# F4280 Technology of thin film deposition and surface treatment 2. Gas Kinetics

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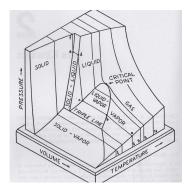


# Outline - chapter 2. Gas Kinetics

- 2.1 Vapors and Gases
- 2.2 Maxwell-Boltzmann Distribution
- 2.3 Ideal-Gas Law
- 2.4 Units of Measurement
- 2.5 Knudsen Equation
- 2.6 Mean Free Path
- 2.7 Knudsen number
- 2.8 Transport Properties

# p-V-T diagram

The possible equilibrium states can be represented in pressure-volume-temperature (p-V-T) space for fixed amount of material (e.g. 1 mol =  $6.02 \times 10^{23}$ ).



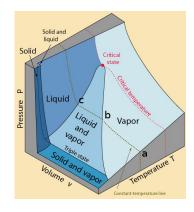
Lines = cuts through the p-V-T surface for fixed  $T \Rightarrow$  relationship between p and  $V_{\rm m}$  (molar volume).

Line a - b - c below the critical point (at  $T_2$ ):

- ▶ point *a*: highest *V* (lowest *p*) vapor phase
- ▶ from point *a* to *b*: reducing  $V \rightarrow$  increasing p
- point b: condensation begins
- from point b to c: V is decreasing at fixed p (b − c line is ⊥ to the p-T plane, p is called saturation vapor pressure p<sub>v</sub> or just vapor pressure)
- point c: condensation completed

If V is abruptly decreased in b-c transition p would be pushed above the line  $b-c \Rightarrow$  **non-equilibrium supersaturated vapors**. Supersaturation is an important drivign force in the nucleation and growth of thin films.

It is important to distinguish between the behaviors of vapors and gases:

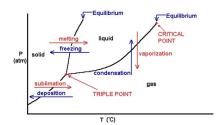


vapors: can be condensed to liquid or solid by compression at fixed  $T \Rightarrow$  below critical point defined by  $p_{\rm c}$ ,  $V_{\rm c}$  and  $T_{\rm c}$ 

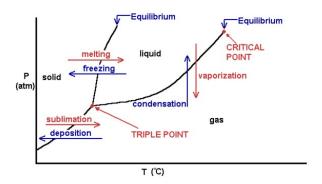
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gases: monotonical decrease of V upon compression ⇒ no distinction between the two phases

Surfaces "liquid-vapor", "solid-vapor" and "solid-liquid" are perpendicular to the p-T plane ⇒ their projection on that plane are lines.

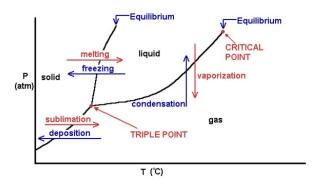


# p-T diagram



- **triple point**: from triple line  $\perp$  to p-T plane
- below T of triple point: liquid-phase region vanishes  $\Rightarrow$  condensation directly to the solid phase, vaporization in this region is sublimation
- pressure along borders of vapor region is vapor pressure  $p_v$

## p-T diagram



- vapor pressure  $p_v$  increases exponentially with T up to  $p_c$
- $p_c$  is well above 1 atm  $\Rightarrow$  deposition of thin films is performed at  $p \ll p_c$ , either  $p > p_v$  (supersaturated vapors) or  $p < p_v$
- first two steps in the deposition (source supply and transport to substrate) should be carried out at  $p < p_v$  to avoid condensation
- condensation should be avoided also during compression in vacuum pumps

### **Maxwell-Boltzmann Distribution**

Distribution of random velocities  $\vec{V}$  in equilibrium state

$$f(\vec{V}) = n \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{mV^2}{2k_{\rm B} T}\right)$$
 (1)

where  $k_{\rm B}=1.38\times 10^{-23}~{\rm m^2~kg~s^{-2}~K^{-1}}$  (or J K<sup>-1</sup>) is the Boltzmann constant, n, T and m are particle density, temperature and mass, respectively.

If the drift velocity is zero we do not need to distinguish between the velocity and random velocity, i.e.  $\vec{v} \equiv \vec{V}$ .

