El-mag

Lekce 2

intenzita elektrického pole v okolí nabité přímky

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{\vec{r}}{|\vec{r}|^3}\, d\vec{S}$$

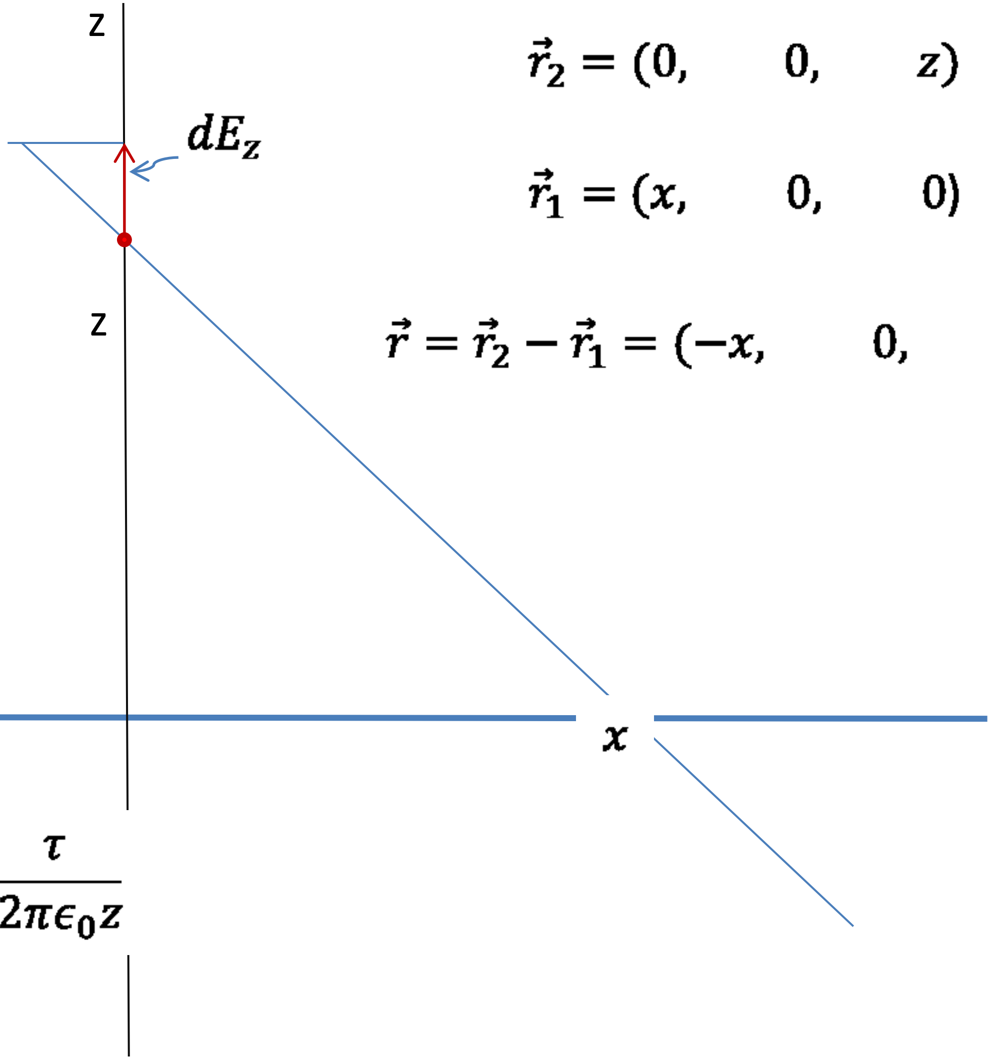
$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_2 = (0, \quad 0, \quad z)$$

$$\vec{r}_1 = (x, \quad 0, \quad 0)$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (-x, \quad 0, \quad z)$$

$$E_z = \frac{\tau}{4\pi\epsilon_0}\int_{-\infty}^{\infty} \frac{z\, dx}{(x^2+z^2)^{3/2}}$$



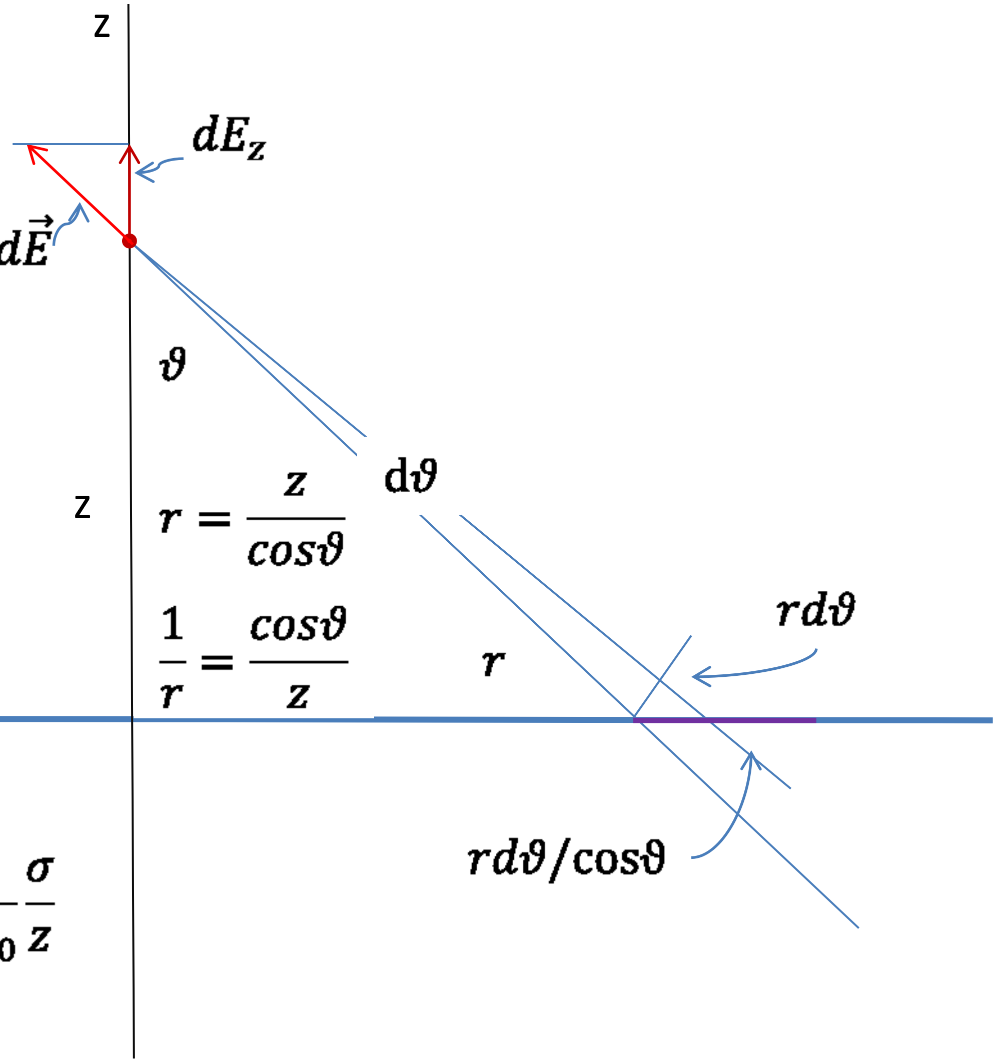
$$E_z = \frac{\tau}{4\pi\epsilon_0 z}\left[\frac{x}{(x^2+z^2)^{1/2}}\right]_{-\infty}^{\infty} = \frac{\tau}{2\pi\epsilon_0 z}$$

intenzita elektrického pole v okolí nabité přímky

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma r d\vartheta/\cos\vartheta}{r^2}\cos\vartheta$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma d\vartheta}{r}$$

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{\sigma d\vartheta}{z}\cos\vartheta$$



$$E_z = 2\int_0^{\pi/2}\frac{1}{4\pi\epsilon_0}\frac{\sigma}{z}\cos\vartheta d\vartheta = \frac{1}{2\pi\epsilon_0}\frac{\sigma}{z}$$

intenzita elektrického pole v okolí nabité roviny

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{\vec{r}}{|\vec{r}|^3}\, d\vec{S}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

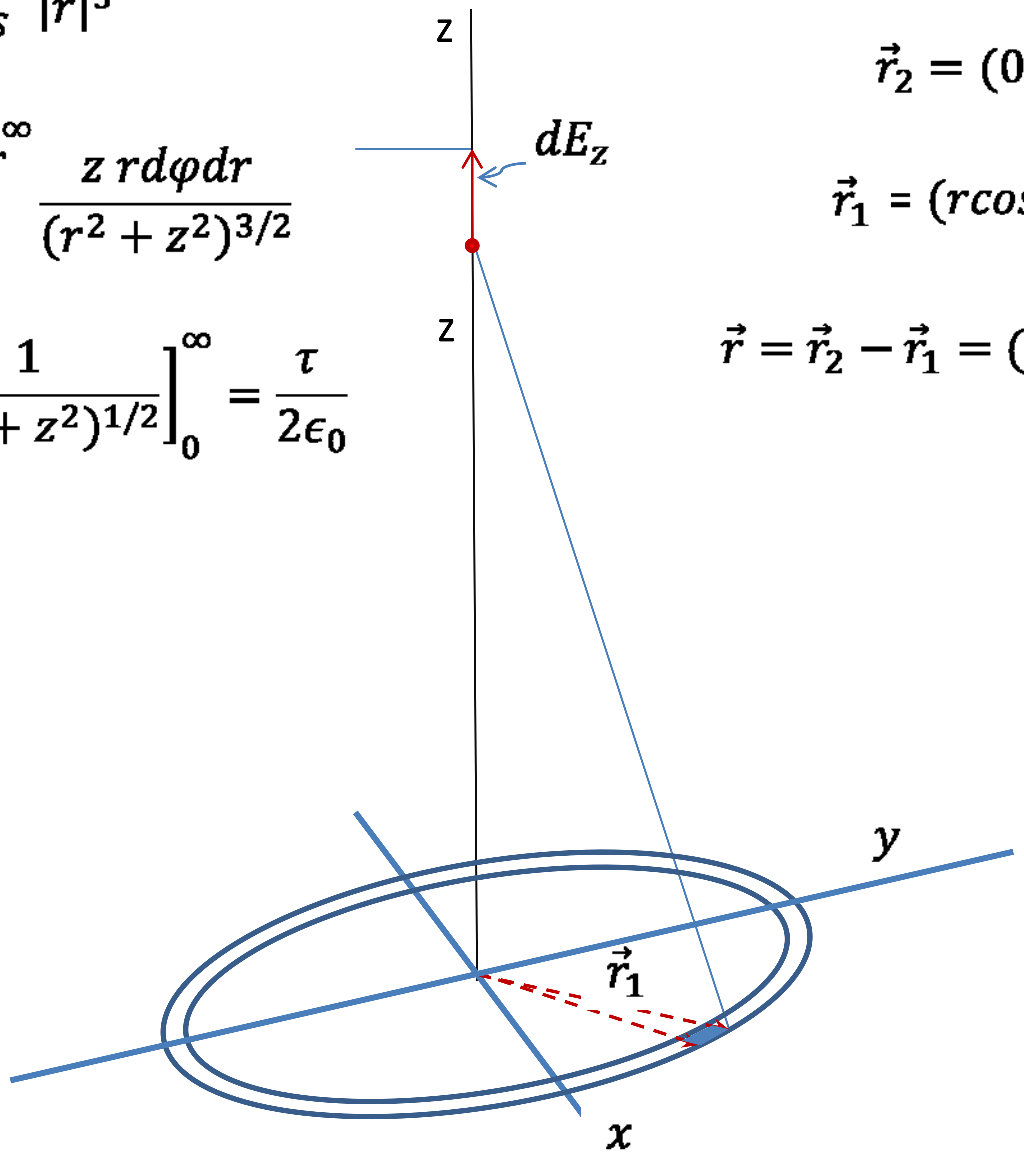
$$\vec{r}_2 = (0, \quad 0, \quad z)$$

$$E_z = \frac{\tau}{4\pi\epsilon_0}\iint_{0,0}^{2\pi,\infty} \frac{z\, r d\varphi dr}{(r^2+z^2)^{3/2}}$$

