Historie astronomie IX.

Vladimír Štefl
Ústav teoretické fyziky a astrofyziky

Vznik a vývoj astrofyziky

Vznik a vývoj astrofyziky

E. Ch. Pickering (1846 – 1919)
A. J. Cannon (1863 – 1941)
spektrální klasifikace hvězd
Harvardský katalog spekter

A. S. Eddington (1882 – 1944)
stavba nitra hvězd
vztah hmotnost – zářivý výkon

S. Chandrasekhar (1910 – 1995)
stavba bílých trpaslíků

H. Bethe (1906 – 2005)
C. Weizsäcker (1912 – 2007)
kvantitativní teorie termonukleárních zdrojů energie hvězd

A. Einstein (1879 – 1955)
speciální teorie relativity

W. A. Fowler (1911 – 1995)
F. Hoyle (1915 – 2001)
E. M. Burbidgeová (1919)
G. R. Burbidge (1925)
vznik chemických prvků

Rozdělení problematiky astrofyziky

I. **Úvod** - Jan Marek, Newton, Fraunhofer, Kirchhoff, Bunsen, Doppler, Zöllner

II. **Hvězdné atmosféry** - Schuster, Saha, Milne, Fowler, Rosseland, Mc Crea, Payene-Gaposchkin, Bowen,

III. **H-R diagram -** Balmer, Hertzsprung, Rosenberg, Russell, Strömgren, Sandage

IV. **Slunce** - Pouillet, Schwabe, Wolf, Carrington, Spörer, Janssen, Lockyer, Maunder, Deslandres, Hale, Young

V. **Hvězdy** - Emden, Eddington, Atkinson, Hourtemans, Weizsacker, Bethe, Salpeter, Fowler, Hoyle, Burbidge - FHB^2, Ledoux, Strömgren, Ževakin, Bakerr, Kippenhahn, Cox, Schwarzschild, Jeans, Hayashi, Heney, Eddington, Adams, Chandrasekhar

VI. **OTR** - Eddington, Adams

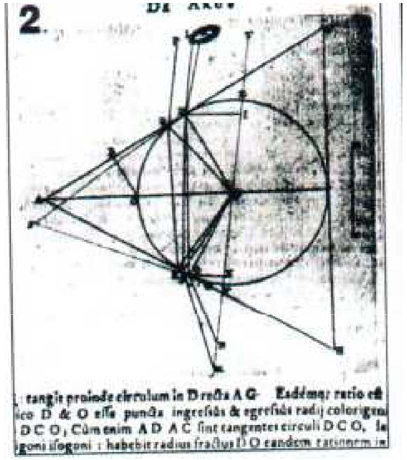
Úvod
Vznik astrofyziky, přehled metod

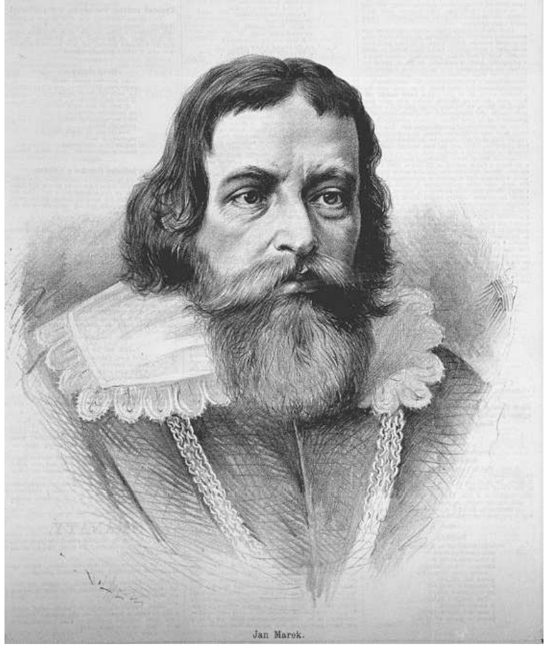
Použití fotografie, fotometrie, spektroskopie

- L. J. M. Daguerre 1789-1851, zavedení fotografie
- A. H. L. Fizeau 1819-1896, fotografie Slunce
- J. B. L. Foucalt 1819-1868, fotografie Slunce
- J. A. Whipple 1822-1891, fotografie Vegy
- H. Draper 1837-1882, fotografie Orionu
- W. Huggins 1824-1910, fotografie komet, hvězd
- J. C. Kapteyn 1851-1922, fotografické desky
- J. K. F. Zöllner 1834-1882, fotometr

Úvod - Jan Marek Marci 1595 - 1667

lékař - fysikus, fyzik, matematik,
profesor lékařské fakulty, hlavní hygienik,
rektor univerzity 1662





O úměrnosti pohybu 1639
mechanika, volný pád,
rázy pružných koulí

Kniha o duze nebeské a původu jejích barev 1648
rozklad slunečního světla skleněným hranolem, vlastnosti hranolového a duhového spektra, výklad vlastností duhy, zbarvení mýdlových bublin, barevný paprsek vycházející z hranolu se při průchodu dalším hranolem už nemění, každá **barva vykazuje jiný úhel lomu, tabulky úhlů**

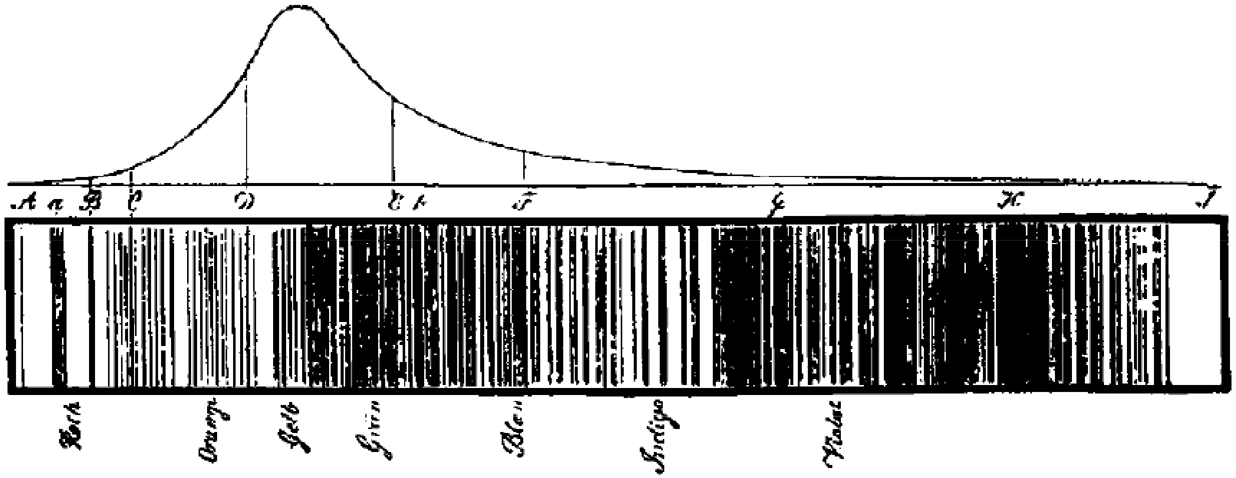
Úvod
Jan Marek Marci

Kniha o duze nebeské a původu jejích barev r. 1648

Informace o prvním pozorování ohybu světla a vzniku barevného spektra na malých otvorech, překážkách a optické mřížce:

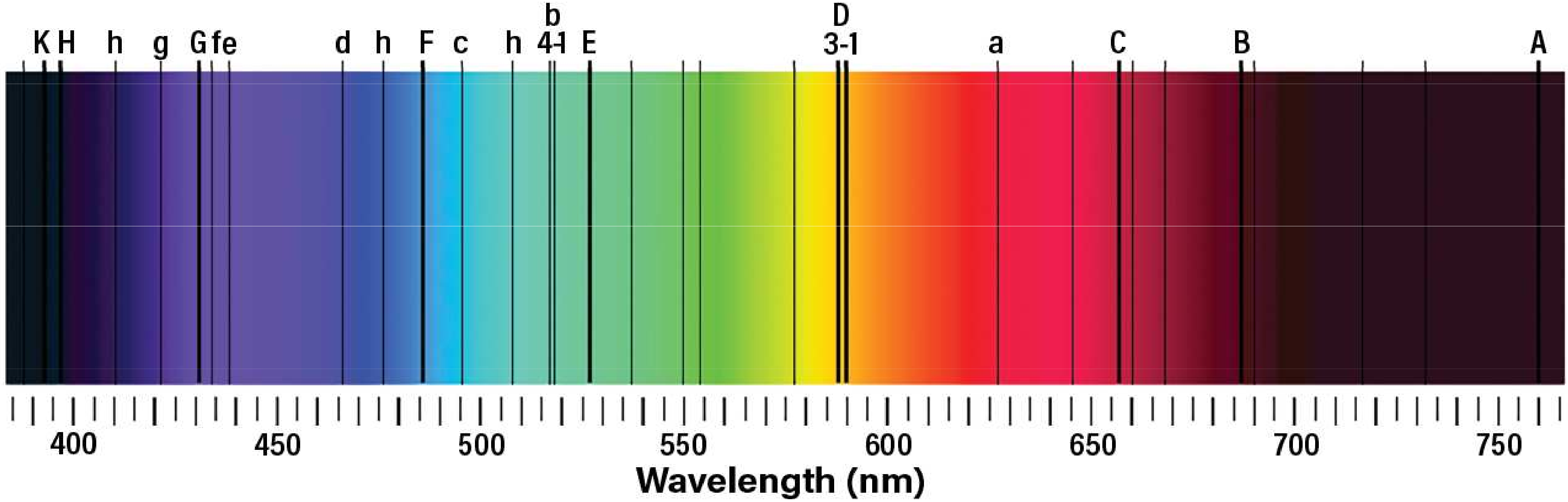
„Jestliže jehla nebo nožík zastíní mezi světelným zdrojem a hranolem libovolnou část zdroje, vidíme okraj stínu barevně. Nebo destička s otvory ve tvaru mřížky či stočená vlákna dají vznik tolika duhám, kolik je otvorů. V obrazu za hranolem , vložíme-li nožík mezi obraz a hranol, vznikne stín s opačnou duhou, ve které při pohybu nožíku mizí hned tyto, hned ony barvy nebo se mění novým míšením barev.“

Úvod
Spektrum Slunce - Fraunhoferovy čáry 1814



Zu Fraunhofer's Abh. Denkschr. 1814_15.

THE SUN'S SPECTRUM



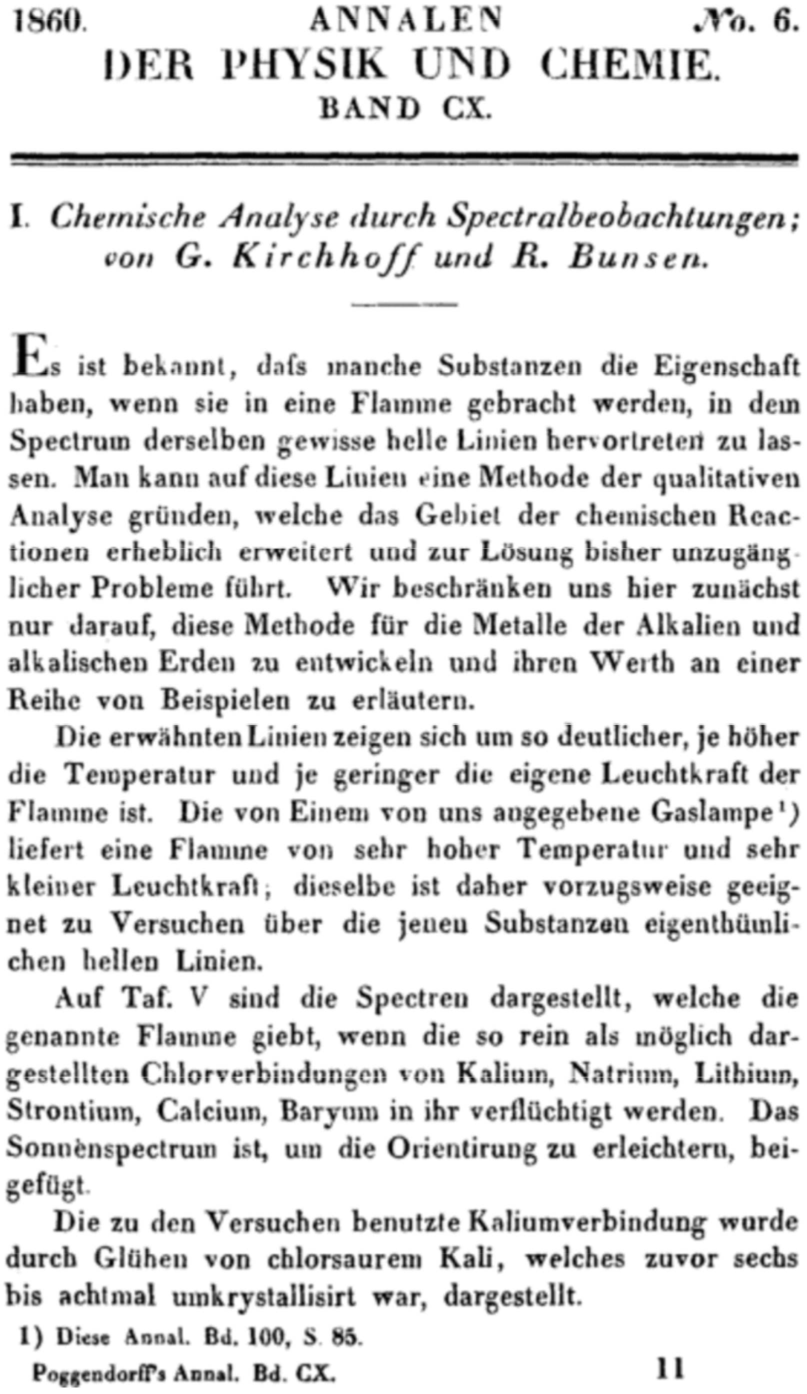
Úvod - Joseph Fraunhofer 1787 - 1826

německý fyzik, optik, konstruktér dalekohledů, strojů na broušení achromatických čoček, studoval sluneční spektra jakož i hvězd, **Fraunhoferovy čáry r. 1814,** proměřil ve spektru Slunce polohy 324 čar, obobné čáry nalezl u Betelgeuse, Prokyonu, Polluxu, Capelly

Úvod: Gustav Kirchhoff 1824 - 1887
Robert Wilhelm Bunsen 1811 - 1899

německý fyzik a chemik, analyzovali různé soli pomocí spektrografu, zabarvení plamene při žíhání, potvrdili, že žlutá čára sodíku splývá s *Fraunhoferovými D* čarami. K odstranění těchto tmavých čar ve spektru Slunce postavili před štěrbinu spektrografu lihový kahan, v jehož plameni žíhali kuchyňskou sůl. Výsledkem bylo ještě větší ztmavení *D čar*. Kirchhoff vyslovil hypotézu, podle níž sodíkový plamen pohlcuje žlutou čáru sodíku ze slunečního spektra. Vlnové délky emisních a absorpčních spektrálních čar jsou charakteristické pro přítomnost prvku bez ohledu na druh sloučeniny, množství prvku a použitý plamen a jeho ťeplotu. Pomocí spektrální analýzy objevili dva prvky - cézium a rubídium. Zjistili, že velký počet Fraunhoferových čar v slunečním spektru je způsobený přítomností železa v atmosféře Slunce. Dále dokázali přítomnost dalších 14 prvků na Slunci. ***Zákon: Vyzařovací schopnost černého tělesa je úměrná jeho pohlcovací schopnosti, těleso tím více pohlcuje záření, čím více je schopno je vyzařovat.***

Úvod - Kirchhoff + Bunsen 1860



1860. ANNALEN №. 6.
DER PHYSIK UND CHEMIE.
BAND CX.

I. *Chemische Analyse durch Spectralbeobachtungen;* *von* G. *Kirchhoff* und R. *Bunsen.*

Es ist bekannt, daſs manche Substanzen die Eigenschaft haben, wenn sie in eine Flamme gebracht werden, in dem Spectrum derselben gewisse helle Linien hervortreten zu lassen. Man kann auf diese Linien eine Methode der qualitativen Analyse gründen, welche das Gebiet der chemischen Reactionen erheblich erweitert und zur Lösung bisher unzugänglicher Probleme führt. Wir beschränken uns hier zunächst nur darauf, diese Methode für die Metalle der Alkalien und alkalischen Erden zu entwickeln und ihren Werth an einer Reihe von Beispielen zu erläutern.

Die erwähnten Linien zeigen sich um so deutlicher, je höher die Temperatur und je geringer die eigene Leuchtkraft der Flamme ist. Die von Einem von uns angegebene Gaslampe[1]) liefert eine Flamme von sehr hoher Temperatur und sehr kleiner Leuchtkraft; dieselbe ist daher vorzugsweise geeignet zu Versuchen über die jenen Substanzen eigenthümlichen hellen Linien.

Auf Taf. V sind die Spectren dargestellt, welche die genannte Flamme giebt, wenn die so rein als möglich dargestellten Chlorverbindungen von Kalium, Natrium, Lithium, Strontium, Calcium, Baryum in ihr verflüchtigt werden. Das Sonnenspectrum ist, um die Orientirung zu erleichtern, beigefügt.

Die zu den Versuchen benutzte Kaliumverbindung wurde durch Glühen von chlorsaurem Kali, welches zuvor sechs bis achtmal umkrystallisirt war, dargestellt.

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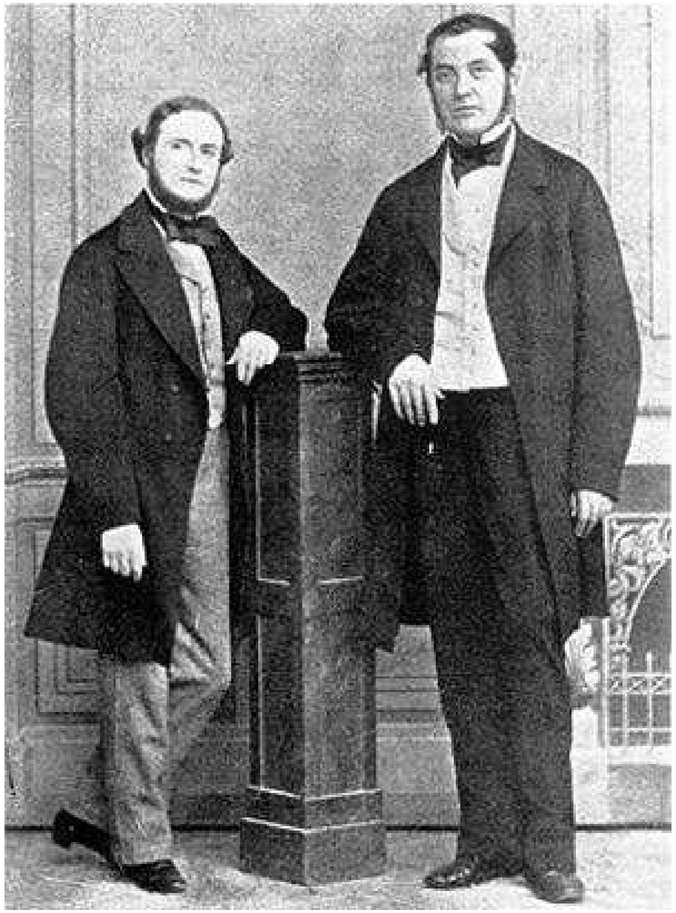
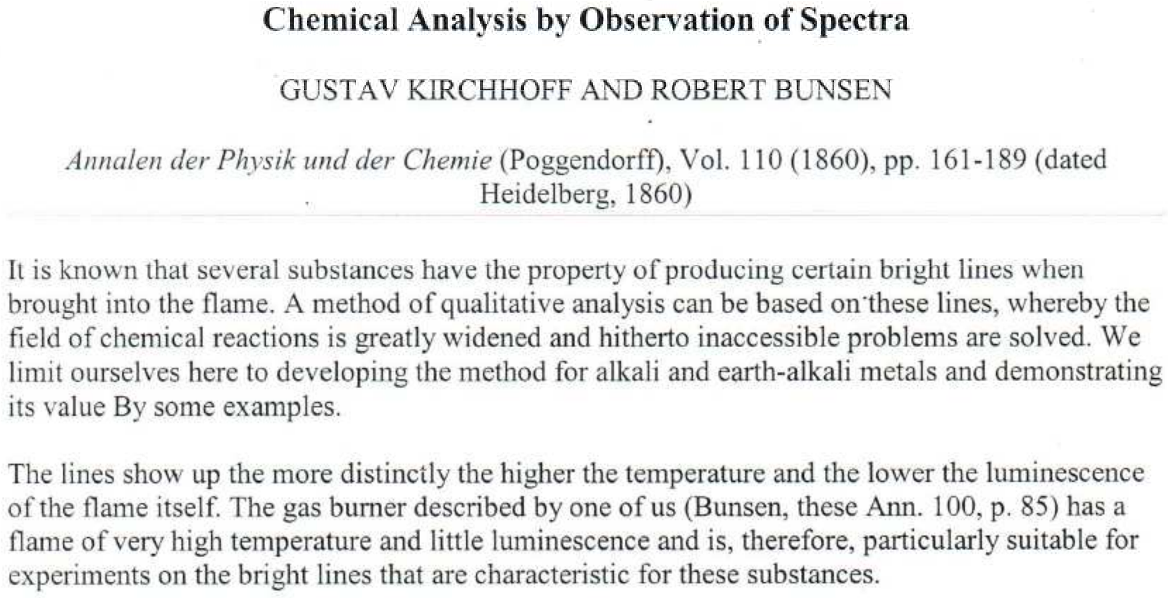
1) Diese Annal. Bd. 100, S. 85.



Úvod Gustav Kirchhoff

německý fyzik, zabývající se elektřinou,spektroskopií,



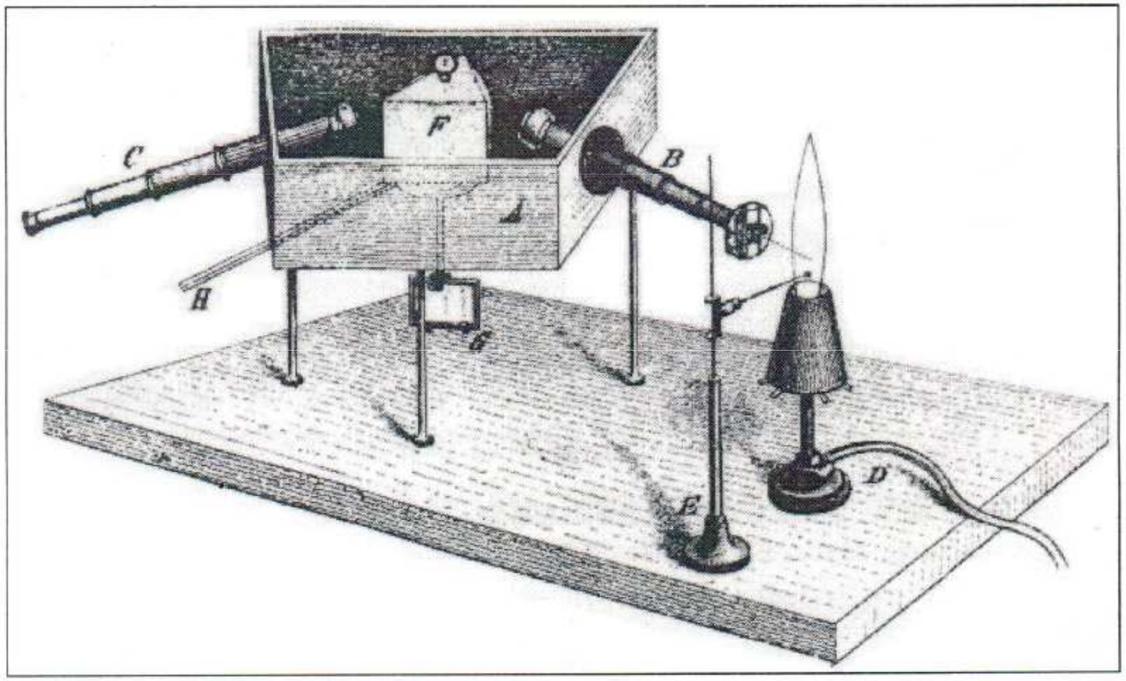
Chemical Analysis by Observation of Spectra

GUSTAV KIRCHHOFF AND ROBERT BUNSEN

Annalen der Physik und der Chemie (Poggendorff), Vol. 110 (1860), pp. 161-189 (dated Heidelberg, 1860)

It is known that several substances have the property of producing certain bright lines when brought into the flame. A method of qualitative analysis can be based on these lines, whereby the field of chemical reactions is greatly widened and hitherto inaccessible problems are solved. We limit ourselves here to developing the method for alkali and earth-alkali metals and demonstrating its value By some examples.

The lines show up the more distinctly the higher the temperature and the lower the luminescence of the flame itself. The gas burner described by one of us (Bunsen, these Ann. 100, p. 85) has a flame of very high temperature and little luminescence and is, therefore, particularly suitable for experiments on the bright lines that are characteristic for these substances.

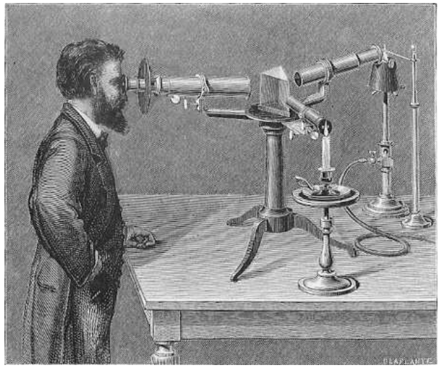


rozlišení tří druhů spekter:

a) ***spojité*** - tekutiny, husté plyny

b) ***emisní*** čárová spektra - poskytují horké plyny

c) ***absorpční*** čárová spektra - ve spojitém spektru jsou tmavé čáry - Fraunhoferovy čáy ve spektru Slunce



Úvod
První spektra hvězd

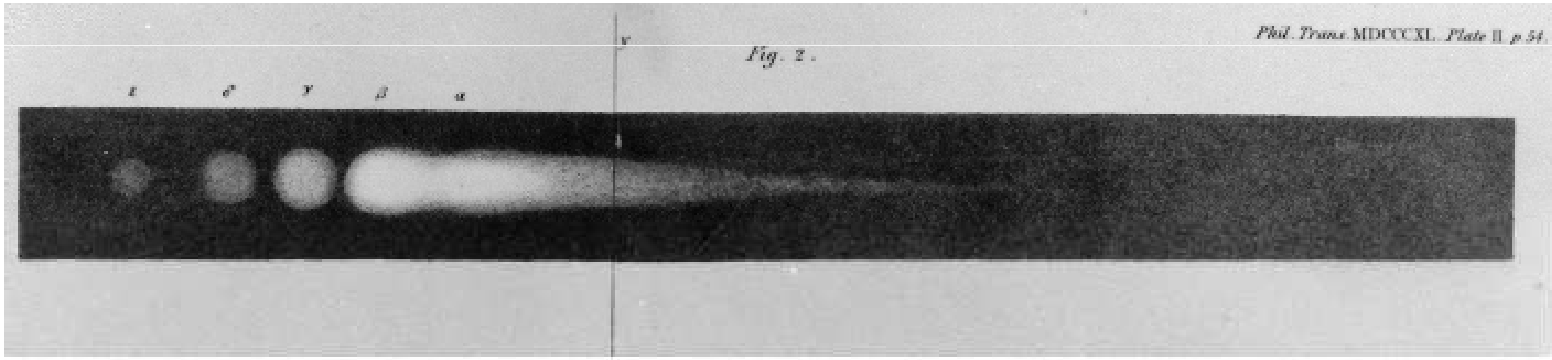


Figure 3.3. Sir John Herschel's infrared absorption bands in the solar spectrum.

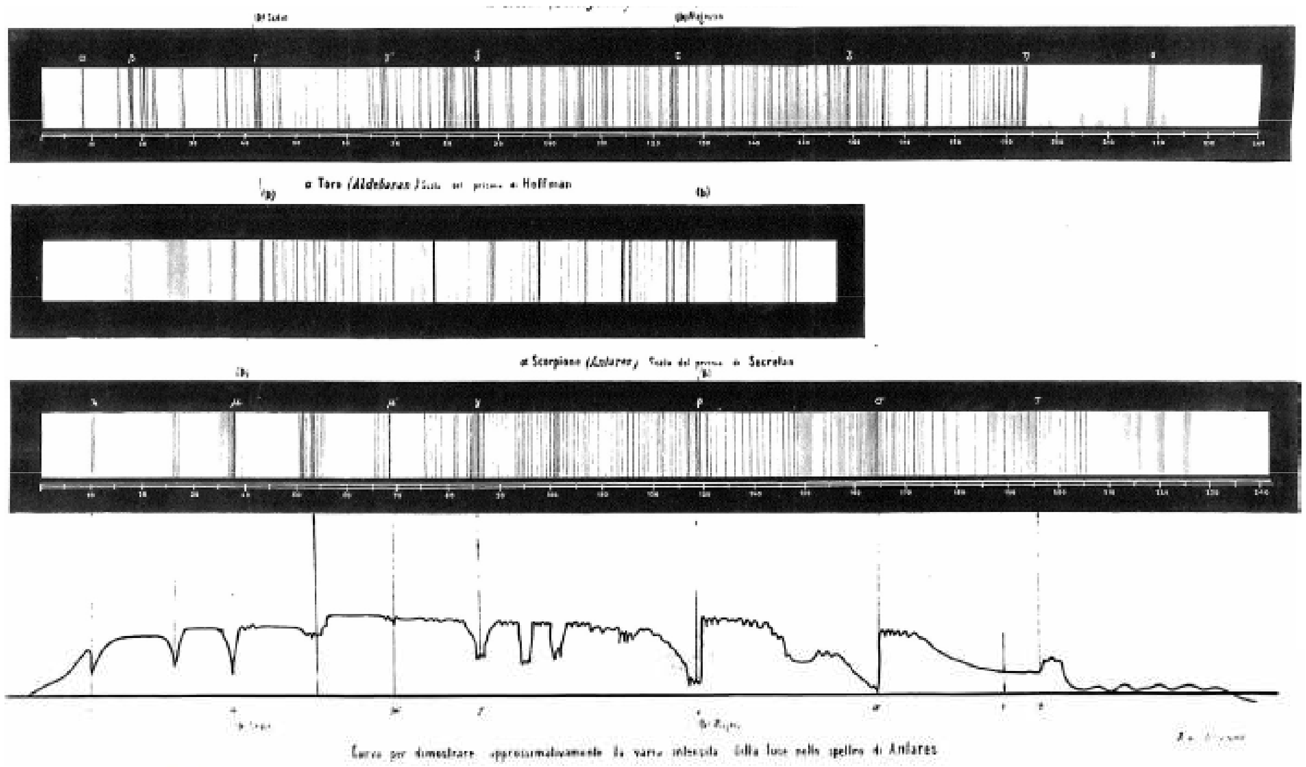


Figure 4.7. Secchi's drawings of the spectra of Betelgeuse, Aldebaran and Antares, 1867.