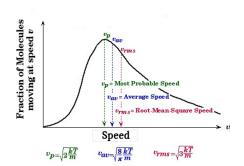
**Maxwell-Boltzmann distribution is isotropic**  $\Rightarrow F(v)$  distribution of speeds  $v \equiv |\vec{v}|$  can be defined by integration of f(v) in spherical coordinates

$$F(v)dv = \int_0^{\pi} \int_0^{2\pi} f(v)v^2 \sin\theta d\phi d\theta dv$$
 (2)

resulting in

$$F(v) = 4\pi v^2 n \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_{\rm B} T}\right)$$
(3)

# Mean (Average) Speed, Molecular Impingement Flux



### Root-mean-square (rms) speed:

$$v_{\rm rms} = \sqrt{\frac{1}{n} \int_0^\infty F(v) v^2 dv} = \sqrt{\frac{3k_{\rm B}T}{m}}$$
 (6)  $\left(\frac{dF(v)}{dv}\right)_{v=v_{\rm D}} = 0 \Rightarrow v_{\rm P} = \sqrt{\frac{2k_{\rm B}T}{m}}$ 

### Mean speed:

$$\langle v \rangle = v_{\rm av} = \frac{1}{n} \int_0^\infty F(v) v dv = \sqrt{\frac{8k_{\rm B}T}{\pi m}}_{(4)}$$

or

$$v_{\rm av} = \sqrt{\frac{8RT}{M}} \tag{5}$$

using molar mass  $M = mN_A$  in kg/mol and gas constant

$$R = k_{\rm B} N_{\rm A} = 8.31 \, {
m Jmol}^{-1} {
m K}^{-1}$$

where  $N_{\rm A} = 6.02 \times 10^{23} \; {\rm mol}^{-1}$  is Avogadro's number

The most probable speed  $v_{\rm p}$ :

$$\left(\frac{dF(v)}{dv}\right)_{v=v_{\rm p}} = 0 \Rightarrow v_{\rm p} = \sqrt{\frac{2k_{\rm B}T}{m}}$$
 (7)

### Ideal-Gas Law

From the definition of pressure for ideal gas (not necessary to consider pressure tensor but only scalar pressure)

$$p = \frac{1}{3}mn\langle \vec{V}_x^2 + \vec{V}_y^2 + \vec{V}_z^2 \rangle = \frac{1}{3}mn\langle \vec{V}^2 \rangle = m \int_V V^2 f(V) d^3 V.$$
 (8)

The ideal-gas law is obtained by integration of (8) using Maxwell-Boltzmann distribution:

$$p = nk_{\rm B}T$$
 or  $\frac{\rho V}{T} = Nk_{\rm B}$  (9)

where N is the number of particles.

Chemists are used to work in molar amounts ( $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$ ):

- ▶ molar concentration  $n_{\rm m} = n/N_{\rm A} \Rightarrow p = n_{\rm m}RT$
- ▶ number of moles  $N_{\rm m} = N/N_{\rm A} \Rightarrow p = N_{\rm m}RT/V$
- ightharpoonup molar volume  $V_{\rm m} = V/N_{\rm A} \Rightarrow p = RT/V_{\rm m}$

### The ideal gas is obeyed if

- the volume of molecules in the gas is much smaller than the volume of the gas
- the cohesive forces between the molecules can be neglected.

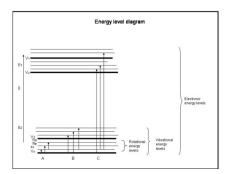
Both assumptions are fulfilled for low  $n \Rightarrow$  always fulfilled for thin film deposition from the vapor phase ( $T \geq T_{\rm room}$  and  $p \leq p_{\rm atm}$ ), i.e. well away from the critical point (most materials  $p_c \gg 1$  atm or if not  $T_c \ll 25$  °C)

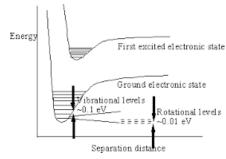
2.3 Ideal-Gas Law

# **Energy Forms Stored by Molecules**

Molecules can store energy in various forms. Their energetic states are quantized (spacing between energy levels  $\Delta E$ )

- ightharpoonup electronic excitations  $\Delta E_{\rm e}$  is highest, transitions between different electronic states are possible only for extremely high T or collision with energetic particle
- ▶ vibrational excitations energy levels correspond to different vibration modes of the molecule,  $\Delta E_v \approx 0.1$  eV (1 eV = 11 600 K)
- ightharpoonup rotational excitations different rotational modes of the molecule,  $\Delta E_{\rm r} \approx 0.01~\text{eV}$
- ▶ translational energy above performed description of molecular random motion  $E_{\rm t}=1/2mV^2$ , no details of inner molecule structure are considered,  $\Delta E_{\rm t}$  negligible at ordinary T.





