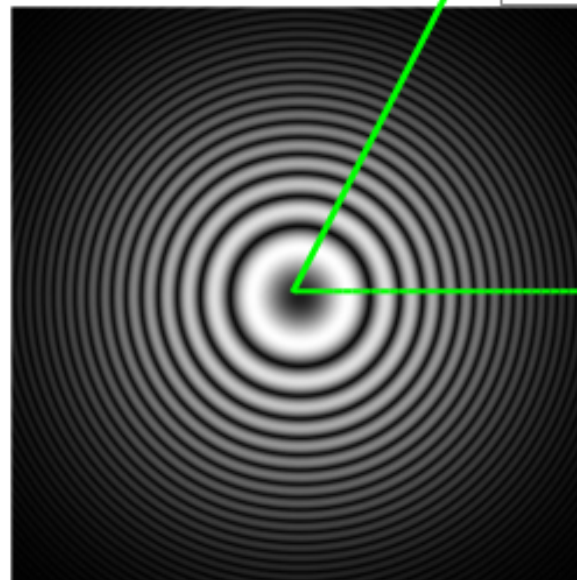
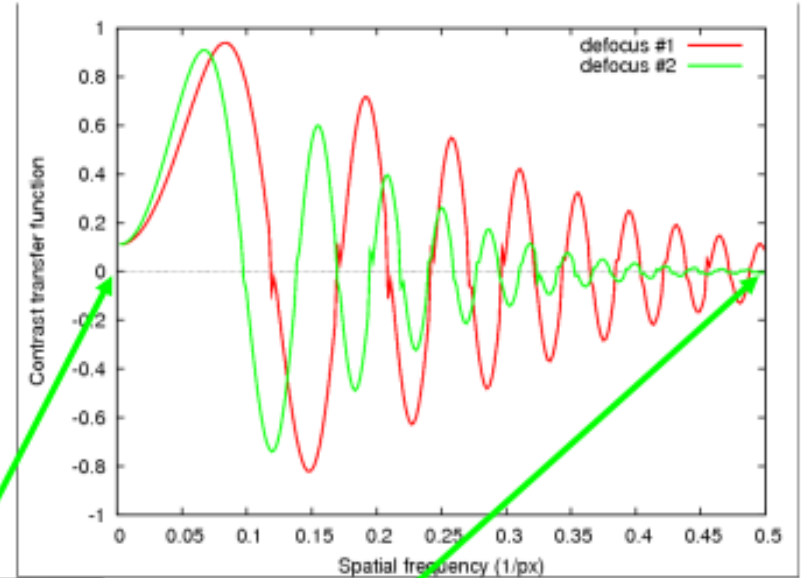
s – spatial frequency