Úvod
Spektrální klasifikace hvězd

Je všeobecně známo, že prvním pokusem o systematickou spektrální klasifikaci hvězd je práce Angela Secchiho, který již v roce 1868 publikoval katalog se 4000 spektry. Rozdělil spektra podle vzhledu do čtyř skupin:

I – bílé hvězdy pouze s čarami vodíku (Sirius, Vega, Altair, Regulus)

II – nažloutlé hvězdy slunečního typu (Arcturus, Capella) se spoustou čar tzv. kovů

III – oranžové hvězdy s absorpčními pásy (Betelgeuze, Mira), proměnné hvězdy

IV – červené hvězdy s absorpčními pásy,

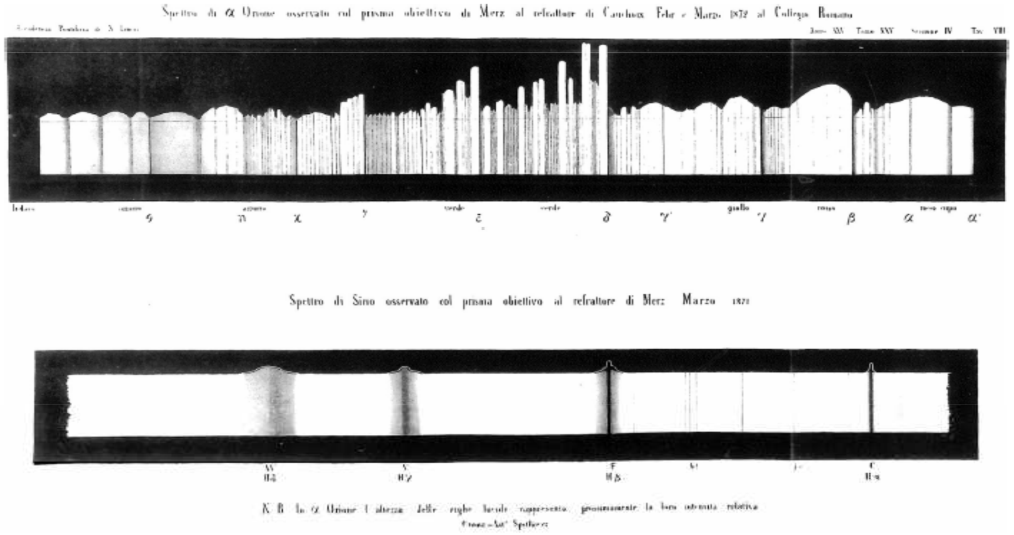
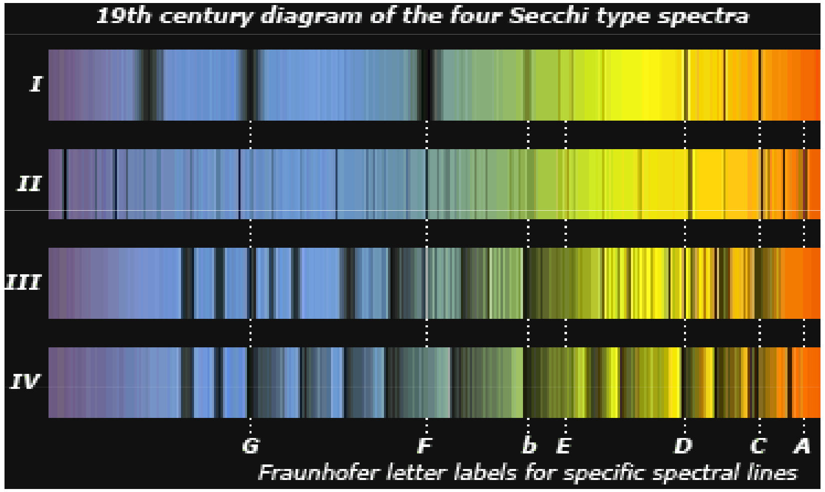
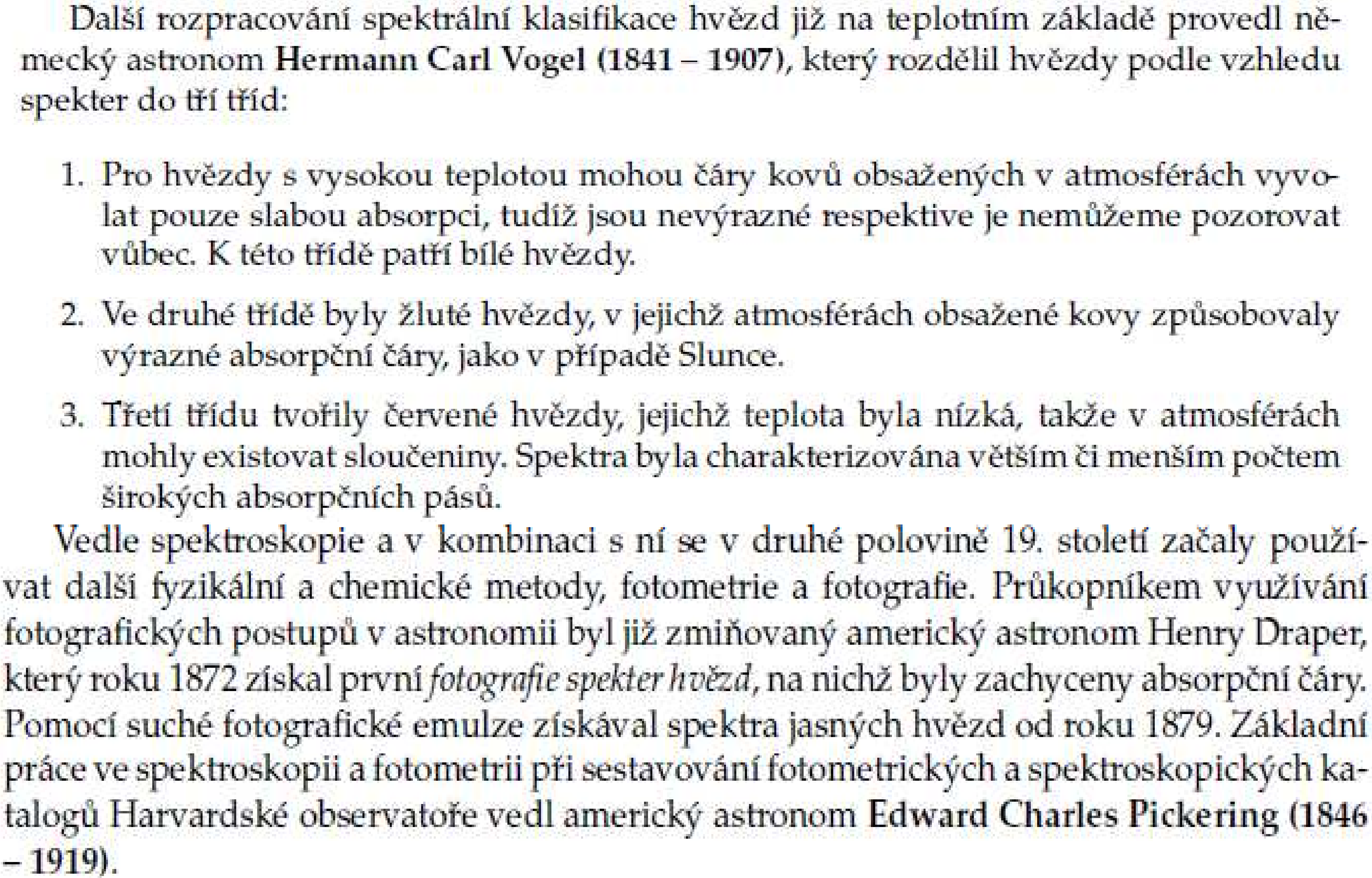


Figure 4.8. Secchi's drawings of the spectra of Betelgeuse and Sirius, 1872.



Úvod
Spektrální klasifikace hvězd



Další rozpracování spektrální klasifikace hvězd již na teplotním základě provedl německý astronom **Hermann Carl Vogel (1841 – 1907)**, který rozdělil hvězdy podle vzhledu spekter do tří tříd:

1. Pro hvězdy s vysokou teplotou mohou čáry kovů obsažených v atmosférách vyvolat pouze slabou absorpci, tudíž jsou nevýrazné respektive je nemůžeme pozorovat vůbec. K této třídě patří bílé hvězdy.
2. Ve druhé třídě byly žluté hvězdy, v jejichž atmosférách obsažené kovy způsobovaly výrazné absorpční čáry, jako v případě Slunce.
3. Třetí třídu tvořily červené hvězdy, jejichž teplota byla nízká, takže v atmosférách mohly existovat sloučeniny. Spektra byla charakterizována větším či menším počtem širokých absorpčních pásů.

Vedle spektroskopie a v kombinaci s ní se v druhé polovině 19. století začaly používat další fyzikální a chemické metody, fotometrie a fotografie. Průkopníkem využívání fotografických postupů v astronomii byl již zmiňovaný americký astronom Henry Draper, který roku 1872 získal první *fotografie spekter hvězd*, na nichž byly zachyceny absorpční čáry. Pomocí suché fotografické emulze získával spektra jasných hvězd od roku 1879. Základní práce ve spektroskopii a fotometrii při sestavování fotometrických a spektroskopických katalogů Harvardské observatoře vedl americký astronom **Edward Charles Pickering (1846 – 1919)**.

Úvod
Spektrální klasifikace hvězd

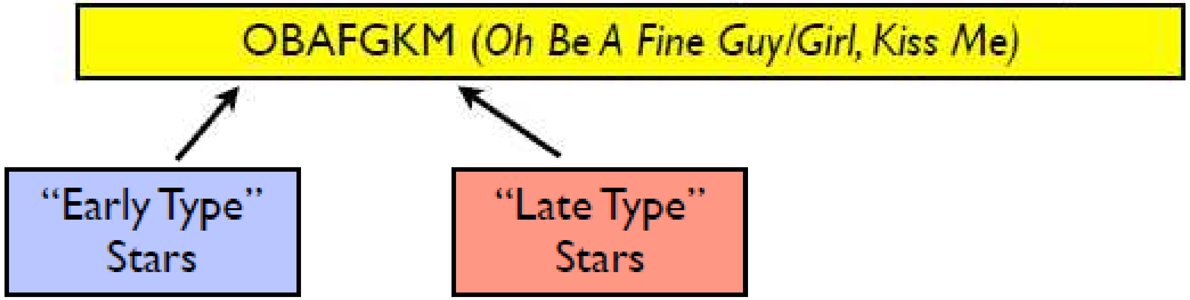
Spectral Classification of Stars

Timeline:

1890s Edward C. Pickering (1846-1919) and Williamina P. Fleming (1857-1911) label spectra alphabetically according to strength of Hydrogen (Balmer) lines, beginning with "A" (strongest).

1890s Antonia Maury (1866-1952) developed a classification scheme based on the "width" of spectral lines. Would place "B" stars before "A" stars.

1901 Annie Cannon (1863-1941), brilliantly combined the above. Rearranged sequence, O before B before A, added decimal divisions (A0...A9) and consolidated classes. Led to classification scheme *still used by astronomers today!*



OBAFGKM (*Oh Be A Fine Guy/Girl, Kiss Me*)

"Early Type" Stars

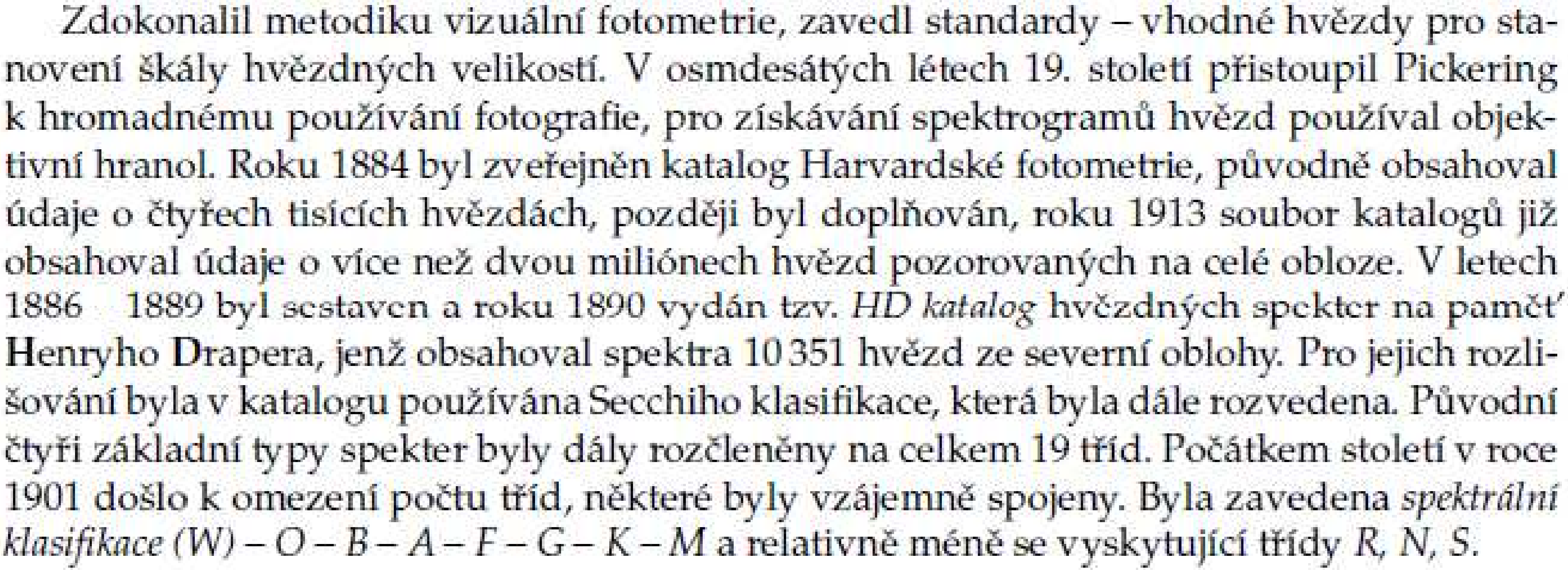
"Late Type" Stars

Úvod
Spektrální klasifikace hvězd

Henry **D**raper 1837-1882 fotografie spekter hvězd 1872

Edward Charles Pickering 1846-1919
HD katalog hvězdných spekter

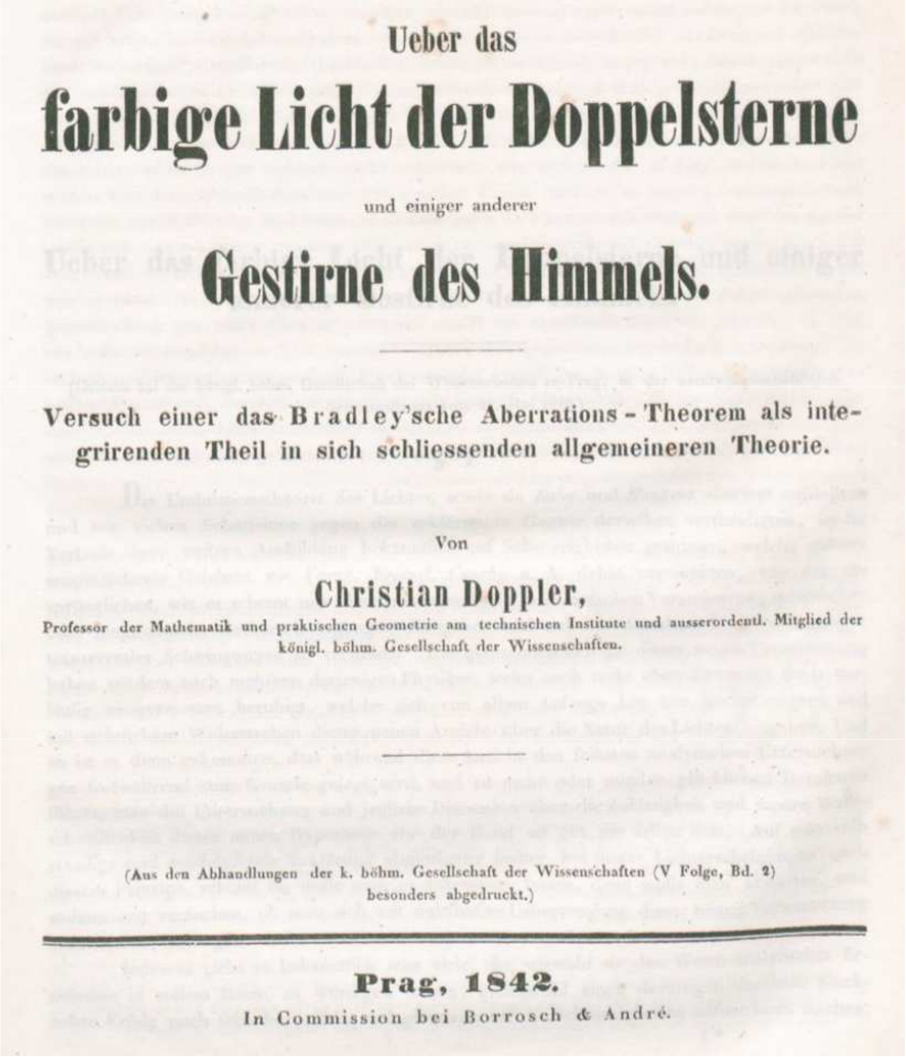
Annie Jump Cannon 1863-1941
Harvardský katalog hvězdných spekter 1918-1924



Zdokonalil metodiku vizuální fotometrie, zavedl standardy – vhodné hvězdy pro stanovení škály hvězdných velikostí. V osmdesátých létech 19. století přistoupil Pickering k hromadnému používání fotografie, pro získávání spektrogramů hvězd používal objektivní hranol. Roku 1884 byl zveřejněn katalog Harvardské fotometrie, původně obsahoval údaje o čtyřech tisících hvězdách, později byl doplňován, roku 1913 soubor katalogů již obsahoval údaje o více než dvou miliónech hvězd pozorovaných na celé obloze. V letech 1886–1889 byl sestaven a roku 1890 vydán tzv. *HD katalog* hvězdných spekter na památ' Henryho Drapera, jenž obsahoval spektra 10 351 hvězd ze severní oblohy. Pro jejich rozlišování byla v katalogu používána Secchiho klasifikace, která byla dále rozvedena. Původní čtyři základní typy spekter byly dály rozčleněny na celkem 19 tříd. Počátkem století v roce 1901 došlo k omezení počtu tříd, některé byly vzájemně spojeny. Byla zavedena *spektrální klasifikace (W) – O – B – A – F – G – K – M* a relativně méně se vyskytující třídy *R, N, S.*

Úvod
Christian Doppler 1803 - 1853

rakouský fyzik a matematik, působil v Praze, ***životopis,***
spis ***O barevném světle dvojhvězd r. 1842***



Ueber das

farbige Licht der Doppelsterne

und einiger anderer

Gestirne des Himmels.

Versuch einer das Bradley'sche Aberrations-Theorem als integrirenden Theil in sich schliessenden allgemeineren Theorie.

Von

Christian Doppler,

Professor der Mathematik und praktischen Geometrie am technischen Institute und ausserordentl. Mitglied der königl. böhm. Gesellschaft der Wissenschaften.

(Aus den Abhandlungen der k. böhm. Gesellschaft der Wissenschaften (V Folge, Bd. 2) besonders abgedruckt.)

Prag, 1842.
In Commission bei Borrosch & André.

Úvod
Christian Doppler

Karolinum v Praze 25. května r. **1842**, zasedání přírodovědné sekce České královské společnosti nauk, pojednání ***O barevném světle dvojhvězd ,*** pět posluchačů, mezi nimi matematik ***Bernard Bolzano 1781-1848,*** který pochopil význam Dopplerova objevu,
téhož roku článek v Pojednání královské české společnosti nauk,
Dopplerův výklad obsahoval chyby:

1. přecenění velikosti radiálních rychlostí složek dvojhvězdy
2. nedocenění intenzity ultrafialové a infračervené části spektra
3. připsání hvězdám libovolnou vlastní barvu

Shrnuto neměl správné o spektrálním složení světla hvězd, o jejich hodnotách rychlosti.

V závěru článku prorocká slova: *,,S přesvědčením očekávám, že jím* (Dopplerovým principem) *bude určována barva nebeských těles, otázka, zda se pohybují, kam a s jakou rychlostí, jaké vzdálenosti nás dělí jeden od druhého, stejně jako i rozřešení mnohých druhých otázek.“*

Úvod
Christian Doppler

ve spisu ***O barevném světle dvojhvězd*** diskutoval, zda světlo je příčná vlna, s éterovými částicemi oscilujícími kolmo ke směru šíření, barva světla je projevem frekvence světelné vlny v oku pozorovatele, pro frekvenční posuv odvodil rovnice, nevycházející z žádného experimentu, v Dopplerově době neměl k dispozici tak rychle se pohybující zdroje světla či zvuku, proto jeho matematické spekulativní odvození vycházelo z dvojhvězd v astronomii tehdy již známých, domníval se, že přirozená barva hvězd je bílá a že při svém pohybu hvězd směrem k nám či od nás se posouvá celé jejich spektrum, barva složek dvojhvězdy by se tak měly periodicky měnit.

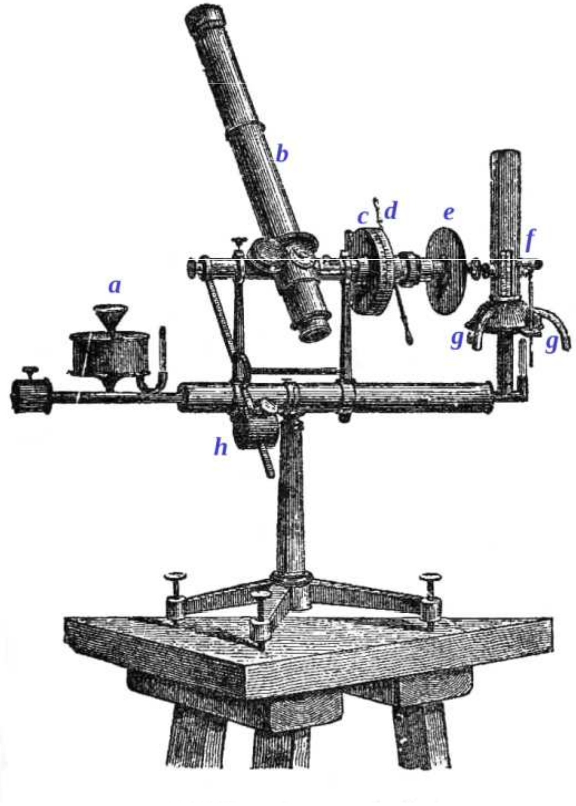
Matematicky zdůvodnil, proč se mění kmitočet záření, tedy barva světla, jestliže se zdroj záření pohybuje.

Doppler mlčky předpokládal, že rychlost světla nezávisí na rychlosti zdroje, tedy éter je nepohyblivý…

Úvod

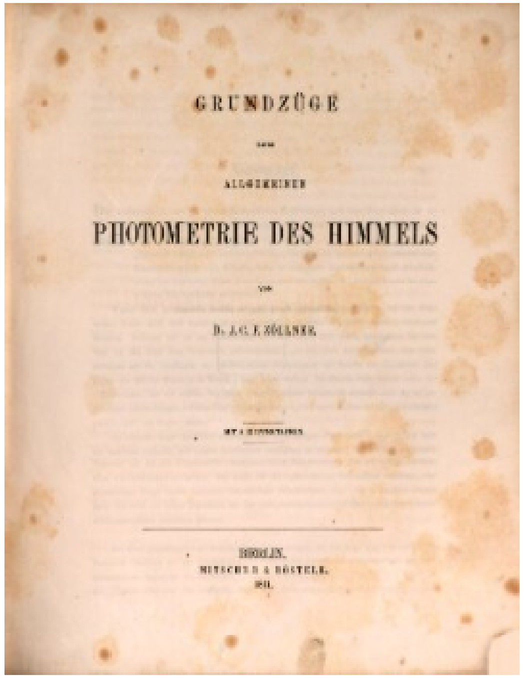
Johann Karl Friedrich Zöllner 1834 - 1882

německý astronom, vyvinul a použil první astronomický fotometr, pozorovanou hvězdu srovnával s umělou, vytvořenou odrazem světla svítilny, zavedl termín ***astrofyzika***



Úvod
Johann Karl Friedrich Zöllner

Základy jedné obecné fotometrie oblohy r. 1861,
Fotometrický výzkum r. 1865



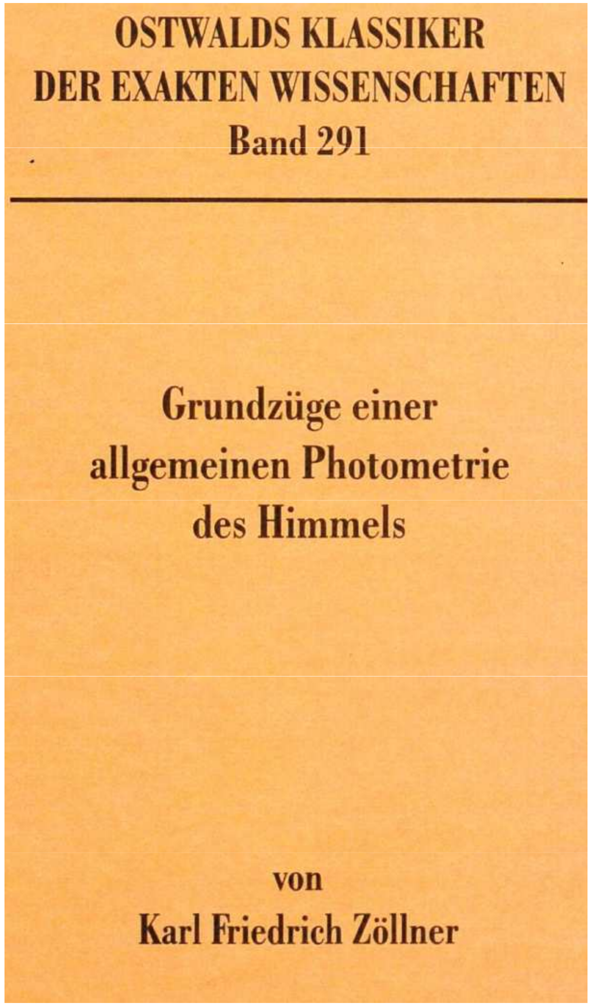
GRUNDZÜGE

ALLGEMEINEN

PHOTOMETRIE DES HIMMELS

D. J. C. F. ZÖLLNER

BERLIN.



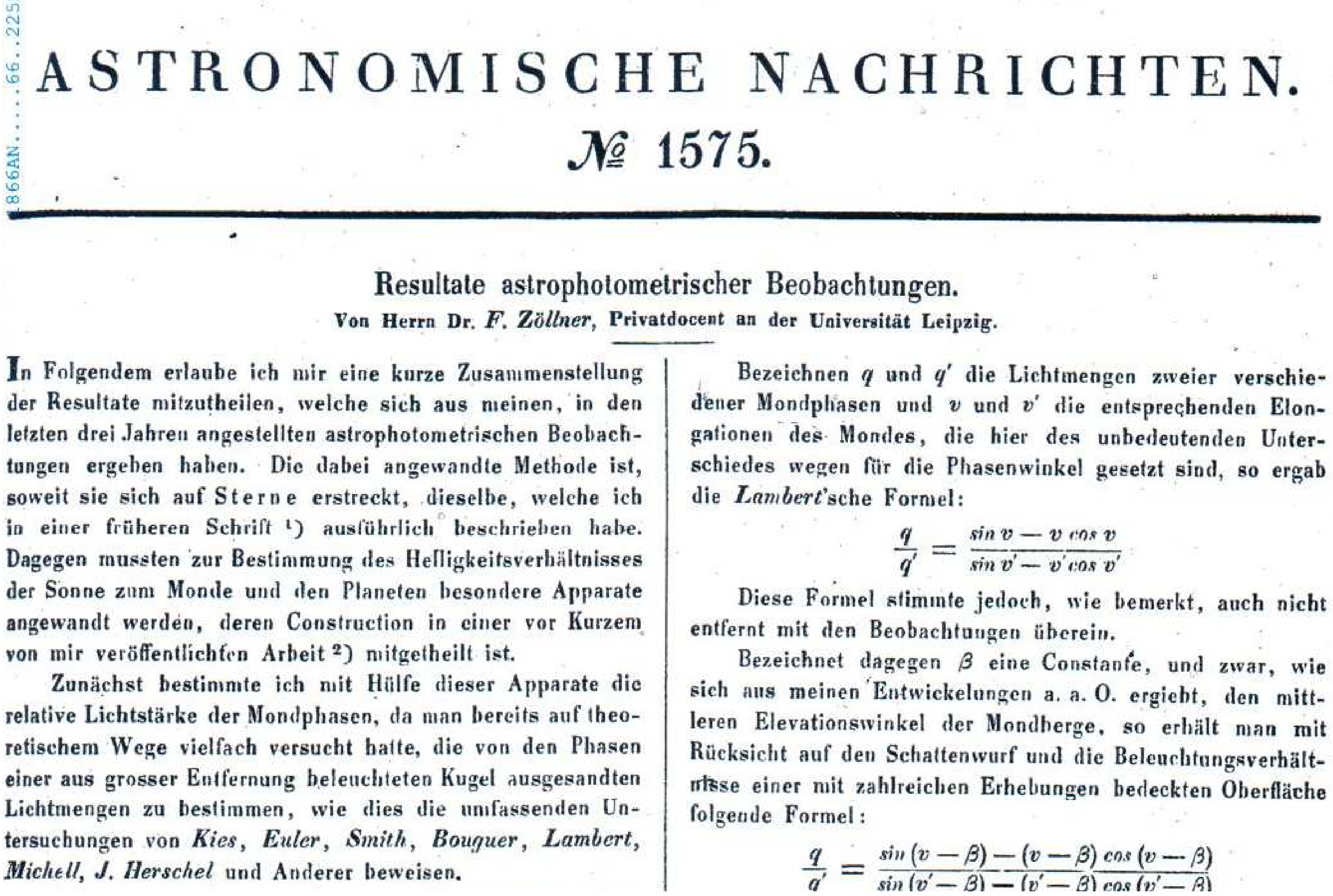
OSTWALDS KLASSIKER
DER EXAKTEN WISSENSCHAFTEN
Band 291

Grundzüge einer
allgemeinen Photometrie
des Himmels

von
Karl Friedrich Zöllner

Úvod
Johann Karl Friedrich Zöllner

Výsledky astrofotometrických pozorování, r. 1866



ASTRONOMISCHE NACHRICHTEN.

№ 1575.

Resultate astrophotometrischer Beobachtungen.

Von Herrn Dr. *F. Zöllner*, Privatdocent an der Universität Leipzig.

In Folgendem erlaube ich mir eine kurze Zusammenstellung der Resultate mitzutheilen, welche sich aus meinen, in den letzten drei Jahren angestellten astrophotometrischen Beobachtungen ergeben haben. Die dabei angewandte Methode ist, soweit sie sich auf Sterne erstreckt, dieselbe, welche ich in einer früheren Schrift [1]) ausführlich beschrieben habe. Dagegen mussten zur Bestimmung des Helligkeitsverhältnisses der Sonne zum Monde und den Planeten besondere Apparate angewandt werden, deren Construction in einer vor Kurzem von mir veröffentlichten Arbeit [2]) mitgetheilt ist.

Zunächst bestimmte ich mit Hülfe dieser Apparate die relative Lichtstärke der Mondphasen, da man bereits auf theoretischem Wege vielfach versucht hatte, die von den Phasen einer aus grosser Entfernung beleuchteten Kugel ausgesandten Lichtmengen zu bestimmen, wie dies die umfassenden Untersuchungen von *Kies*, *Euler*, *Smith*, *Bouguer*, *Lambert*, *Michell*, *J. Herschel* und Anderer beweisen.

Bezeichnen q und q' die Lichtmengen zweier verschiedener Mondphasen und v und v' die entsprechenden Elongationen des Mondes, die hier des unbedeutenden Unterschiedes wegen für die Phasenwinkel gesetzt sind, so ergab die *Lambert*'sche Formel:

$$\frac{q}{q'} = \frac{\sin v - v \cos v}{\sin v' - v' \cos v'}$$

Diese Formel stimmte jedoch, wie bemerkt, auch nicht entfernt mit den Beobachtungen überein.