# **Energy Content of Gas**

From definition of absolute temperature - the mean thermal energy kT/2 belongs to each translational degree of freedom and molecular translation energy is

$$E_{\rm t} = \frac{3}{2} k_{\rm B} T \tag{10}$$

 $\Rightarrow$  equipartition theorem of classical statistical mechanics. Classical statistical treatment assumes very close quantized energy levels of molecules, i.e. approximated as a continuum. It is a good assumption for translational energy when  $T \gg 0$  K.

- For atomic gases,  $E_t$  is total kinetic energy content.
- For molecular gases,  $E_r$  is added at ordinary T and  $E_v$  at very high T:

**Molar heat capacity at constant volume**  $c_V$  (for molecular gas) [J/(mol.K)] - increase of total kinetic energy for increasing T:

$$c_{\rm V} = \frac{\mathrm{d}E_{\rm m}}{\mathrm{d}T}N_{\rm A} = \frac{\mathrm{d}(E_{\rm t} + E_{\rm r} + E_{\rm v})}{\mathrm{d}T}N_{\rm A} \tag{11}$$

for atomic gases

for small diatomic molecules at room T

$$c_{\mathrm{V}}=rac{3}{2}R=rac{3}{2}kN_{\mathrm{A}}$$

$$c_{\rm V} = \frac{5}{2} R$$

 two rotational degrees of freedom are excited but vibrational ones are not

# **Energy Content of Gas**

The heat capacity of any gas is larger when measured at constant pressure  $c_{\rm p}$  - heat input is doing  $p{\rm d}\,V$  work on the surroundings in addition to adding kinetic energy to the molecules:

$$c_{\rm p} = c_{\rm V} + R \tag{12}$$

We can write from thermodynamics

$$c_{\rm V} = \left(\frac{\partial U_{\rm m}}{\partial T}\right)_{V} \tag{13}$$

where  $U_{\mathrm{m}}$  is internal energy per mol  $U_{\mathrm{m}}=E_{\mathrm{m}}N_{\mathrm{A}}$  and

$$c_{\rm p} = \left(\frac{\partial H_{\rm m}}{\partial T}\right)_{p} \tag{14}$$

where  $H_{
m m}$  is enthalpy per mol  $H_{
m m}=U_{
m m}+
ho V_{
m m} \Rightarrow$ 

$$\left(\frac{\partial H_{\rm m}}{\partial T}\right)_{\rho} = \left(\frac{\partial U_{\rm m}}{\partial T}\right)_{\rho} + \rho \left(\frac{\partial V_{\rm m}}{\partial T}\right)_{\rho} \tag{15}$$

giving  $c_{
m p} = c_{
m V} + R$ 

### 2.4 Units of Measurement

SI units?!

1 Torr = 133 Pa = 1 mm Hg

1 bar = 750 Torr =  $1.0 \times 10^5$  Pa = 0.99 atm (standard atmosphere)

The "standard" conditions of T and p (stp) are  $0^{\circ}$ C and 1 atm (760 Torr). From ideal gas law at stp  $V_{\rm m} = 22400 \, {\rm cm}^3$ .

These conditions are different from standard conditions to which thermodynamic data are referenced: 25°C and 1 bar.

In gas supply monitoring - the term "mass" flow rate measured in standard cm<sup>3</sup> per minute (second or liters per minute): sccm, sccs, slm. Standard means 0°C and 1 atm.