C_s – spherical aberration

λ – electron wavelength

z – defocus

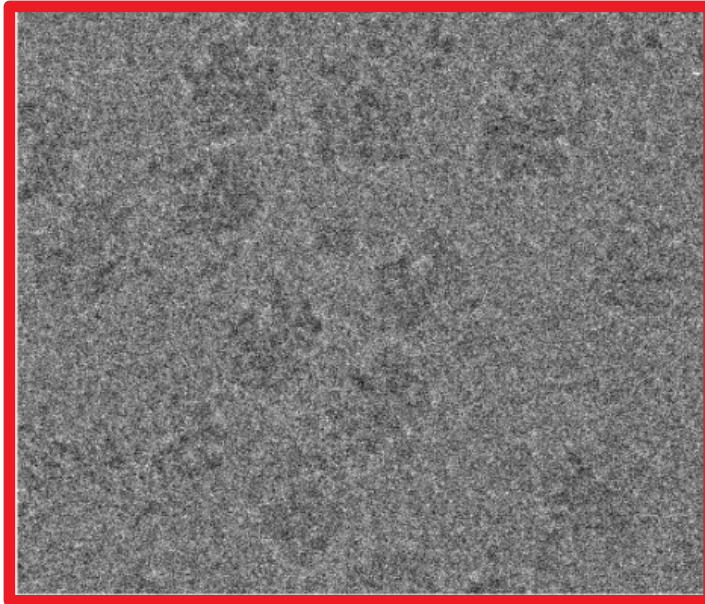
1D profile



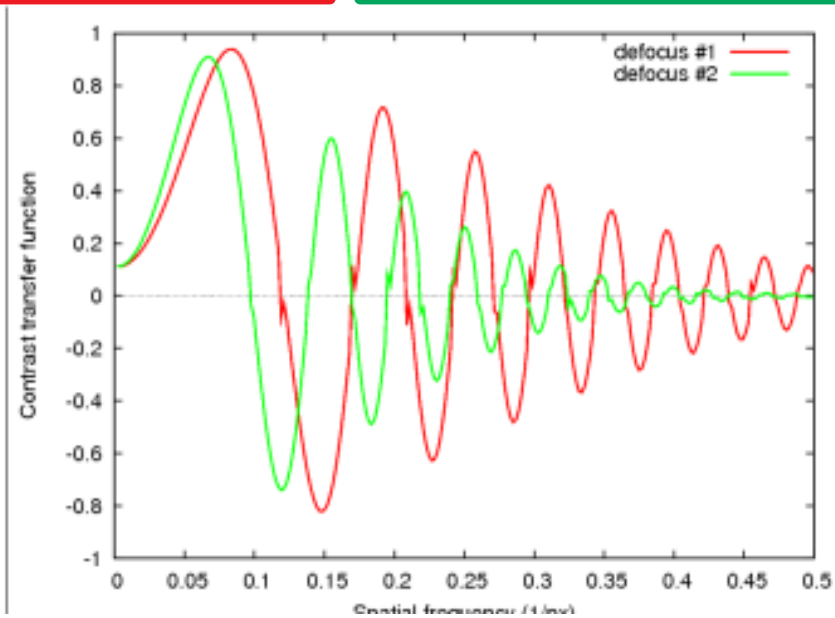
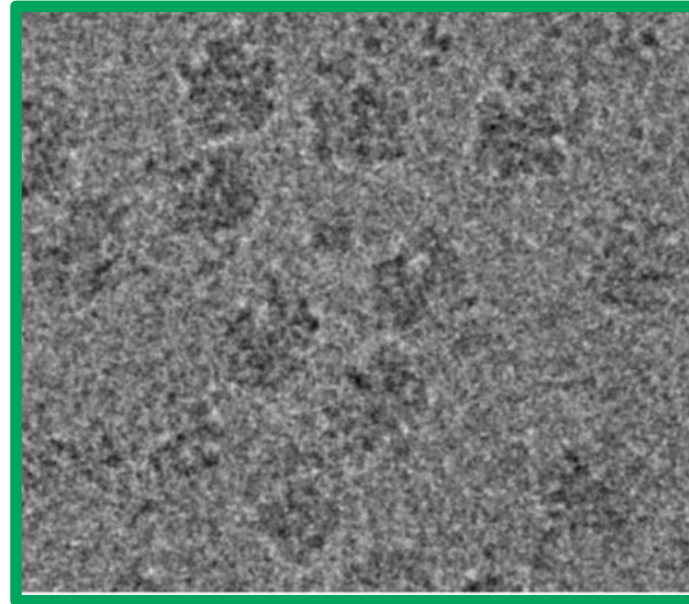
2D power spectrum
 $G(X)$

Image formation

Low defocus



High defocus



Parameters required for 3D reconstruction

Two translational:

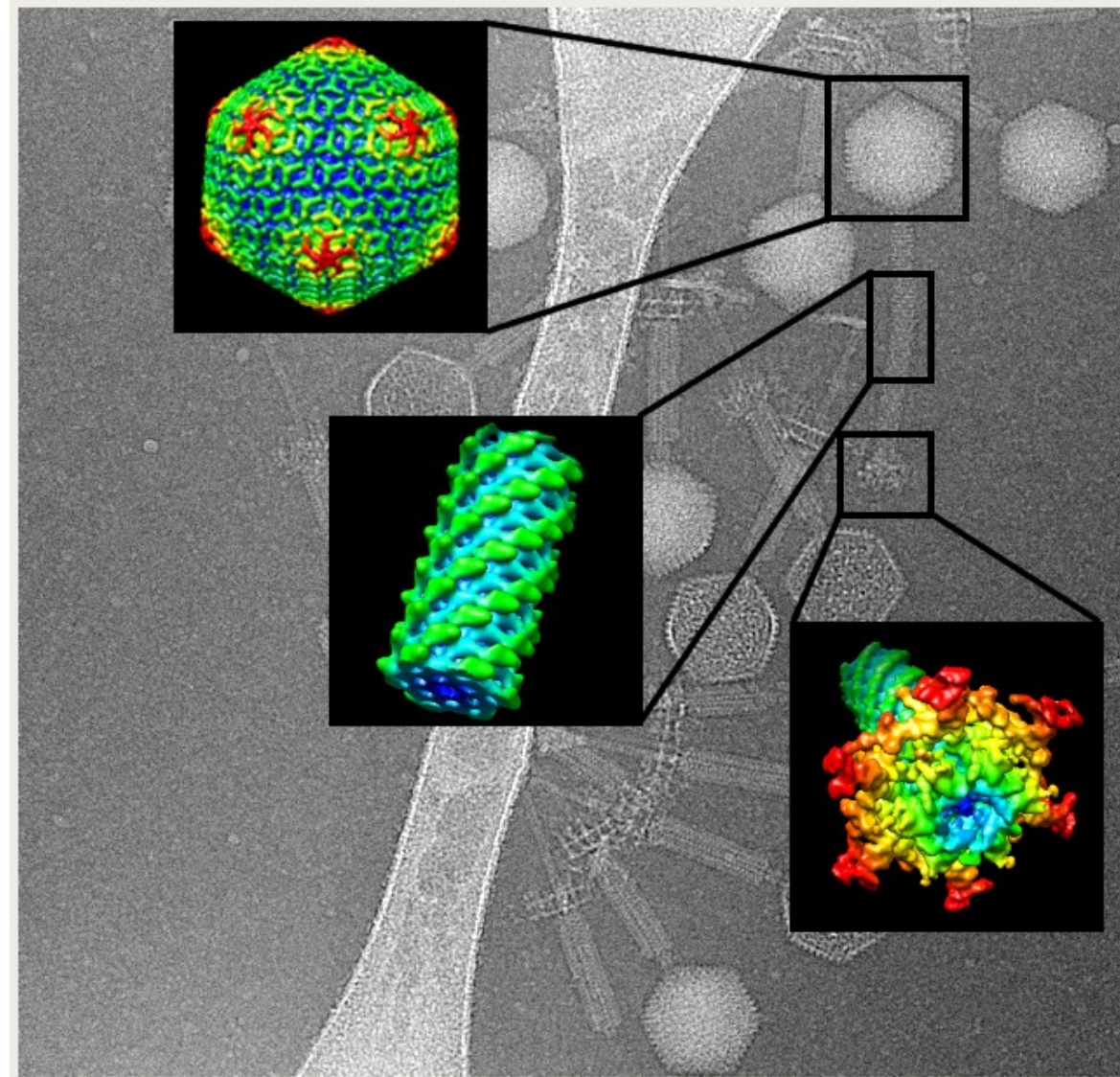
- ✓ Δx
- ✓ Δy

Three orientational
(Euler angles):