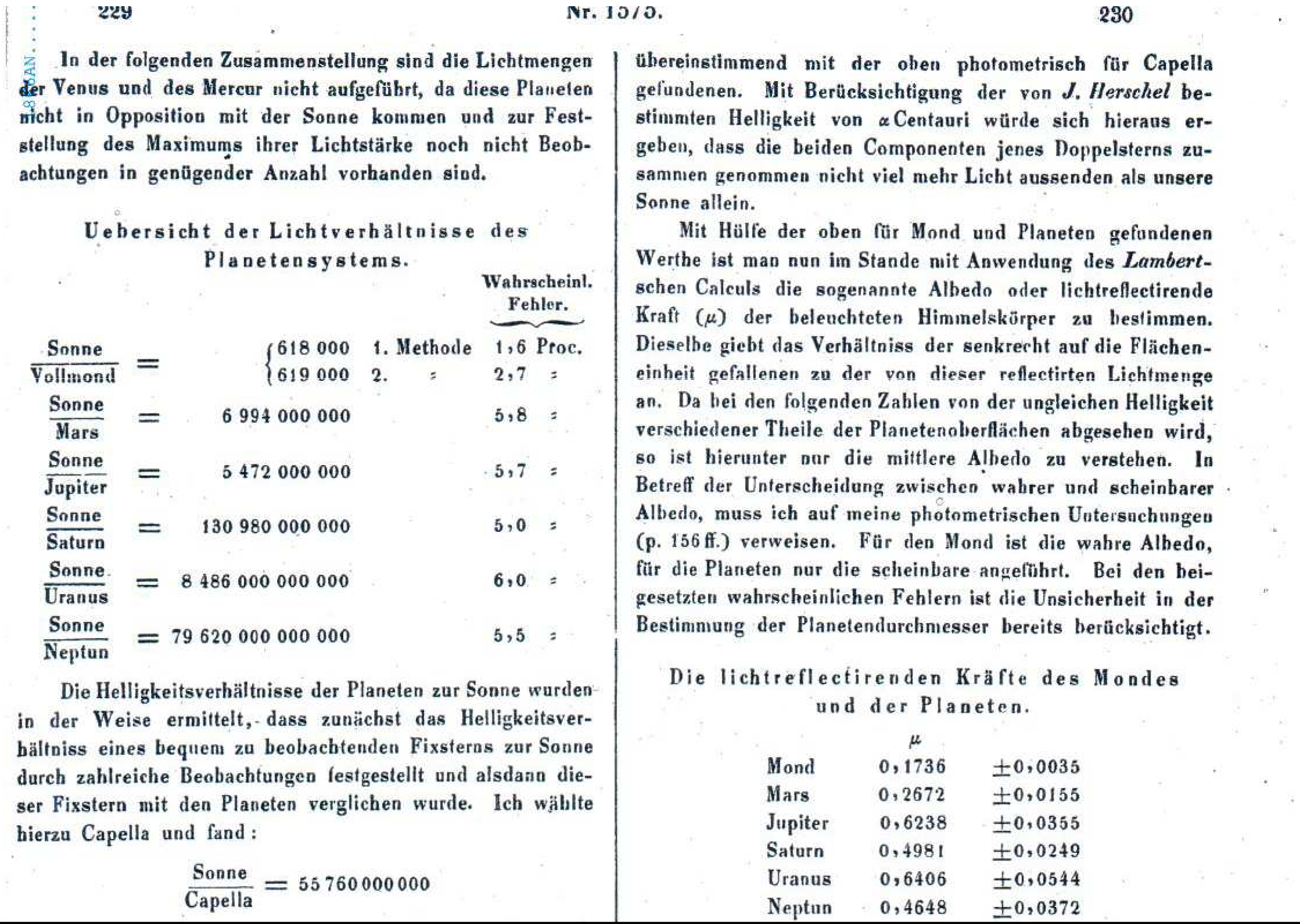
Bezeichnet dagegen β eine Constante, und zwar, wie sich aus meinen Entwickelungen a. a. O. ergiebt, den mittleren Elevationswinkel der Mondberge, so erhält man mit Rücksicht auf den Schattenwurf und die Beleuchtungsverhältnisse einer mit zahlreichen Erhebungen bedeckten Oberfläche folgende Formel:

$$\frac{q}{q'} = \frac{\sin(v-\beta) - (v-\beta)\cos(v-\beta)}{\sin(v'-\beta) - (v'-\beta)\cos(v'-\beta)}$$

Úvod

Johann Karl Friedrich Zöllner

Výsledky astrofotometrických pozorování, r. 1866



In der folgenden Zusammenstellung sind die Lichtmengen der Venus und des Mercur nicht aufgeführt, da diese Planeten nicht in Opposition mit der Sonne kommen und zur Feststellung des Maximums ihrer Lichtstärke noch nicht Beobachtungen in genügender Anzahl vorhanden sind.

Uebersicht der Lichtverhältnisse des Planetensystems.

				Wahrscheinl. Fehler.
$\frac{\text{Sonne}}{\text{Vollmond}}$	=	618 000	1. Methode	1,6 Proc.
		619 000	2. ⸗	2,7 ⸗
$\frac{\text{Sonne}}{\text{Mars}}$	=	6 994 000 000		5,8 ⸗
$\frac{\text{Sonne}}{\text{Jupiter}}$	=	5 472 000 000		5,7 ⸗
$\frac{\text{Sonne}}{\text{Saturn}}$	=	130 980 000 000		5,0 ⸗
$\frac{\text{Sonne}}{\text{Uranus}}$	=	8 486 000 000 000		6,0 ⸗
$\frac{\text{Sonne}}{\text{Neptun}}$	=	79 620 000 000 000		5,5 ⸗

Die Helligkeitsverhältnisse der Planeten zur Sonne wurden in der Weise ermittelt, dass zunächst das Helligkeitsverhältniss eines bequem zu beobachtenden Fixsterns zur Sonne durch zahlreiche Beobachtungen festgestellt und alsdann dieser Fixstern mit den Planeten verglichen wurde. Ich wählte hierzu Capella und fand:

$$\frac{\text{Sonne}}{\text{Capella}} = 55\,760\,000\,000$$

übereinstimmend mit der oben photometrisch für Capella gefundenen. Mit Berücksichtigung der von *J. Herschel* bestimmten Helligkeit von α Centauri würde sich hieraus ergeben, dass die beiden Componenten jenes Doppelsterns zusammen genommen nicht viel mehr Licht aussenden als unsere Sonne allein.

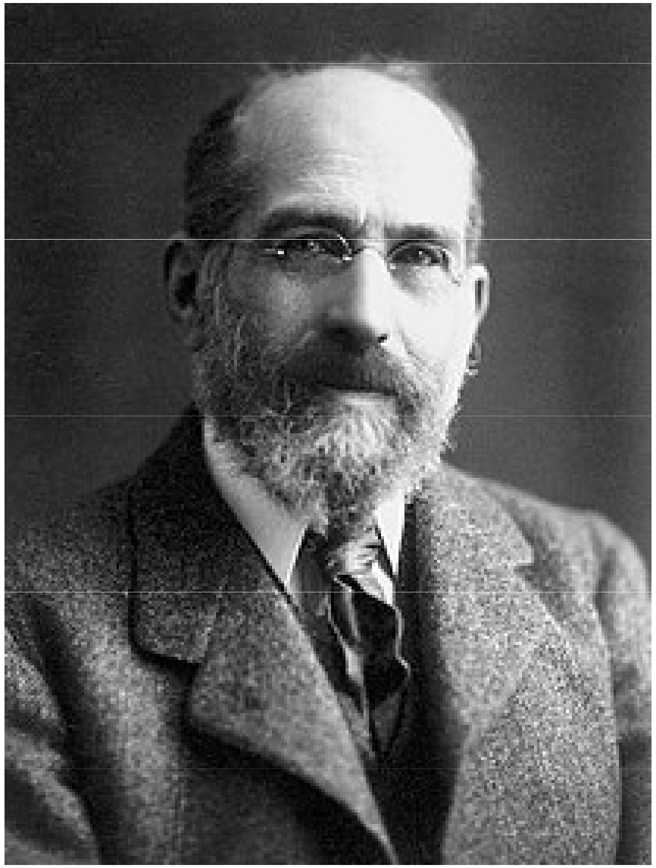
Mit Hülfe der oben für Mond und Planeten gefundenen Werthe ist man nun im Stande mit Anwendung des *Lambert*schen Calculs die sogenannte Albedo oder lichtreflectirende Kraft (μ) der beleuchteten Himmelskörper zu bestimmen. Dieselbe giebt das Verhältniss der senkrecht auf die Flächeneinheit gefallenen zu der von dieser reflectirten Lichtmenge an. Da bei den folgenden Zahlen von der ungleichen Helligkeit verschiedener Theile der Planetenoberflächen abgesehen wird, so ist hierunter nur die mittlere Albedo zu verstehen. In Betreff der Unterscheidung zwischen wahrer und scheinbarer Albedo, muss ich auf meine photometrischen Untersuchungen (p. 156 ff.) verweisen. Für den Mond ist die wahre Albedo, für die Planeten nur die scheinbare angeführt. Bei den beigesetzten wahrscheinlichen Fehlern ist die Unsicherheit in der Bestimmung der Planetendurchmesser bereits berücksichtigt.

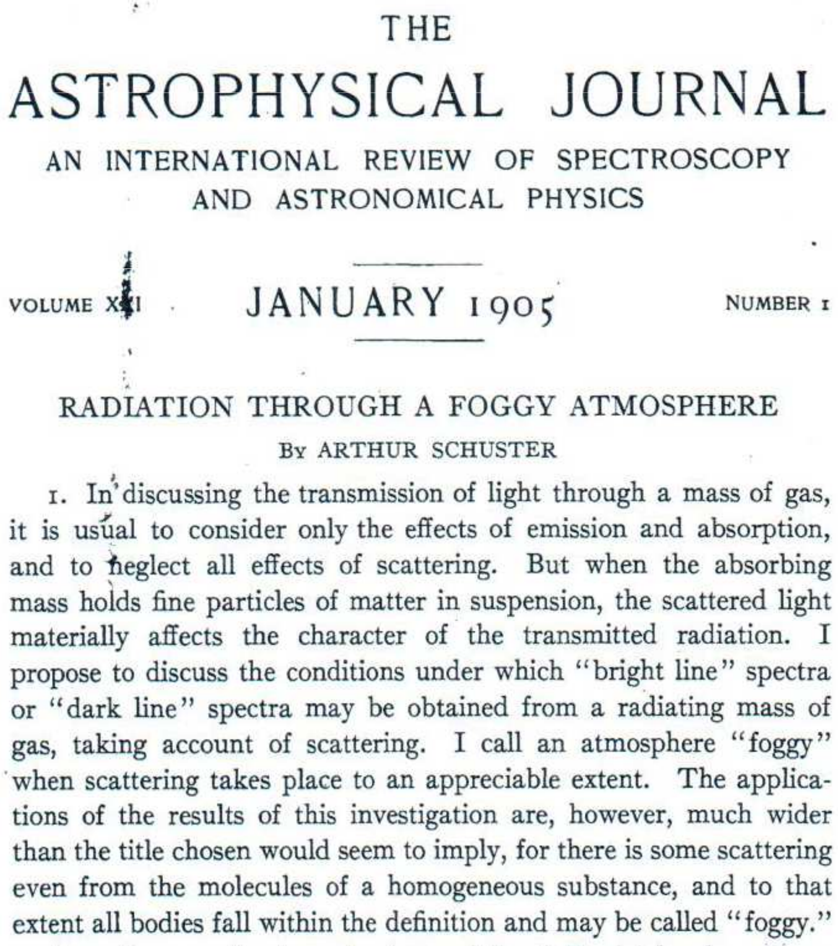
Die lichtreflectirenden Kräfte des Mondes und der Planeten.

	μ	
Mond	0,1736	±0,0035
Mars	0,2672	±0,0155
Jupiter	0,6238	±0,0355
Saturn	0,4981	±0,0249
Uranus	0,6406	±0,0544
Neptun	0,4648	±0,0372

Hvězdné atmosféry

anglický astrofyzik německého původu **Arthur Schuster 1851 - 1934** ***Průchod záření mlhavou atmosférou, r. 1905,*** popis přenosu záření pro rozptylující prostředí, rozvoj teorie po fyzikální a matematické stránce pro popis přenosu záření absorpčním a emisním prostředím, záření z nitra hvězd prochází vrstvou o velmi nízké hustotě





THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXI JANUARY 1905 NUMBER 1

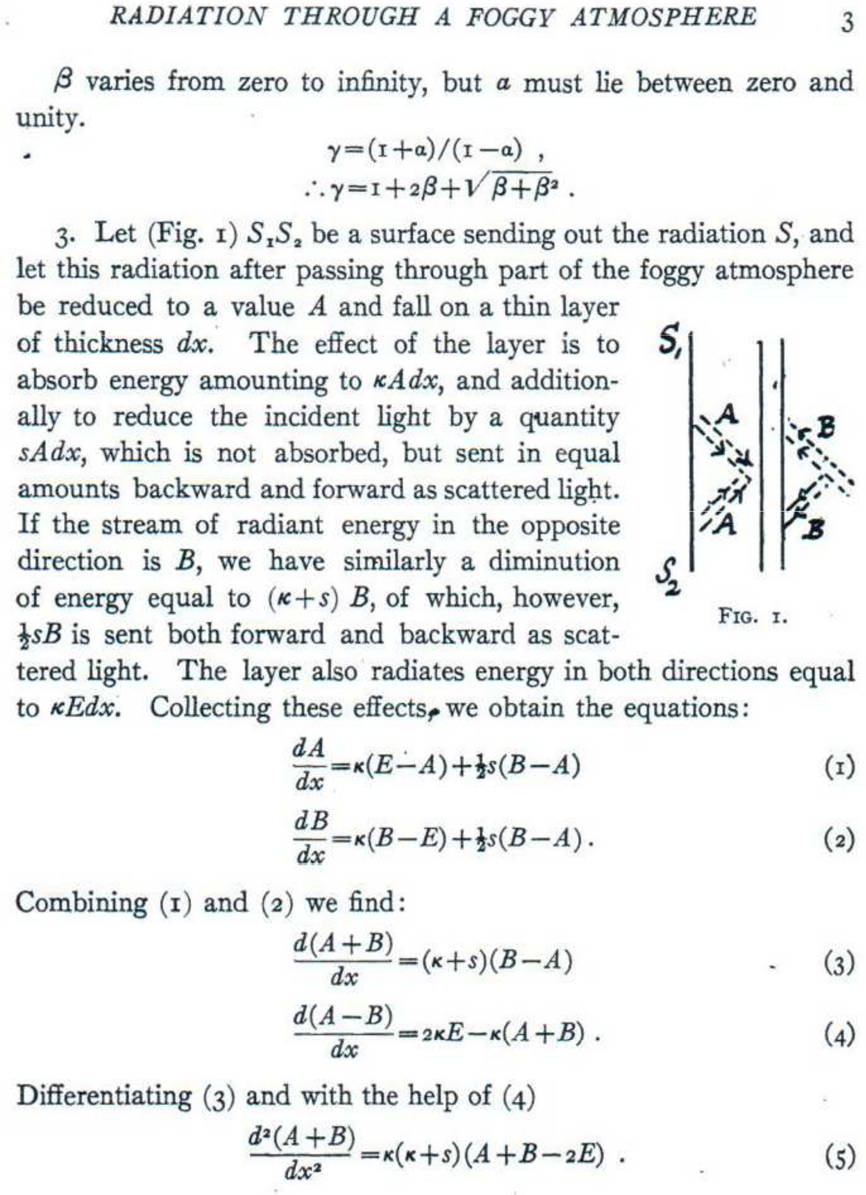
RADIATION THROUGH A FOGGY ATMOSPHERE

By ARTHUR SCHUSTER

1. In discussing the transmission of light through a mass of gas, it is usual to consider only the effects of emission and absorption, and to neglect all effects of scattering. But when the absorbing mass holds fine particles of matter in suspension, the scattered light materially affects the character of the transmitted radiation. I propose to discuss the conditions under which "bright line" spectra or "dark line" spectra may be obtained from a radiating mass of gas, taking account of scattering. I call an atmosphere "foggy" when scattering takes place to an appreciable extent. The applications of the results of this investigation are, however, much wider than the title chosen would seem to imply, for there is some scattering even from the molecules of a homogeneous substance, and to that extent all bodies fall within the definition and may be called "foggy."

Hvězdné atmosféry
Přenos záření

Průchod záření mlhavou atmosférou, r. 1905



β varies from zero to infinity, but α must lie between zero and unity.

$$\gamma=(1+\alpha)/(1-\alpha)\ ,$$
$$\therefore \gamma=1+2\beta+\sqrt{\beta+\beta^2}\ .$$

3. Let (Fig. 1) S_1S_2 be a surface sending out the radiation S, and let this radiation after passing through part of the foggy atmosphere be reduced to a value A and fall on a thin layer of thickness dx. The effect of the layer is to absorb energy amounting to $\kappa A dx$, and additionally to reduce the incident light by a quantity $sAdx$, which is not absorbed, but sent in equal amounts backward and forward as scattered light. If the stream of radiant energy in the opposite direction is B, we have similarly a diminution of energy equal to $(\kappa+s)\,B$, of which, however, $\frac{1}{2}sB$ is sent both forward and backward as scattered light. The layer also radiates energy in both directions equal to $\kappa E dx$. Collecting these effects, we obtain the equations:

FIG. 1.

$$\frac{dA}{dx}=\kappa(E-A)+\tfrac{1}{2}s(B-A) \qquad (1)$$

$$\frac{dB}{dx}=\kappa(B-E)+\tfrac{1}{2}s(B-A)\ . \qquad (2)$$

Combining (1) and (2) we find:

$$\frac{d(A+B)}{dx}=(\kappa+s)(B-A) \qquad (3)$$

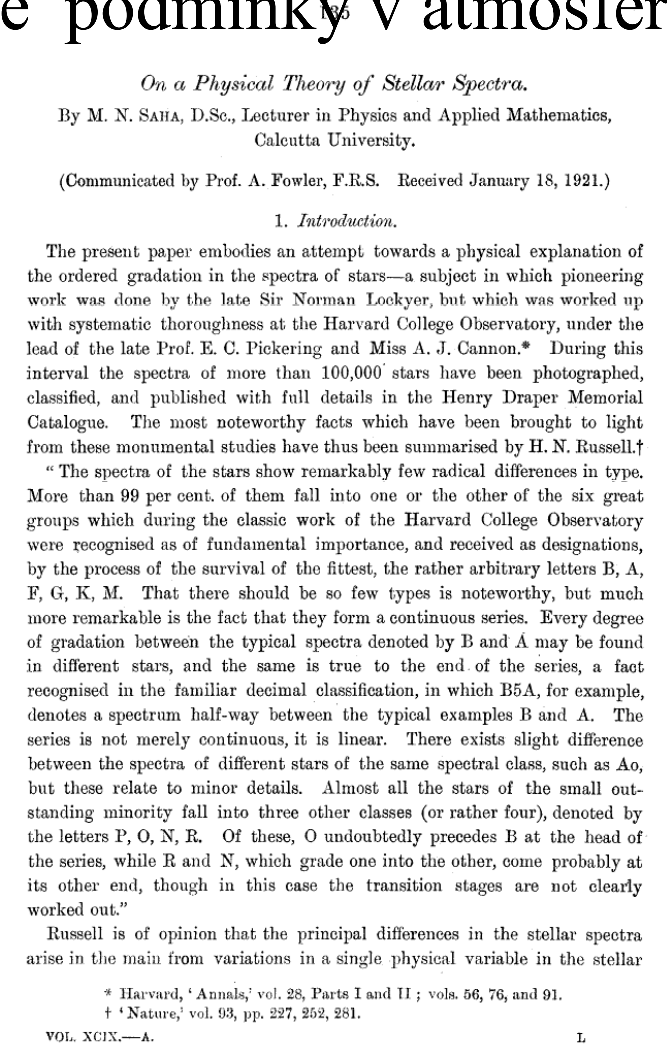
$$\frac{d(A-B)}{dx}=2\kappa E-\kappa(A+B)\ . \qquad (4)$$

Differentiating (3) and with the help of (4)

$$\frac{d^2(A+B)}{dx^2}=\kappa(\kappa+s)(A+B-2E)\ . \qquad (5)$$

Hvězdné atmosféry
Meghnad Saha 1893 - 1956

indický astrofyzik, ***r. 1920 – O fyzikální teorii spektra hvězd - Sahova rovnice ionizace***, fyzikální a chemické podmínky v atmosférách hvězd



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On a Physical Theory of Stellar Spectra.

By M. N. Saha, D.Sc., Lecturer in Physics and Applied Mathematics,
Calcutta University.

(Communicated by Prof. A. Fowler, F.R.S. Received January 18, 1921.)

1. *Introduction.*

The present paper embodies an attempt towards a physical explanation of the ordered gradation in the spectra of stars—a subject in which pioneering work was done by the late Sir Norman Lockyer, but which was worked up with systematic thoroughness at the Harvard College Observatory, under the lead of the late Prof. E. C. Pickering and Miss A. J. Cannon.* During this interval the spectra of more than 100,000 stars have been photographed, classified, and published with full details in the Henry Draper Memorial Catalogue. The most noteworthy facts which have been brought to light from these monumental studies have thus been summarised by H. N. Russell.†

"The spectra of the stars show remarkably few radical differences in type. More than 99 per cent. of them fall into one or the other of the six great groups which during the classic work of the Harvard College Observatory were recognised as of fundamental importance, and received as designations, by the process of the survival of the fittest, the rather arbitrary letters B, A, F, G, K, M. That there should be so few types is noteworthy, but much more remarkable is the fact that they form a continuous series. Every degree of gradation between the typical spectra denoted by B and A may be found in different stars, and the same is true to the end of the series, a fact recognised in the familiar decimal classification, in which B5A, for example, denotes a spectrum half-way between the typical examples B and A. The series is not merely continuous, it is linear. There exists slight difference between the spectra of different stars of the same spectral class, such as Ao, but these relate to minor details. Almost all the stars of the small outstanding minority fall into three other classes (or rather four), denoted by the letters P, O, N, R. Of these, O undoubtedly precedes B at the head of the series, while R and N, which grade one into the other, come probably at its other end, though in this case the transition stages are not clearly worked out."

Russell is of opinion that the principal differences in the stellar spectra arise in the main from variations in a single physical variable in the stellar

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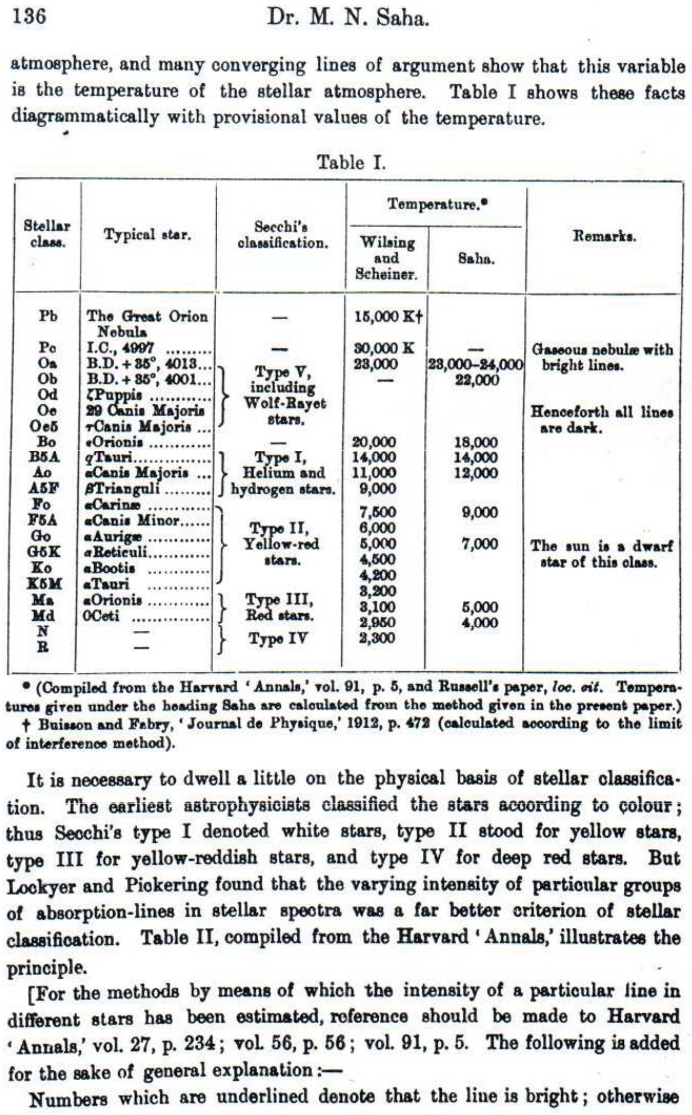
* Harvard, 'Annals,' vol. 28, Parts I and II; vols. 56, 76, and 91.

† 'Nature,' vol. 93, pp. 227, 252, 281.

VOL. XCIX.—A. L

Hvězdné atmosféry - Meghnad Saha

r. 1920 - O fyzikální teorii spektra hvězd - Sahova rovnice ionizace, excitační a ionizační stav plazmatu je řízen zářivým polem



atmosphere, and many converging lines of argument show that this variable is the temperature of the stellar atmosphere. Table I shows these facts diagrammatically with provisional values of the temperature.

Table I.

Stellar class.	Typical star.	Secchi's classification.	Temperature.*		Remarks.
			Wilsing and Scheiner.	Saha.	
Pb	The Great Orion Nebula	—	15,000 K†		
Pc	I.C., 4997	—	30,000 K	—	Gaseous nebulæ with bright lines.
Oa	B.D. + 35°, 4013	Type V, including Wolf-Rayet stars.	23,000	23,000–24,000	
Ob	B.D. + 35°, 4001		—	22,000	
Od	ζPuppis				
Oe	29 Canis Majoris				Henceforth all lines are dark.
Oe5	τCanis Majoris				
Bo	εOrionis	—	20,000	18,000	
B5A	ηTauri	Type I, Helium and hydrogen stars.	14,000	14,000	
Ao	αCanis Majoris		11,000	12,000	
A5F	βTrianguli		9,000		
Fo	αCarinæ	Type II, Yellow-red stars.	7,500	9,000	
F5A	αCanis Minor		6,000		
Go	αAurigæ		5,000	7,000	The sun is a dwarf star of this class.
G5K	αReticuli		4,500		
Ko	αBootis		4,200		
K5M	αTauri		3,200		
Ma	αOrionis	Type III, Red stars.	3,100	5,000	
Md	οCeti		2,950	4,000	
N	—	Type IV	2,300		
R	—				

* (Compiled from the Harvard 'Annals,' vol. 91, p. 5, and Russell's paper, *loc. cit.* Temperatures given under the heading Saha are calculated from the method given in the present paper.)

† Buisson and Fabry, 'Journal de Physique,' 1912, p. 472 (calculated according to the limit of interference method).

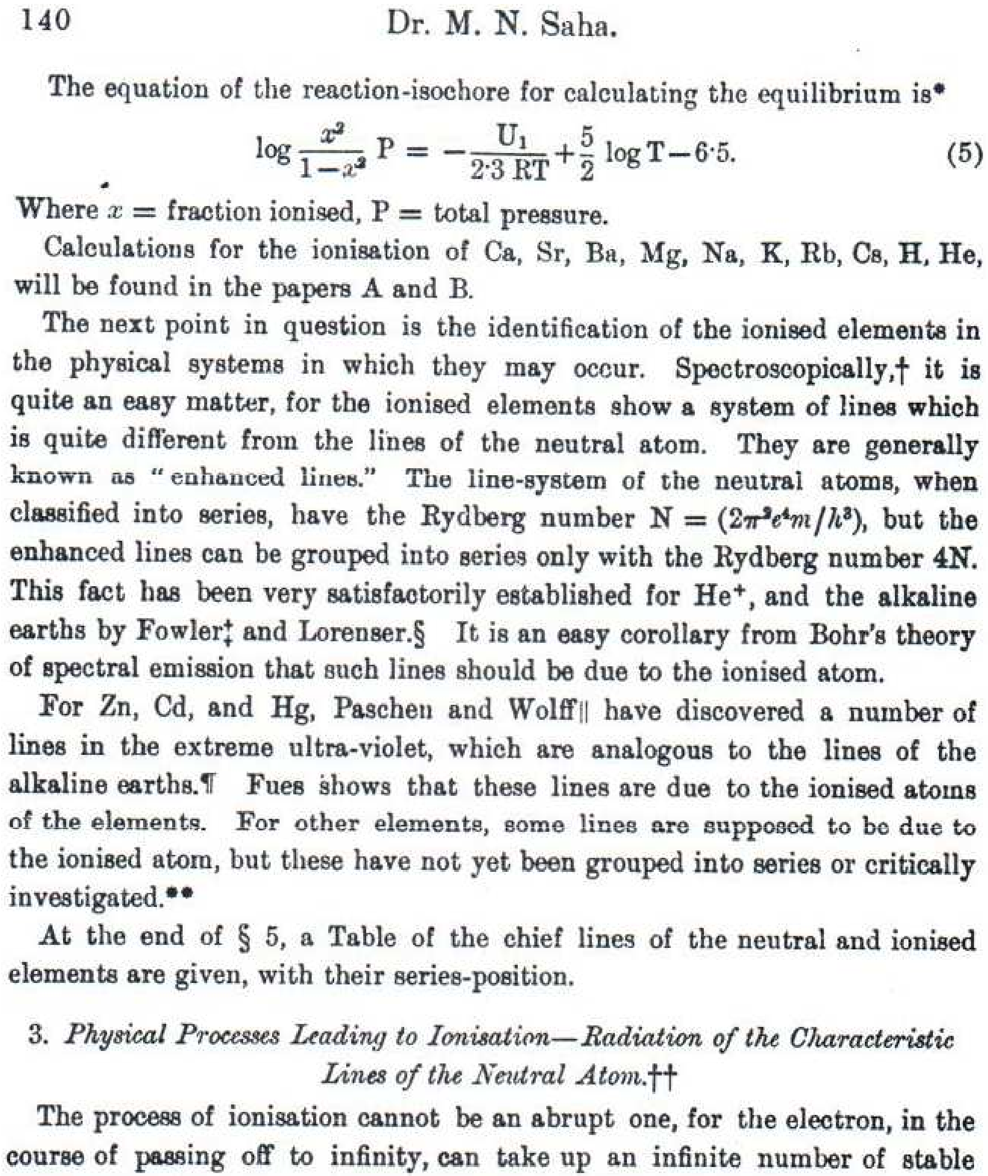
It is necessary to dwell a little on the physical basis of stellar classification. The earliest astrophysicists classified the stars according to colour; thus Secchi's type I denoted white stars, type II stood for yellow stars, type III for yellow-reddish stars, and type IV for deep red stars. But Lockyer and Pickering found that the varying intensity of particular groups of absorption-lines in stellar spectra was a far better criterion of stellar classification. Table II, compiled from the Harvard 'Annals,' illustrates the principle.

[For the methods by means of which the intensity of a particular line in different stars has been estimated, reference should be made to Harvard 'Annals,' vol. 27, p. 234; vol. 56, p. 56; vol. 91, p. 5. The following is added for the sake of general explanation:—

Numbers which are underlined denote that the line is bright; otherwise

Hvězdné atmosféry - Meghnad Saha

r. 1920 – O fyzikální teorii spektra hvězd - Sahova rovnice ionizace, rovnice pro stav ionizační rovnováhy, postupná ionizace v závislosti na teplotě a hustotě v atmosférách hvězd



The equation of the reaction-isochore for calculating the equilibrium is[*]

$$\log \frac{x^2}{1-x^2} P = -\frac{U_1}{2\cdot 3\,RT} + \frac{5}{2}\log T - 6\cdot 5. \qquad (5)$$

Where x = fraction ionised, P = total pressure.

Calculations for the ionisation of Ca, Sr, Ba, Mg, Na, K, Rb, Cs, H, He, will be found in the papers A and B.

The next point in question is the identification of the ionised elements in the physical systems in which they may occur. Spectroscopically,[†] it is quite an easy matter, for the ionised elements show a system of lines which is quite different from the lines of the neutral atom. They are generally known as "enhanced lines." The line-system of the neutral atoms, when classified into series, have the Rydberg number $N = (2\pi^2 e^4 m/h^3)$, but the enhanced lines can be grouped into series only with the Rydberg number 4N. This fact has been very satisfactorily established for He$^+$, and the alkaline earths by Fowler[‡] and Lorenser.[§] It is an easy corollary from Bohr's theory of spectral emission that such lines should be due to the ionised atom.

For Zn, Cd, and Hg, Paschen and Wolff[||] have discovered a number of lines in the extreme ultra-violet, which are analogous to the lines of the alkaline earths.[¶] Fues shows that these lines are due to the ionised atoms of the elements. For other elements, some lines are supposed to be due to the ionised atom, but these have not yet been grouped into series or critically investigated.[**]

At the end of § 5, a Table of the chief lines of the neutral and ionised elements are given, with their series-position.