$$\vec{r}_1 = (r\cos\varphi, \ r\sin\varphi, \quad 0)$$

$$E_z = \frac{2\pi\,\tau z}{4\pi\epsilon_0}\left[-\frac{1}{(r^2+z^2)^{1/2}}\right]_0^\infty = \frac{\tau}{2\epsilon_0}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (-r\cos\varphi\,, -r\sin\varphi, \ z)$$



Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{\vec{r}}{|\vec{r}|^3}\, d\vec{S}$$

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_S \frac{(\vec{d}-\vec{R})}{\left|\vec{d}-\vec{R}\right|^3}\, d\vec{S}$$

$$\vec{r} = \vec{d} - \vec{R}$$

$$r^2 = (\vec{d}-\vec{R})(\vec{d}-\vec{R})$$

$$r^2 = d^2 + R^2 - 2d\,R\cos\vartheta$$

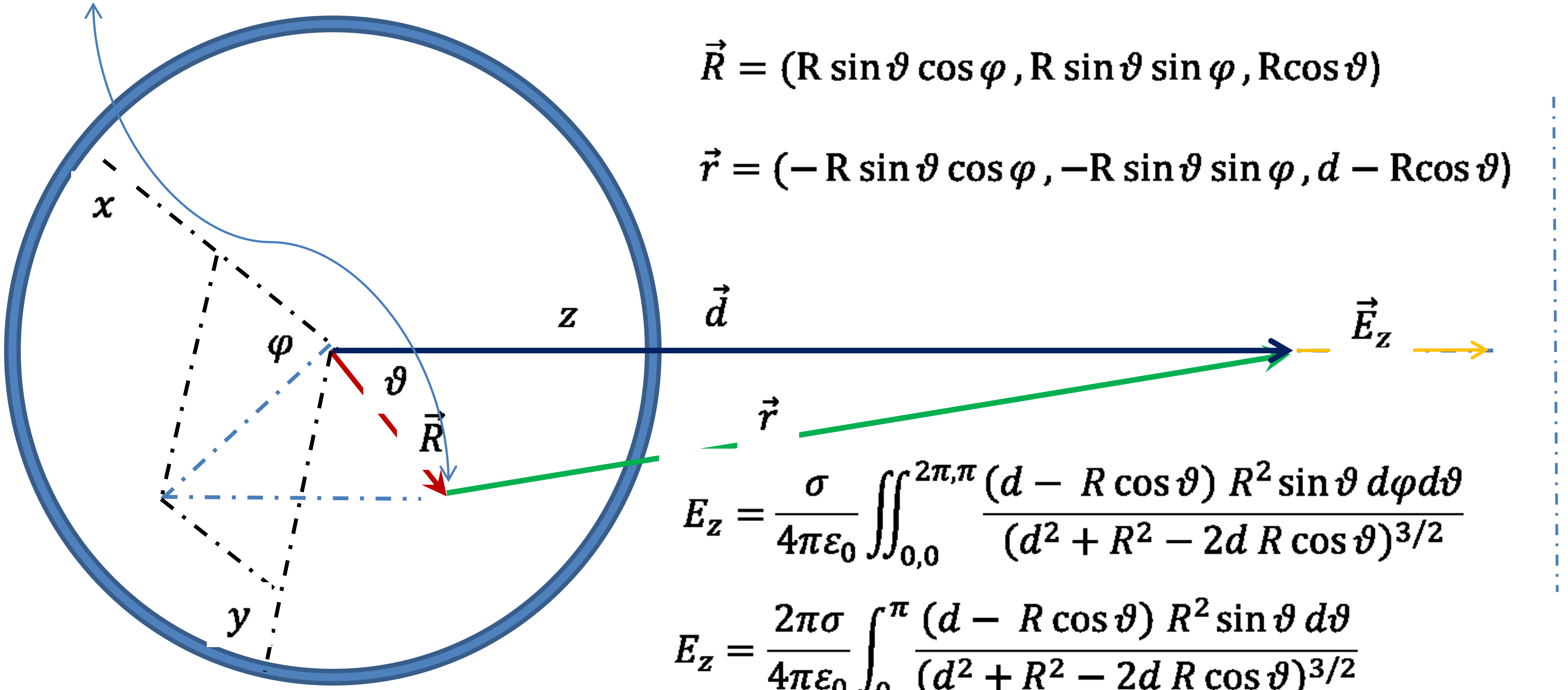
$$r^3 = \left|\vec{d}-\vec{R}\right|^3 = (d^2 + R^2 - 2d\,R\cos\vartheta\ )^{3/2}$$

$$dS = R^2 \sin\vartheta\, d\vartheta\, d\varphi$$

$$\vec{d} = (0, \quad 0, \quad d)$$

$$\vec{R} = (\mathrm{R}\sin\vartheta\cos\varphi\,, \mathrm{R}\sin\vartheta\sin\varphi\,, \mathrm{R}\cos\vartheta)$$

$$\vec{r} = (-\mathrm{R}\sin\vartheta\cos\varphi\,, -\mathrm{R}\sin\vartheta\sin\varphi\,, d - \mathrm{R}\cos\vartheta)$$



$$E_z = \frac{\sigma}{4\pi\varepsilon_0}\iint_{0,0}^{2\pi,\pi} \frac{(d - \ R\cos\vartheta)\ R^2 \sin\vartheta\, d\varphi d\vartheta}{(d^2 + R^2 - 2d\,R\cos\vartheta)^{3/2}}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{(d - \ R\cos\vartheta)\ R^2 \sin\vartheta\, d\vartheta}{(d^2 + R^2 - 2d\,R\cos\vartheta)^{3/2}}$$

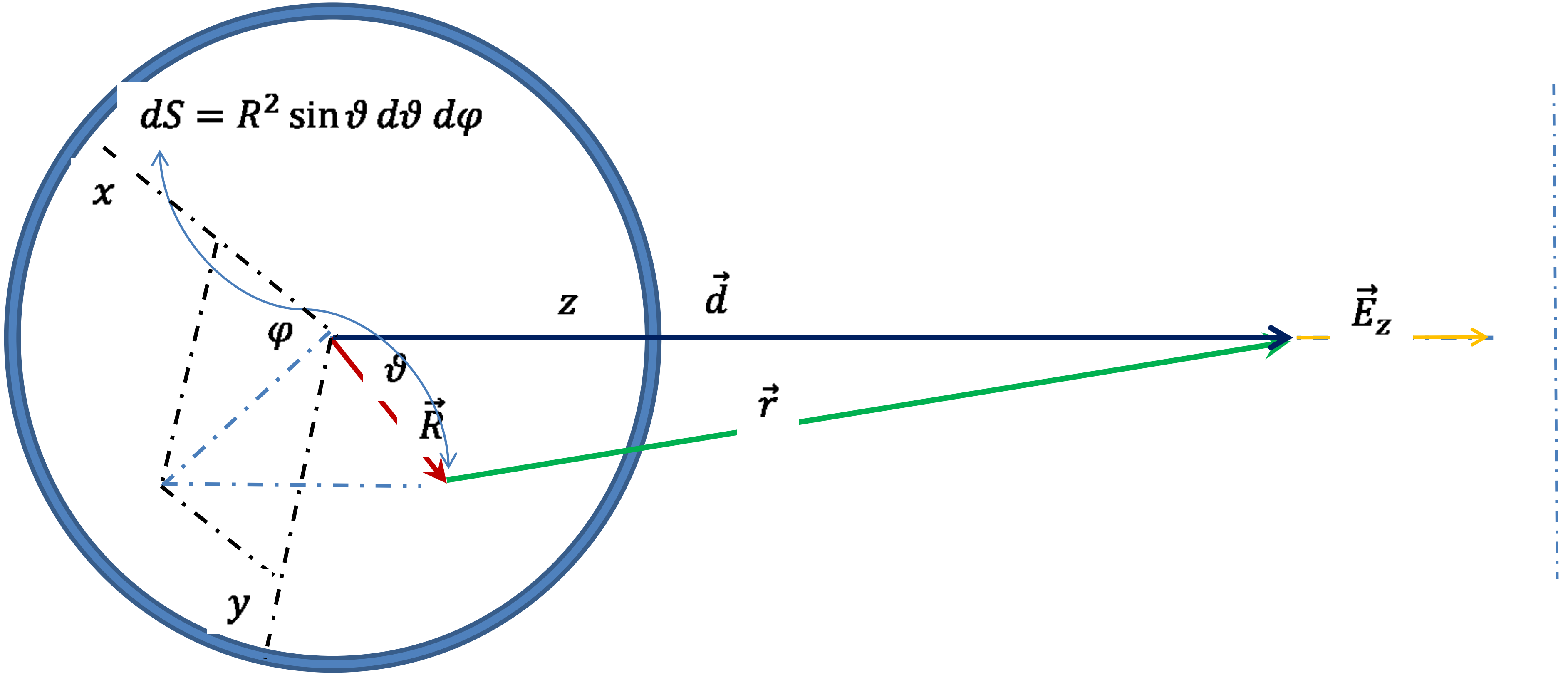
Intenzita elektrického pole v okolí homogenně nabité koulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi}\frac{(d - R\cos\vartheta)\,R^2\sin\vartheta\,d\vartheta}{(d^2 + R^2 - 2d\,R\cos\vartheta)^{3/2}}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi}\frac{R^3\left(\frac{d}{R} - \cos\vartheta\right)\sin\vartheta\,d\vartheta}{R^3\left(\left(\frac{d}{R}\right)^2 + 1 - 2\frac{d}{R}\cos\vartheta\right)^{3/2}}$$

$$\frac{d}{R} = \xi$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi}\frac{(\xi - \cos\vartheta)\sin\vartheta\,d\vartheta}{\left((\xi)^2 + 1 - 2\xi\cos\vartheta\right)^{3/2}}$$