# 2.5 Knudsen Equation

The molecular impingement flux at a surface is a fundamental determinant of film deposition rate:

$$\Gamma = n\langle v \cos \theta \rangle = \int_0^\infty \int_0^{\pi/2} \int_0^{2\pi} f(v) v^3 \cos \theta \sin \theta d\phi d\theta dv \tag{16}$$

Substituting Maxwell-Boltzmann distribution

$$\Gamma = n \left(\frac{k_{\rm B}T}{2\pi m}\right)^{1/2} = \frac{1}{4} n v_{\rm av} \tag{17}$$

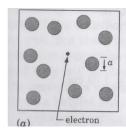
and using ideal gas law

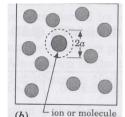
$$\Gamma = \rho \left(\frac{1}{2\pi kTm}\right)^{1/2} = \rho N_A \left(\frac{1}{2\pi RTM}\right)^{1/2} \tag{18}$$

where  $M = mN_A$  and  $R = kN_A$  (M is molar mass)

Calculate molecular impinging flux for  $CO_2$  molecules (44 a. u., 330 pm), 25  $^{\circ}$ C, 10<sup>-3</sup> Pa. Considering the molecule diameter of 330 pm calculate monolayer deposition rate considering all impinging molecules stick to the surface.

### F4280 Technologie depozice a povrchových úprav: 2.6 Mean Free Path





Unless T is extremely high, p is the main determinant of I.  $I \approx 1/p$ .

#### Mean free path

$$\lambda = \frac{1}{\sigma_{\rm m} n} \tag{19}$$

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electrons travelling through gas: electrons are much smaller than molecules  $\Rightarrow$  collision cross section  $\sigma_{\rm m}$  is just projected area of the gas molecule

$$\lambda_{\rm e} = \frac{1}{\pi/4a^2n} \tag{20}$$

It's approximation,  $\sigma_{\rm m}$  is function of el. energy

ions travelling through gas: similar diameter

$$\lambda_{\rm i} = \frac{1}{\pi a^2 n} \tag{21}$$

molecule-molecule collisions: "target" particles are not steady (comparable velocities) ⇒ mean speed of mutual approach is  $\sqrt{2}v_{\rm av}$  rather than  $v_{\rm av}$  (on average they approach each other at 90°)  $\Rightarrow$  it shortens / by  $\sqrt{2}$ 

$$\lambda_{\rm m} = \frac{1}{\sqrt{2}\pi a^2 n} \tag{22}$$

### 2.7 Knudsen number

It is worth remembering that the mean free path at 1 Pa and room  $\mathcal{T}$  is about 1 cm for small molecules.

The order of magnitude of *I* is very important in film deposition, because it determines whether the process is operating in the high-vacuum or the fluid-flow regime. The regime is determined by the **Knudsen number**:

$$Kn = \lambda/L$$
 (23)

where L is a characteristic dimension in the process, e.g. distance between the source and the substrate,  $\lambda$  is the mean free path.

- ► For *Kn* > 1, the process is in high-vacuum regime (molecular flow regime).
- For  $Kn \ll 0.01$ , the process is in fluid flow regime.

Intermediate values of Kn constitute a transition regime where the equations applicable to either of limiting regimes are not strictly valid.

Plasma processes often operate in the transition regime. High-vacuum processes require  $p < 10^{-2}$  Pa for typical chamber sizes to ensure Kn > 1.

# 2.8 Transport Properties

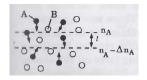
```
quantify the rate of transport of mans (diffusion)
                                momentum (viscous shear) / through a
                                energy ( head conduction) ) fluid
 Transport to be discussed occurs by random moderular modion
  through a gas which has no bulk flow in the direction of the
  hangeord.
  Man and head can be also hanglooked of bulk flow but it is not
  discussed here.
  Table 2.1 (p. 26) summarizes the quantities and eggs. It imbude also it enough (for of the hamp of charge) because it's helpful analogy.
   Transport is always described by an eg. of the form
                      (flux of A) = - (preportional factor) x (grad in
   general form is 3D but we will give it in 1D
   for simplicity.
   Example for electrical current /jx: jx (A/m²) = S. dv gred of
                                             el conductivity el potential
            familiar ohm's law
                                        = 0 el. resistivity
```