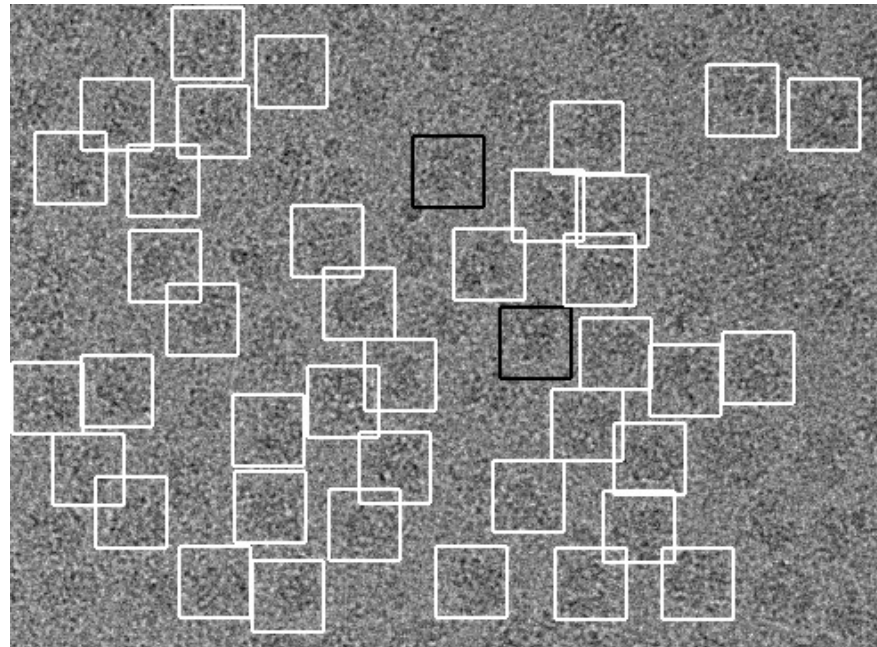
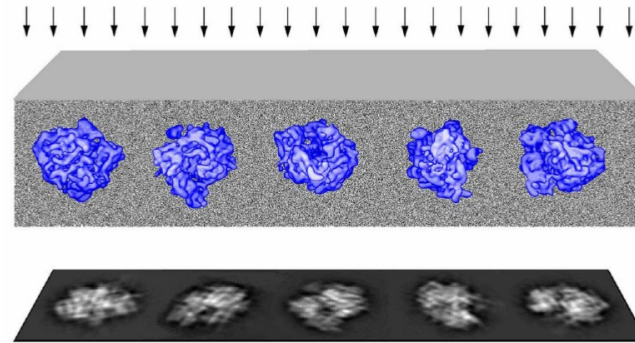
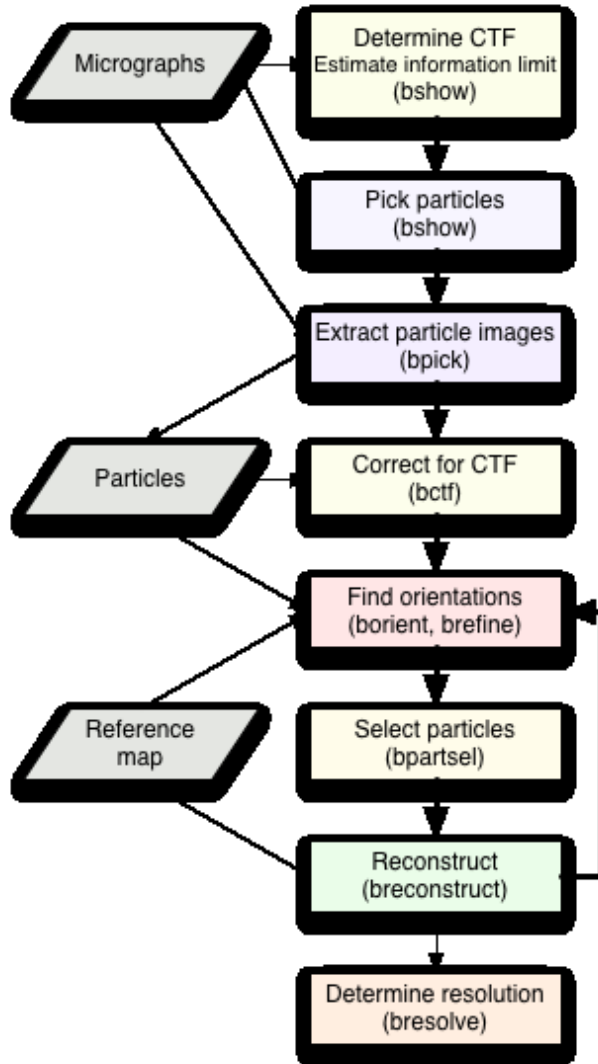
- ✓ ϕ (about z axis)
- ✓ θ (about y)
- ✓ ψ (about new z)

These are determined in 2D.