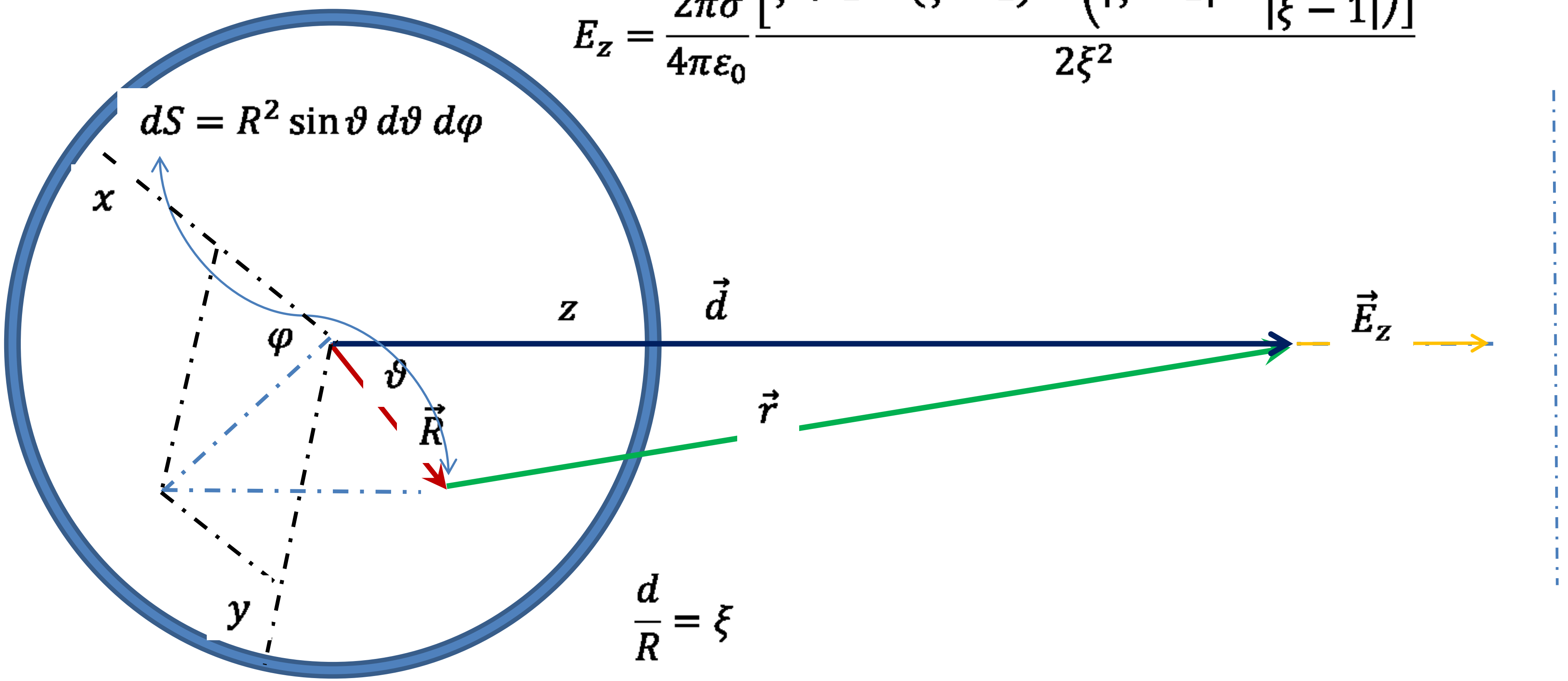


Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\pi}\frac{(\xi - \cos\vartheta)\sin\vartheta\, d\vartheta}{(\xi^2+1-2\xi\cos\vartheta)^{3/2}}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi}$$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{\left[\xi+1-(\xi-1)-\left(|\xi-1|-\frac{\xi^2-1}{|\xi-1|}\right)\right]}{2\xi^2}$$

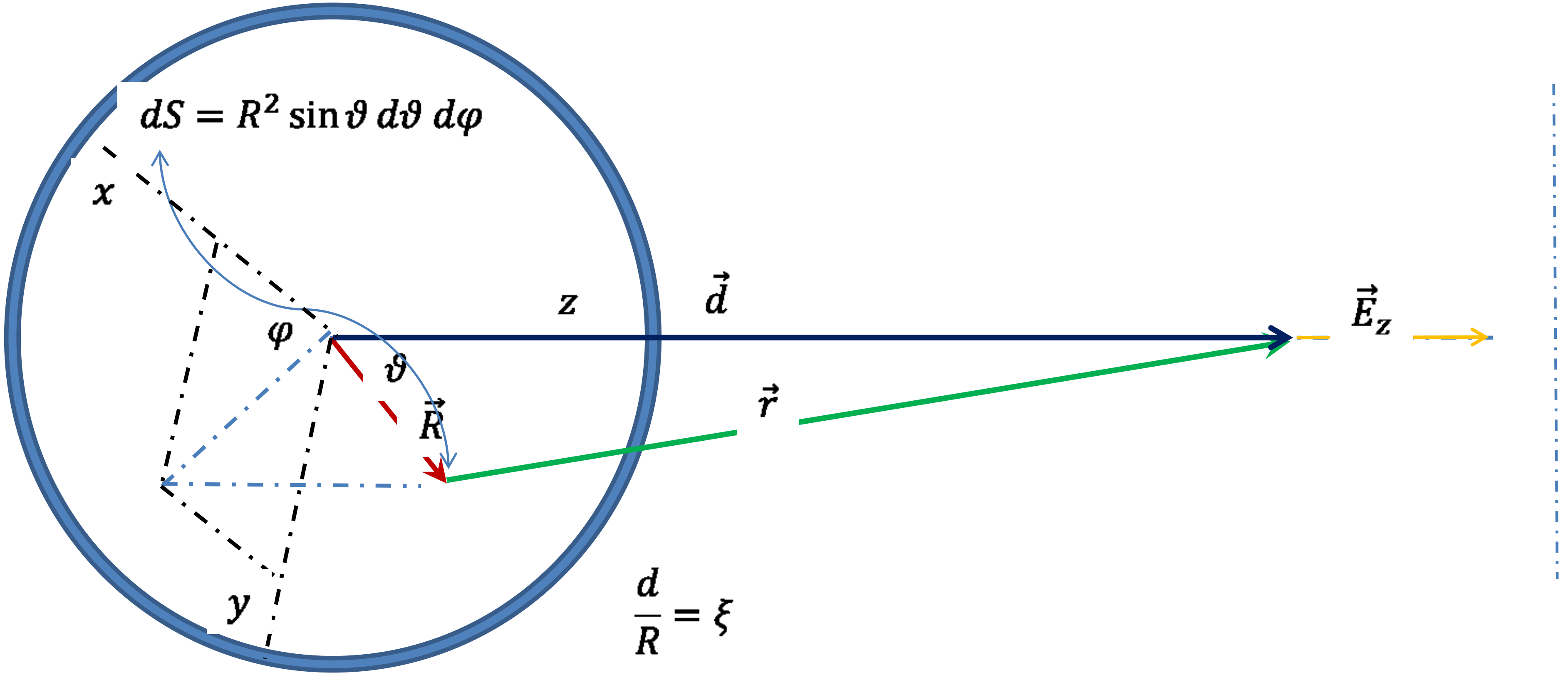


Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0} \frac{\left[\xi + 1 - (\xi - 1) - \left(|\xi - 1| - \frac{\xi^2 - 1}{|\xi - 1|}\right)\right]}{2\xi^2}$$

$\xi > 1$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0} \frac{[\xi + 1 - \xi + 1 - \xi + 1 + \xi + 1]}{2\xi^2} = \frac{4\pi\sigma}{4\pi\varepsilon_0 \xi^2} = \frac{4\pi\sigma R^2}{4\pi\varepsilon_0 d^2} = \frac{q}{4\pi\varepsilon_0 d^2}$$

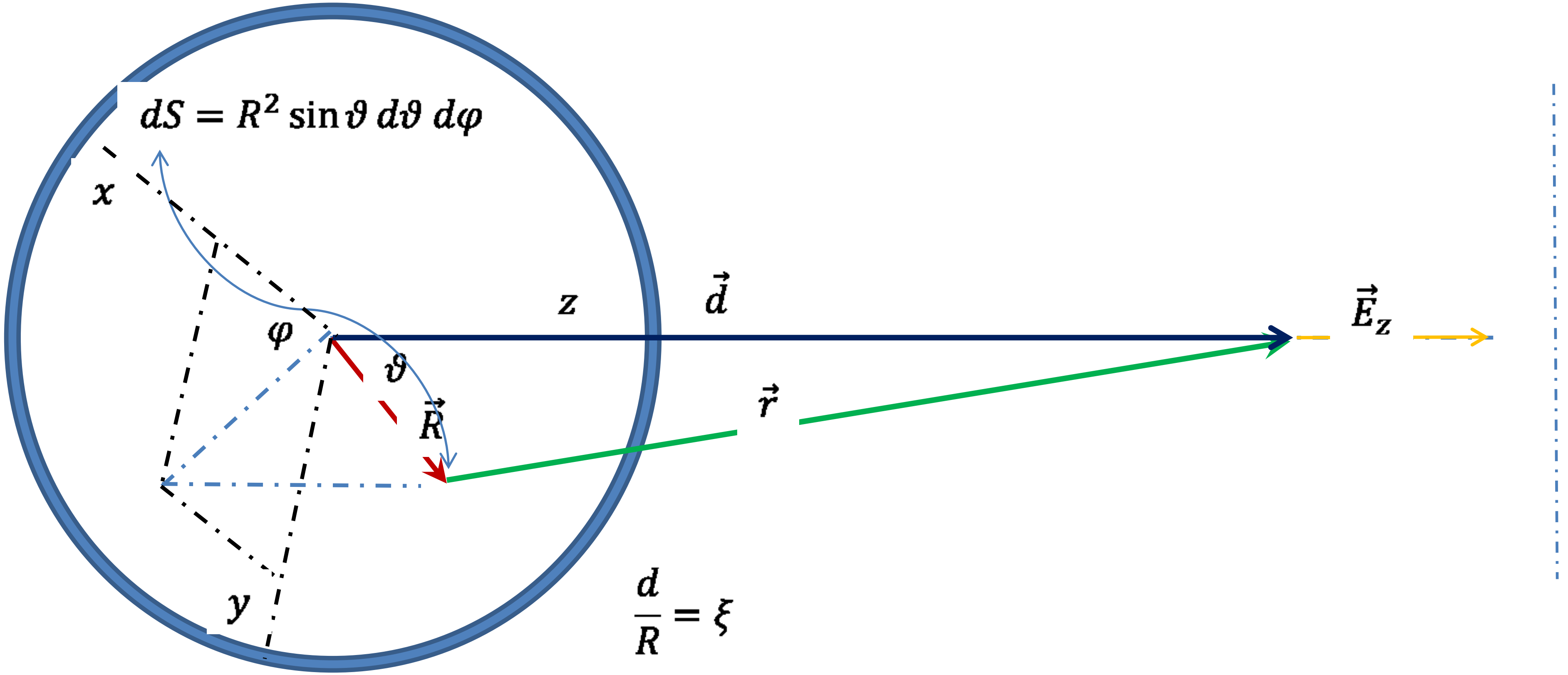


Intenzita elektrického pole v okolí homogenně nabité kulové sféry

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0} \frac{\left[\xi + 1 - (\xi - 1) - \left(|\xi - 1| - \frac{\xi^2 - 1}{|\xi - 1|}\right)\right]}{2\xi^2}$$