3. *Physical Processes Leading to Ionisation—Radiation of the Characteristic Lines of the Neutral Atom.*[††]

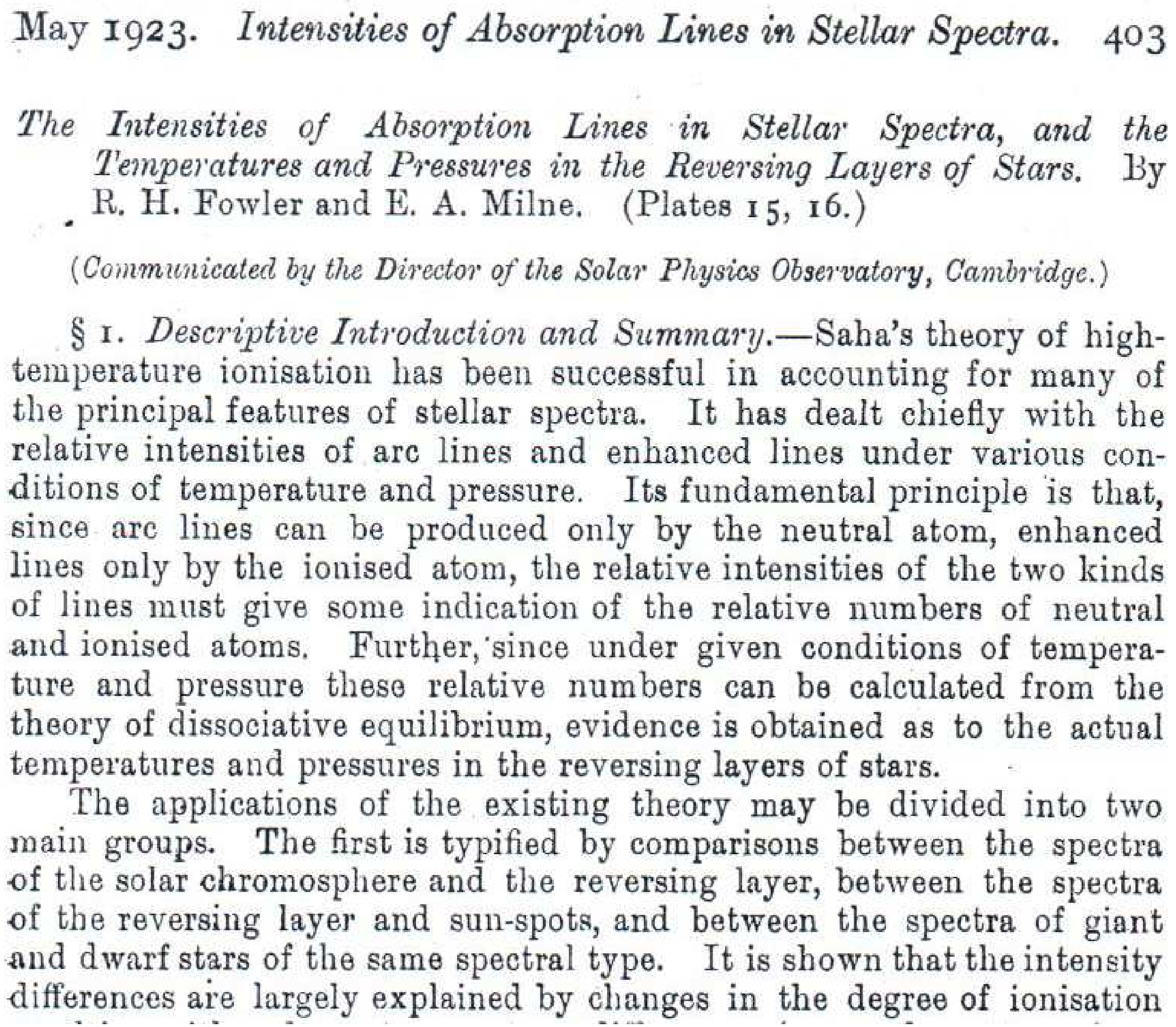
The process of ionisation cannot be an abrupt one, for the electron, in the course of passing off to infinity, can take up an infinite number of stable

Hvězdné atmosféry

anglický astrofyzik, matematik **Edward Arthur Milne 1896 - 1950**

anglický fyzik, chemik, astrofyzik **Ralp Howard Fowler 1889 - 1944**

Intenzity absorpčních čar ve hvězdném spektru, teploty a tlaky v převracejích vrstvách hvězd, r. 1923



May 1923. *Intensities of Absorption Lines in Stellar Spectra.* 403

The Intensities of Absorption Lines in Stellar Spectra, and the Temperatures and Pressures in the Reversing Layers of Stars. By R. H. Fowler and E. A. Milne. (Plates 15, 16.)

(*Communicated by the Director of the Solar Physics Observatory, Cambridge.*)

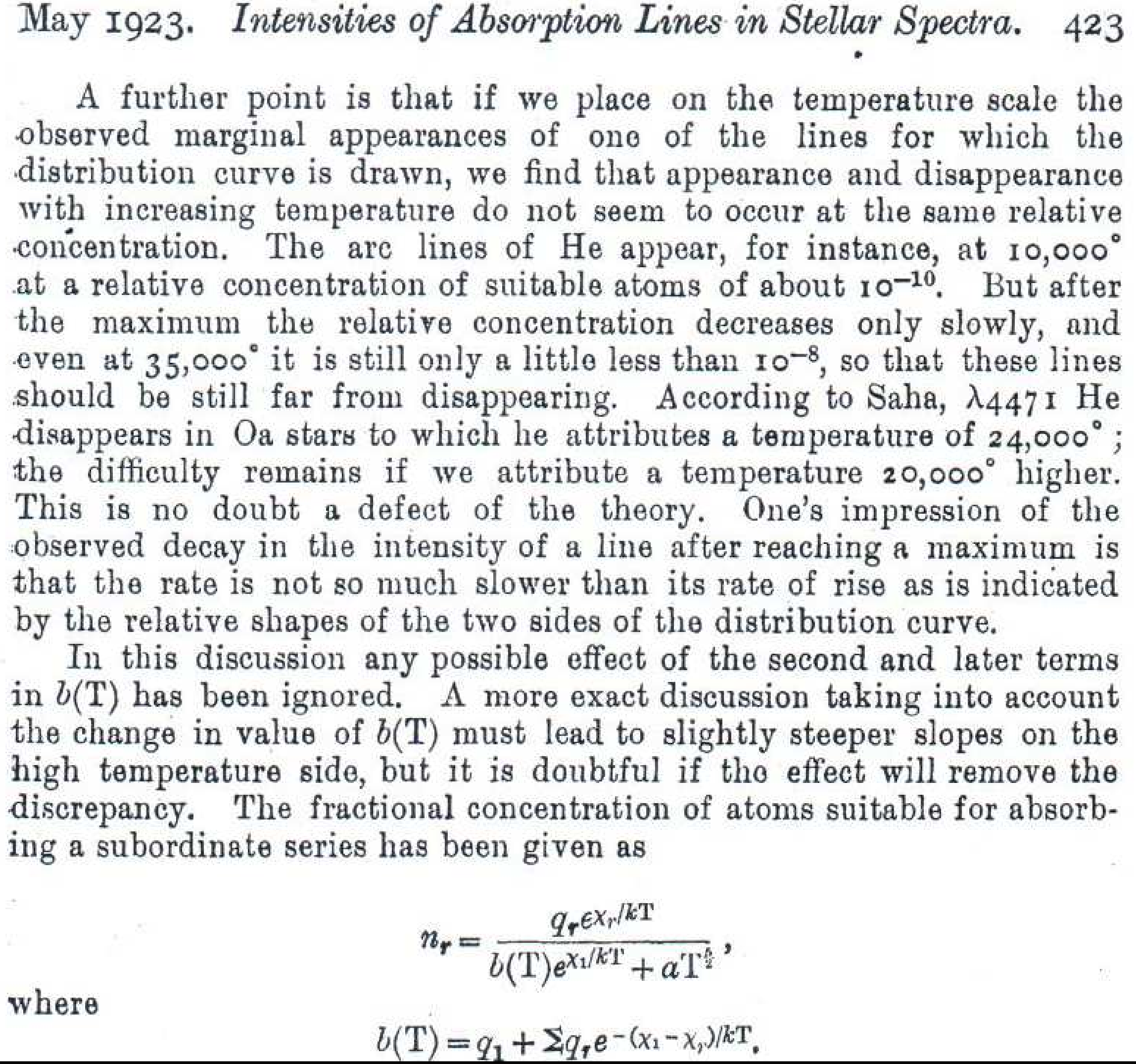
§ 1. *Descriptive Introduction and Summary.*—Saha's theory of high-temperature ionisation has been successful in accounting for many of the principal features of stellar spectra. It has dealt chiefly with the relative intensities of arc lines and enhanced lines under various conditions of temperature and pressure. Its fundamental principle is that, since arc lines can be produced only by the neutral atom, enhanced lines only by the ionised atom, the relative intensities of the two kinds of lines must give some indication of the relative numbers of neutral and ionised atoms. Further, since under given conditions of temperature and pressure these relative numbers can be calculated from the theory of dissociative equilibrium, evidence is obtained as to the actual temperatures and pressures in the reversing layers of stars.

The applications of the existing theory may be divided into two main groups. The first is typified by comparisons between the spectra of the solar chromosphere and the reversing layer, between the spectra of the reversing layer and sun-spots, and between the spectra of giant and dwarf stars of the same spectral type. It is shown that the intensity differences are largely explained by changes in the degree of ionisation

Hvězdné atmoésféry

anglický astrofyzik, matematik **Edward Arthur Milne 1896 - 1950**
anglický fyzik, chemik, astrofyzik **Ralp Howard Fowler 1889 - 1944**

Intenzity absorpčních čar ve hvězdném spektru, teploty a tlaky v převracejích vrstvách hvězd, r. 1923, ionizace – statistická mechanika



May 1923. *Intensities of Absorption Lines in Stellar Spectra.* 423

A further point is that if we place on the temperature scale the observed marginal appearances of one of the lines for which the distribution curve is drawn, we find that appearance and disappearance with increasing temperature do not seem to occur at the same relative concentration. The arc lines of He appear, for instance, at 10,000° at a relative concentration of suitable atoms of about 10^{-10}. But after the maximum the relative concentration decreases only slowly, and even at 35,000° it is still only a little less than 10^{-8}, so that these lines should be still far from disappearing. According to Saha, λ4471 He disappears in Oa stars to which he attributes a temperature of 24,000°; the difficulty remains if we attribute a temperature 20,000° higher. This is no doubt a defect of the theory. One's impression of the observed decay in the intensity of a line after reaching a maximum is that the rate is not so much slower than its rate of rise as is indicated by the relative shapes of the two sides of the distribution curve.

In this discussion any possible effect of the second and later terms in $b(\mathrm{T})$ has been ignored. A more exact discussion taking into account the change in value of $b(\mathrm{T})$ must lead to slightly steeper slopes on the high temperature side, but it is doubtful if the effect will remove the discrepancy. The fractional concentration of atoms suitable for absorbing a subordinate series has been given as

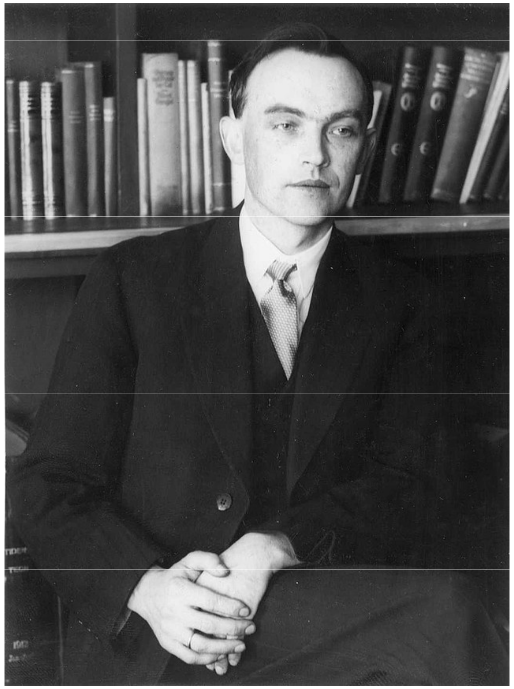
$$n_r = \frac{q_r e^{\chi_r/k\mathrm{T}}}{b(\mathrm{T})e^{\chi_1/k\mathrm{T}} + a\mathrm{T}^{\frac{5}{2}}},$$

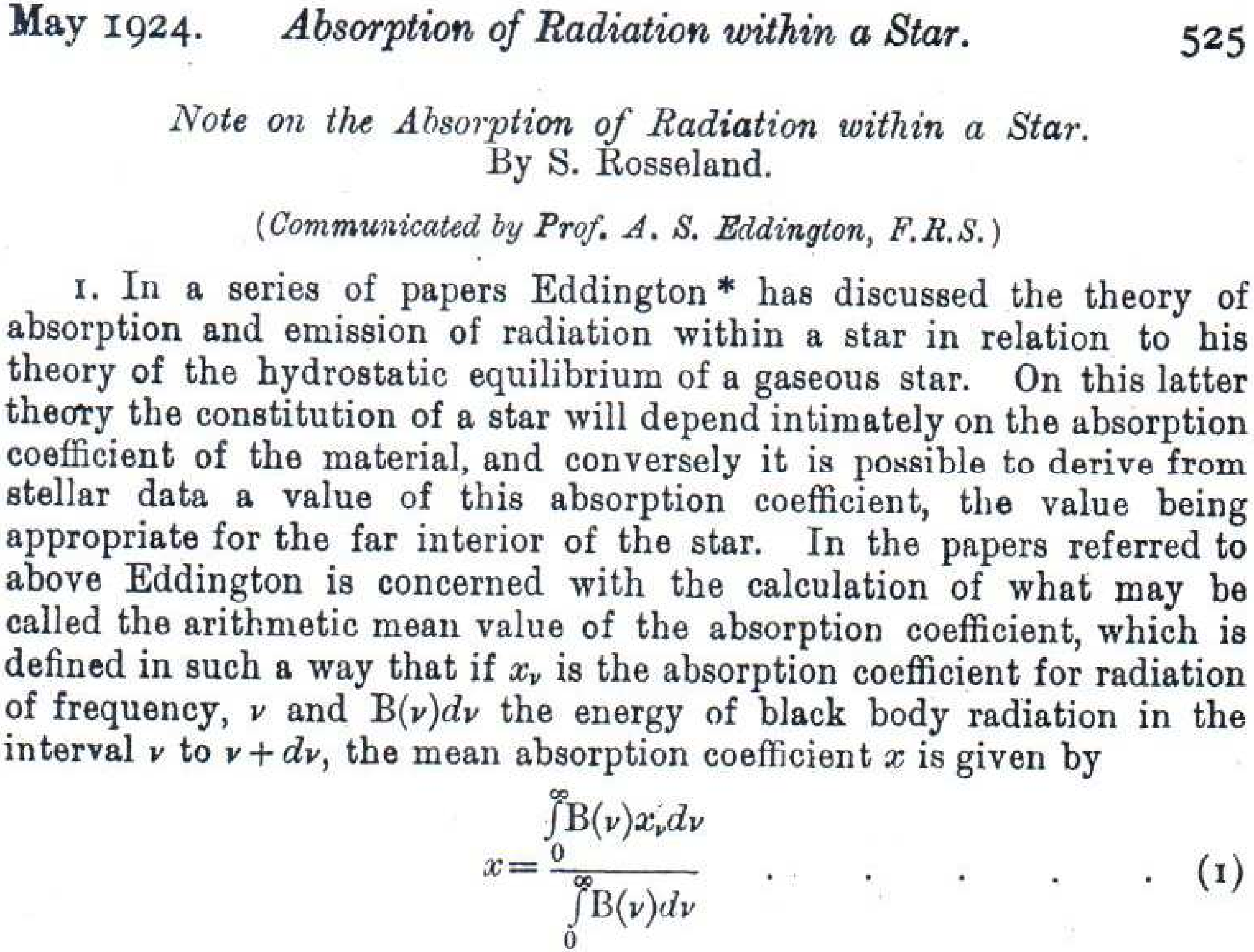
where

$$b(\mathrm{T}) = q_1 + \Sigma q_r e^{-(\chi_1 - \chi_r)/k\mathrm{T}}.$$

Hvězdné atmosféry
Svein Rosseland 1894 - 1985

norský astrofyzik, ***Absorpce záření ve hvězdách r. 1924,*** přenos záření v nitru hvězd, difúzní proces s průměrnou hodnotou opacity, využíval ho dále Eddington





May 1924. *Absorption of Radiation within a Star.* 525

Note on the Absorption of Radiation within a Star.
By S. Rosseland.

(*Communicated by Prof. A. S. Eddington, F.R.S.*)

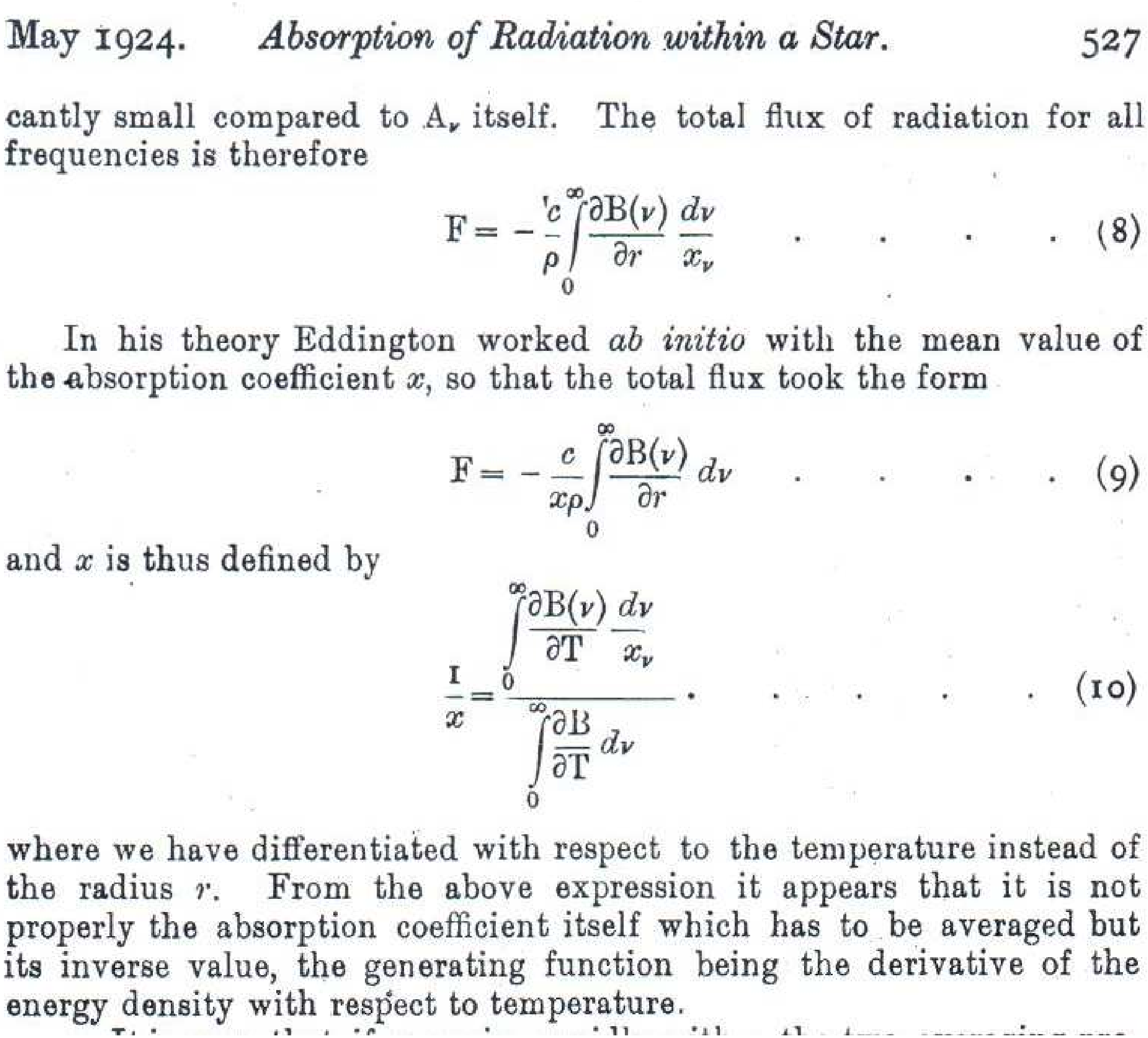
1. In a series of papers Eddington * has discussed the theory of absorption and emission of radiation within a star in relation to his theory of the hydrostatic equilibrium of a gaseous star. On this latter theory the constitution of a star will depend intimately on the absorption coefficient of the material, and conversely it is possible to derive from stellar data a value of this absorption coefficient, the value being appropriate for the far interior of the star. In the papers referred to above Eddington is concerned with the calculation of what may be called the arithmetic mean value of the absorption coefficient, which is defined in such a way that if x_ν is the absorption coefficient for radiation of frequency, ν and $B(\nu)d\nu$ the energy of black body radiation in the interval ν to $\nu+d\nu$, the mean absorption coefficient x is given by

$$x=\frac{\int_0^\infty B(\nu)x_\nu d\nu}{\int_0^\infty B(\nu)d\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hvězdné atmosféry

Svein Rosseland: *Absorpce záření ve hvězdách 1924*

Rosselandova střední opacita



cantly small compared to A_ν itself. The total flux of radiation for all frequencies is therefore

$$F = -\frac{c}{\rho}\int_0^\infty \frac{\partial B(\nu)}{\partial r}\frac{d\nu}{x_\nu} \quad . \quad . \quad . \quad . \quad (8)$$

In his theory Eddington worked *ab initio* with the mean value of the absorption coefficient x, so that the total flux took the form

$$F = -\frac{c}{x\rho}\int_0^\infty \frac{\partial B(\nu)}{\partial r}\,d\nu \quad . \quad . \quad . \quad . \quad (9)$$

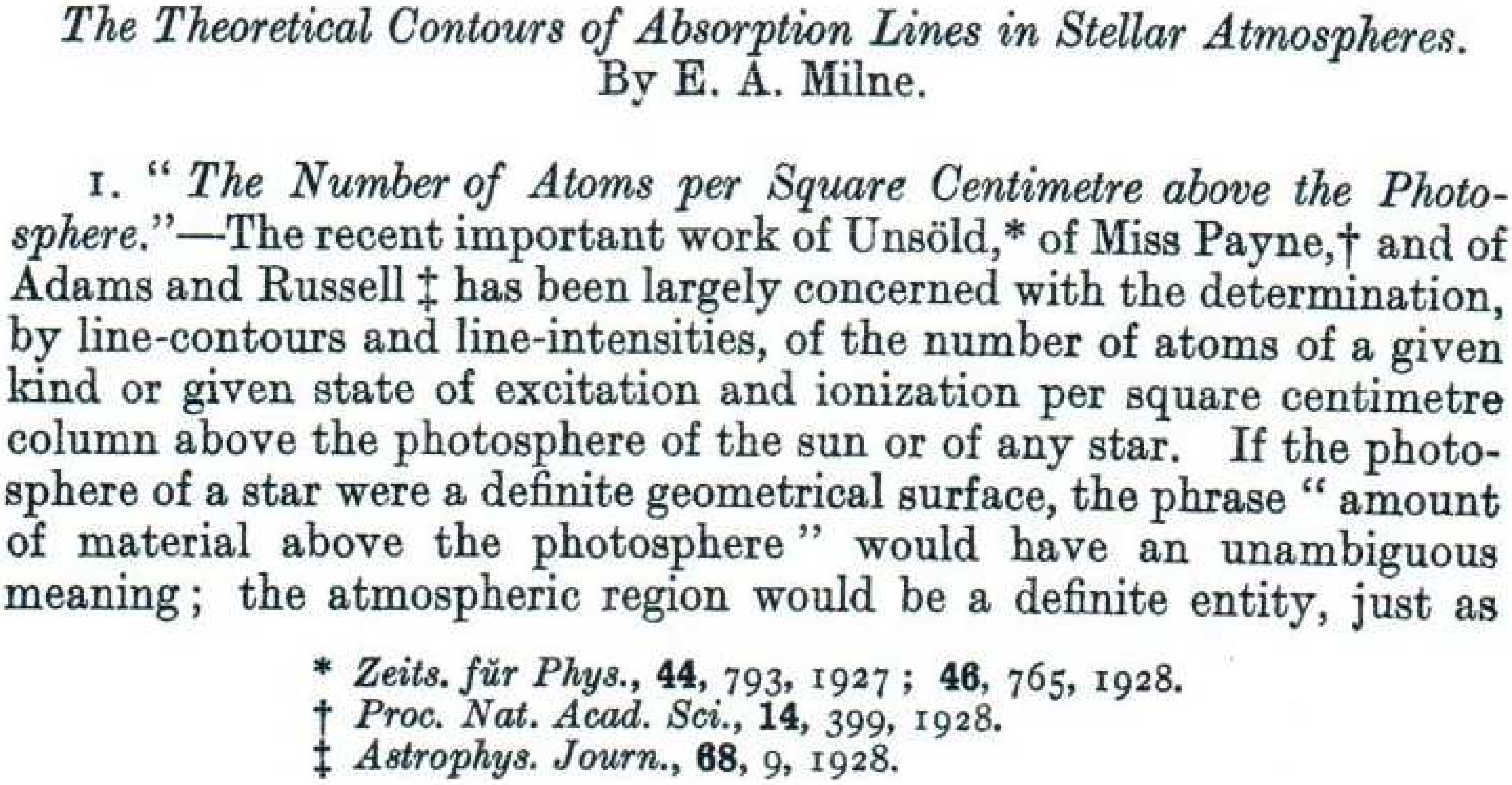
and x is thus defined by

$$\frac{1}{x} = \frac{\displaystyle\int_0^\infty \frac{\partial B(\nu)}{\partial T}\frac{d\nu}{x_\nu}}{\displaystyle\int_0^\infty \frac{\partial B}{\partial T}\,d\nu} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where we have differentiated with respect to the temperature instead of the radius r. From the above expression it appears that it is not properly the absorption coefficient itself which has to be averaged but its inverse value, the generating function being the derivative of the energy density with respect to temperature.

Hvězdné atmosféry

anglický astrofyzik, matematik **Edward Arthur Milne 1896 - 1950**
Teoretický profil absorpčních čar v hvězdných atmosférách r. 1928,
důsledky zářivé rovnováhy, rozložení teploty ve vznikajícím spektru, předpoklad šedé atmosféry, lokální termodynamická rovnováha



The Theoretical Contours of Absorption Lines in Stellar Atmospheres.
By E. A. Milne.

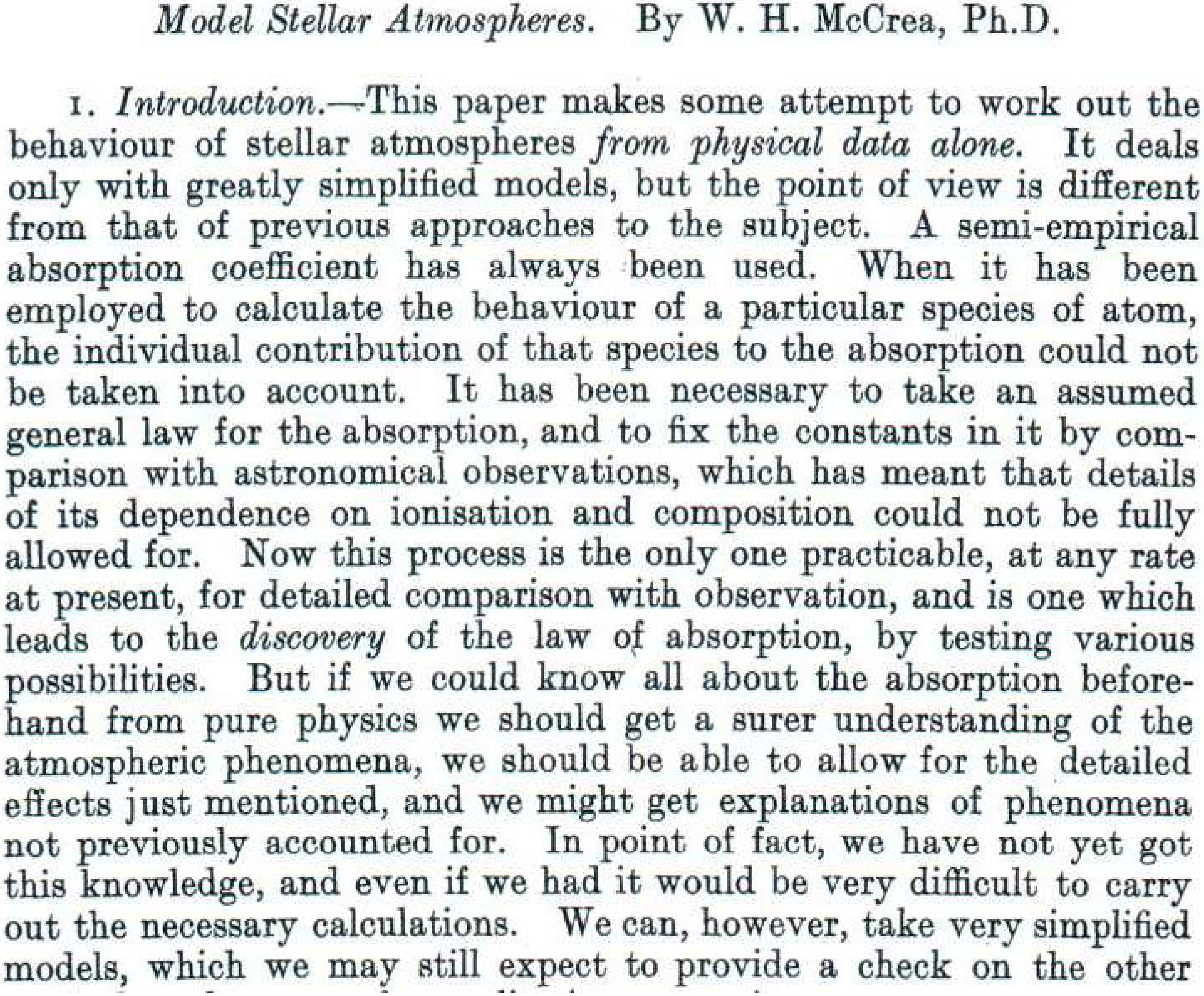
1. "*The Number of Atoms per Square Centimetre above the Photosphere.*"—The recent important work of Unsöld,[*] of Miss Payne,[†] and of Adams and Russell [‡] has been largely concerned with the determination, by line-contours and line-intensities, of the number of atoms of a given kind or given state of excitation and ionization per square centimetre column above the photosphere of the sun or of any star. If the photosphere of a star were a definite geometrical surface, the phrase "amount of material above the photosphere" would have an unambiguous meaning; the atmospheric region would be a definite entity, just as

---

[*] *Zeits. für Phys.*, **44**, 793, 1927; **46**, 765, 1928.
[†] *Proc. Nat. Acad. Sci.*, **14**, 399, 1928.
[‡] *Astrophys. Journ.*, **68**, 9, 1928.