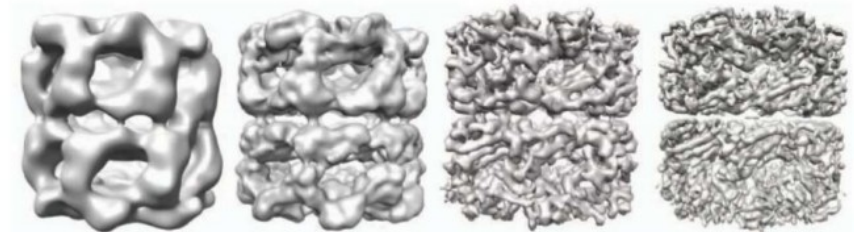
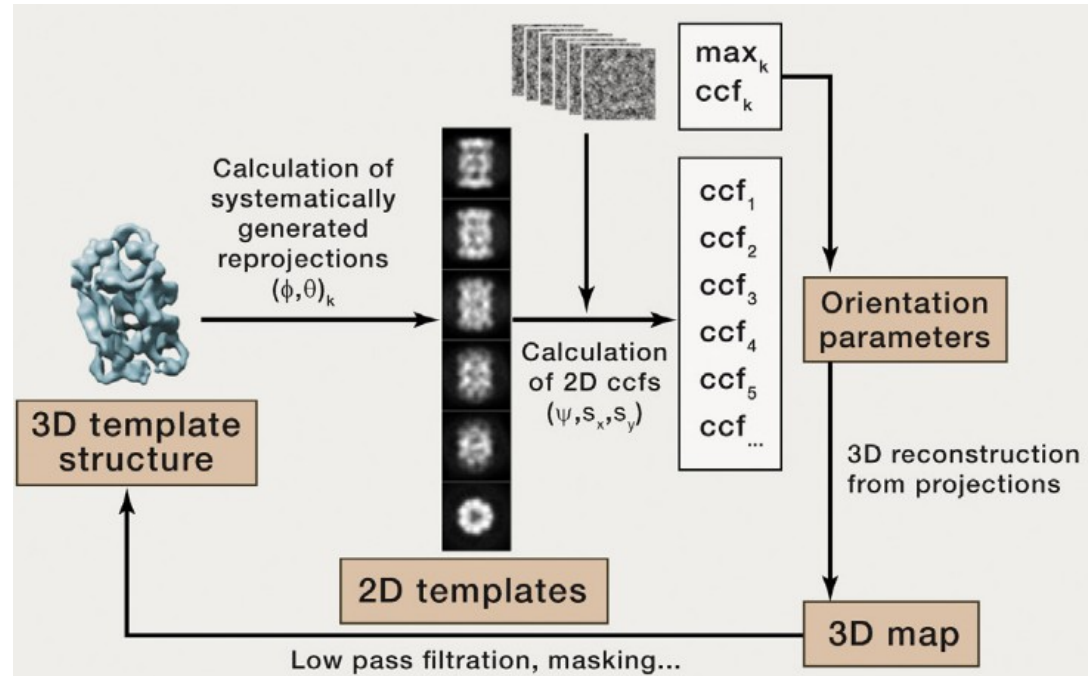
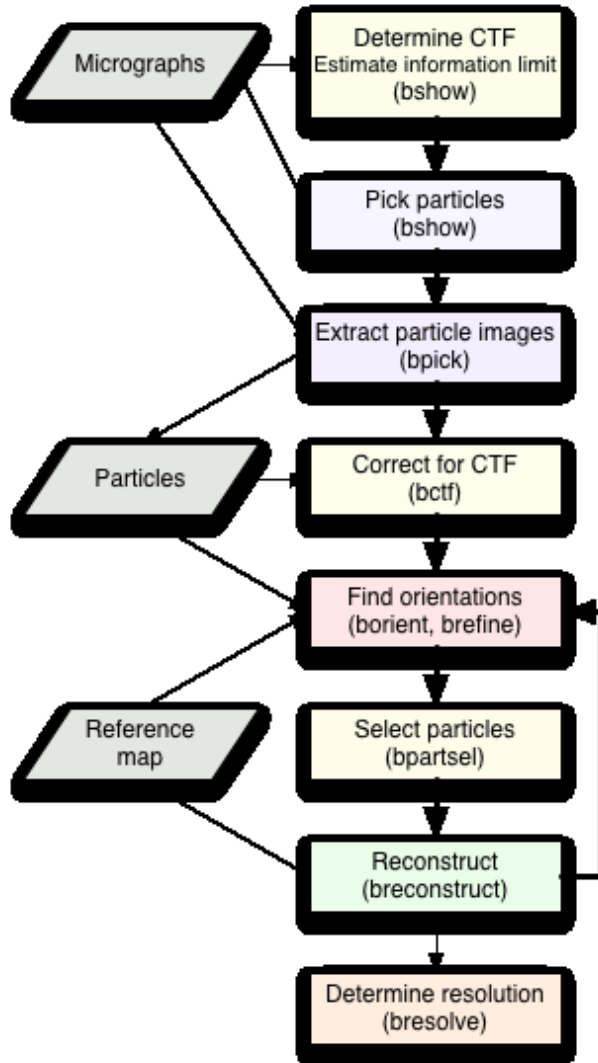
These are determined in 3D.