$\xi < 1$

$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0} \frac{[\xi + 1 - \xi + 1 + \xi - 1 - \xi - 1]}{2\xi^2} = 0$$

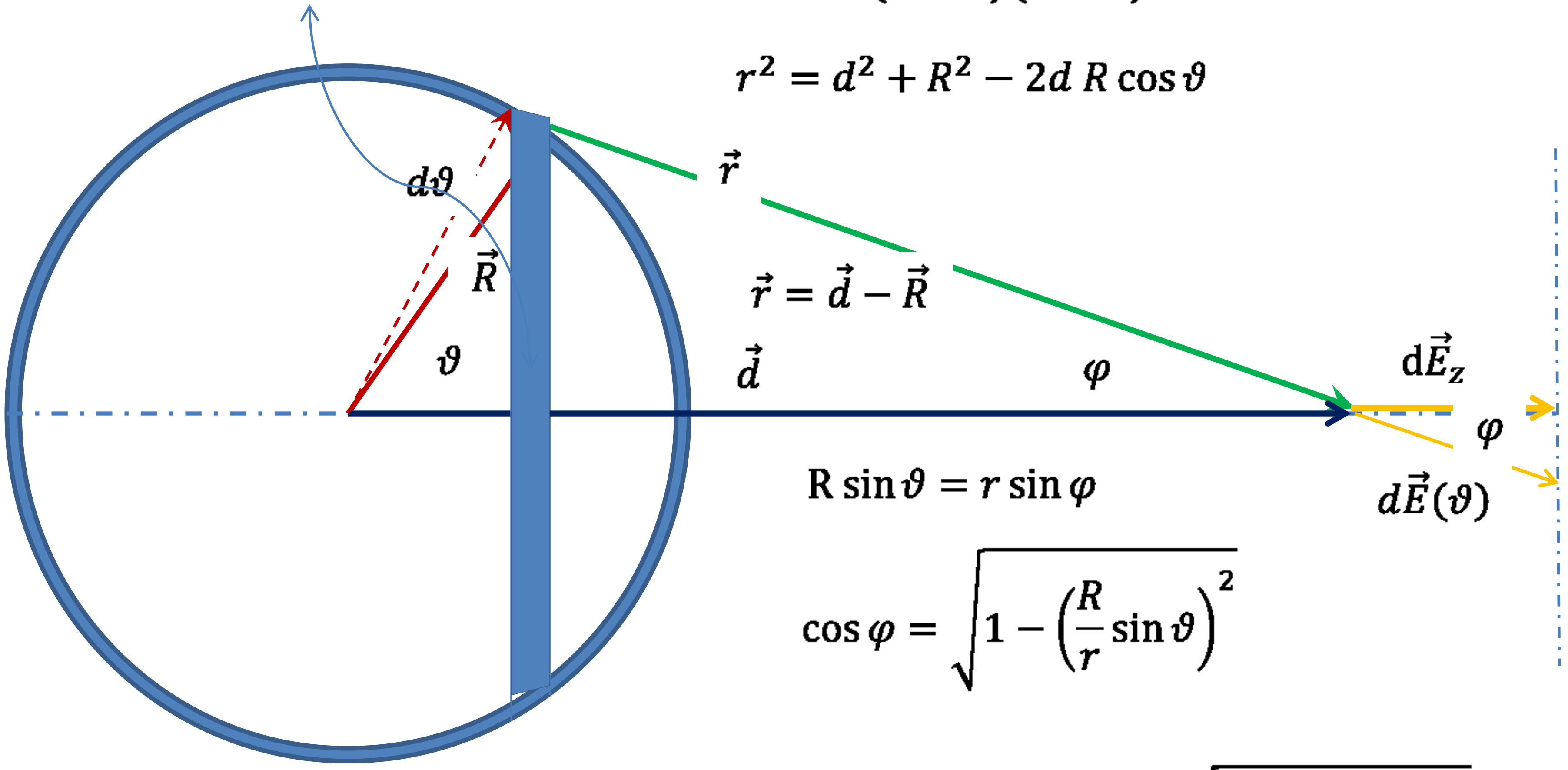


Intenzita elektrického pole v okolí homogenně nabité koulové sféry.
Jiný způsob

$$dS = 2\pi R^2 \sin\vartheta \, d\vartheta$$

$$r^2 = \vec{r} * \vec{r} = (\vec{d} - \vec{R})(\vec{d} - \vec{R}) = d^2 + R^2 - 2\,\vec{d}\,\vec{R}$$

$$r^2 = d^2 + R^2 - 2d\,R\cos\vartheta$$



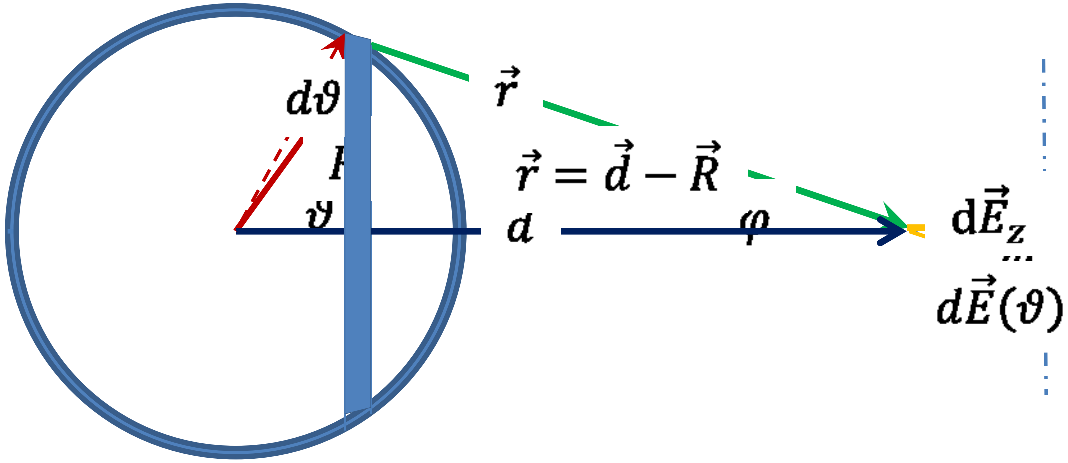
$$\vec{r} = \vec{d} - \vec{R}$$

$$\mathrm{R}\sin\vartheta = r\sin\varphi$$

$$\cos\varphi = \sqrt{1 - \left(\frac{R}{r}\sin\vartheta\right)^2}$$

$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi R^2 \sin\vartheta \, d\vartheta}{d^2 + R^2 - 2d\,R\cos\vartheta}\sqrt{1 - \left(\frac{R}{r}\sin\vartheta\right)^2}$$

Intenzita elektrického pole v okolí homogenně nabité koulové sféry.
Jiný způsob

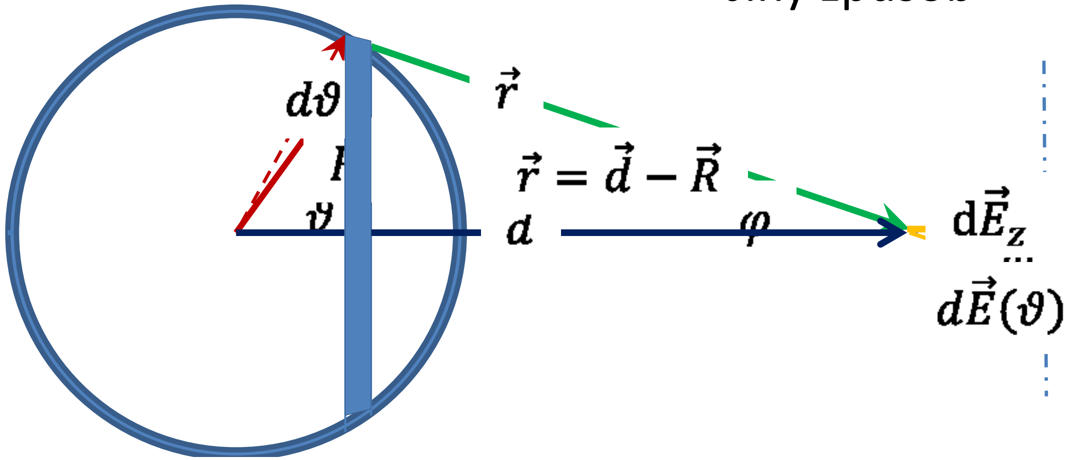


$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{\sigma 2\pi R^2 \sin\vartheta \, d\vartheta}{d^2 + R^2 - 2d\,R\cos\vartheta} \sqrt{1 - \left(\frac{R}{r}\sin\vartheta\right)^2}$$

$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{\sigma 2\pi R^2 \sin\vartheta \, d\vartheta}{d^2 + R^2 - 2d\,R\cos\vartheta} \sqrt{1 - \frac{\mathrm{R}^2 \sin\vartheta^2}{d^2 + R^2 - 2d\,R\cos\vartheta}}$$

$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta \, d\vartheta}{\left(\frac{d}{R}\right)^2 + 1 - 2\,\frac{d}{R}\cos\vartheta} \sqrt{1 - \frac{\sin\vartheta^2}{\left(\frac{d}{R}\right)^2 + 1 - 2\,\frac{d}{R}\cos\vartheta}}$$