Hvězdné atmosféry
William Hunter McCrea 1904 - 1999

irský astrofyzik, ***Model hvězdné atmosféry r. 1931***

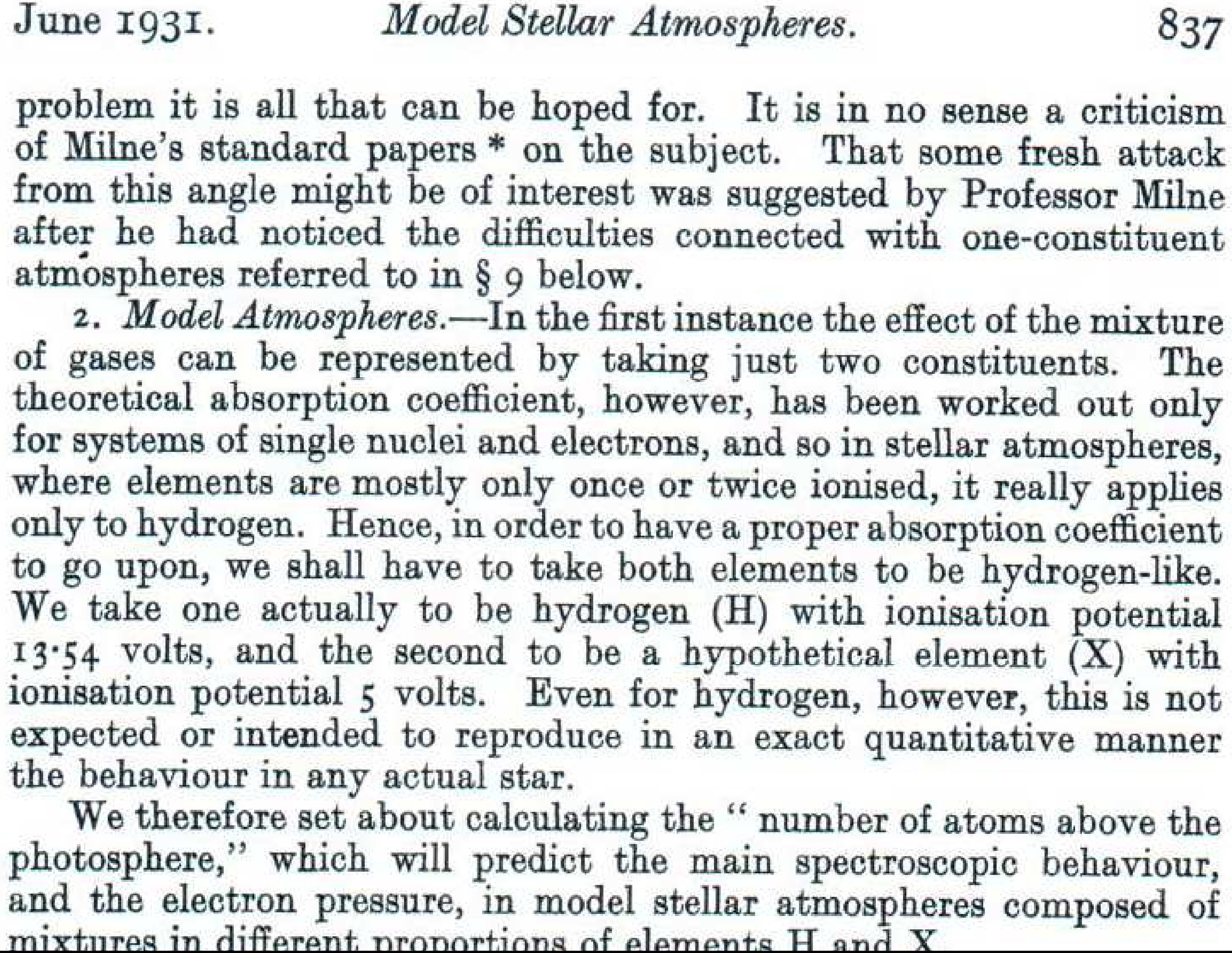


Model Stellar Atmospheres. By W. H. McCrea, Ph.D.

1. *Introduction.*—This paper makes some attempt to work out the behaviour of stellar atmospheres *from physical data alone.* It deals only with greatly simplified models, but the point of view is different from that of previous approaches to the subject. A semi-empirical absorption coefficient has always been used. When it has been employed to calculate the behaviour of a particular species of atom, the individual contribution of that species to the absorption could not be taken into account. It has been necessary to take an assumed general law for the absorption, and to fix the constants in it by comparison with astronomical observations, which has meant that details of its dependence on ionisation and composition could not be fully allowed for. Now this process is the only one practicable, at any rate at present, for detailed comparison with observation, and is one which leads to the *discovery* of the law of absorption, by testing various possibilities. But if we could know all about the absorption beforehand from pure physics we should get a surer understanding of the atmospheric phenomena, we should be able to allow for the detailed effects just mentioned, and we might get explanations of phenomena not previously accounted for. In point of fact, we have not yet got this knowledge, and even if we had it would be very difficult to carry out the necessary calculations. We can, however, take very simplified models, which we may still expect to provide a check on the other

Hvězdné atmosféry
William Hunter McCrea

Model hvězdné atmosféry r. 1931, zahrnutí excitace a ionizace



June 1931. *Model Stellar Atmospheres.* 837

problem it is all that can be hoped for. It is in no sense a criticism of Milne's standard papers * on the subject. That some fresh attack from this angle might be of interest was suggested by Professor Milne after he had noticed the difficulties connected with one-constituent atmospheres referred to in § 9 below.

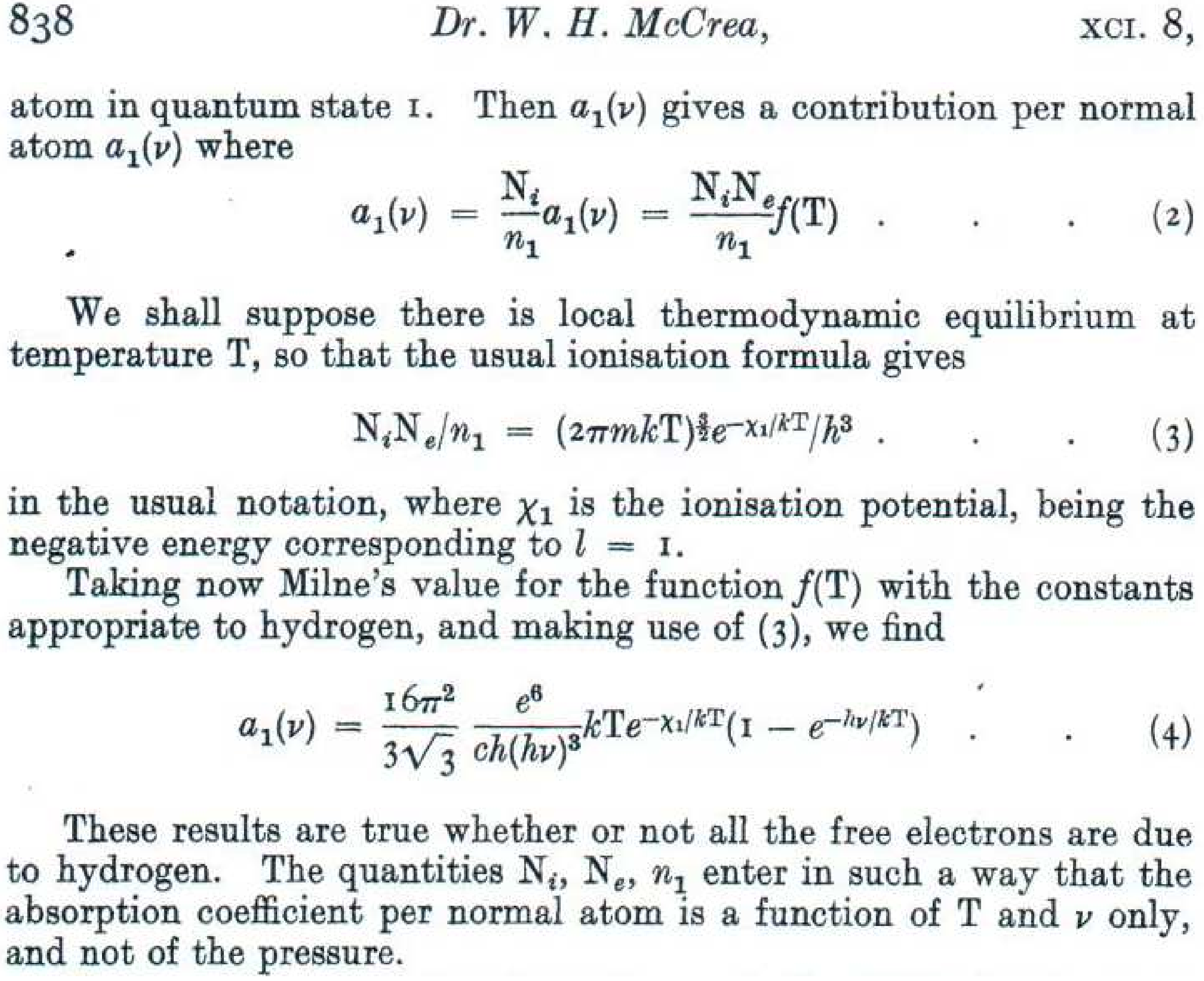
2. *Model Atmospheres.*—In the first instance the effect of the mixture of gases can be represented by taking just two constituents. The theoretical absorption coefficient, however, has been worked out only for systems of single nuclei and electrons, and so in stellar atmospheres, where elements are mostly only once or twice ionised, it really applies only to hydrogen. Hence, in order to have a proper absorption coefficient to go upon, we shall have to take both elements to be hydrogen-like. We take one actually to be hydrogen (H) with ionisation potential 13·54 volts, and the second to be a hypothetical element (X) with ionisation potential 5 volts. Even for hydrogen, however, this is not expected or intended to reproduce in an exact quantitative manner the behaviour in any actual star.

We therefore set about calculating the "number of atoms above the photosphere," which will predict the main spectroscopic behaviour, and the electron pressure, in model stellar atmospheres composed of mixtures in different proportions of elements H and X

Hvězdné atmosféry

William Hunter McCrea

Model hvězdné atmosféry r. 1931



atom in quantum state I. Then $a_1(\nu)$ gives a contribution per normal atom $a_1(\nu)$ where

$$a_1(\nu) = \frac{N_i}{n_1} a_1(\nu) = \frac{N_i N_e}{n_1} f(T) \quad . \quad . \quad . \quad (2)$$

We shall suppose there is local thermodynamic equilibrium at temperature T, so that the usual ionisation formula gives

$$N_i N_e / n_1 = (2\pi m k T)^{\frac{3}{2}} e^{-\chi_1/kT} / h^3 \quad . \quad . \quad . \quad (3)$$

in the usual notation, where χ_1 is the ionisation potential, being the negative energy corresponding to $l = 1$.

Taking now Milne's value for the function $f(T)$ with the constants appropriate to hydrogen, and making use of (3), we find

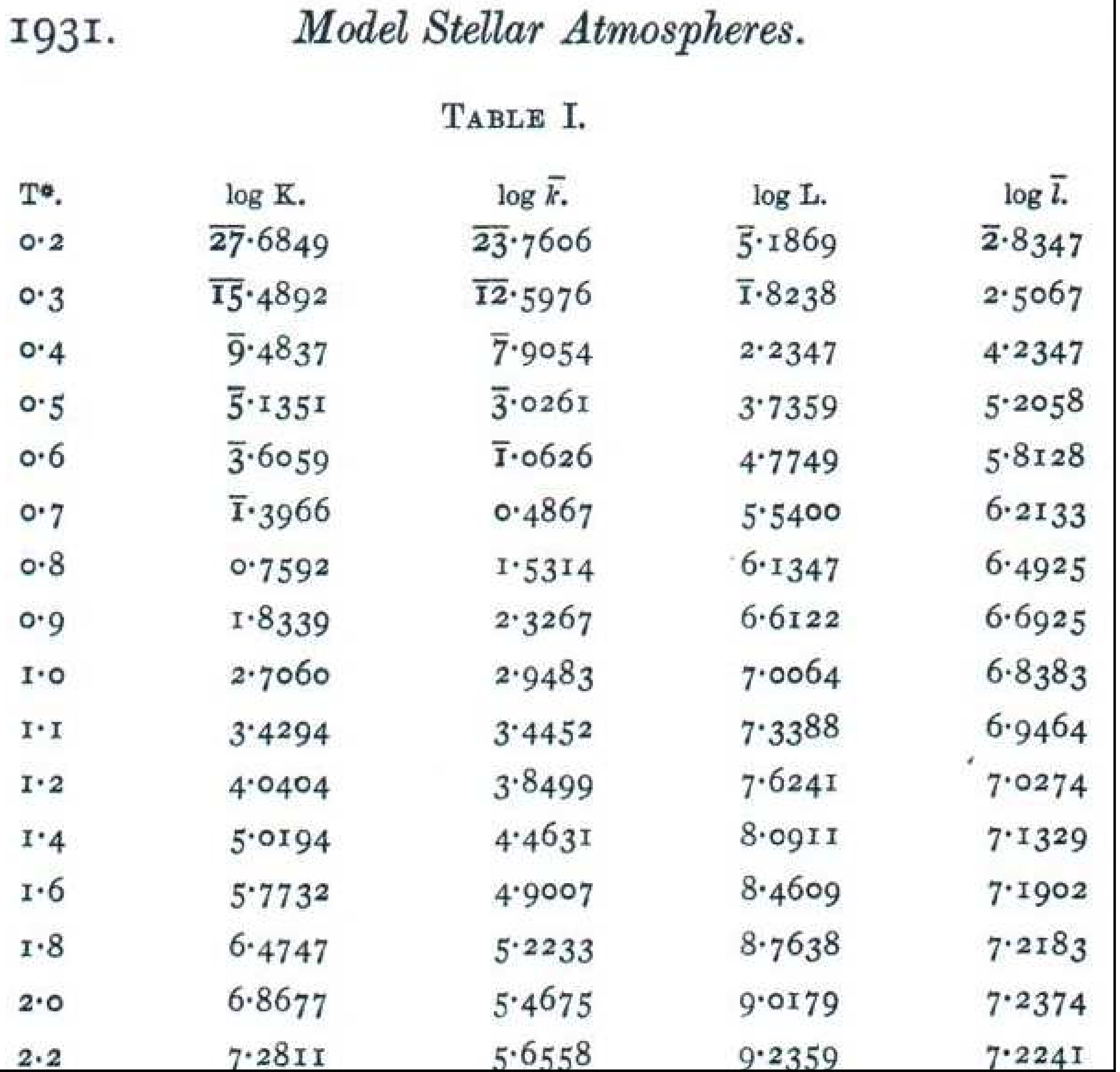
$$a_1(\nu) = \frac{16\pi^2}{3\sqrt{3}} \frac{e^6}{ch(h\nu)^3} kT e^{-\chi_1/kT} (1 - e^{-h\nu/kT}) \quad . \quad . \quad . \quad (4)$$

These results are true whether or not all the free electrons are due to hydrogen. The quantities N_i, N_e, n_1 enter in such a way that the absorption coefficient per normal atom is a function of T and ν only, and not of the pressure.

Hvězdné atmosféry

William Hunter McCrea

Model hvězdné atmosféry r. 1931



1931. *Model Stellar Atmospheres.*

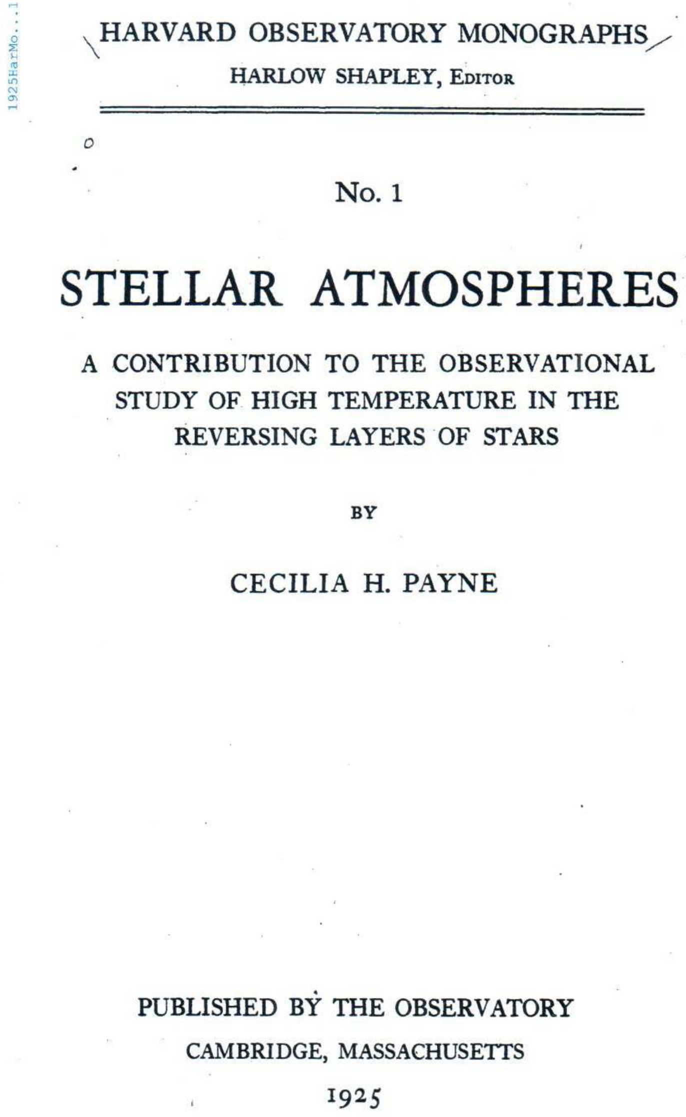
TABLE I.

T°.	log K.	log $\bar{k}$.	log L.	log $\bar{l}$.
0·2	$\overline{27}$·6849	$\overline{23}$·7606	$\overline{5}$·1869	$\overline{2}$·8347
0·3	$\overline{15}$·4892	$\overline{12}$·5976	$\overline{1}$·8238	2·5067
0·4	$\overline{9}$·4837	$\overline{7}$·9054	2·2347	4·2347
0·5	$\overline{5}$·1351	$\overline{3}$·0261	3·7359	5·2058
0·6	$\overline{3}$·6059	$\overline{1}$·0626	4·7749	5·8128
0·7	$\overline{1}$·3966	0·4867	5·5400	6·2133
0·8	0·7592	1·5314	6·1347	6·4925
0·9	1·8339	2·3267	6·6122	6·6925
1·0	2·7060	2·9483	7·0064	6·8383
1·1	3·4294	3·4452	7·3388	6·9464
1·2	4·0404	3·8499	7·6241	7·0274
1·4	5·0194	4·4631	8·0911	7·1329
1·6	5·7732	4·9007	8·4609	7·1902
1·8	6·4747	5·2233	8·7638	7·2183
2·0	6·8677	5·4675	9·0179	7·2374
2·2	7·2811	5·6558	9·2359	7·2241

Hvězdné atmosféry

Cecilie Helene Payene - Gaposhkin 1900 – 1979,

Hvězdné atmosféry r. 1925, určení chemického složení hvězd, první profesorka - žena na Harvardově univerzitě



HARVARD OBSERVATORY MONOGRAPHS
HARLOW SHAPLEY, EDITOR

No. 1

STELLAR ATMOSPHERES

A CONTRIBUTION TO THE OBSERVATIONAL STUDY OF HIGH TEMPERATURE IN THE REVERSING LAYERS OF STARS

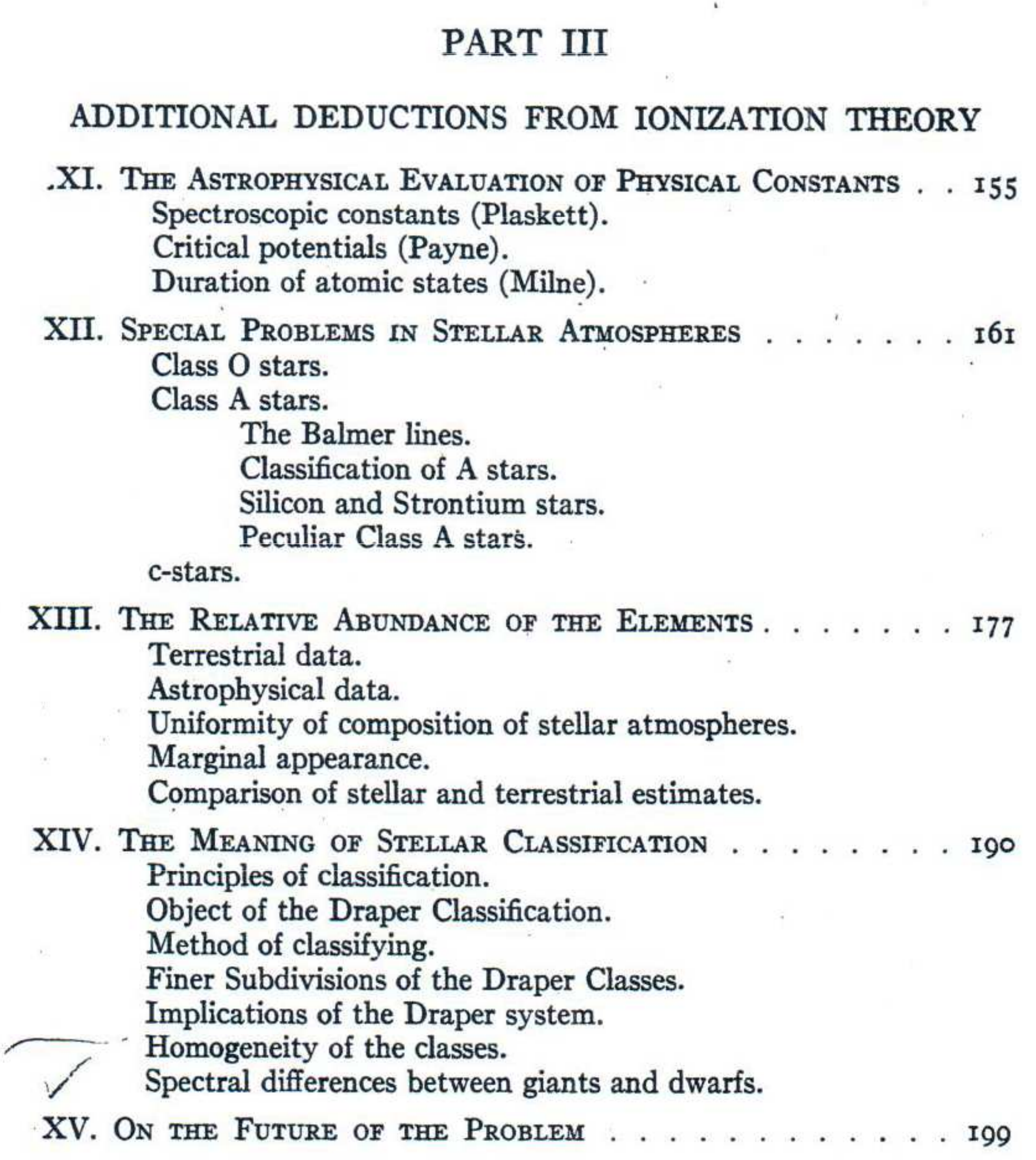
BY

CECILIA H. PAYNE

PUBLISHED BY THE OBSERVATORY
CAMBRIDGE, MASSACHUSETTS
1925

Hvězdné atmosféry

Cecilie Helene Payene - Gaposhkin, ***Hvězdné atmosféry r. 1925***

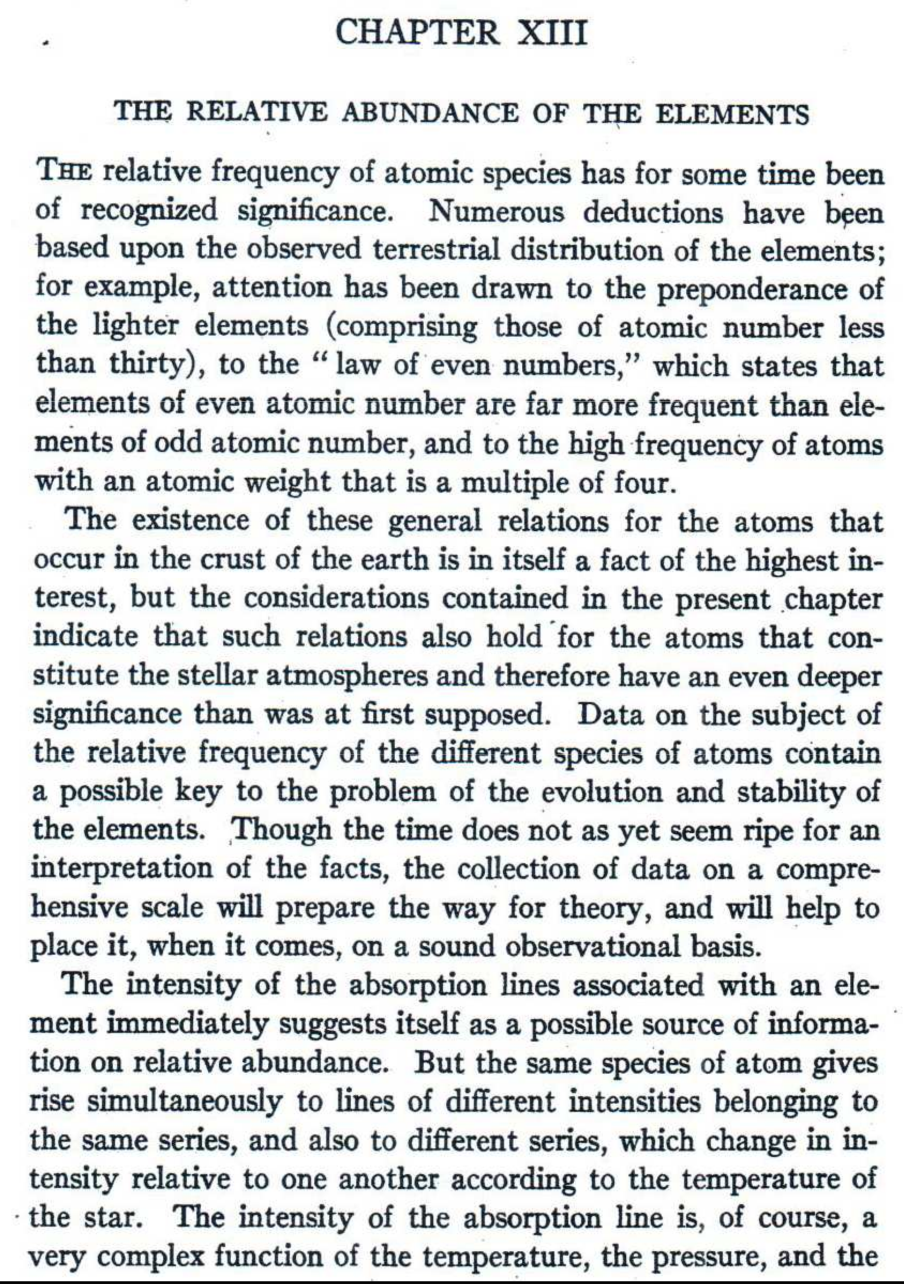


PART III

ADDITIONAL DEDUCTIONS FROM IONIZATION THEORY

Hvězdné atmosféry

Cecilie Helene Payene – Gaposhkin, *Hvězdné atmosféry r. 1925*,



CHAPTER XIII

THE RELATIVE ABUNDANCE OF THE ELEMENTS

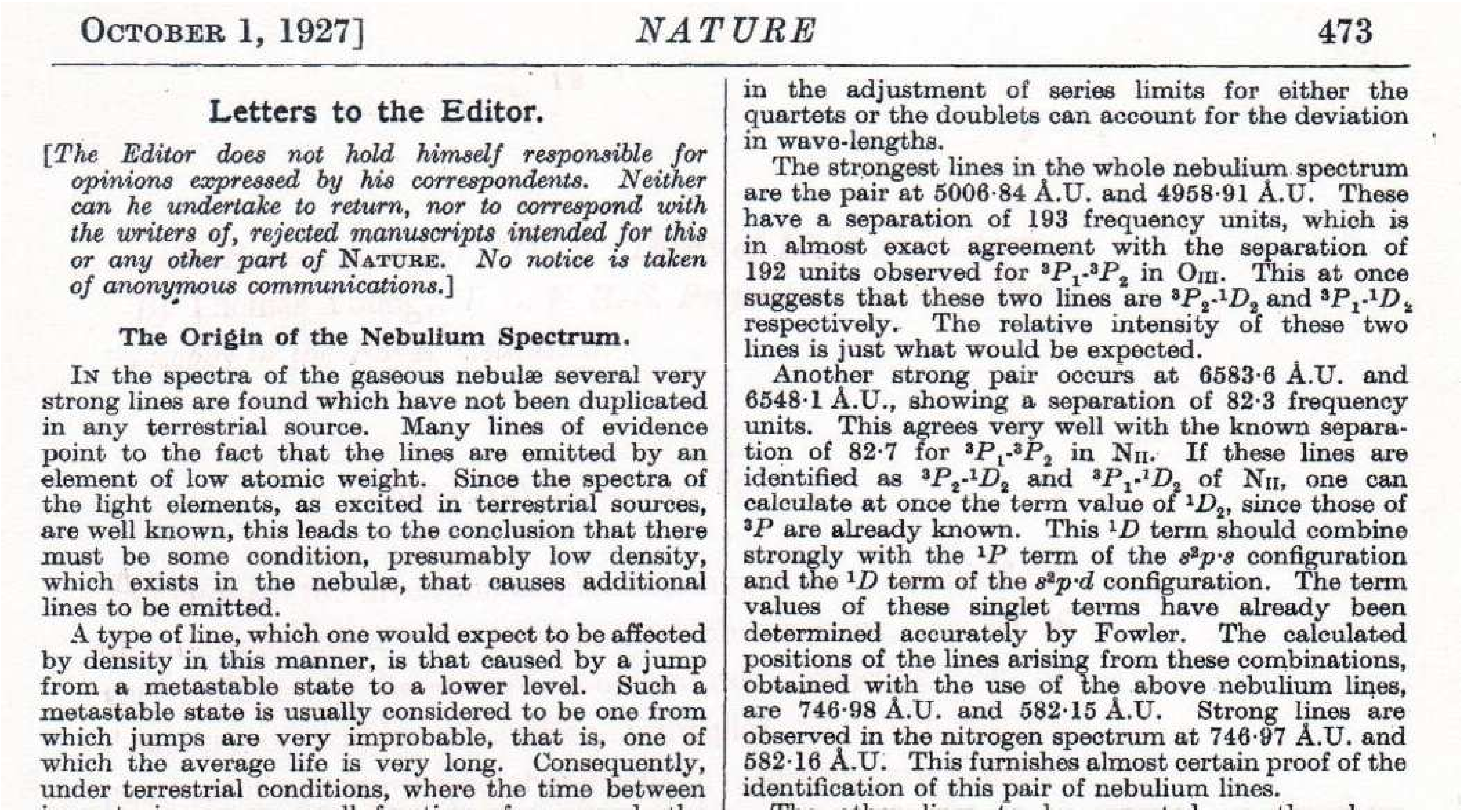
THE relative frequency of atomic species has for some time been of recognized significance. Numerous deductions have been based upon the observed terrestrial distribution of the elements; for example, attention has been drawn to the preponderance of the lighter elements (comprising those of atomic number less than thirty), to the " law of even numbers," which states that elements of even atomic number are far more frequent than elements of odd atomic number, and to the high frequency of atoms with an atomic weight that is a multiple of four.

The existence of these general relations for the atoms that occur in the crust of the earth is in itself a fact of the highest interest, but the considerations contained in the present chapter indicate that such relations also hold for the atoms that constitute the stellar atmospheres and therefore have an even deeper significance than was at first supposed. Data on the subject of the relative frequency of the different species of atoms contain a possible key to the problem of the evolution and stability of the elements. Though the time does not as yet seem ripe for an interpretation of the facts, the collection of data on a comprehensive scale will prepare the way for theory, and will help to place it, when it comes, on a sound observational basis.

The intensity of the absorption lines associated with an element immediately suggests itself as a possible source of information on relative abundance. But the same species of atom gives rise simultaneously to lines of different intensities belonging to the same series, and also to different series, which change in intensity relative to one another according to the temperature of the star. The intensity of the absorption line is, of course, a very complex function of the temperature, the pressure, and the

Hvězdné atmosféry

americký astrofyzik **Ira Sprague Bowen 1898 - 1973**,
Původ spektra nebulia r. 1927, **„nebulium“ -** dvakrát ionizovaný kyslík



Letters to the Editor.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, nor to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

The Origin of the Nebulium Spectrum.

IN the spectra of the gaseous nebulæ several very strong lines are found which have not been duplicated in any terrestrial source. Many lines of evidence point to the fact that the lines are emitted by an element of low atomic weight. Since the spectra of the light elements, as excited in terrestrial sources, are well known, this leads to the conclusion that there must be some condition, presumably low density, which exists in the nebulæ, that causes additional lines to be emitted.