## 2.8 Transport Properties - Overview

ransported quantity	Describing equation	Proportionality factor	
		Derivation from elementary kinetic theory	Typical value at 300 K, 1 atm
Mass	Diffusing flux = $J_A\left(\frac{mc}{cm^28}\right) = -D_{AB}\left(\frac{dn_A}{dx}\right)$ (Fick's law)	$\begin{split} & \text{Diffusivity} = \\ & \text{D}_{AB}\!\!\left(\!\frac{cm^2}{s}\!\right) = \frac{1}{4}\delta l \approx \frac{T^{7/4}\!\!\left(\!\frac{1}{M_A}\!+\!\frac{1}{M_B}\!\right)^{\!1/2}}{p\left(a_A\!+\!a_B\!\right)^2} \end{split}$	Ar-Ar: 0.19 cm <sup>2</sup> /s Ar-He: 0.72
Momentum	Shear stress = $\tau(N/m^2) = \eta \frac{du}{dx}$	Viscosity = $\eta(\text{Poise})^{\dagger} = \frac{1}{4} \mathbf{n}  \mathbf{m}  \hat{\mathbf{c}}  l \simeq \frac{\sqrt{MT}}{a^2}$	Ar: 2.26×10 <sup>-4</sup> Poise <sup>†</sup> He: 2.02×10 <sup>-4</sup>
Energy (heat)	Conductive heat flux = $\Phi\Big(\frac{W}{cm^2}\Big) = -K_T\frac{dT}{dx}$ (Fourier's law)	$\begin{split} & \text{Thermal conductivity} = \\ & K_T\!\left(\!\frac{W}{cm \cdot K}\right) = \frac{1}{2} n\! \left(\!\frac{c_\nu}{N_A}\!\right) \! \tilde{c} l \approx \sqrt{\frac{T}{M}} \; \frac{c_\nu}{\alpha^2} \end{split}$	Ar: 0.176 mW/cm-K He: 1.52
Charge	Current density = $j\left(\frac{A}{cm^2}\right) = \frac{-1}{\rho} \frac{dV}{dx} = -s \frac{dV}{dx}$ (Ohm's law)		molt authorized in the control of th

Table 2.1 Gas transport properties from the book by Donald L. Smith, Thin-Film Deposition: Principles & Practice, McGraw-Hill 1995.

### 2.8.1 Diffusion



Molecular diffusion is demonstrated using mixture of molecules A (black) and B (white). Consider, the concentration of black molecules A is decreasing from  $n_A$  to  $n_A - \Delta n_A$  in the x-direction over a distance of one mean free path  $\lambda \Rightarrow$ Diffusion of A occurs in the direction of decreasing  $n_A$ .

A rough estimate of diffusion flux can be made by calculating the net flux through an imaginary slab of thickness  $\lambda$  (I in D. Smith book), using

$$\Gamma = rac{1}{4} n \langle v \rangle \ {
m where} \ \langle v \rangle = \sqrt{rac{8 k T_{
m B}}{\pi m}}$$

for the fluxes in opposite directions  $\downarrow$  and  $\uparrow \Rightarrow \Gamma_{\rm A} = \Gamma(x) - \Gamma(x+\lambda) = 1/4\Delta n_{\rm A} \langle v \rangle$ 

Since 
$$\Delta n_{\rm A}=\lambda\,rac{-{
m d}n_{\rm A}}{{
m d}\,{
m x}}$$
, we have  $\Gamma_{\rm A}=-rac{1}{4}\langle v
angle\lambdarac{{
m d}n_{\rm A}}{{
m d}\,{
m x}}\Rightarrow$ 

$$\Gamma_{\rm A} = -D_{\rm AB} \frac{\mathrm{d} n_{\rm A}}{\mathrm{dx}}$$

 $\Gamma_{\rm A} = -D_{\rm AB} \frac{{\rm d}n_{\rm A}}{{\rm d}n_{\rm A}}$  Fik's law,  $D_{\rm AB}$  diffusion coefficient of A through B

Inserting expression for  $\langle v \rangle$  and molecule-molecule mean free path  $\lambda = \frac{1}{\sqrt{2\pi} \sigma^2 n}$  we find

$$D_{
m AB} \sim rac{T^{3/2}}{\sqrt{m}\,a^2p}$$

Empirically, it should be  $T^{7/4}$ , and m and a are averaged to account for A-B mixture, see next slide.