Single particle analysis

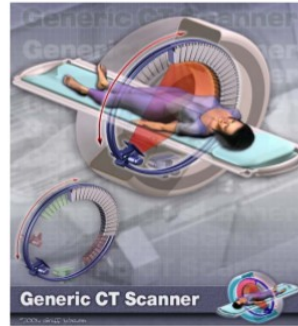


Single particle analysis

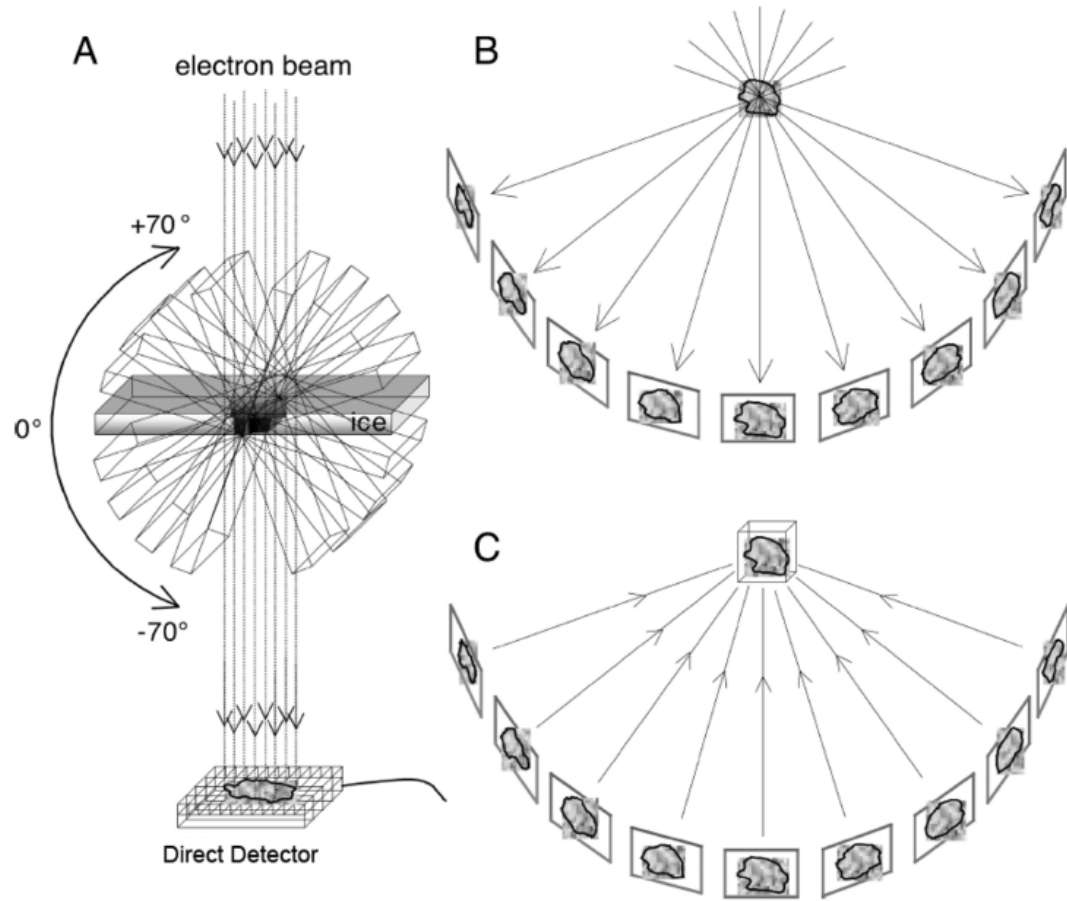
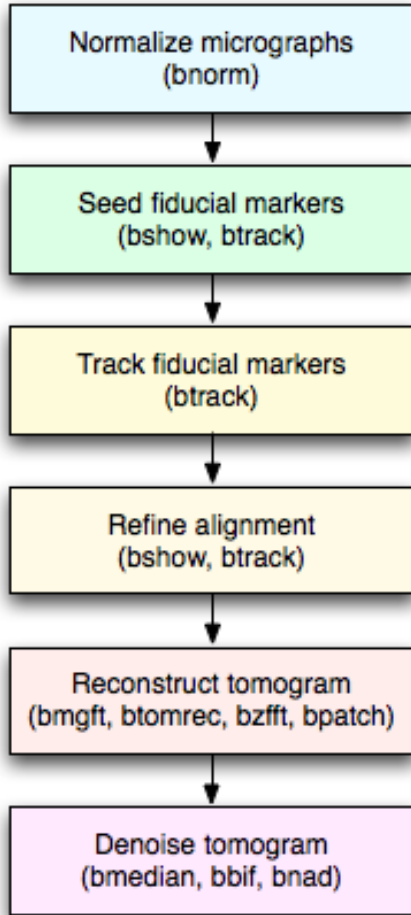