Intenzita elektrického pole v okolí homogenně nabité koulové sféry.
Jiný způsob



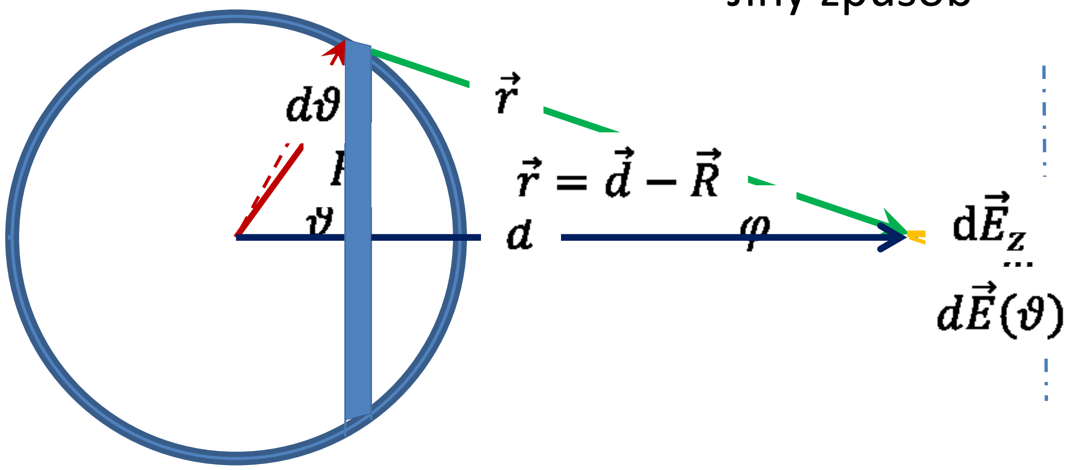
$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta \, d\vartheta}{\left(\frac{d}{R}\right)^2 + 1 - 2\,\frac{d}{R}\cos\vartheta} \sqrt{1 - \frac{\sin\vartheta^2}{\left(\frac{d}{R}\right)^2 + 1 - 2\,\frac{d}{R}\cos\vartheta}}$$

$$\alpha = \frac{d}{R}$$

$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta \, d\vartheta}{\propto^2 + 1 - 2 \propto \cos\vartheta} \sqrt{1 - \frac{\sin\vartheta^2}{\propto^2 + 1 - 2 \propto \cos\vartheta}}$$

$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta \, d\vartheta}{\propto^2 + 1 - 2 \propto \cos\vartheta} \sqrt{\frac{\propto^2 + 1 - 2 \propto \cos\vartheta - \sin\vartheta^2}{\propto^2 + 1 - 2 \propto \cos\vartheta}}$$

Intenzita elektrického pole v okolí homogenně nabité koulové sféry.
Jiný způsob



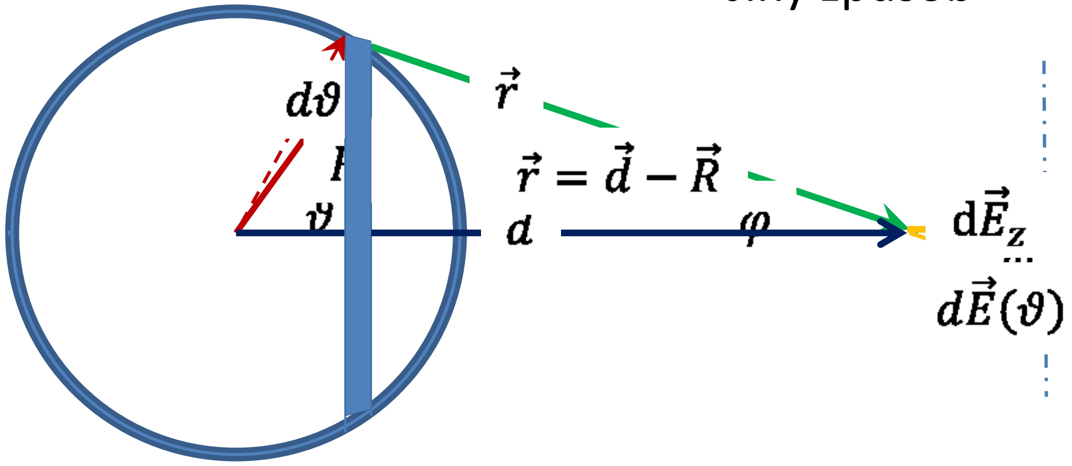
$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta}{\alpha^2 + 1 - 2\alpha\cos\vartheta} \sqrt{\frac{\alpha^2 + 1 - 2\alpha\cos\vartheta - \sin\vartheta^2}{\alpha^2 + 1 - 2\alpha\cos\vartheta}}\, d\vartheta \qquad \alpha = \frac{d}{R}$$

$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta}{\alpha^2 + 1 - 2\alpha\cos\vartheta} \sqrt{\frac{\alpha^2 - 2\alpha\cos\vartheta + \cos\vartheta^2}{\alpha^2 + 1 - 2\alpha\cos\vartheta}}\, d\vartheta$$

$$\sqrt{\alpha^2 - 2\alpha\cos\vartheta + \cos\vartheta^2} = \sqrt{(\alpha - \cos(\vartheta))^2} = \alpha - \cos(\vartheta)$$

$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta\,(\alpha - \cos(\vartheta))}{(\alpha^2 + 1 - 2\alpha\cos\vartheta)^{3/2}}\, d\vartheta$$

Intenzita elektrického pole v okolí homogenně nabité koulové sféry.
Jiný způsob



$$\alpha = \frac{d}{R}$$

$$dE_z = \frac{\sigma}{2\varepsilon_0} \frac{\sin\vartheta\,(\propto - \cos(\vartheta)\,)}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta =$$

$$= \frac{\sigma}{2\varepsilon_0} \left(\frac{\alpha * \sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}} - \frac{\sin\vartheta * \cos\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}} \right) d\vartheta$$

$$E_z = \frac{\sigma}{2\varepsilon_0} \int_0^{\pi} \left(\frac{\alpha * \sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}} - \frac{\sin\vartheta * \cos\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}} \right) d\vartheta$$

$$E_z = \frac{\sigma \propto}{2\varepsilon_0} \int_0^{\pi} \frac{\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta - \frac{\sigma}{2\varepsilon_0} \int_0^{\pi} \frac{\cos\vartheta \sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta$$

$$E_z = \frac{\sigma \propto}{2\varepsilon_0} \int_0^{\pi} \frac{\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta - \frac{\sigma}{2\varepsilon_0} \int_0^{\pi} \frac{cos\vartheta \sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta$$

$$\int_0^{\pi} \frac{\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta = \left[\frac{1}{\alpha} \frac{1}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\right]_0^{\pi} =$$

$$= \frac{1}{\alpha}\left(\frac{1}{(\propto^2 + 1 + 2 \propto)^{\frac{1}{2}}} - \frac{1}{(\propto^2 + 1 - 2 \propto)^{\frac{1}{2}}}\right) = \frac{1}{\alpha}\left(\frac{1}{\propto + 1} - \frac{1}{\propto - 1}\right) = -\frac{2}{\alpha} \frac{1}{\alpha^2 - 1}$$

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{(\vec{d}-\vec{R})}{|\vec{d}-\vec{R}|^3}\, d\vec{S} \qquad \vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_0^{\pi} \frac{\vec{r}}{|\vec{r}|^3}\, d\vec{S}$$

$$\int_0^{\pi} \frac{\cos\vartheta * \sin\vartheta}{(\propto^2 +1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta \; =$$

$$= \left[\frac{1}{\alpha}\frac{\cos\vartheta}{(\propto^2 +1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\right]_0^{\pi} + \frac{1}{\alpha}\int_0^{\pi} \frac{\sin\vartheta}{(\propto^2 +1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\, d\vartheta$$

$$\left[\frac{1}{\alpha}\frac{\cos\vartheta}{(\propto^2 +1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\right]_0^{\pi} = -\frac{1}{\alpha}\left(\frac{1}{\alpha+1} + \frac{1}{\alpha-1}\right) = -\frac{2}{\alpha^2-1}$$

$$\int_0^{\pi} \frac{\sin\vartheta}{(\propto^2 +1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\, d\vartheta = \frac{1}{\alpha}\left[-(\propto^2 +1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}\right]^{\pi} = \frac{1}{\alpha}\big((\alpha-1) - (\alpha+1)\big) = -\frac{2}{\alpha}$$

$$\int_0^{\pi} \frac{\cos\vartheta * \sin\vartheta}{(\propto^2 +1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta \; = -\frac{2}{\alpha^2-1} - \frac{2}{\alpha}$$

$$E_z = \frac{\sigma \propto}{2\varepsilon_0}\int_0^{\pi}\frac{\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta \; - \frac{\sigma}{2\varepsilon_0}\int_0^{\pi}\frac{cos\vartheta\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta$$

$$\int_0^{\pi}\frac{\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta \; = -\frac{2}{\alpha}\frac{1}{\alpha^2 - 1}$$

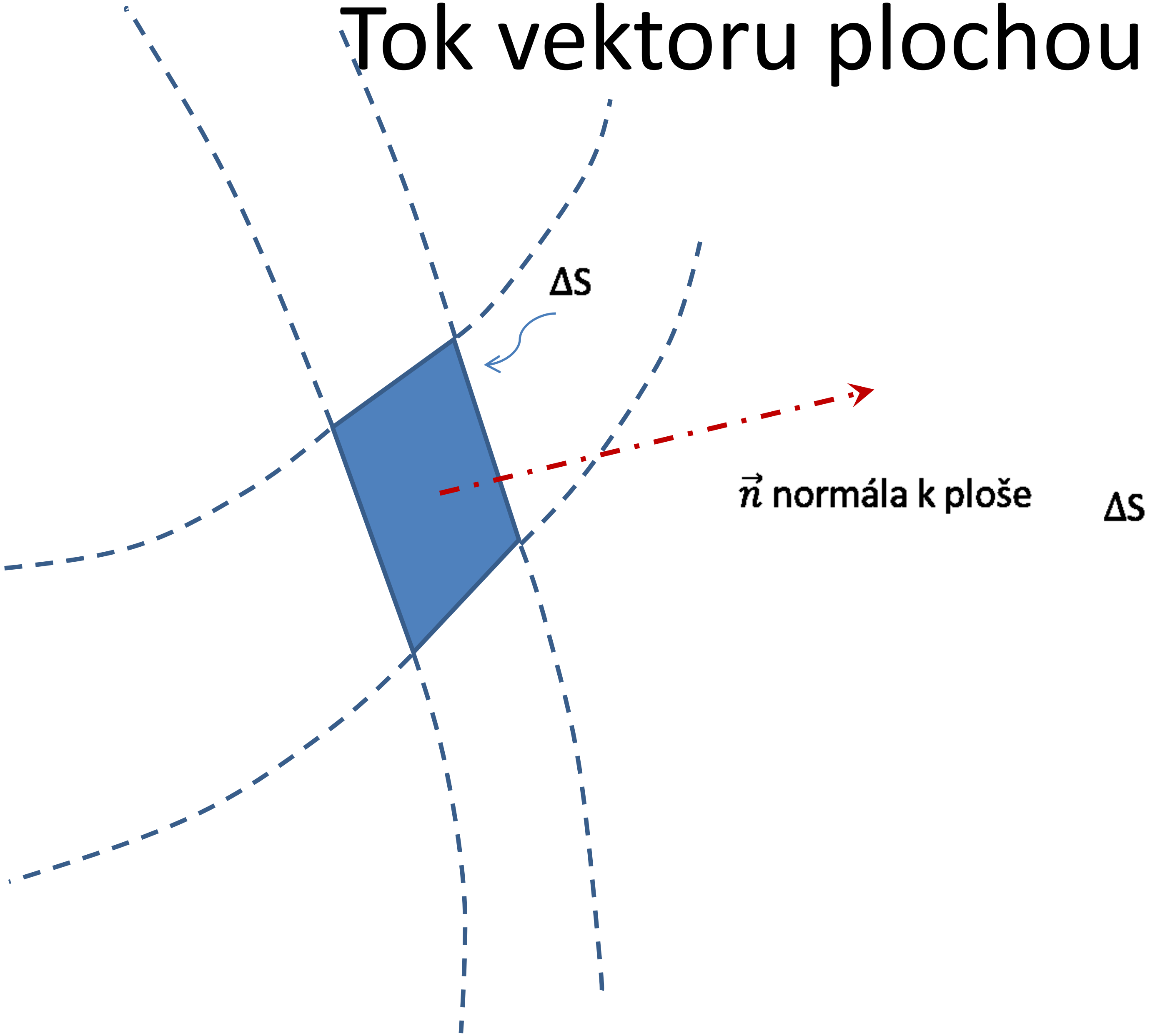
$$\int_0^{\pi}\frac{\cos\vartheta * \sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{3/2}}\, d\vartheta \; =$$

$$= \left[\frac{1}{\alpha}\frac{\cos\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\right]_0^{\pi} + \frac{1}{\alpha}\int_0^{\pi}\frac{\sin\vartheta}{(\propto^2 + 1 - 2 \propto \cos\vartheta)^{\frac{1}{2}}}\, d\vartheta \; = -\frac{2}{\alpha^2 - 1} - \frac{2}{\alpha^2}$$