A type of line, which one would expect to be affected by density in this manner, is that caused by a jump from a metastable state to a lower level. Such a metastable state is usually considered to be one from which jumps are very improbable, that is, one of which the average life is very long. Consequently, under terrestrial conditions, where the time between

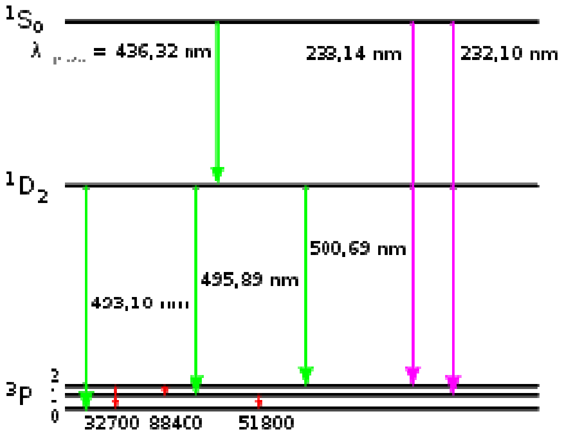
in the adjustment of series limits for either the quartets or the doublets can account for the deviation in wave-lengths.

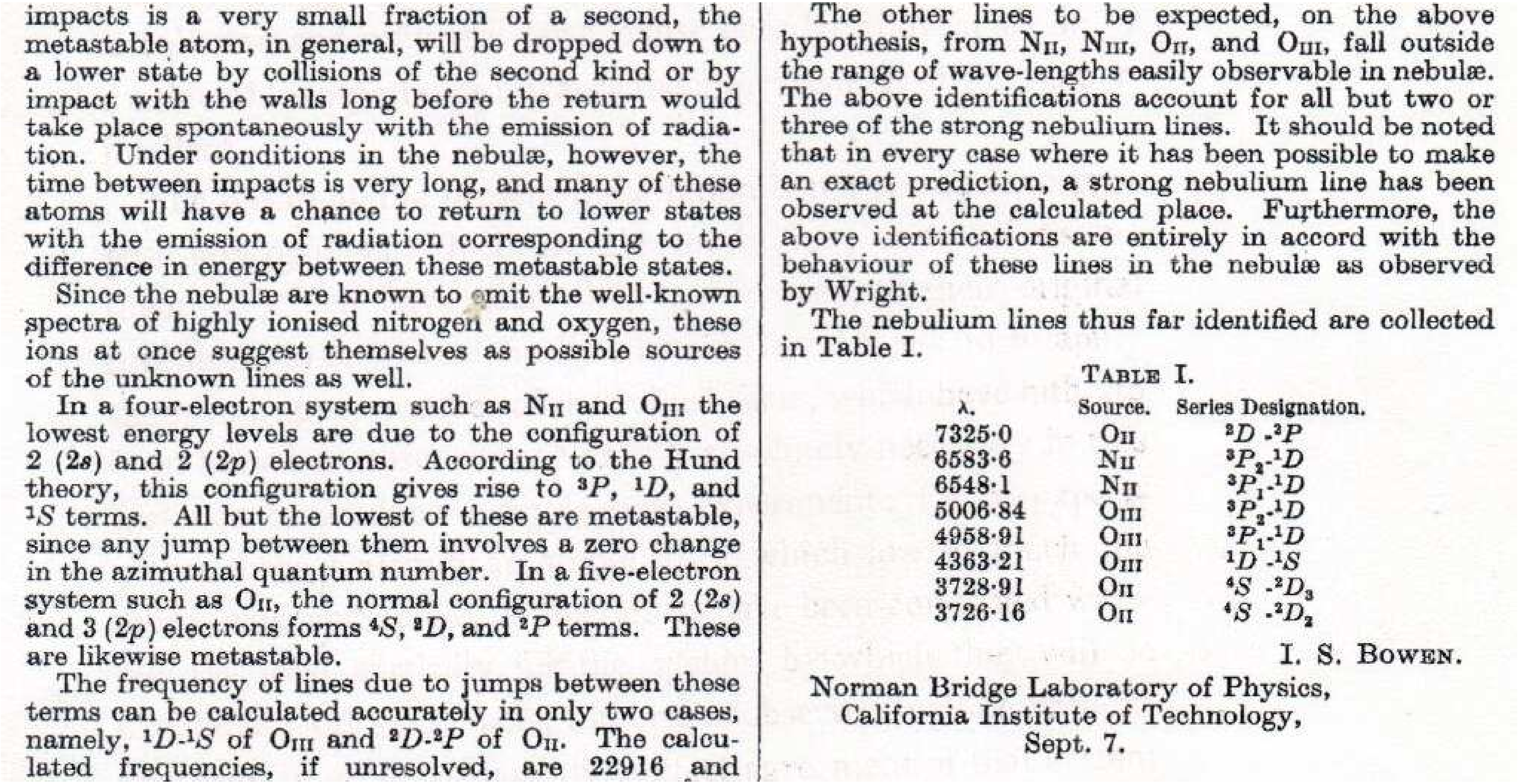
The strongest lines in the whole nebulium spectrum are the pair at 5006·84 Å.U. and 4958·91 Å.U. These have a separation of 193 frequency units, which is in almost exact agreement with the separation of 192 units observed for $^3P_1$-$^3P_2$ in O_{III}. This at once suggests that these two lines are $^3P_2$-$^1D_2$ and $^3P_1$-$^1D_2$ respectively. The relative intensity of these two lines is just what would be expected.

Another strong pair occurs at 6583·6 Å.U. and 6548·1 Å.U., showing a separation of 82·3 frequency units. This agrees very well with the known separation of 82·7 for $^3P_1$-$^3P_2$ in N_{II}. If these lines are identified as $^3P_2$-$^1D_2$ and $^3P_1$-$^1D_2$ of N_{II}, one can calculate at once the term value of $^1D_2$, since those of ^3P are already known. This ^1D term should combine strongly with the ^1P term of the $s^2p\cdot s$ configuration and the ^1D term of the $s^2p\cdot d$ configuration. The term values of these singlet terms have already been determined accurately by Fowler. The calculated positions of the lines arising from these combinations, obtained with the use of the above nebulium lines, are 746·98 Å.U. and 582·15 Å.U. Strong lines are observed in the nitrogen spectrum at 746·97 Å.U. and 582·16 Å.U. This furnishes almost certain proof of the identification of this pair of nebulium lines.

Hvězdné atmosféry

Původ spektra nebulia r. 1927, zakázané čáry





impacts is a very small fraction of a second, the metastable atom, in general, will be dropped down to a lower state by collisions of the second kind or by impact with the walls long before the return would take place spontaneously with the emission of radiation. Under conditions in the nebulæ, however, the time between impacts is very long, and many of these atoms will have a chance to return to lower states with the emission of radiation corresponding to the difference in energy between these metastable states.

Since the nebulæ are known to emit the well-known spectra of highly ionised nitrogen and oxygen, these ions at once suggest themselves as possible sources of the unknown lines as well.

In a four-electron system such as $\mathrm{N_{II}}$ and $\mathrm{O_{III}}$ the lowest energy levels are due to the configuration of 2 (2*s*) and 2 (2*p*) electrons. According to the Hund theory, this configuration gives rise to ^3P, ^1D, and ^1S terms. All but the lowest of these are metastable, since any jump between them involves a zero change in the azimuthal quantum number. In a five-electron system such as $\mathrm{O_{II}}$, the normal configuration of 2 (2*s*) and 3 (2*p*) electrons forms ^4S, ^2D, and ^2P terms. These are likewise metastable.

The frequency of lines due to jumps between these terms can be calculated accurately in only two cases, namely, ^1D-^1S of $\mathrm{O_{III}}$ and ^2D-^2P of $\mathrm{O_{II}}$. The calculated frequencies, if unresolved, are 22916 and

The other lines to be expected, on the above hypothesis, from $\mathrm{N_{II}}$, $\mathrm{N_{III}}$, $\mathrm{O_{II}}$, and $\mathrm{O_{III}}$, fall outside the range of wave-lengths easily observable in nebulæ. The above identifications account for all but two or three of the strong nebulium lines. It should be noted that in every case where it has been possible to make an exact prediction, a strong nebulium line has been observed at the calculated place. Furthermore, the above identifications are entirely in accord with the behaviour of these lines in the nebulæ as observed by Wright.

The nebulium lines thus far identified are collected in Table I.

TABLE I.

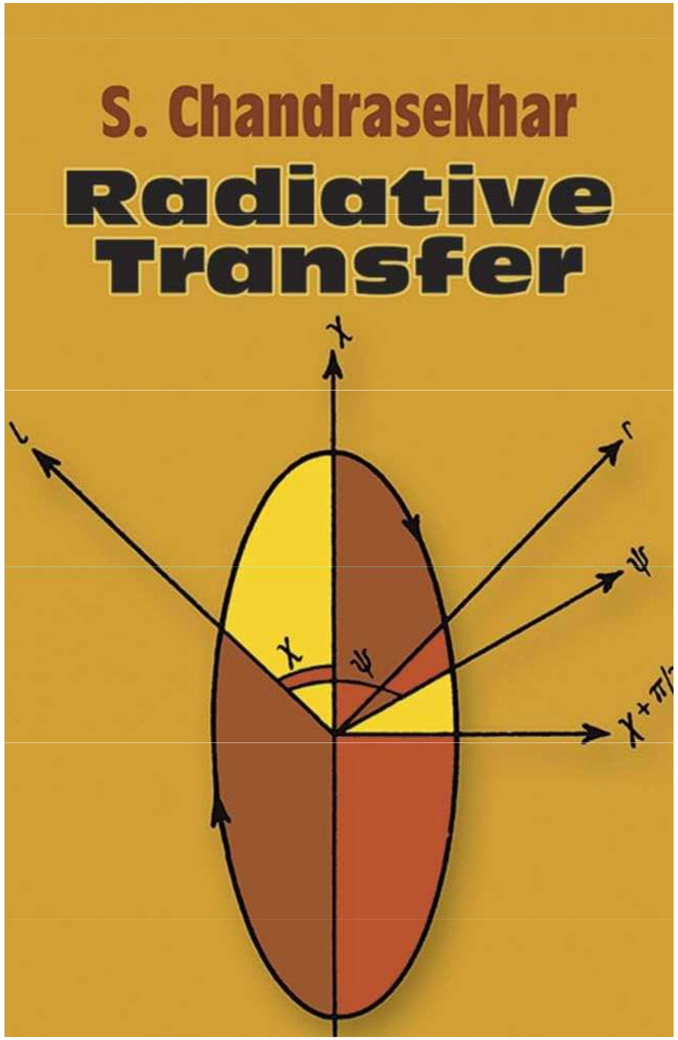
λ.	Source.	Series Designation.
7325·0	$\mathrm{O_{II}}$	^2D -^2P
6583·6	$\mathrm{N_{II}}$	$^3P_2$-^1D
6548·1	$\mathrm{N_{II}}$	$^3P_1$-^1D
5006·84	$\mathrm{O_{III}}$	$^3P_2$-^1D
4958·91	$\mathrm{O_{III}}$	$^3P_1$-^1D
4363·21	$\mathrm{O_{III}}$	^1D -^1S
3728·91	$\mathrm{O_{II}}$	^4S -$^2D_3$
3726·16	$\mathrm{O_{II}}$	^4S -$^2D_2$

I. S. BOWEN.

Norman Bridge Laboratory of Physics,
California Institute of Technology,
Sept. 7.

Hvězdné atmosféry
Subrahmanyan Chandrasekhar 1910 - 1995

americký astrofyzik indického původu, ***Přenos záření r. 1960,*** shrnutí výsledků řešení rovnice zářivé rovnováhy

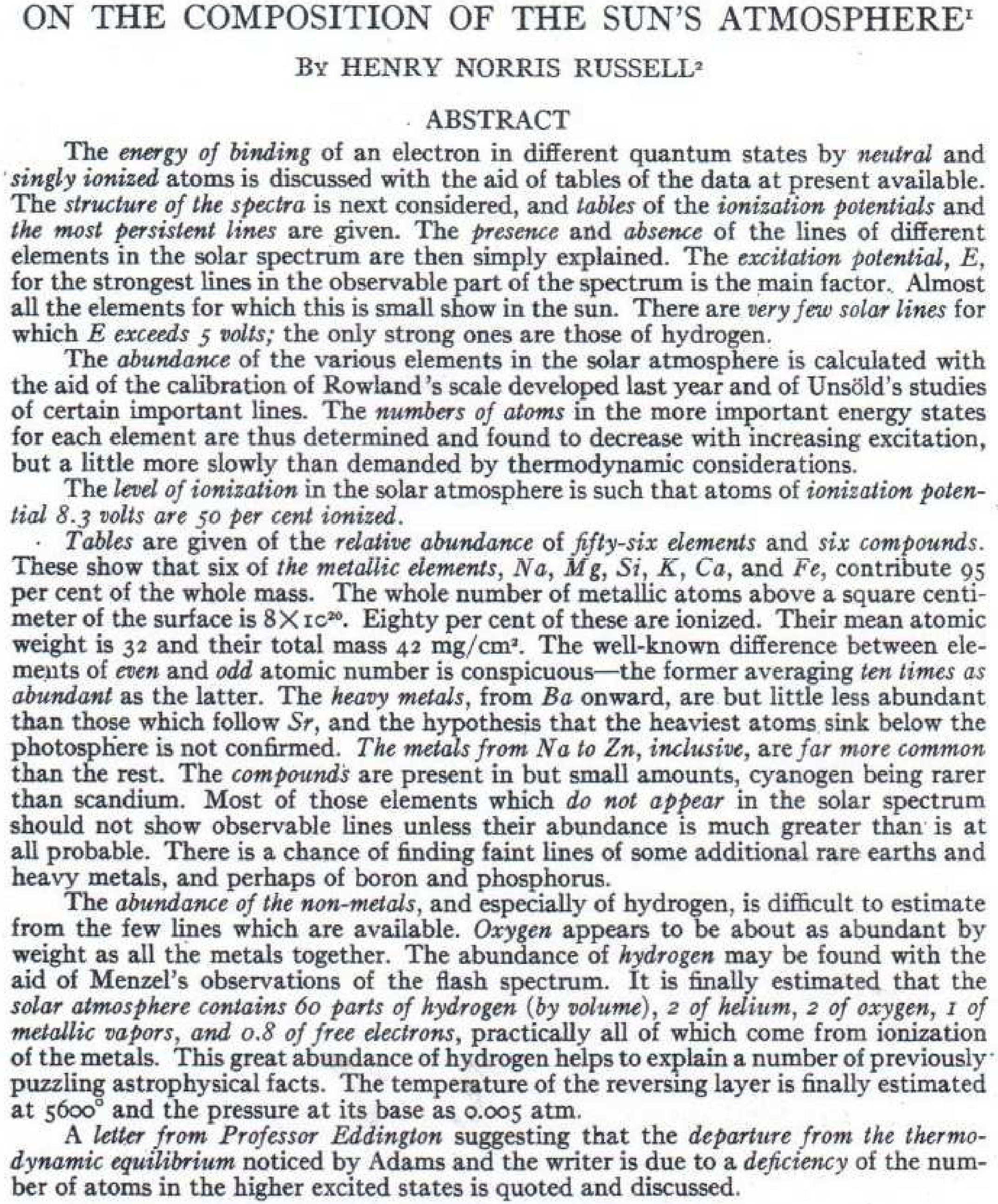


S. Chandrasekhar

Radiative Transfer

Hvězdné atmosféry

americký astrofyzik **Henry Norris Russell 1910 - 1995,** ***O složení sluneční atmosféry r. 1929***



ON THE COMPOSITION OF THE SUN'S ATMOSPHERE[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

The *energy of binding* of an electron in different quantum states by *neutral* and *singly ionized* atoms is discussed with the aid of tables of the data at present available. The *structure of the spectra* is next considered, and *tables* of the *ionization potentials* and *the most persistent lines* are given. The *presence* and *absence* of the lines of different elements in the solar spectrum are then simply explained. The *excitation potential, E,* for the strongest lines in the observable part of the spectrum is the main factor. Almost all the elements for which this is small show in the sun. There are *very few solar lines* for which *E exceeds 5 volts;* the only strong ones are those of hydrogen.

The *abundance* of the various elements in the solar atmosphere is calculated with the aid of the calibration of Rowland's scale developed last year and of Unsöld's studies of certain important lines. The *numbers of atoms* in the more important energy states for each element are thus determined and found to decrease with increasing excitation, but a little more slowly than demanded by thermodynamic considerations.

The *level of ionization* in the solar atmosphere is such that atoms of *ionization potential 8.3 volts are 50 per cent ionized.*

Tables are given of the *relative abundance* of *fifty-six elements* and *six compounds.* These show that six of *the metallic elements, Na, Mg, Si, K, Ca,* and *Fe,* contribute 95 per cent of the whole mass. The whole number of metallic atoms above a square centimeter of the surface is 8×10^{20}. Eighty per cent of these are ionized. Their mean atomic weight is 32 and their total mass 42 mg/cm^2. The well-known difference between elements of *even* and *odd* atomic number is conspicuous—the former averaging *ten times as abundant* as the latter. The *heavy metals,* from *Ba* onward, are but little less abundant than those which follow *Sr,* and the hypothesis that the heaviest atoms sink below the photosphere is not confirmed. *The metals from Na to Zn, inclusive,* are *far more common* than the rest. The *compounds* are present in but small amounts, cyanogen being rarer than scandium. Most of those elements which *do not appear* in the solar spectrum should not show observable lines unless their abundance is much greater than is at all probable. There is a chance of finding faint lines of some additional rare earths and heavy metals, and perhaps of boron and phosphorus.

The *abundance of the non-metals,* and especially of hydrogen, is difficult to estimate from the few lines which are available. *Oxygen* appears to be about as abundant by weight as all the metals together. The abundance of *hydrogen* may be found with the aid of Menzel's observations of the flash spectrum. It is finally estimated that the *solar atmosphere contains 60 parts of hydrogen (by volume), 2 of helium, 2 of oxygen, 1 of metallic vapors, and 0.8 of free electrons,* practically all of which come from ionization of the metals. This great abundance of hydrogen helps to explain a number of previously puzzling astrophysical facts. The temperature of the reversing layer is finally estimated at 5600° and the pressure at its base as 0.005 atm.

A *letter from Professor Eddington* suggesting that the *departure from the thermodynamic equilibrium* noticed by Adams and the writer is due to a *deficiency* of the number of atoms in the higher excited states is quoted and discussed.

Hvězdné atmosféry

O složení sluneční atmosféry r. 1929

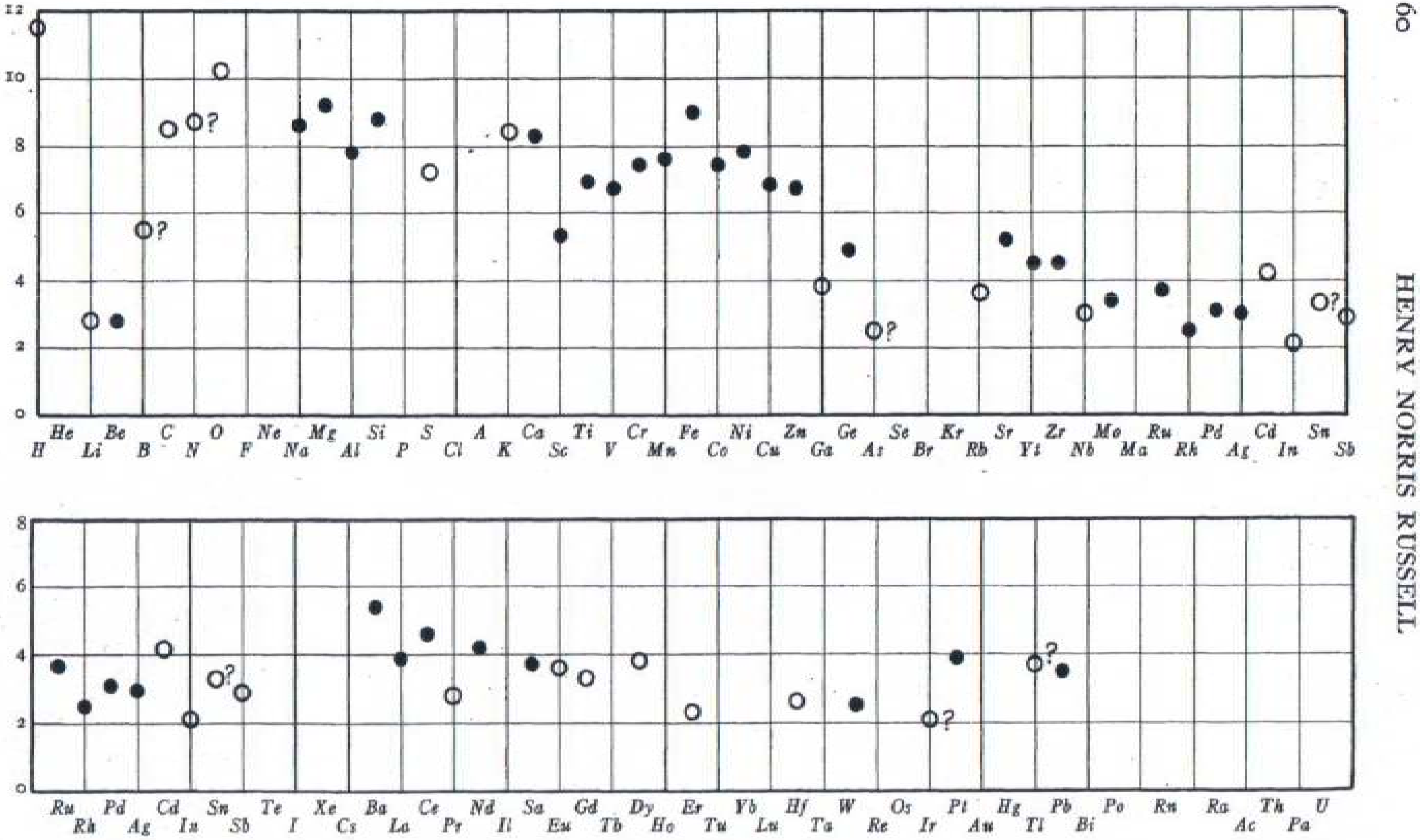


FIG. 3.—Values of log Q, where Q represents the total mass of the atoms or molecules of an element per unit area of the sun's surface.

Hvězdné atmosféry - Johann Jacob Balmer 1825 - 1898

švýcarský matematik v Basileji **r. 1885** si všiml, že čáry vodíkového spektra se zhušťují k hraně s vlnovou délkou blízkou 365 nm, napsal si vlnové délky všech čar a snažil se v nich uhodnout nějakou pravidelnost. Objevil vztah, který souhlasil s experimentálními údaji,

$$\lambda_m = 364.56 \frac{m^2}{m^2 - 4} \,[\mathrm{nm}], \qquad \text{kde } m = 3(\mathrm{H}_\alpha), 4(\mathrm{H}_\beta), 5(\mathrm{H}_\gamma), 6(\mathrm{H}_\delta)$$

Později se zavedlo $\frac{1}{\lambda_m} = R\left[\frac{1}{2^2} - \frac{1}{m^2}\right]$, kde $m > 2$ je přirozené číslo

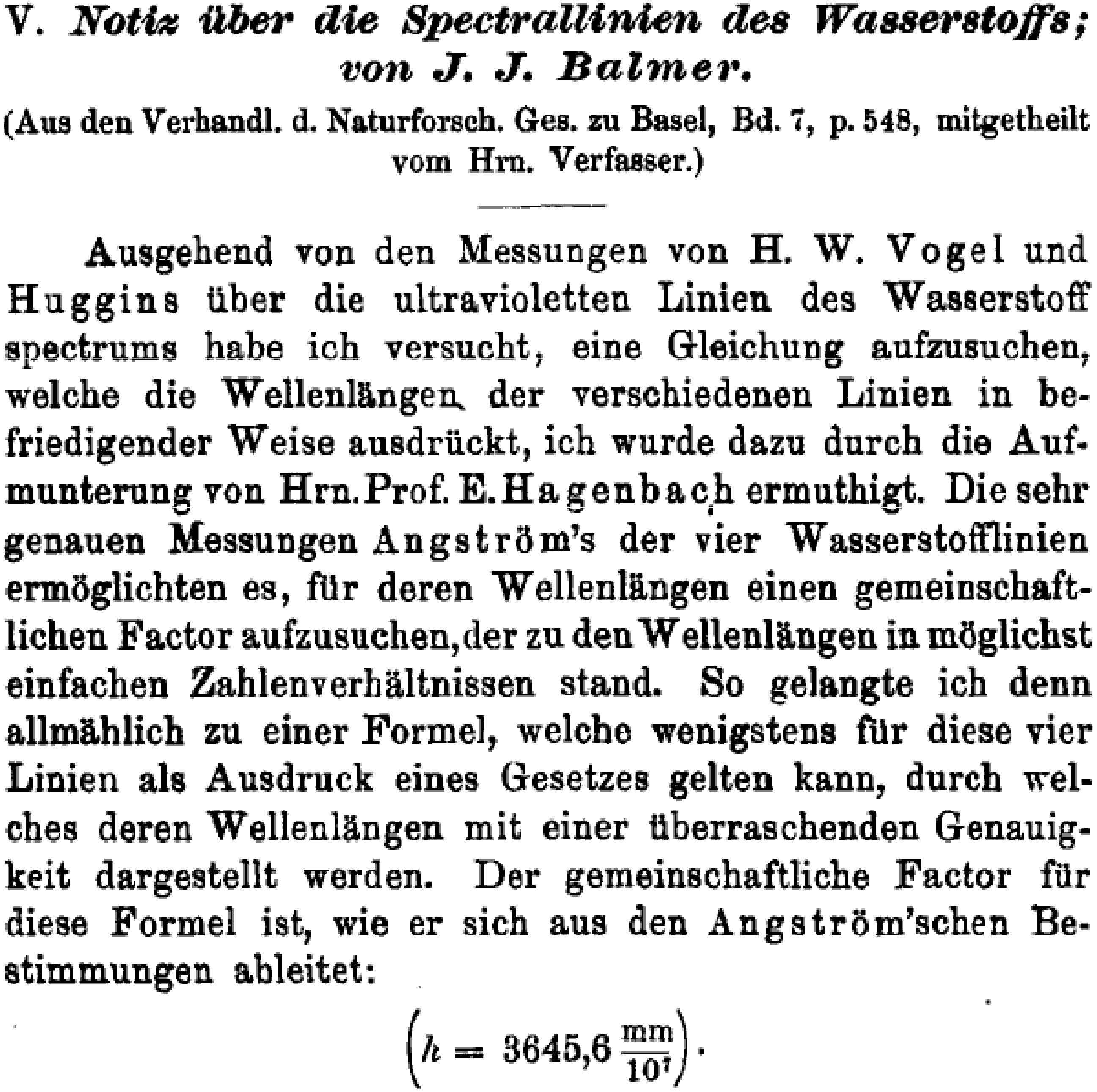
a $R = \frac{4}{364.56 \cdot 10^{-9}} = 1.097213 \cdot 10^7 \ \mathrm{m}^{-1}$ t.zv. *Rydbergova konstanta.*

zobecněný Balmerův vztah $\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$, kde n, $m > n$ jsou přirozená čísla

studia v Basileji, Karlsruhe, Berlín, doktorská disertace 1849 o cykloidách, soukromý docent deskriptivní geometrie na univerzitě v Basileji.

Hvězdné atmosféry - Johann Jacob Balmer

článek Poznámky o spektrálních čarách vodíku



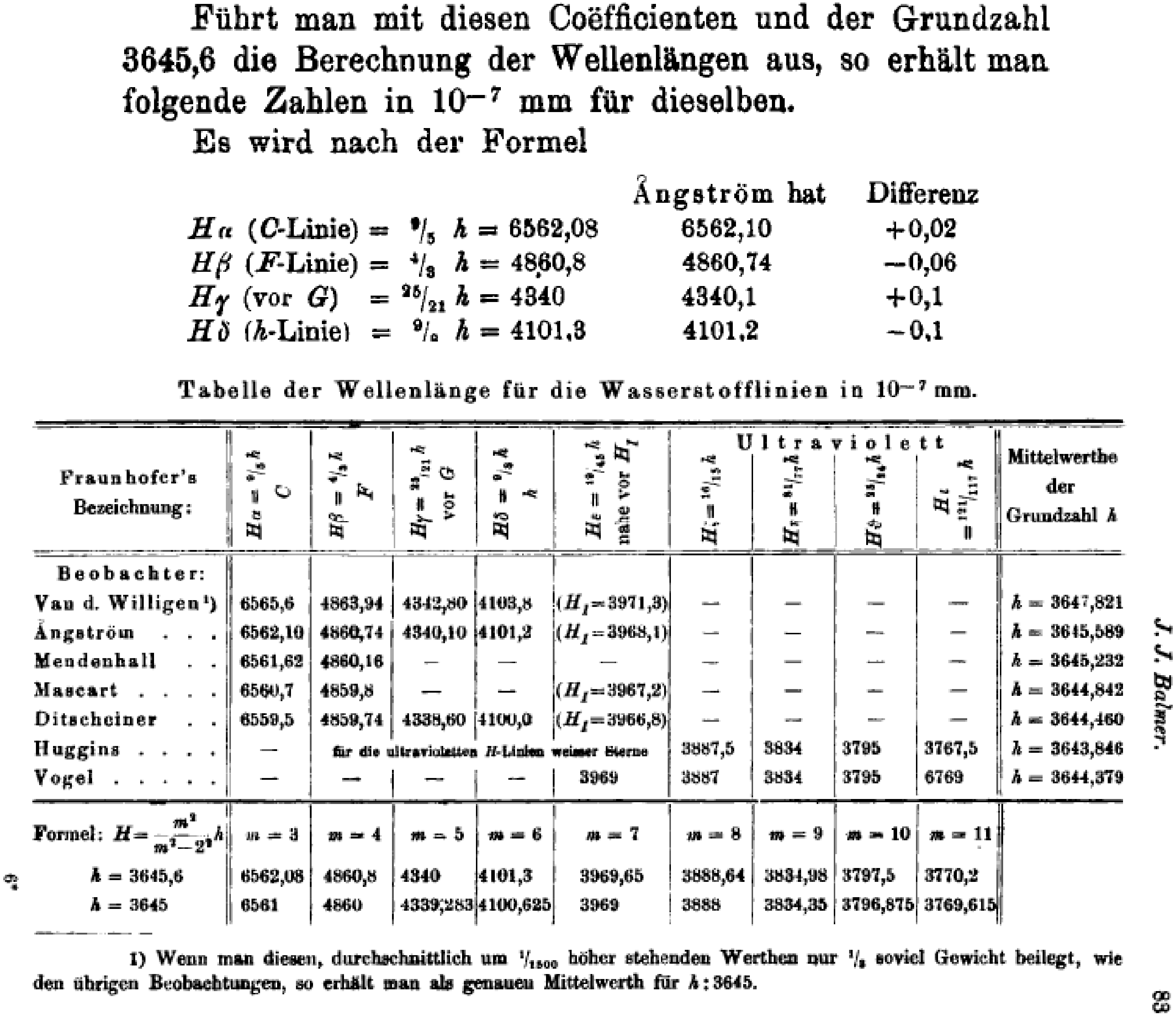
V. *Notiz über die Spectrallinien des Wasserstoffs;*
von J. J. Balmer.

(Aus den Verhandl. d. Naturforsch. Ges. zu Basel, Bd. 7, p. 548, mitgetheilt vom Hrn. Verfasser.)

Ausgehend von den Messungen von H. W. Vogel und Huggins über die ultravioletten Linien des Wasserstoffspectrums habe ich versucht, eine Gleichung aufzusuchen, welche die Wellenlängen der verschiedenen Linien in befriedigender Weise ausdrückt, ich wurde dazu durch die Aufmunterung von Hrn. Prof. E. Hagenbach ermuthigt. Die sehr genauen Messungen Angström's der vier Wasserstofflinien ermöglichten es, für deren Wellenlängen einen gemeinschaftlichen Factor aufzusuchen, der zu den Wellenlängen in möglichst einfachen Zahlenverhältnissen stand. So gelangte ich denn allmählich zu einer Formel, welche wenigstens für diese vier Linien als Ausdruck eines Gesetzes gelten kann, durch welches deren Wellenlängen mit einer überraschenden Genauigkeit dargestellt werden. Der gemeinschaftliche Factor für diese Formel ist, wie er sich aus den Angström'schen Bestimmungen ableitet:

$$\left(h = 3645{,}6\,\frac{\text{mm}}{10^7}\right).$$

Johann Jacob Balmer



Führt man mit diesen Coëfficienten und der Grundzahl 3645,6 die Berechnung der Wellenlängen aus, so erhält man folgende Zahlen in 10^{-7} mm für dieselben.