### 2.8.1 Diffusion

$$D_{AB} = \frac{T^{+1/4} \left(\frac{1}{m_A} + \frac{1}{m_B}\right)^{1/2}}{\rho \left(\alpha_A + \alpha_B\right)^2} \text{ but } D_{AB} \text{ has }$$

$$\left\{\frac{m^2}{3}\right\} = \frac{\rho \left(\alpha_A + \alpha_B\right)^2}{\rho \left(\alpha_B + \alpha_B\right)^2} \text{ obtaining originally }$$

$$\left\{\frac{m^2}{3}\right\} = \frac{1}{\rho \left(\alpha_A + \alpha_B\right)^2} \text{ obtaining originally }$$

Typical value at 300K, 1alon Ar-Ar: 0,19 cm2/s Ar-He: 0,72 cm/s

and defindent of  $D_{AB}$  on p, T can be integrabled using empirically determined values and our expression D(p,T) with accuracy of a juder two.

Dependence of DAB on p is particularly important, since it can be lower than talm by many orders of magnitude in depos. mocesses => 1 DAB.

when p is reduced so far that Km > 1 ( $Kn = \frac{\lambda}{L}$ )

(molecular flow regime)

objusion no longer vicurs. Indead, molecules have from wall to wall in molecular flow, without encountering each other in the gas place.

# 2.8.2 Viscosity

I gas a vinority is the result of molecular memeritum transport along a gradient in bulk flow velocity is. (drift velocity)

The in gradient is along the x axis, perpendicular to it. This situation is encountered in fluid flowa boundary (u must go to sero) Superimposed is the random moles molion at relocity (v).

The random motion causes moterules to centinually cross up and down between means reparalled by A. Those moving unward will gain memeralum made upon colliding with molecules in the forter flow beam => drag force on the flow hearn at x ( Exomologici rila!)

# 2.8.2 Viscosity

Similarly, there moving downward will event an ascelerating force on the slower flow dream at  $x+\lambda$  viskban teams at  $x+\lambda$  viskban teams and experite in a deady-tale - viscous dream therese T (N/m²) = rate of momentum transper per unit area  $\pm$  for  $x=\lambda$  dress T (N/m²) = rate of momentum transper per unit area  $\pm$  for  $x=\lambda$  = 1 =

Two surprises: 1.  $\eta$  1 with  $\tau$  (expecte to liquid behavious)

2.  $\eta \neq \text{fancl.}(r)$ 

as in the case of diffusion, vinosity has no meaning for Kn >1

### 2.8.3 Heat transfer

Gaseous heat conduction occurs by transfer of energy in molecular collisions downward along a gradient in molecular kinetic energy  $E_{\rm m}$ . Equation for the **heat flux**  $\Phi$  is analogous to that for momentum transfer, with  $m\Delta u$  replaced by  $2\Delta E_{\rm m}$ 

$$\Phi = \frac{1}{4} n \langle v \rangle 2\lambda \frac{\mathrm{d}E_{\mathrm{m}}}{\mathrm{d}x} \tag{24}$$

with the units  $J/(s.m^2)$  or  $W/m^2$ .

We can substitute T for  $E_{\rm m}$  using Eq. (11) for molar heat capacity at constant volume

$$\frac{\mathrm{d}E_{\mathrm{m}}}{\mathrm{d}x} = \frac{\mathrm{d}E_{\mathrm{m}}}{\mathrm{d}T} \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{c_{\mathrm{V}}}{N_{\mathrm{A}}} \frac{\mathrm{d}T}{\mathrm{d}x}$$
(25)

Thus, we obtain Fourier's law

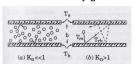
$$\Phi = -K_{\rm T} \frac{\mathrm{d}T}{\mathrm{d}x} \tag{26}$$

where thermal conducitivty  $K_{\rm T}\sim \sqrt{T/m}\,c_{\rm V}/a^2$ . Small, light molecules generally have higher  $K_{\rm T}$  although this trend is sometimes reversed by the higher  $c_{\rm V}$  of more complex molecules, which have more rot. and vibr. modes of energy storage.

Like  $\eta$ ,  $K_{\rm T}$  is independent of p for the same reason: as  $p\downarrow$ , molecular flux  $\Gamma\downarrow$  but  $\lambda\uparrow$ . However, the situation changes for so low p that the gas is in molecular regime (see next slides).