Electron tomography

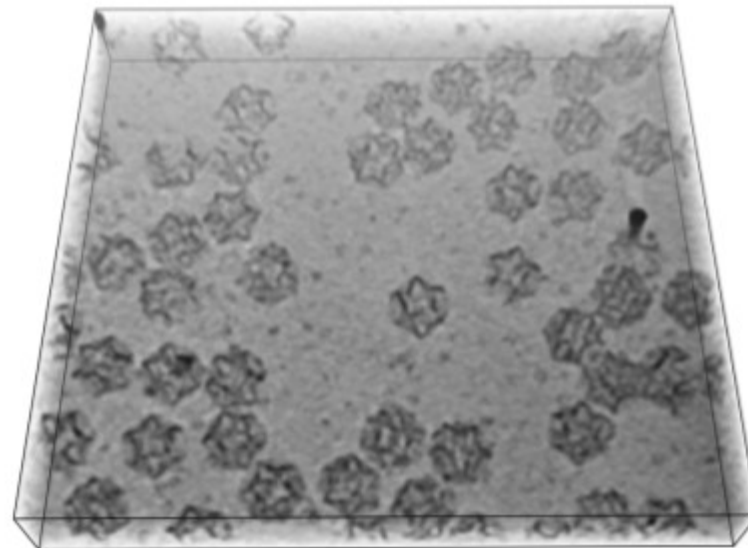
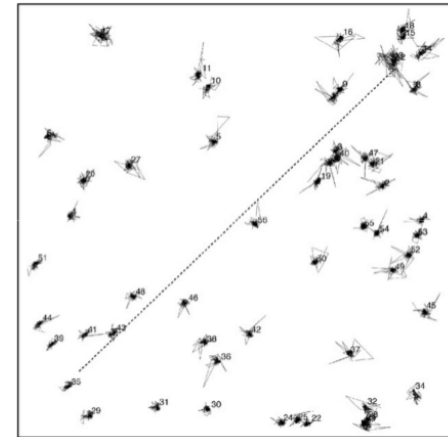
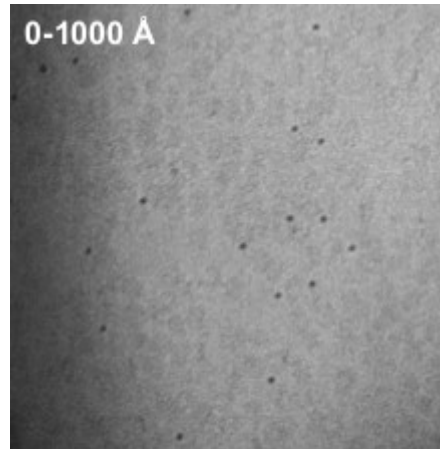
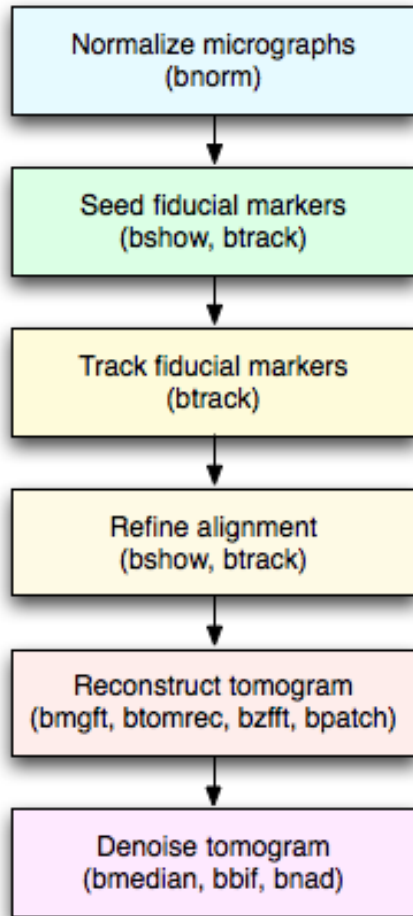
Computer Tomography