Es wird nach der Formel

		Ångström hat	Differenz
$H\alpha$ (*C*-Linie)	$= {}^9/_5\ h = 6562,08$	6562,10	+0,02
$H\beta$ (*F*-Linie)	$= {}^4/_3\ h = 4860,8$	4860,74	−0,06
$H\gamma$ (vor *G*)	$= {}^{25}/_{21}\ h = 4340$	4340,1	+0,1
$H\delta$ (*h*-Linie)	$= {}^9/_8\ h = 4101,3$	4101,2	−0,1

Tabelle der Wellenlänge für die Wasserstofflinien in 10^{-7} mm.

Fraunhofer's Bezeichnung:	$H\alpha = {}^9/_5 h$ C	$H\beta = {}^4/_3 h$ F	$H\gamma = {}^{25}/_{21} h$ vor G	$H\delta = {}^9/_8 h$ h	$H\varepsilon = {}^{49}/_{45} h$ nahe vor H_I	Ultraviolett $H\zeta = {}^{16}/_{15} h$	$H\eta = {}^{81}/_{77} h$	$H\vartheta = {}^{25}/_{24} h$	$H\iota = {}^{121}/_{117} h$	Mittelwerthe der Grundzahl h
Beobachter:										
Van d. Willigen[1]	6565,6	4863,94	4342,80	4103,8	$(H_I = 3971,3)$	—	—	—	—	$h = 3647,821$
Ångström . . .	6562,10	4860,74	4340,10	4101,2	$(H_I = 3968,1)$	—	—	—	—	$h = 3645,589$
Mendenhall . .	6561,62	4860,16	—	—	—	—	—	—	—	$h = 3645,232$
Mascart	6560,7	4859,8	—	—	$(H_I = 3967,2)$	—	—	—	—	$h = 3644,842$
Ditscheiner . .	6559,5	4859,74	4338,60	4100,0	$(H_I = 3966,8)$	—	—	—	—	$h = 3644,460$
Huggins	—	für die ultravioletten *H*-Linien weisser Sterne				3887,5	3834	3795	3767,5	$h = 3643,846$
Vogel	—	—	—	—	3969	3887	3834	3795	6769	$h = 3644,379$
Formel: $H = \frac{m^2}{m^2 - 2^2} h$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$m = 9$	$m = 10$	$m = 11$	
$h = 3645,6$	6562,08	4860,8	4340	4101,3	3969,65	3888,64	3834,98	3797,5	3770,2	
$h = 3645$	6561	4860	4339,283	4100,625	3969	3888	3834,35	3796,875	3769,615	

---

[1] Wenn man diesen, durchschnittlich um $^1/_{1500}$ höher stehenden Werthen nur $^1/_2$ soviel Gewicht beilegt, wie den übrigen Beobachtungen, so erhält man als genauen Mittelwerth für h: 3645.

Hvězdné atmosféry

Spektrální série čar atom vodíku

pro m = 1 - série Lymanova (ultrafialová část spektra), objevena 1904
Theodore Lyman 1874-1954

pro m = 2 - série Balmerova (viditelná část spektra), objevena 1885
Johann Jacob Balmer 1825-1898

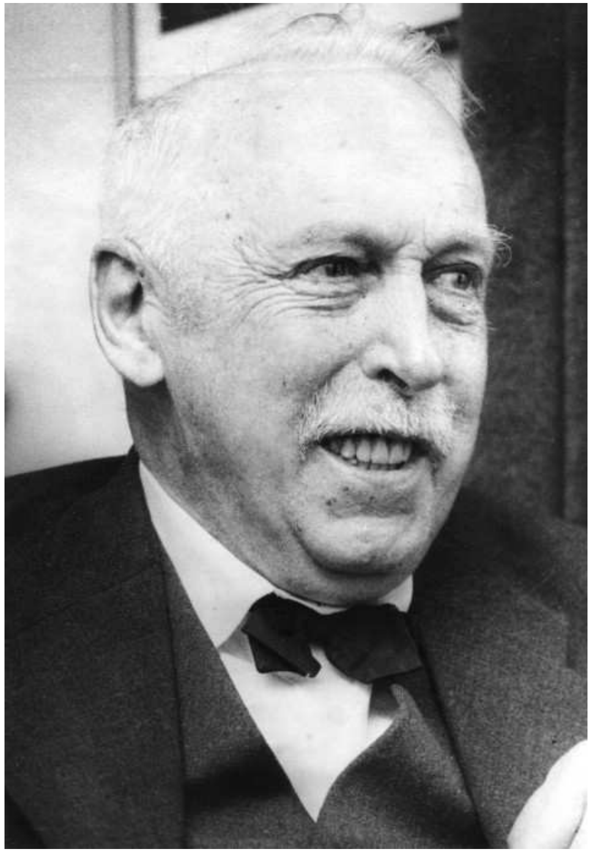
pro m = 3 - série Paschenova (infračervená část spektra), objevena 1908
Friedrich Paschen 1865-1947

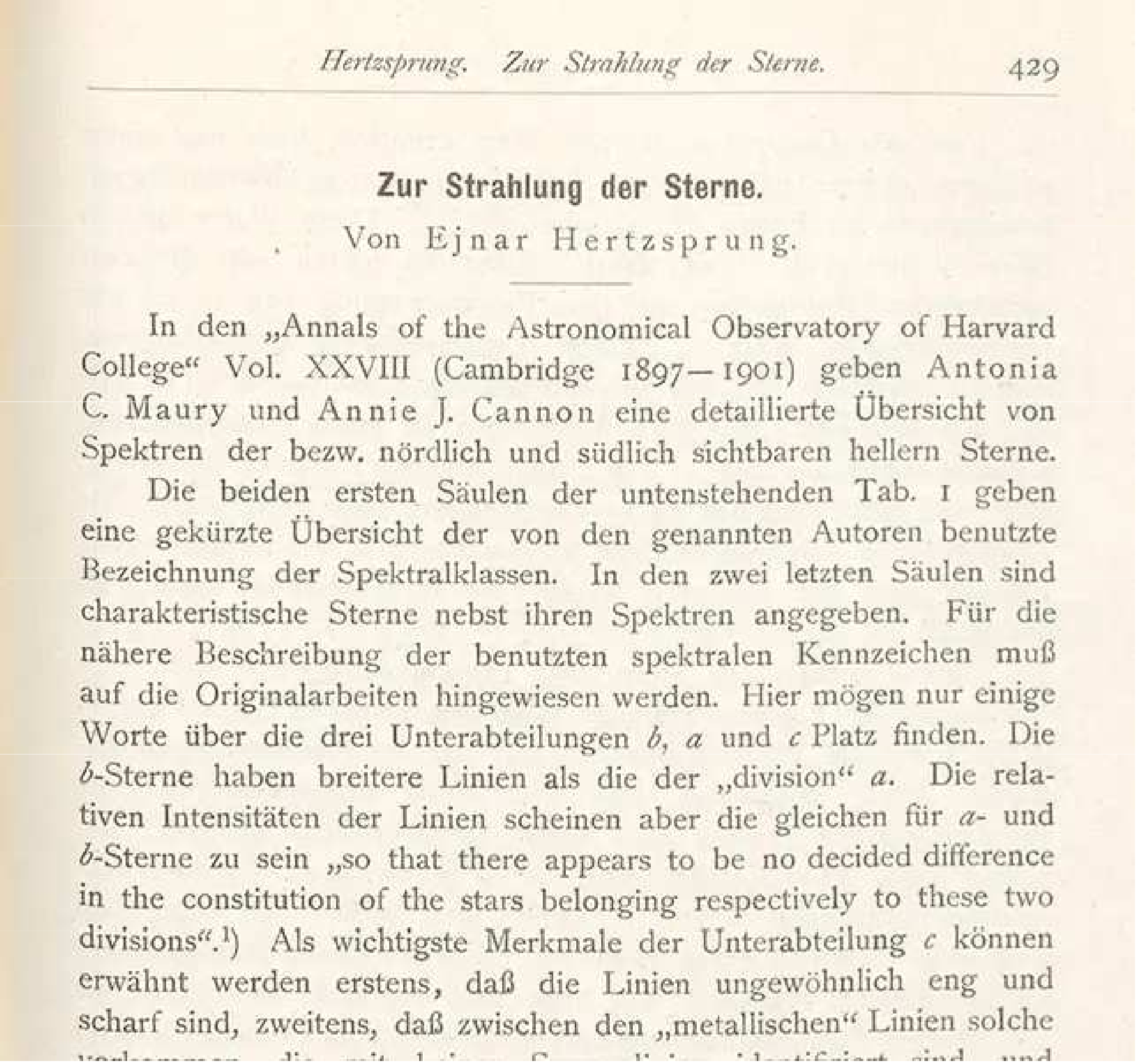
pro m = 4 - série Brackettova (infračervená část spektra), objevena 1922
Frederick Summer Brackett 1896-1988

pro m = 5 - série Pfundova (infračervená část spektra), objevena 1924
August Herman Pfund 1879-1949

H – R diagram

dánský astronom **Ejnar Hertzsprung 1873-1967**, ***O záření hvězd, r. 1905,*** v tabulkové podobě provedl rozdělení hvězd na posloupnost trpaslíků a obrů, ***Zur Stralung Der Sterne. Zeitschrift Für Wissenschaftliche Photographie, 3 (1905)***





Hertzsprung. Zur Strahlung der Sterne. 429

Zur Strahlung der Sterne.

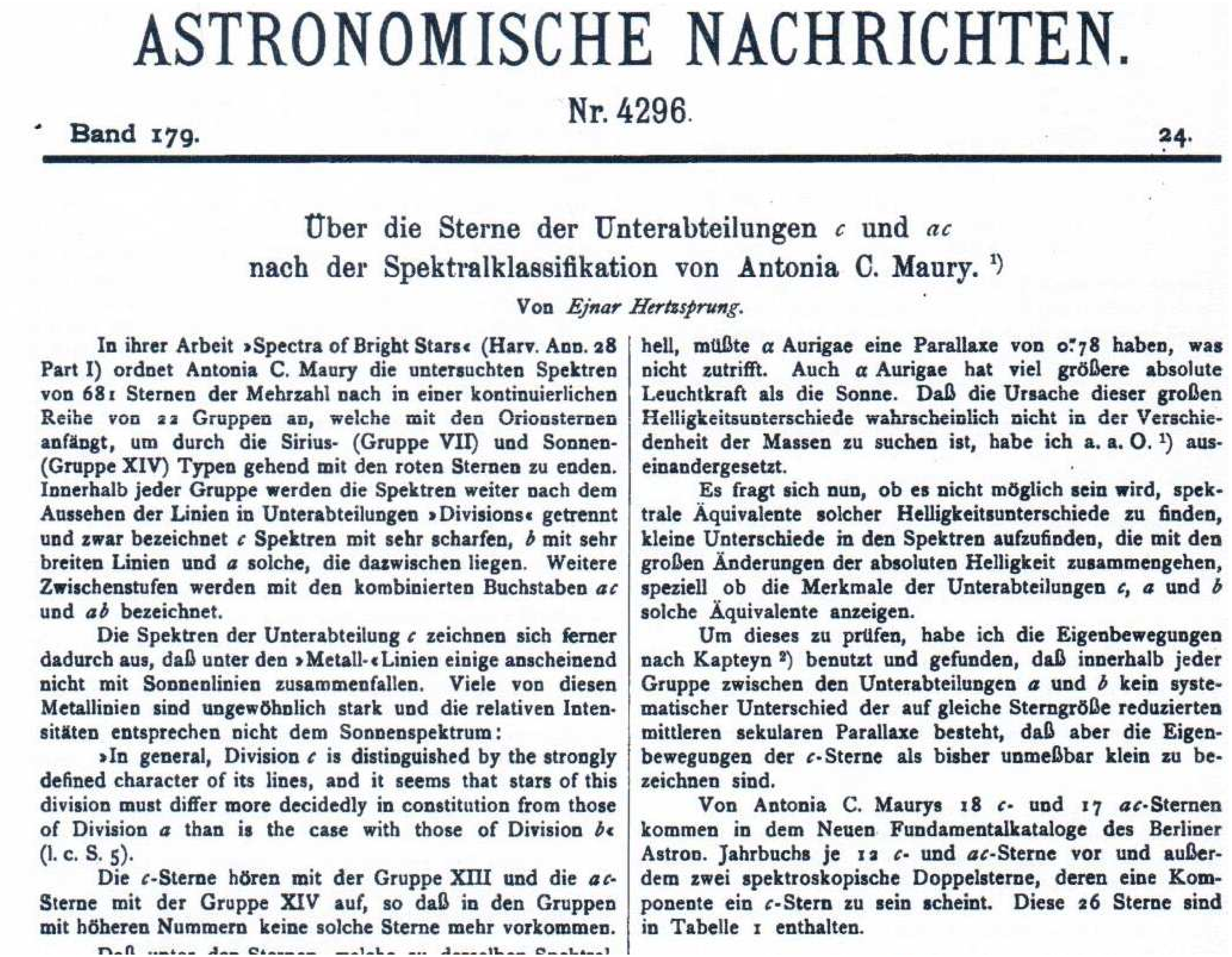
Von Ejnar Hertzsprung.

In den „Annals of the Astronomical Observatory of Harvard College“ Vol. XXVIII (Cambridge 1897—1901) geben Antonia C. Maury und Annie J. Cannon eine detaillierte Übersicht von Spektren der bezw. nördlich und südlich sichtbaren hellern Sterne.

Die beiden ersten Säulen der untenstehenden Tab. 1 geben eine gekürzte Übersicht der von den genannten Autoren benutzte Bezeichnung der Spektralklassen. In den zwei letzten Säulen sind charakteristische Sterne nebst ihren Spektren angegeben. Für die nähere Beschreibung der benutzten spektralen Kennzeichen muß auf die Originalarbeiten hingewiesen werden. Hier mögen nur einige Worte über die drei Unterabteilungen *b*, *a* und *c* Platz finden. Die *b*-Sterne haben breitere Linien als die der „division“ *a*. Die relativen Intensitäten der Linien scheinen aber die gleichen für *a*- und *b*-Sterne zu sein „so that there appears to be no decided difference in the constitution of the stars belonging respectively to these two divisions“.¹) Als wichtigste Merkmale der Unterabteilung *c* können erwähnt werden erstens, daß die Linien ungewöhnlich eng und scharf sind, zweitens, daß zwischen den „metallischen“ Linien solche

H – R diagram

dánský astronom **Ejnar Hertzsprung,** ***O hvězdách skupiny c a ac spektrální klasifikace A. C. Maury (1866-1952), r. 1909***



ASTRONOMISCHE NACHRICHTEN.

Nr. 4296.

Band 179. 24.

Über die Sterne der Unterabteilungen *c* und *ac* nach der Spektralklassifikation von Antonia C. Maury. 1)

Von *Ejnar Hertzsprung.*

In ihrer Arbeit »Spectra of Bright Stars« (Harv. Ann. 28 Part I) ordnet Antonia C. Maury die untersuchten Spektren von 681 Sternen der Mehrzahl nach in einer kontinuierlichen Reihe von 22 Gruppen an, welche mit den Orionsternen anfängt, um durch die Sirius- (Gruppe VII) und Sonnen- (Gruppe XIV) Typen gehend mit den roten Sternen zu enden. Innerhalb jeder Gruppe werden die Spektren weiter nach dem Aussehen der Linien in Unterabteilungen »Divisions« getrennt und zwar bezeichnet *c* Spektren mit sehr scharfen, *b* mit sehr breiten Linien und *a* solche, die dazwischen liegen. Weitere Zwischenstufen werden mit den kombinierten Buchstaben *ac* und *ab* bezeichnet.

Die Spektren der Unterabteilung *c* zeichnen sich ferner dadurch aus, daß unter den »Metall-«Linien einige anscheinend nicht mit Sonnenlinien zusammenfallen. Viele von diesen Metallinien sind ungewöhnlich stark und die relativen Intensitäten entsprechen nicht dem Sonnenspektrum:

»In general, Division *c* is distinguished by the strongly defined character of its lines, and it seems that stars of this division must differ more decidedly in constitution from those of Division *a* than is the case with those of Division *b*« (l. c. S. 5).

Die *c*-Sterne hören mit der Gruppe XIII und die *ac*-Sterne mit der Gruppe XIV auf, so daß in den Gruppen mit höheren Nummern keine solche Sterne mehr vorkommen.

Daß unter den Sternen, welche zu derselben Spektral-

hell, müßte *α* Aurigae eine Parallaxe von 0″78 haben, was nicht zutrifft. Auch *α* Aurigae hat viel größere absolute Leuchtkraft als die Sonne. Daß die Ursache dieser großen Helligkeitsunterschiede wahrscheinlich nicht in der Verschiedenheit der Massen zu suchen ist, habe ich a. a. O. 1) auseinandergesetzt.

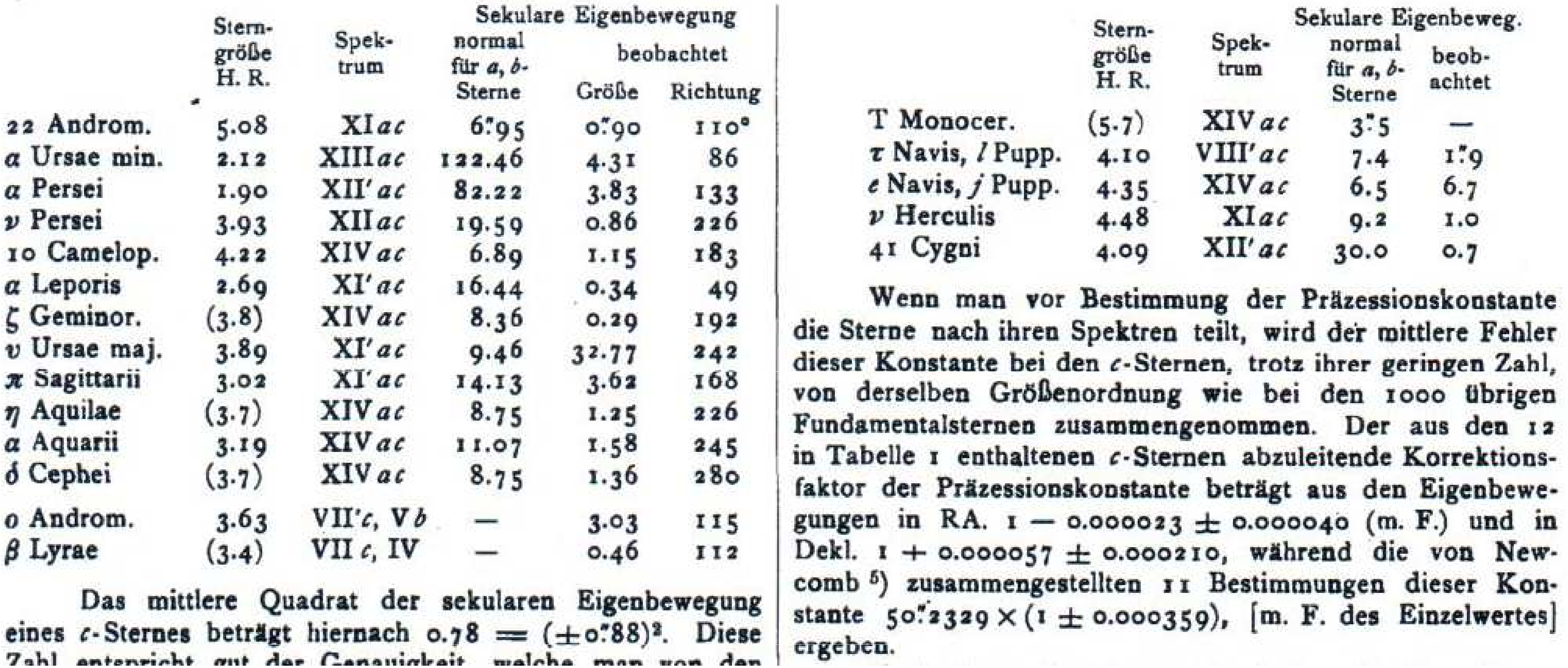
Es fragt sich nun, ob es nicht möglich sein wird, spektrale Äquivalente solcher Helligkeitsunterschiede zu finden, kleine Unterschiede in den Spektren aufzufinden, die mit den großen Änderungen der absoluten Helligkeit zusammengehen, speziell ob die Merkmale der Unterabteilungen *c*, *a* und *b* solche Äquivalente anzeigen.

Um dieses zu prüfen, habe ich die Eigenbewegungen nach Kapteyn 2) benutzt und gefunden, daß innerhalb jeder Gruppe zwischen den Unterabteilungen *a* und *b* kein systematischer Unterschied der auf gleiche Sterngröße reduzierten mittleren sekularen Parallaxe besteht, daß aber die Eigenbewegungen der *c*-Sterne als bisher unmeßbar klein zu bezeichnen sind.

Von Antonia C. Maurys 18 *c*- und 17 *ac*-Sternen kommen in dem Neuen Fundamentalkataloge des Berliner Astron. Jahrbuchs je 12 *c*- und *ac*-Sterne vor und außerdem zwei spektroskopische Doppelsterne, deren eine Komponente ein *c*-Stern zu sein scheint. Diese 26 Sterne sind in Tabelle 1 enthalten.

H – R diagram

O hvězdách skupiny c a ac spektrální klasifikace A.C.Maury, r. 1909



	Stern-größe H. R.	Spek-trum	Sekulare Eigenbewegung normal für *a*, *b*-Sterne	beobachtet Größe	beobachtet Richtung
22 Androm.	5.08	XI *ac*	6″.95	0″.90	110°
α Ursae min.	2.12	XIII *ac*	122.46	4.31	86
α Persei	1.90	XII′ *ac*	82.22	3.83	133
ν Persei	3.93	XII *ac*	19.59	0.86	226
10 Camelop.	4.22	XIV *ac*	6.89	1.15	183
α Leporis	2.69	XI′ *ac*	16.44	0.34	49
ζ Geminor.	(3.8)	XIV *ac*	8.36	0.29	192
υ Ursae maj.	3.89	XI′ *ac*	9.46	32.77	242
π Sagittarii	3.02	XI′ *ac*	14.13	3.62	168
η Aquilae	(3.7)	XIV *ac*	8.75	1.25	226
α Aquarii	3.19	XIV *ac*	11.07	1.58	245
δ Cephei	(3.7)	XIV *ac*	8.75	1.36	280
ο Androm.	3.63	VII′ *c*, V *b*	—	3.03	115
β Lyrae	(3.4)	VII *c*, IV	—	0.46	112

Das mittlere Quadrat der sekularen Eigenbewegung eines *c*-Sternes beträgt hiernach $0.78 = (\pm 0''.88)^2$. Diese Zahl entspricht gut der Genauigkeit, welche man von den

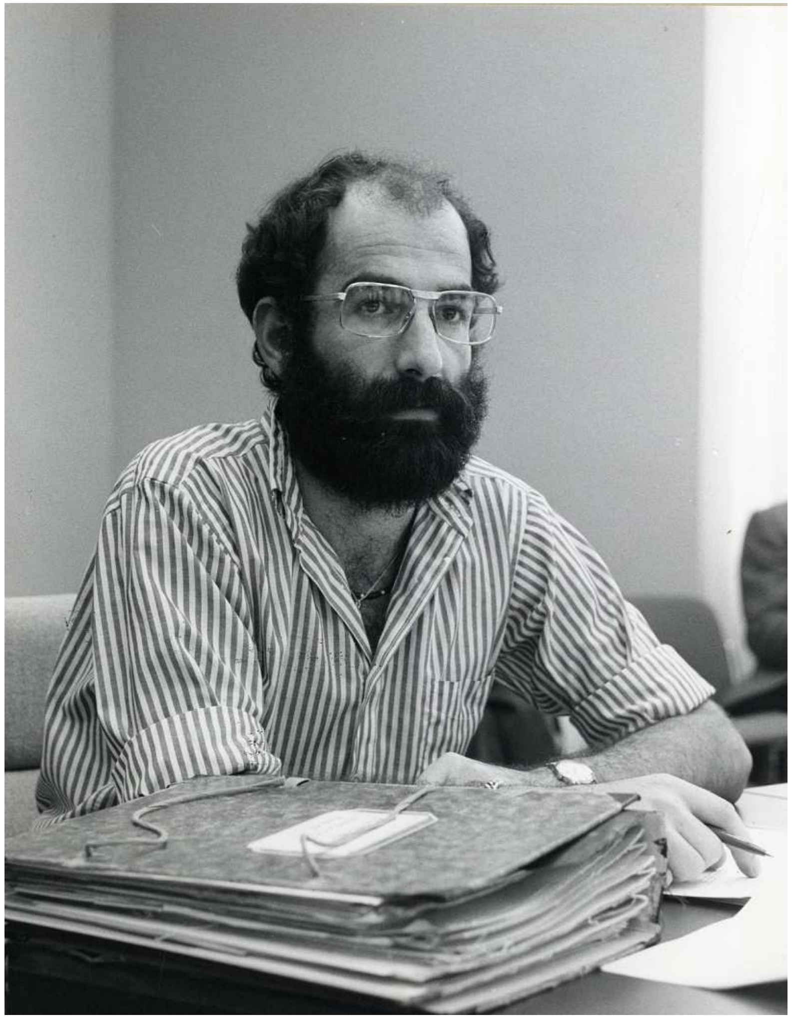
	Stern-größe H. R.	Spek-trum	Sekulare Eigenbeweg. normal für *a*, *b*-Sterne	beob-achtet
T Monocer.	(5.7)	XIV *ac*	3″.5	—
τ Navis, *l* Pupp.	4.10	VIII′ *ac*	7.4	1″.9
ε Navis, *j* Pupp.	4.35	XIV *ac*	6.5	6.7
ν Herculis	4.48	XI *ac*	9.2	1.0
41 Cygni	4.09	XII′ *ac*	30.0	0.7

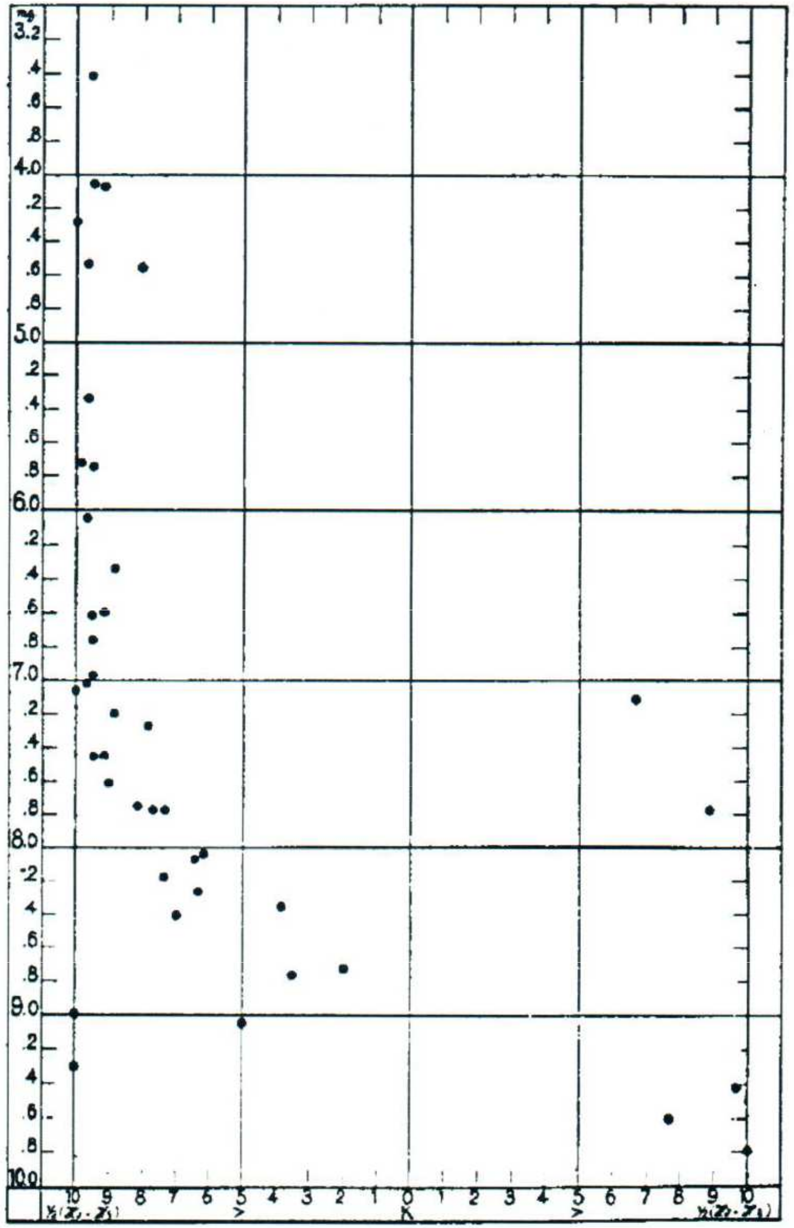
Wenn man vor Bestimmung der Präzessionskonstante die Sterne nach ihren Spektren teilt, wird der mittlere Fehler dieser Konstante bei den *c*-Sternen, trotz ihrer geringen Zahl, von derselben Größenordnung wie bei den 1000 übrigen Fundamentalsternen zusammengenommen. Der aus den 12 in Tabelle 1 enthaltenen *c*-Sternen abzuleitende Korrektionsfaktor der Präzessionskonstante beträgt aus den Eigenbewegungen in RA. $1 - 0.000023 \pm 0.000040$ (m. F.) und in Dekl. $1 + 0.000057 \pm 0.000210$, während die von Newcomb [5]) zusammengestellten 11 Bestimmungen dieser Konstante $50''.2329 \times (1 \pm 0.000359)$, [m. F. des Einzelwertes] ergeben.

H – R diagram

německý astronom **Hans Oswald Rosenberg 1879 – 1940**

Vztah mezi jasností a spektrálním typem v Plejádách, r. 1910





Obrázek 1.4: Rosenbergův diagram z [18]. Na vodorovné ose je spektrální třída a na vertikální pozorovaná hvězdná velikost.