### 2.8.3 Heat transfer - molecular regime

Heat transfer by gas conduction between two parallel plates:



 $\dots$  common situation in the film deposition, where one plate is a heated platform at temperature  $\mathcal{T}_h$ , and the other is a substrate being raised to  $\mathcal{T}_s$  by the heat transfer from the platform.

For the gap distance *b*, the **Knudsen number is**  $Kn = \lambda/b$ .

At the higher *p* where  $Kn \ll 1$  (fluid flow), the heat flux is (using  $K_{\rm T} \sim \sqrt{T/m} \, c_{\rm V}/a^2$ )

$$\Phi = -K_{\rm T} \frac{\mathrm{d}T}{\mathrm{d}x} = \frac{K_{\rm T}}{b} (T_{\rm h} - T_{\rm s}) \tag{27}$$

For Kn > 1 (molecular flow), gas molecules are bouncing back and forth from plate to

plate without encountering any collisions  $\Rightarrow$  use of  $K_T$  (bulk fluid property) is no longer appropriate. Instead, the heat flux between the plates is proportional to the flux of molecules across the gap ( $\Gamma$ ) times the heat carried per molecule (using Eq. (25)):

$$\Phi = \Gamma \gamma' \frac{c_{\rm V}}{N_{\rm A}} (T_{\rm h} - T_{\rm s}) \equiv h_{\rm c} (T_{\rm h} - T_{\rm s})$$
 (28)

where  $\gamma'$  is the thermal accommodation factor ( $\approx$  unity except for He) and  $h_c$  is the heat transfer coefficient (W K/m²) given as  $h_c = \sqrt{N_{\rm A}/(2\pi R)}\,p/\sqrt{mT}\,\gamma'\,c_{\rm V}$ 

Note that  $h_{\rm c}$  appears, rather than  $K_{\rm T}$ , whenever heat transfer is taking place across an interface rather than through a bulk fluid or other material.

### 2.8.3 Heat transfer - heat transfer to substrate

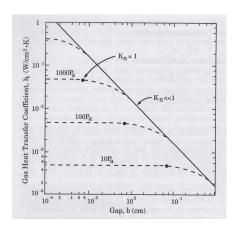
There are two fundamental difference between the heat flux in the case of fluid  $Kn \ll 1$  and molecular Kn > 1 regimes:

- Φ is inversely proportional to b for fluid whereas independent of b for molecular regime
- Φ is independent of p for fluid whereas proportional to p for molecular regime

One important conclusion which can be drawn for low p is that the heat transfer to a substrate from a platform can be increased by increasing p, but only if the gap is kept small enough that Kn > 1.

Helium is often chosen to improve the heat transfer because of its high  $K_{\rm T}$ , but in fact it is not the best choice when Kn>1 because of the thermal accommodation factor in Eq. (28)

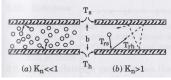
- for discussion of  $\gamma'$  see the next slide.



From Eq. (28), the best choice for a heat-transfer gas is one having low molecular mass to give high  $\Gamma$ , while also having many rotational modes to give high  $c_{\rm V}$ . Choices will usually be limited by process chemistry.

### 2.8.3 Heat transfer - thermal accommodation coefficient

In molecular flow regime Kn > 1 (right figure), consider the molecule approaching the heated platform.



It has the temperature  $T_{\rm rs}$  acquired when reflected from the substrate. Upon being reflected from the platform, it will have temperature  $T_{\rm rh}$ .

The thermal accommodation coefficient  $\gamma$  is defined as

$$\gamma = \frac{T_{\rm rs} - T_{\rm rh}}{T_{\rm rs} - T_{\rm h}}.$$
 (29)

It represents the degree to which the molecule accommodates itself to the temperature  $\mathcal{T}_{\rm h}$  of the surface from which it is reflected.

For most molecule-surface combinations,  $\gamma$  is close to unity, but for He it is 0.1–0.4, depending on the surface.

If  $\gamma$  is less than unity and is the same at both surfaces, the **overall reduction in the heat** flux represented by  $\gamma'$  is

$$\gamma' = \frac{\gamma}{2 - \gamma} \tag{30}$$