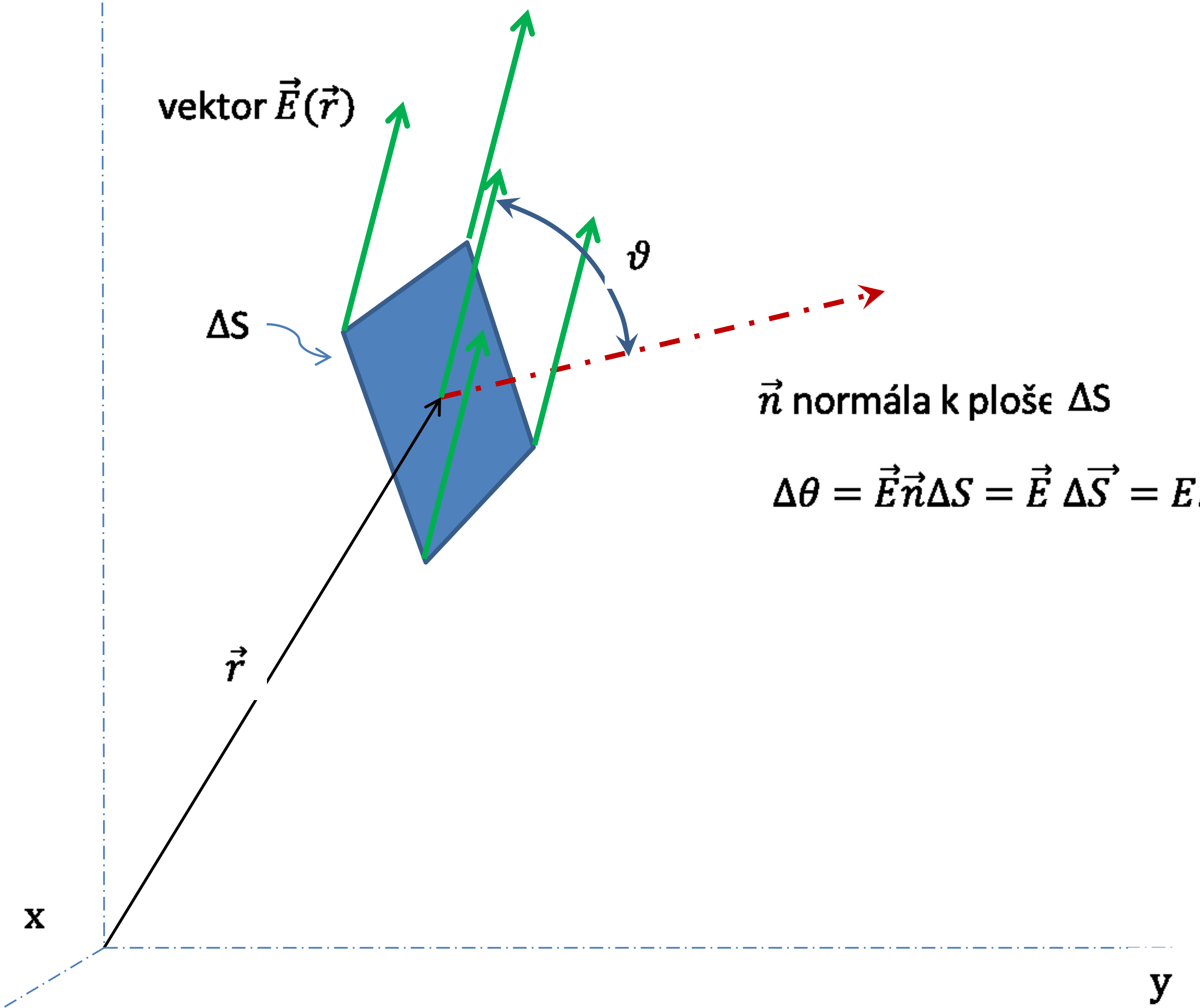
$$E_z = \frac{\sigma}{2\varepsilon_0}\left(-\frac{2}{\alpha^2 - 1} + \frac{2}{\alpha^2 - 1} + \frac{2}{\alpha^2}\right) = \frac{\sigma}{\varepsilon_0}\frac{1}{\alpha^2} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{d^2} \qquad \alpha = \frac{d}{R}$$

$$\sigma = \frac{Q}{4\pi R^2} \qquad E_z = \frac{\frac{Q}{4\pi R^2}}{\varepsilon_0}\frac{R^2}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{d^2}$$