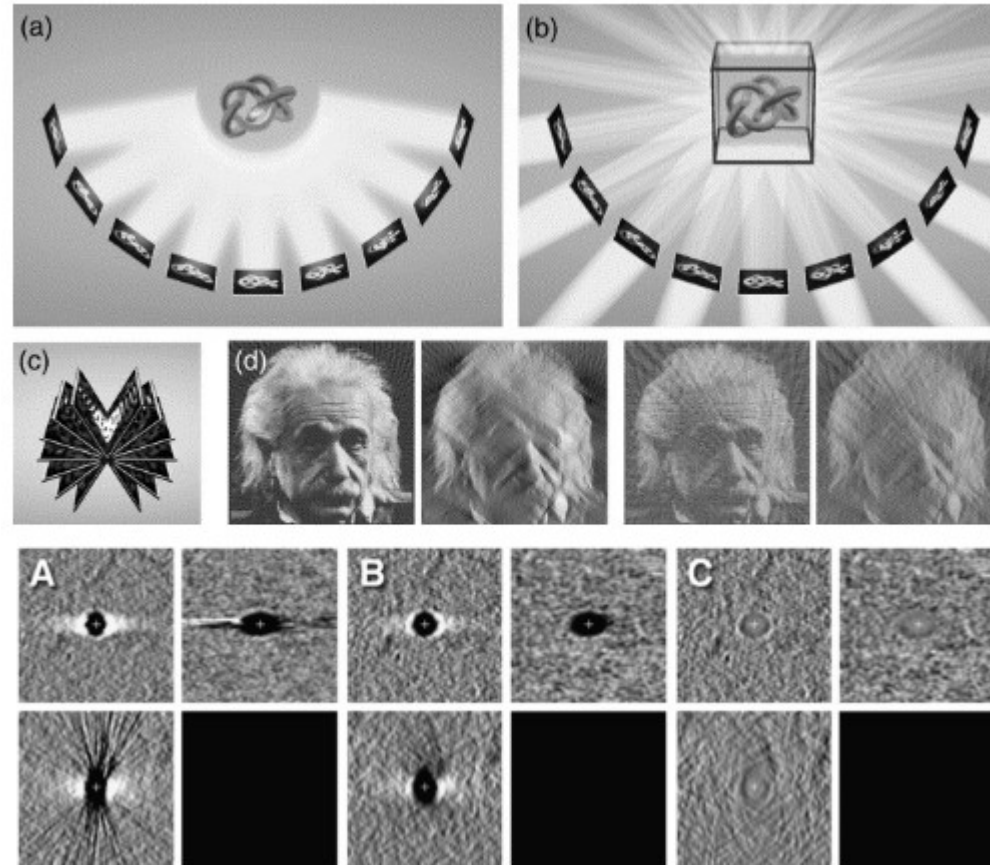
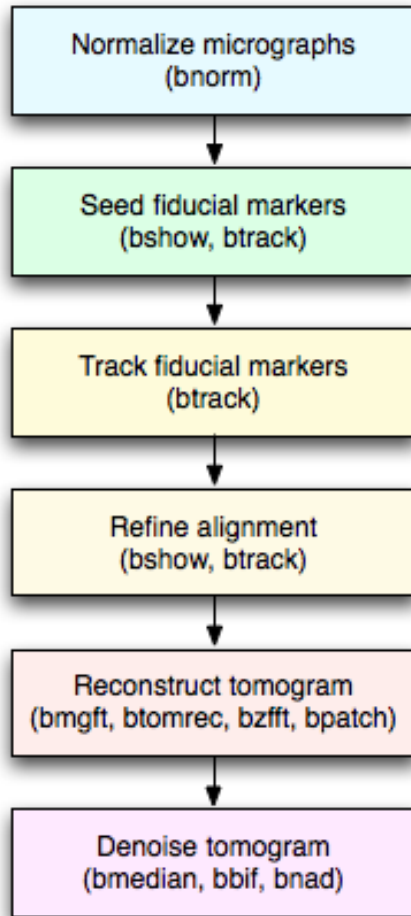
Electron Tomography



Electron tomography



Electron tomography



Thank you for attention

jiri.novacek@ceitec.muni.cz