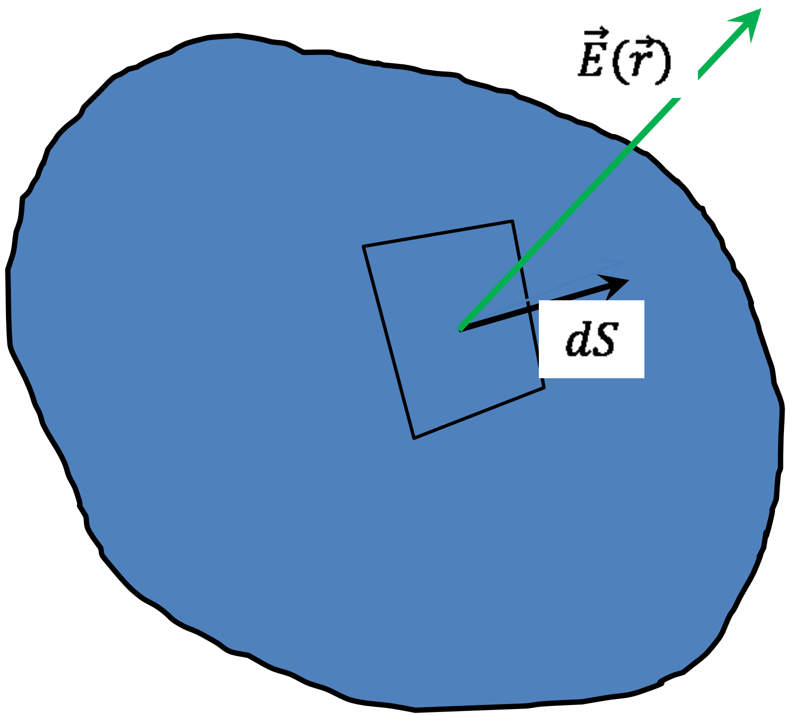
Tok vektoru plochou



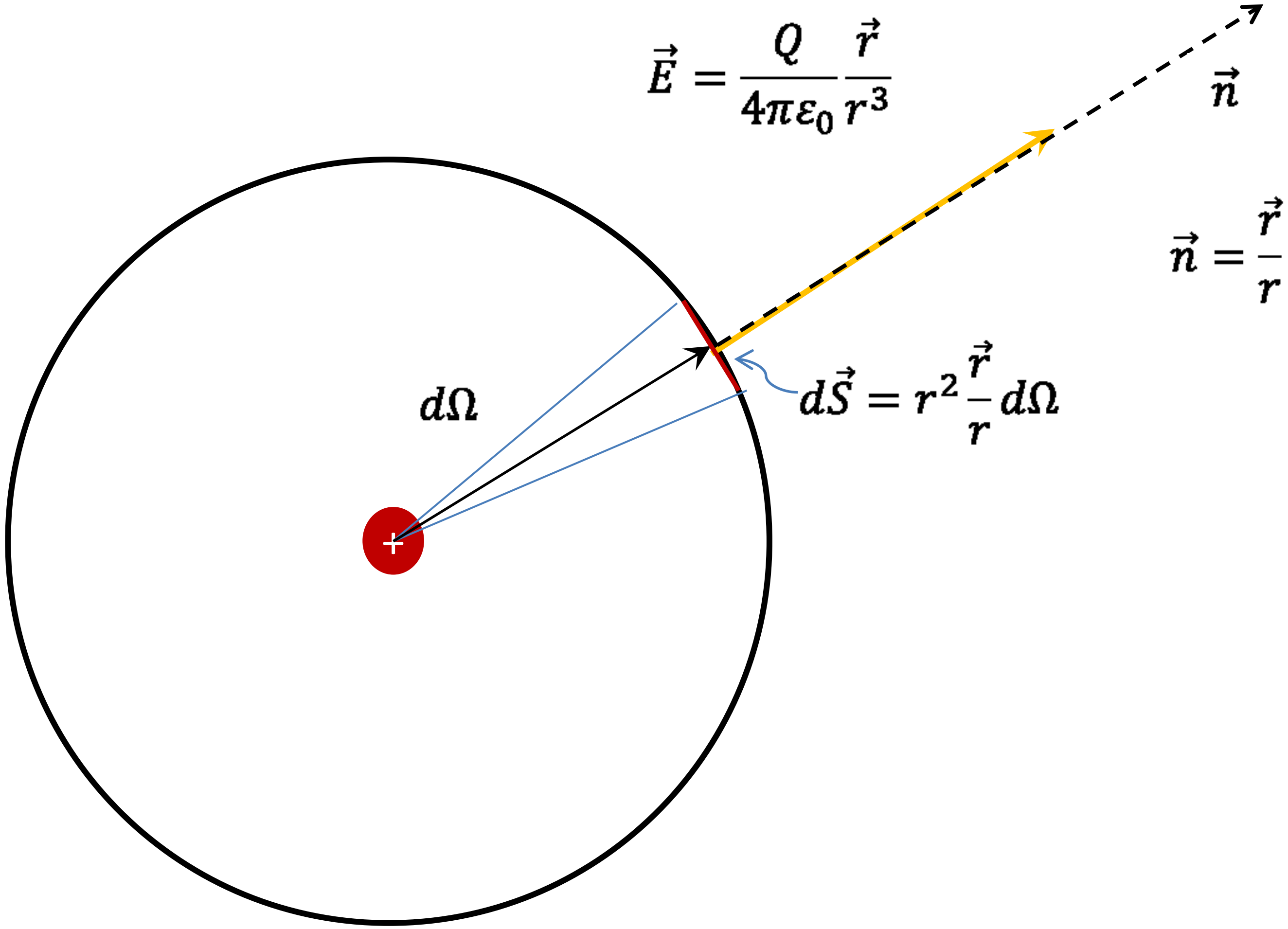
Tok vektoru plochou



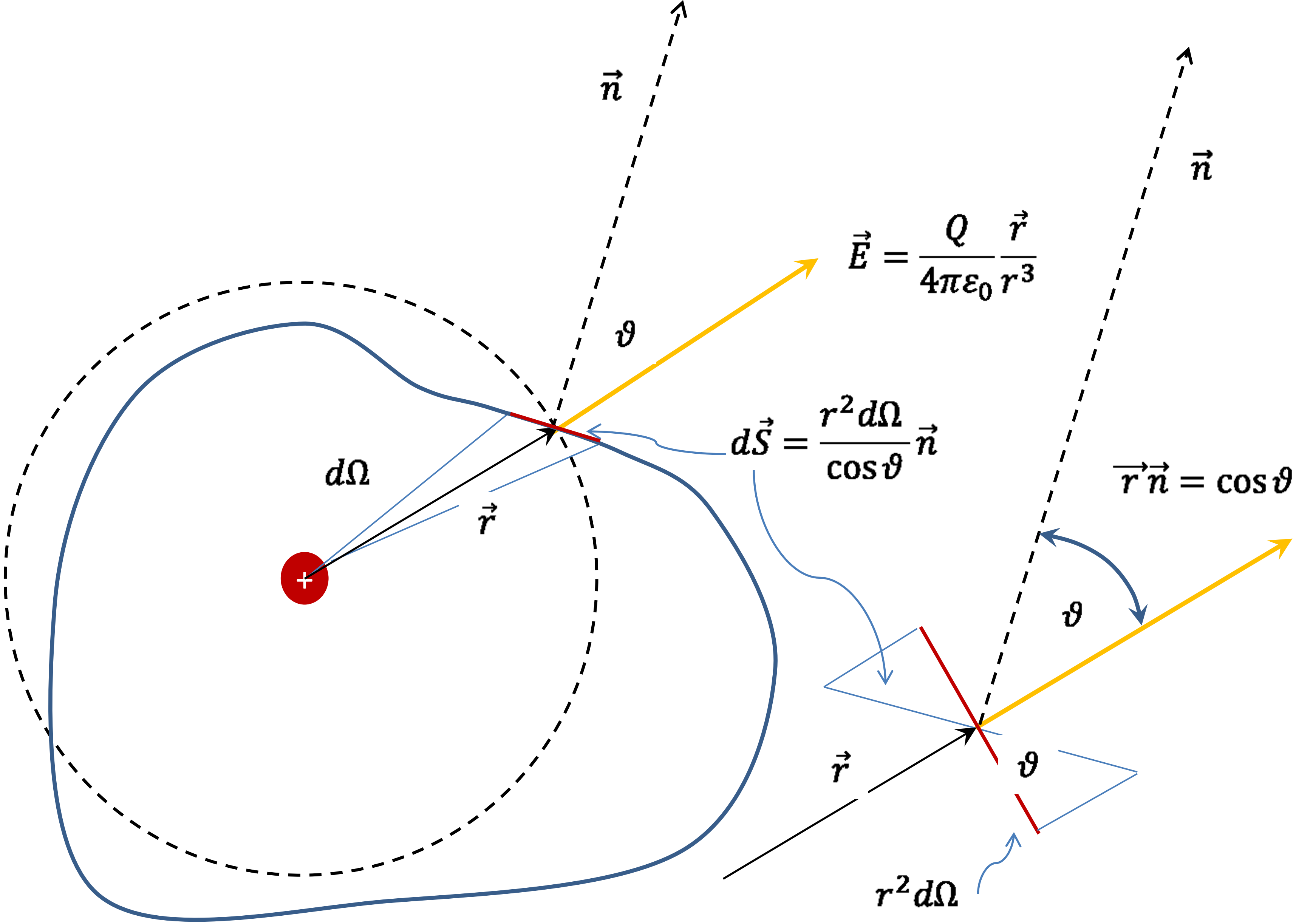
$$\Delta\theta = \vec{E}\vec{n}\Delta S = \vec{E}\ \Delta\vec{S} = E\Delta S\cos\vartheta$$



$$\oiint \vec{E}(\vec{r})dS = \frac{Q}{\varepsilon_0}$$

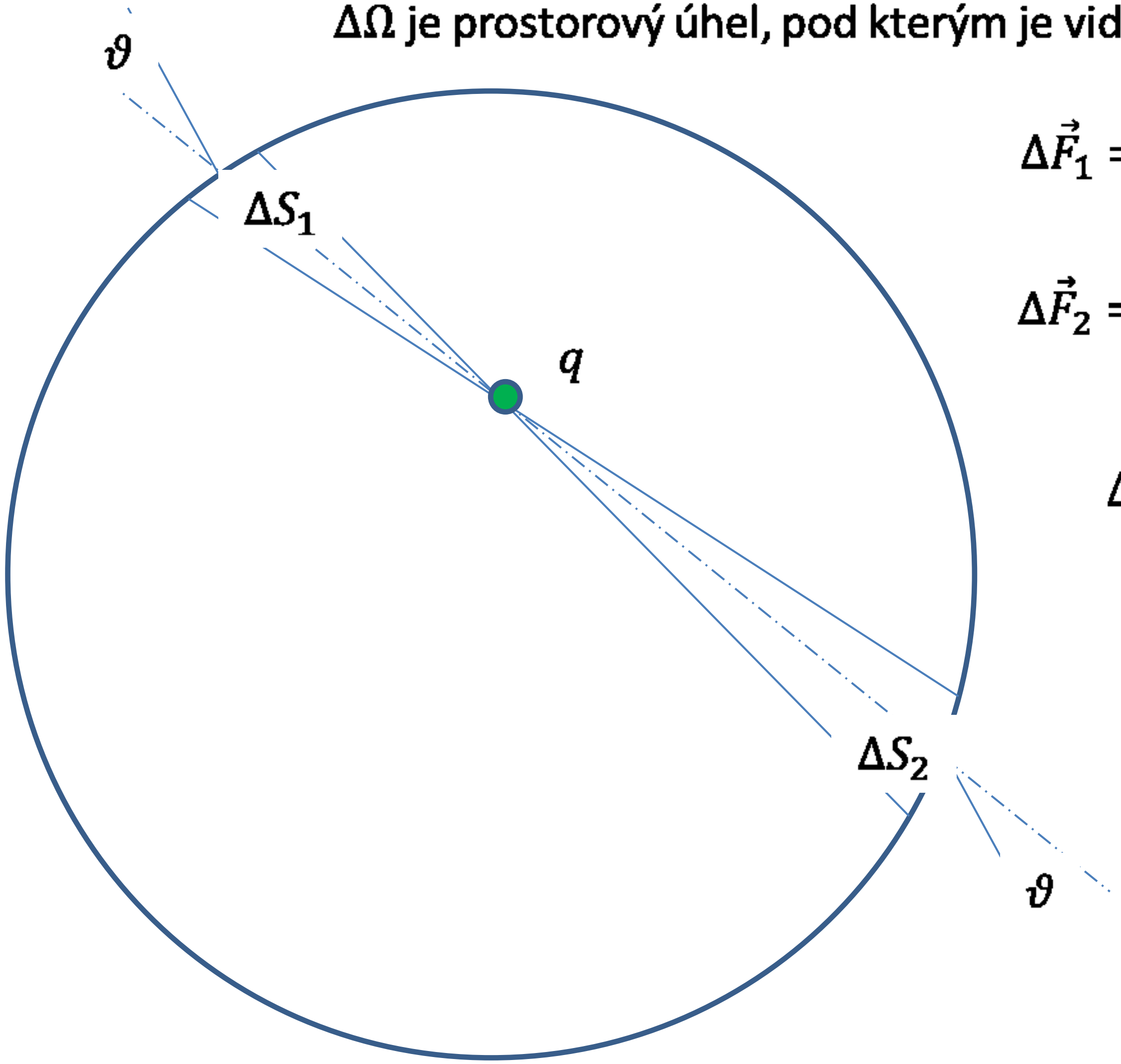


$$\oiint \vec{E}(\vec{r})d\vec{S} = \oiint \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}d\vec{S} = \int_0^{4\pi}\frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3} * r^2\frac{\vec{r}}{r}d\Omega = \frac{Q}{\varepsilon_0}$$



$$\oiint \vec{E}(\vec{r})d\vec{S} = \oiint \frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}\frac{r^2 d\Omega}{\cos\vartheta}\vec{n} = \int_0^{4\pi}\frac{Q}{4\pi\varepsilon_0 r}\frac{1}{\cos\vartheta}\vec{r}\vec{n}\,d\Omega = \int_0^{4\pi}\frac{Q}{4\pi\varepsilon_0}\frac{1}{\cos\vartheta}\cos\vartheta\,d\Omega = \frac{Q}{\varepsilon_0}$$

Síla působící na náboj uvnitř rovnoměrně nabité kulové sféry.



$\Delta\Omega$ je prostorový úhel, pod kterým je vidět ploška ΔS_1, ΔS_2

$$\Delta\vec{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\Delta S_1\cos(\vartheta)}{r^2}\vec{e}$$

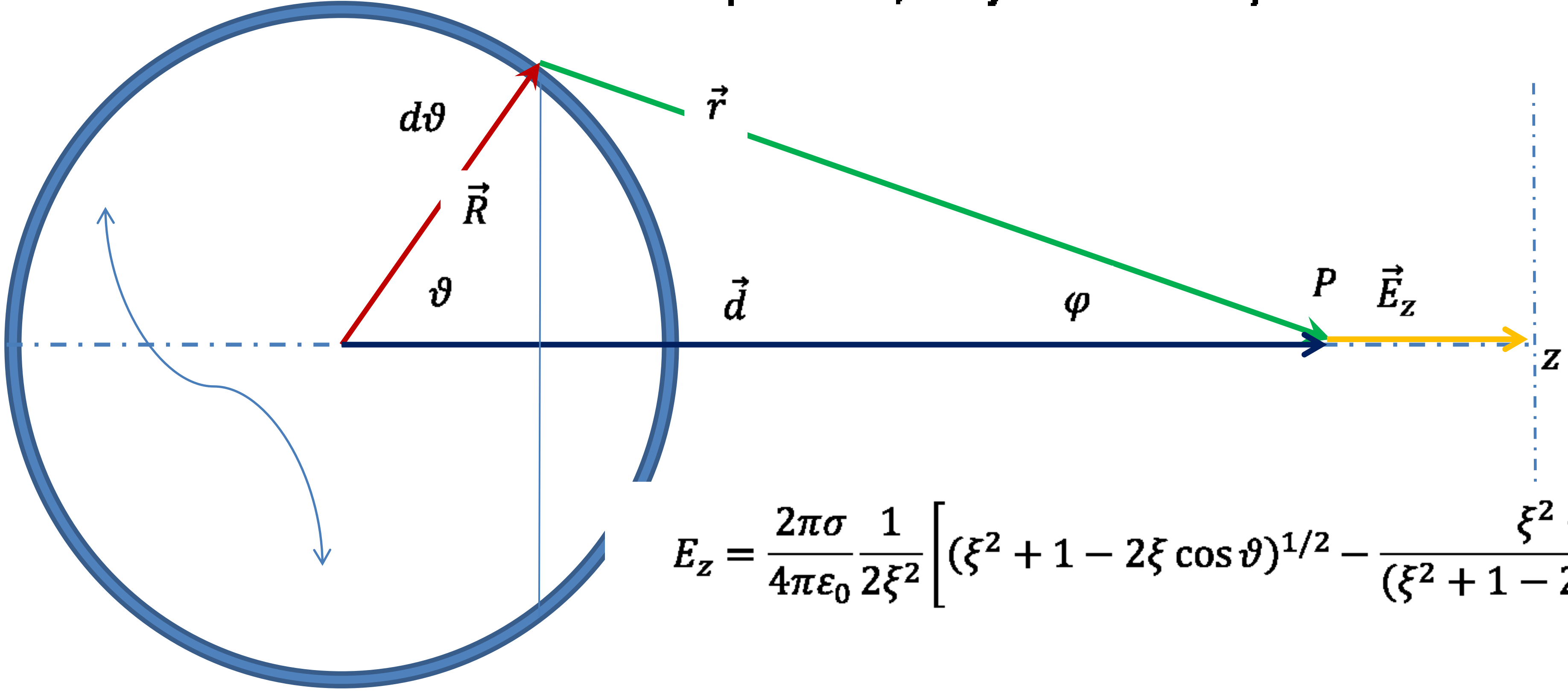
$$\Delta\vec{F}_2 = -\frac{1}{4\pi\varepsilon_0}\frac{\sigma\Delta S_2\cos(\vartheta)}{r^2}\vec{e}$$

$$\Delta\Omega = \frac{\sigma\Delta S\cos(\vartheta)}{r^2}$$

$$\Delta F = \frac{\sigma\Delta\Omega}{4\pi\varepsilon_0}$$

$$\Delta\vec{F}_1 + \Delta\vec{F}_2 = 0$$

Příklad 1: určete pro jaký úhel ϑ_0 bude el. pole v bodě P od každé z obou částí homogenně nabité sféry vymezené kružnicí, která je určena úhlem ϑ_0 poloviční, než je od celé sféry.

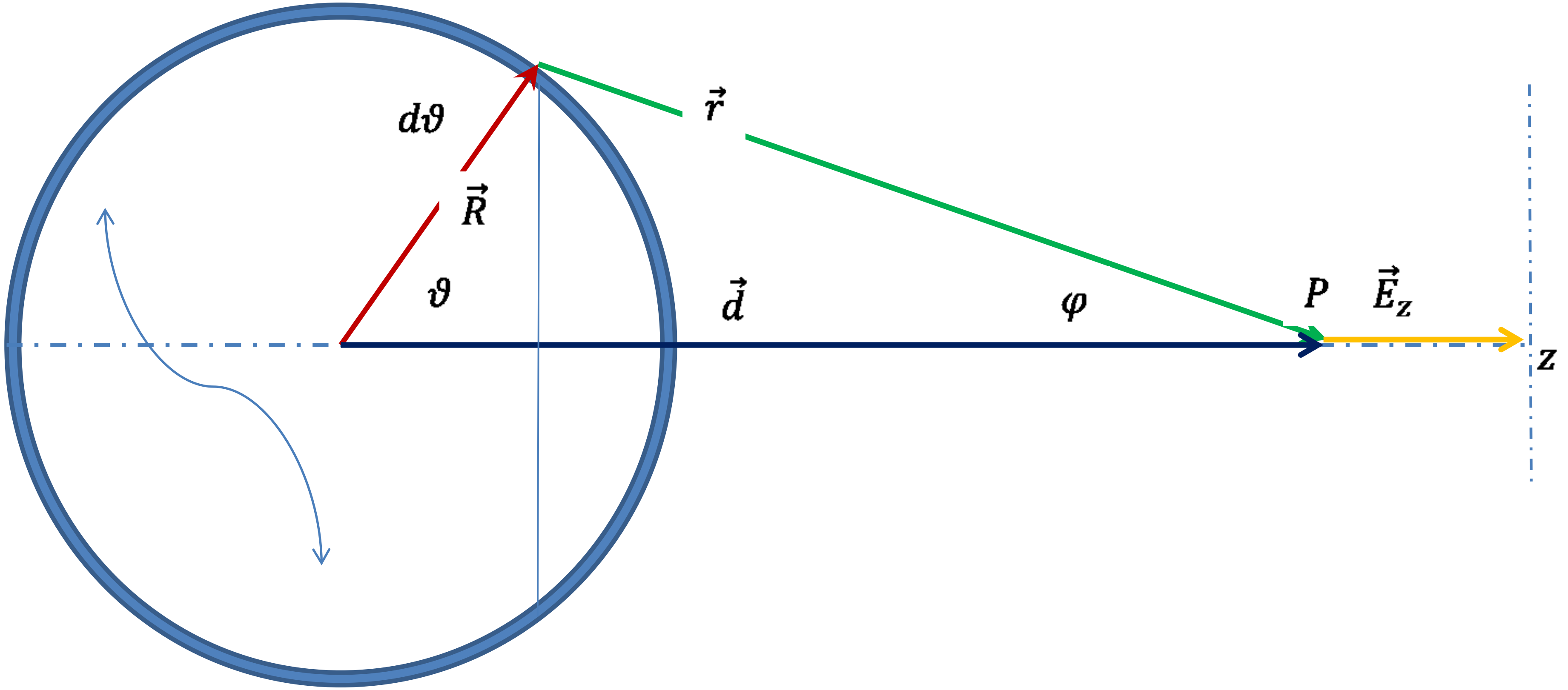


$$E_z = \frac{2\pi\sigma}{4\pi\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi}$$

$$\frac{E_z}{2} = \frac{\sigma}{4\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi} = \frac{\sigma}{2\varepsilon_0}\frac{1}{2\xi^2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\vartheta_0}$$

$$\frac{1}{2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi} = \left[(\xi^2+1-2\xi\cos\vartheta)^{1/2} - \frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\vartheta_0}$$

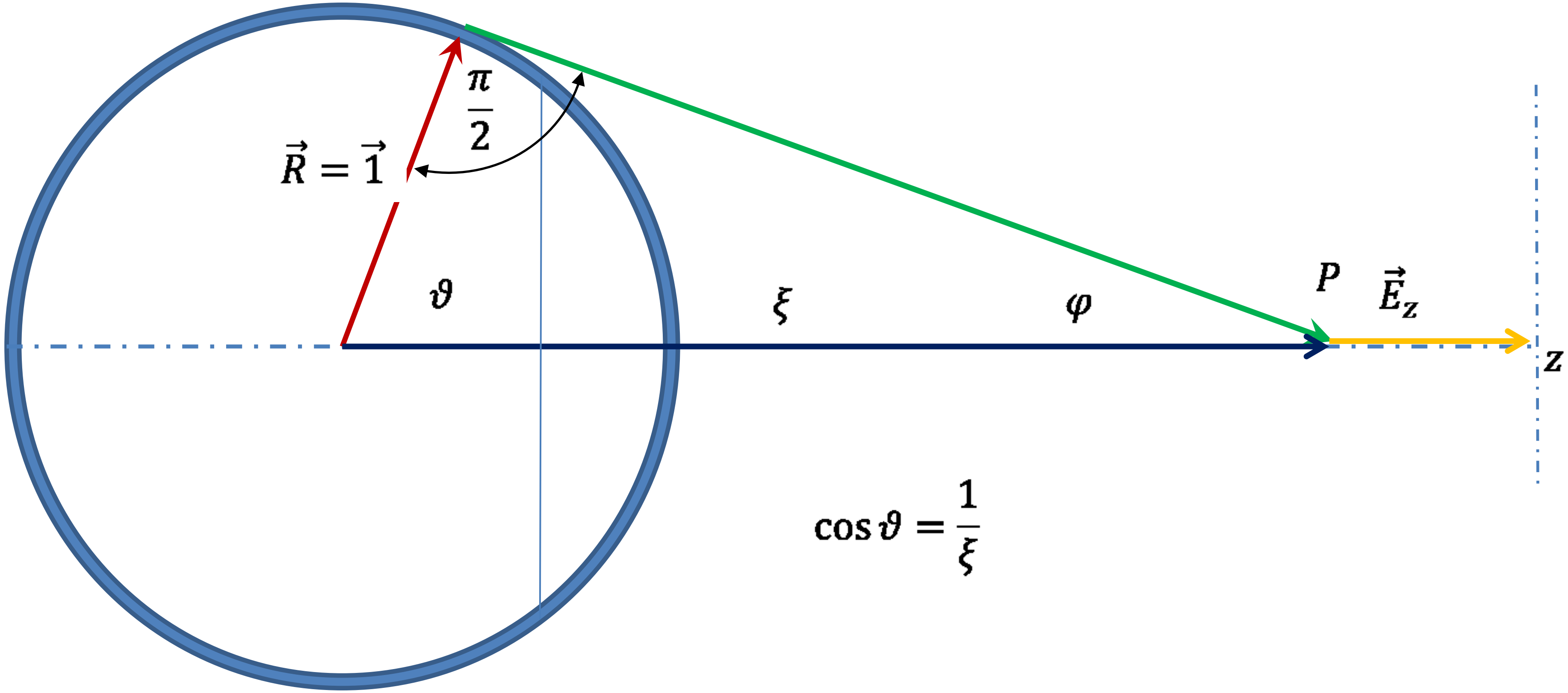
Příklad 1, řešení



$$\frac{1}{2}\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2}-\frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\pi}=\left[(\xi^2+1-2\xi\cos\vartheta)^{1/2}-\frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{1/2}}\right]_0^{\vartheta_0}$$

$$1+1=(\xi^2+1-2\xi\cos\vartheta)^{1/2}-\frac{\xi^2-1}{(\xi^2+1-2\xi\cos\vartheta)^{\frac{1}{2}}}+2 \qquad \cos\vartheta=\frac{1}{\xi}$$

Příklad 1, řešení



$$\cos\vartheta = \frac{1}{\xi}$$