H – R diagram

Nad užitím fotografické efektivní vlnové délky k určení odpovídající barvy, r. 1911

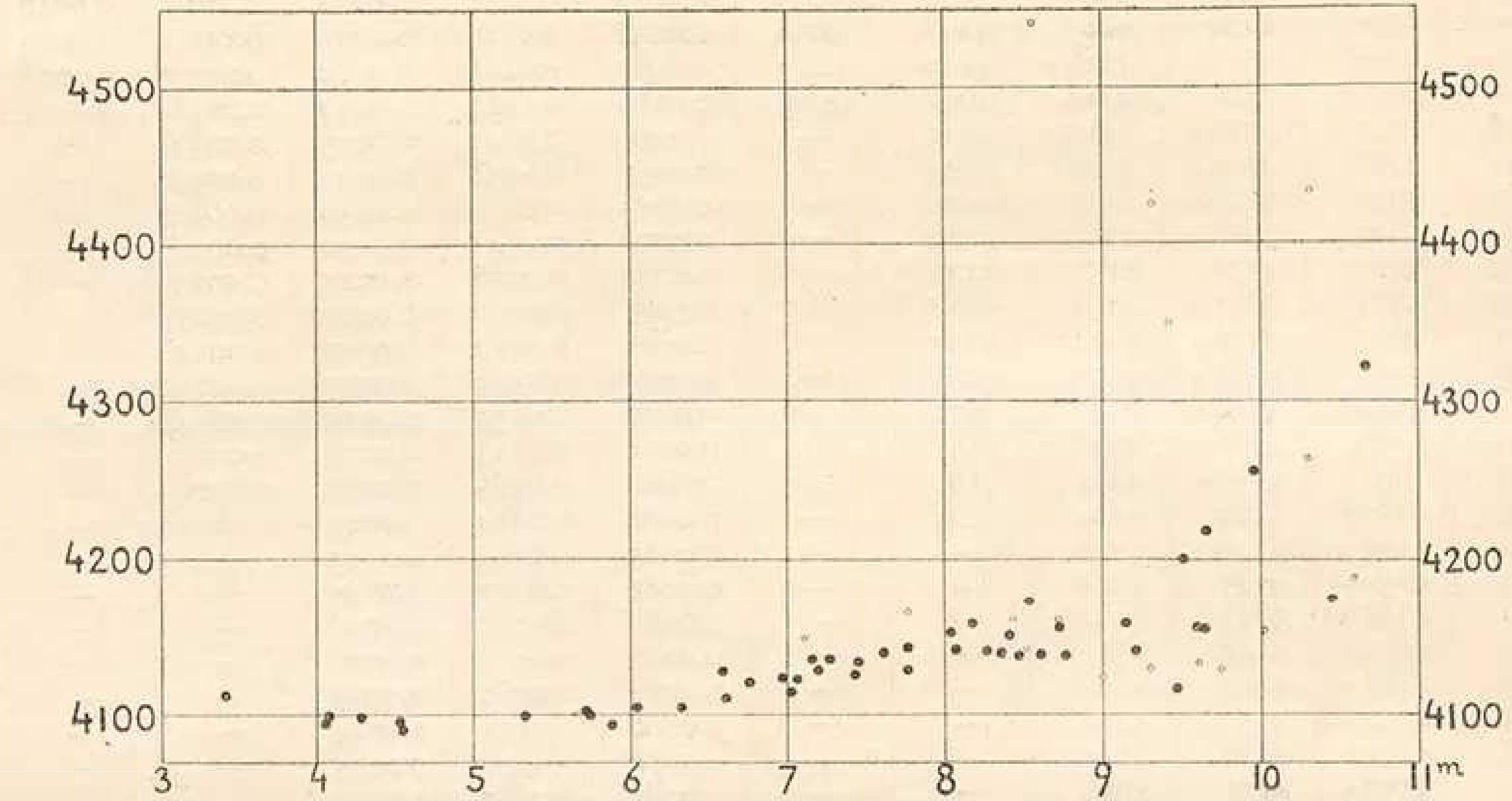


Fig. 4. Plejaden.

Abszisse: Photographische Sterngröße m_t.

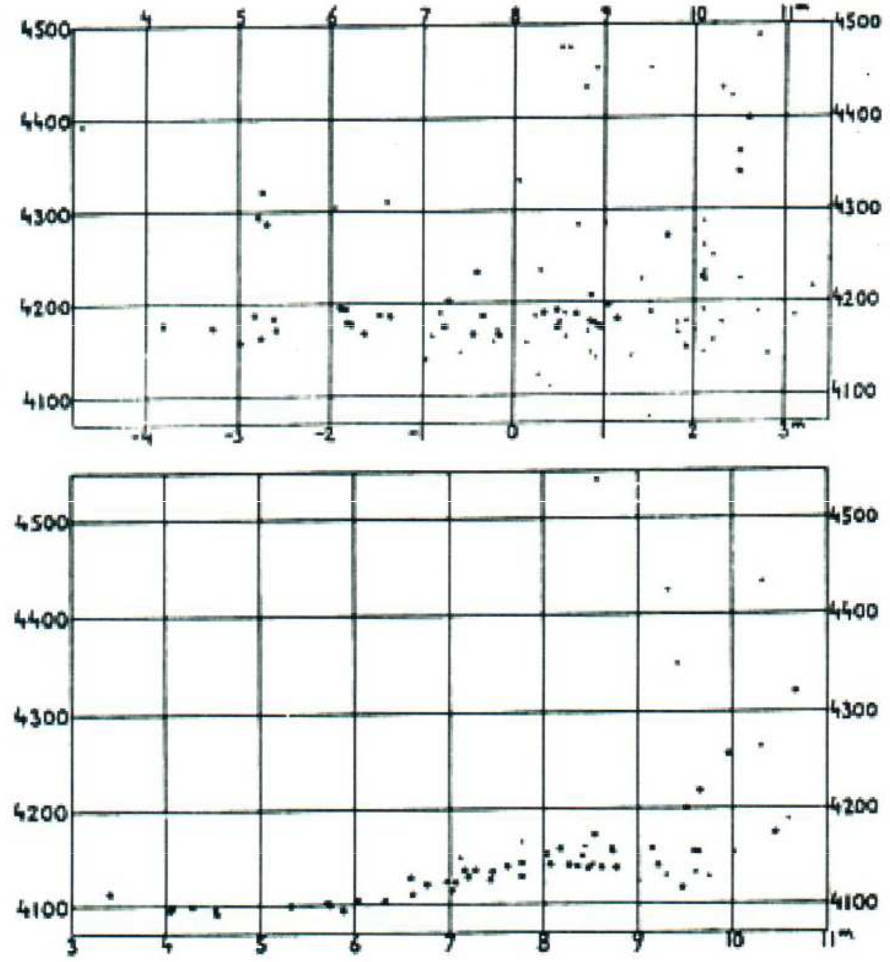
Ordinate: Farbenäquivalent λ_c der effektiven Wellenlängen in Å auf die Bildstärke $\varphi(d) = 8^{m}_{\cdot}2$ reduziert.

• physische Mitglieder der Plejaden.

∘ andere Sterne derselben Gegend.

H – R diagram

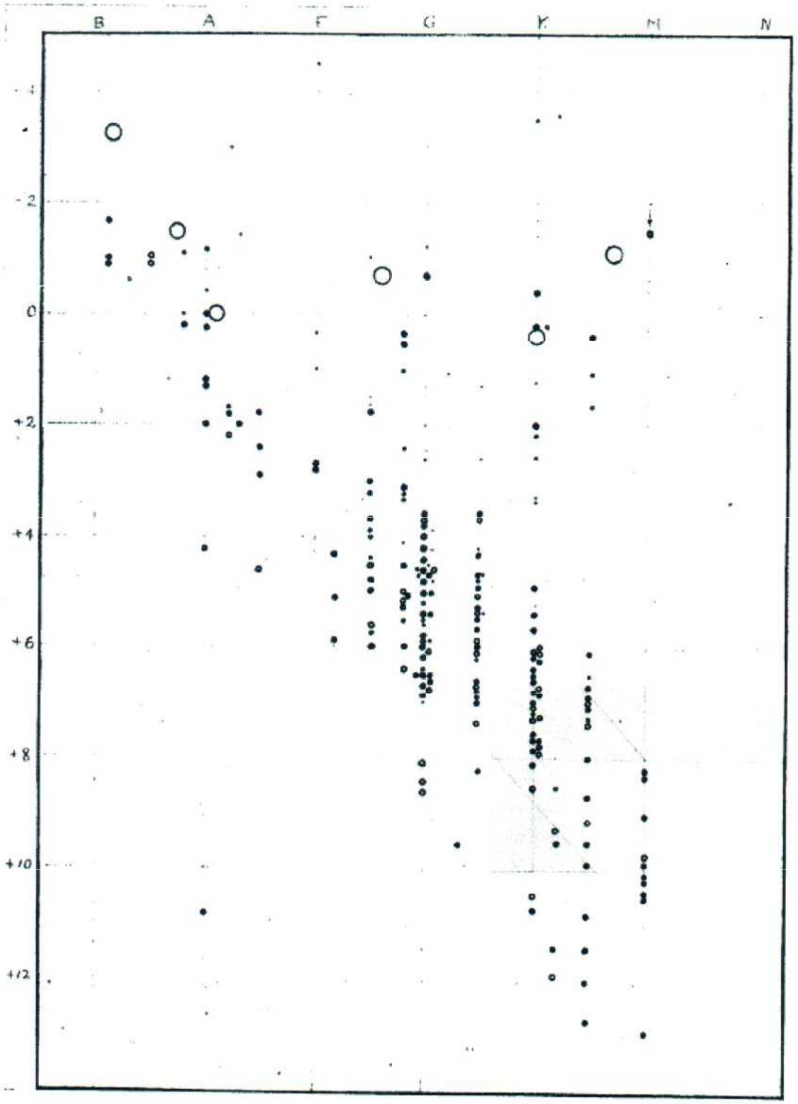
dánský astronom **Ejnar Hertzsprung,** ***Nad užitím fotografické efektivní vlnové délky k určení odpovídající barvy, r. 1911***



Obrázek 1.1: Hertzsprungův diagram z [8]. Na svislých osách je efektivní vlnová délka v Å. Osa začínající číslem -4 znázorňuje absolutní hvězdnou velikost. Zbylé osy x začínající 4 (horní obrázek horní osa), příp. 3 (spodní obrázek dolní osa) znázorňují pozorovanou hvězdnou velikost.

H – R diagram

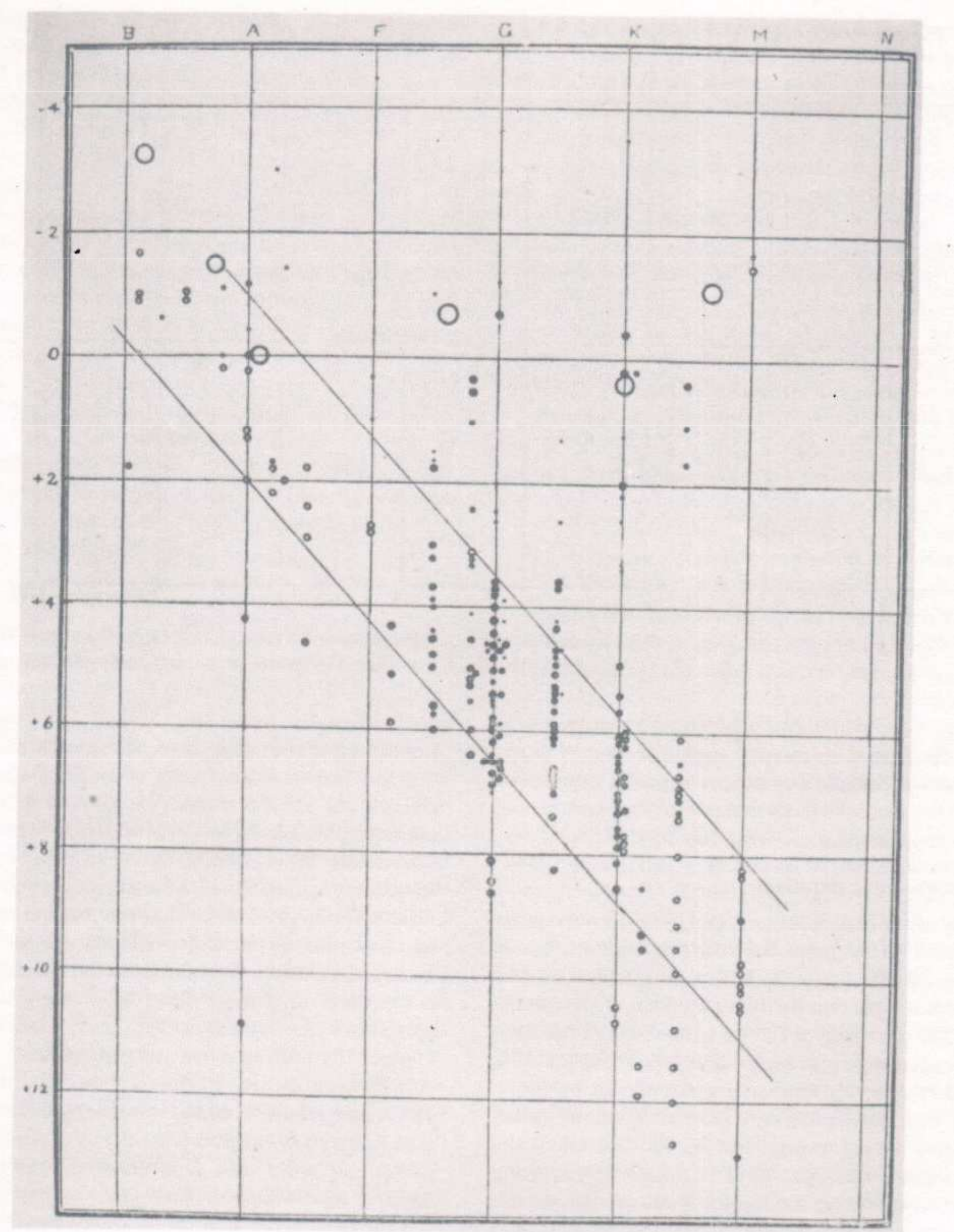
americký astrofyzik **Henry Norris Russell 1910 - 1995,** ***Vztah mezi spektrem a dalšími charakteristikami hvězd, r. 1914***



Obrázek 1.2: Russellův HR diagram pro hvězdy z [15]. Na vodorovné ose jsou spektrální třídy, na vertikální absolutní hvězdná velikost.

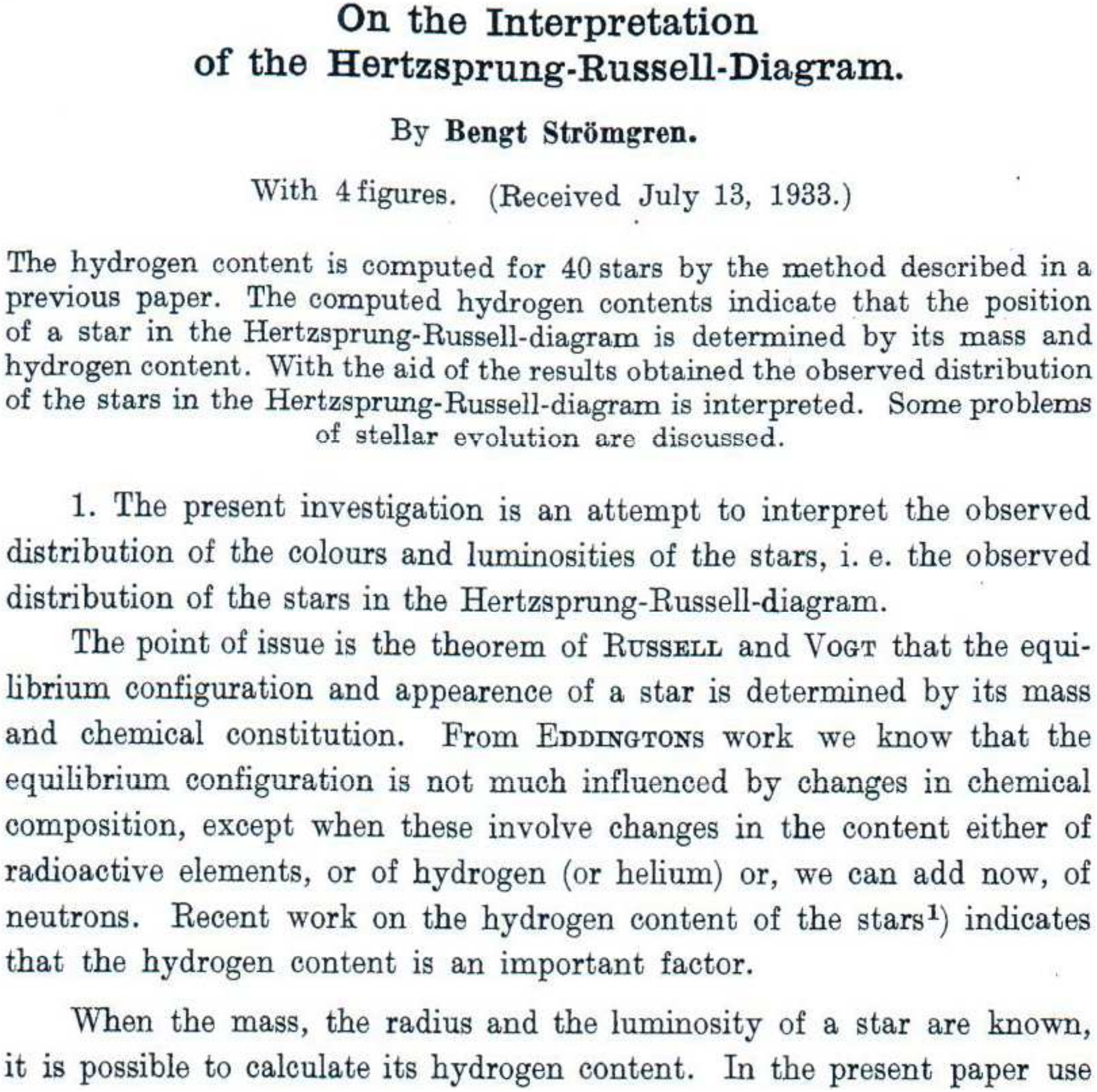
H – R diagram

Vztah mezi spektrem a dalšími charakteristikami hvězd, r. 1914



H - R diagram

dánský astrofyzik **Bengt Georg Daniel Strömgren 1908-1987,** ***O interpretaci H-R diagramu r. 1933,*** odchod hvězd z hlavní posloupnosti, úhel sklonu závisí na obsahu vodíku



On the Interpretation of the Hertzsprung-Russell-Diagram.

By **Bengt Strömgren.**

With 4 figures. (Received July 13, 1933.)

The hydrogen content is computed for 40 stars by the method described in a previous paper. The computed hydrogen contents indicate that the position of a star in the Hertzsprung-Russell-diagram is determined by its mass and hydrogen content. With the aid of the results obtained the observed distribution of the stars in the Hertzsprung-Russell-diagram is interpreted. Some problems of stellar evolution are discussed.

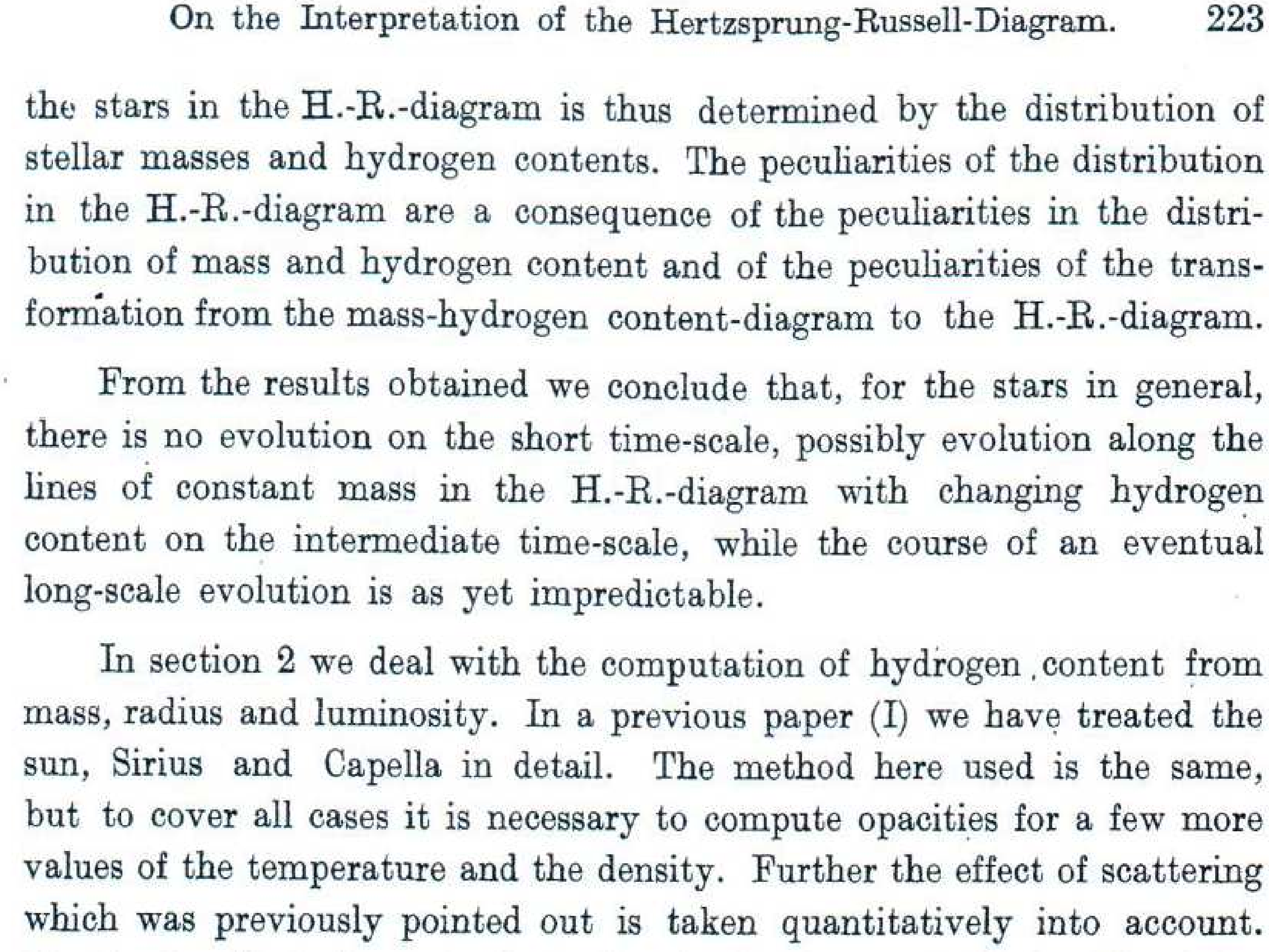
1. The present investigation is an attempt to interpret the observed distribution of the colours and luminosities of the stars, i. e. the observed distribution of the stars in the Hertzsprung-Russell-diagram.

The point of issue is the theorem of Russell and Vogt that the equilibrium configuration and appearence of a star is determined by its mass and chemical constitution. From Eddingtons work we know that the equilibrium configuration is not much influenced by changes in chemical composition, except when these involve changes in the content either of radioactive elements, or of hydrogen (or helium) or, we can add now, of neutrons. Recent work on the hydrogen content of the stars[1]) indicates that the hydrogen content is an important factor.

When the mass, the radius and the luminosity of a star are known, it is possible to calculate its hydrogen content. In the present paper use

H - R diagram

O interpretaci H-R diagramu r. 1933, odchod hvězd z hlavní posloupnosti, úhel sklonu závisí na obsahu vodíku



On the Interpretation of the Hertzsprung-Russell-Diagram. 223

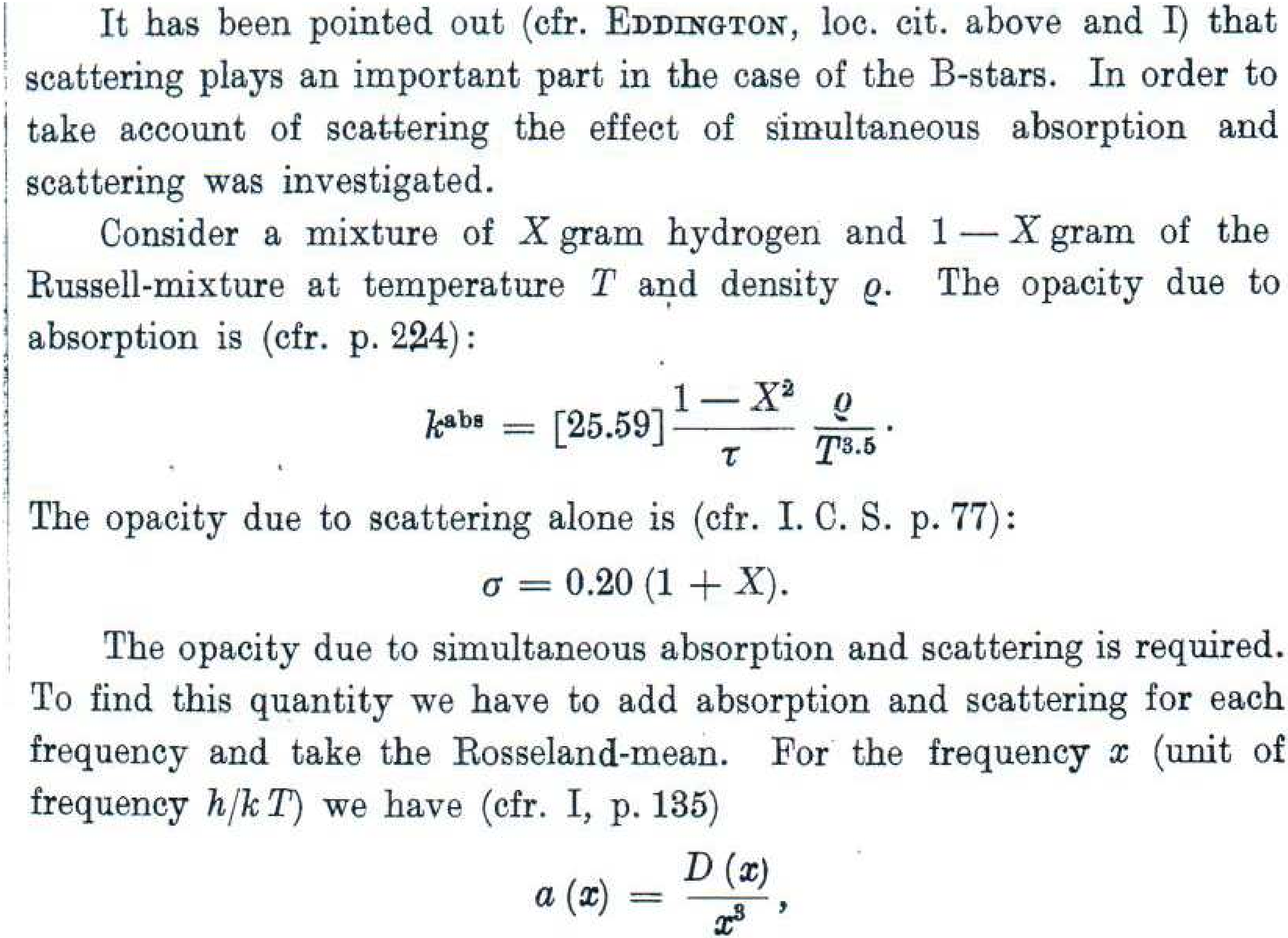
the stars in the H.-R.-diagram is thus determined by the distribution of stellar masses and hydrogen contents. The peculiarities of the distribution in the H.-R.-diagram are a consequence of the peculiarities in the distribution of mass and hydrogen content and of the peculiarities of the transformation from the mass-hydrogen content-diagram to the H.-R.-diagram.

From the results obtained we conclude that, for the stars in general, there is no evolution on the short time-scale, possibly evolution along the lines of constant mass in the H.-R.-diagram with changing hydrogen content on the intermediate time-scale, while the course of an eventual long-scale evolution is as yet impredictable.

In section 2 we deal with the computation of hydrogen content from mass, radius and luminosity. In a previous paper (I) we have treated the sun, Sirius and Capella in detail. The method here used is the same, but to cover all cases it is necessary to compute opacities for a few more values of the temperature and the density. Further the effect of scattering which was previously pointed out is taken quantitatively into account.

H - R diagram

O interpretaci H-R diagramu r. 1933, odchod hvězd z hlavní posloupnosti, úhel sklonu závisí na obsahu vodíku



It has been pointed out (cfr. Eddington, loc. cit. above and I) that scattering plays an important part in the case of the B-stars. In order to take account of scattering the effect of simultaneous absorption and scattering was investigated.

Consider a mixture of X gram hydrogen and $1-X$ gram of the Russell-mixture at temperature T and density ϱ. The opacity due to absorption is (cfr. p. 224):

$$k^{\text{abs}} = [25.59]\frac{1-X^2}{\tau}\frac{\varrho}{T^{3.5}}.$$

The opacity due to scattering alone is (cfr. I. C. S. p. 77):

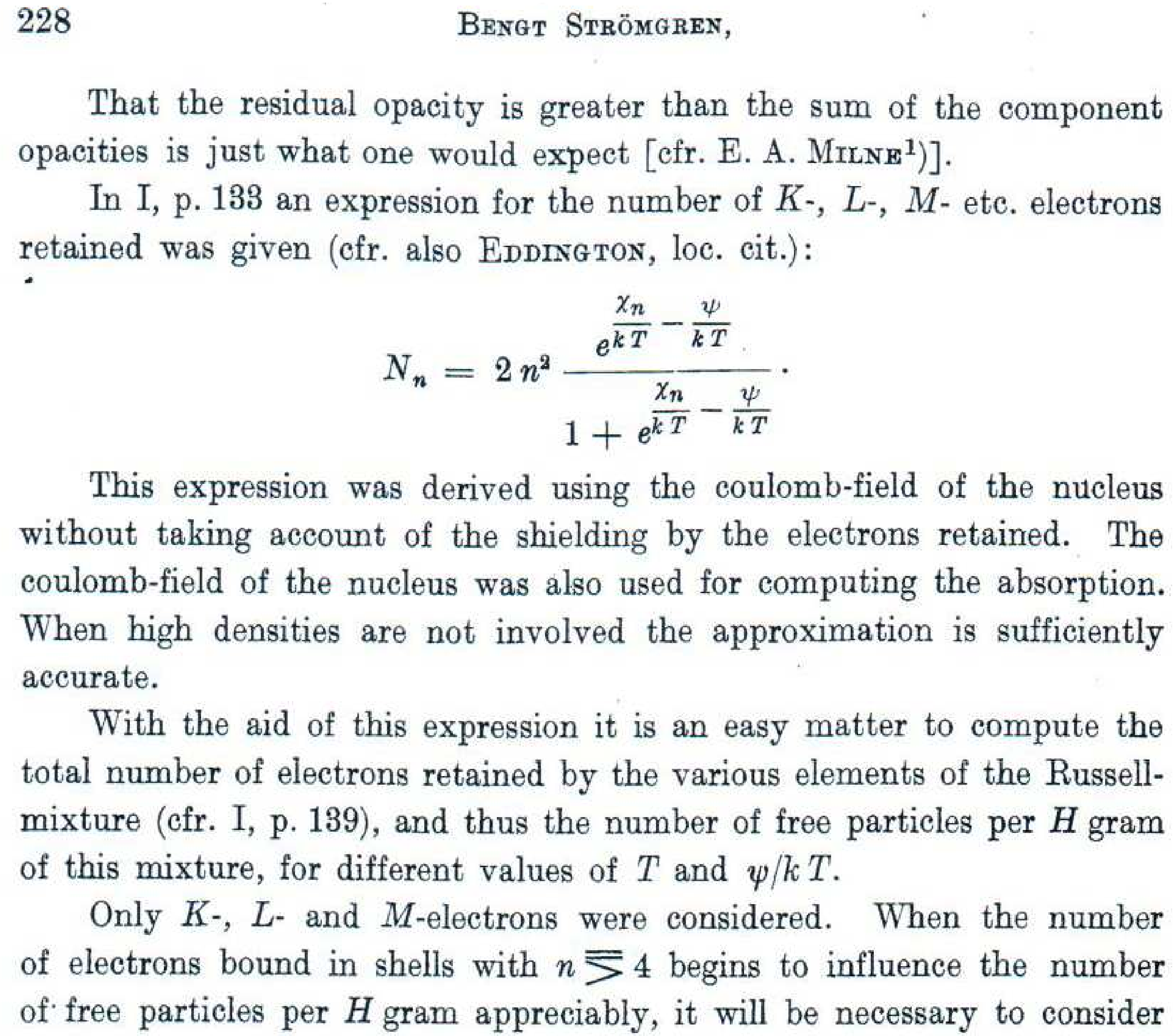
$$\sigma = 0.20\,(1+X).$$

The opacity due to simultaneous absorption and scattering is required. To find this quantity we have to add absorption and scattering for each frequency and take the Rosseland-mean. For the frequency x (unit of frequency h/kT) we have (cfr. I, p. 135)

$$a\,(x) = \frac{D\,(x)}{x^3},$$

H - R diagram

O interpretaci H-R diagramu r. 1933, odchod hvězd z hlavní posloupnosti, úhel sklonu závisí na obsahu vodíku



That the residual opacity is greater than the sum of the component opacities is just what one would expect [cfr. E. A. MILNE[1])].

In I, p. 133 an expression for the number of K-, L-, M- etc. electrons retained was given (cfr. also EDDINGTON, loc. cit.):

$$N_n = 2\,n^2 \frac{e^{\frac{\chi_n}{kT} - \frac{\psi}{kT}}}{1 + e^{\frac{\chi_n}{kT} - \frac{\psi}{kT}}}.$$

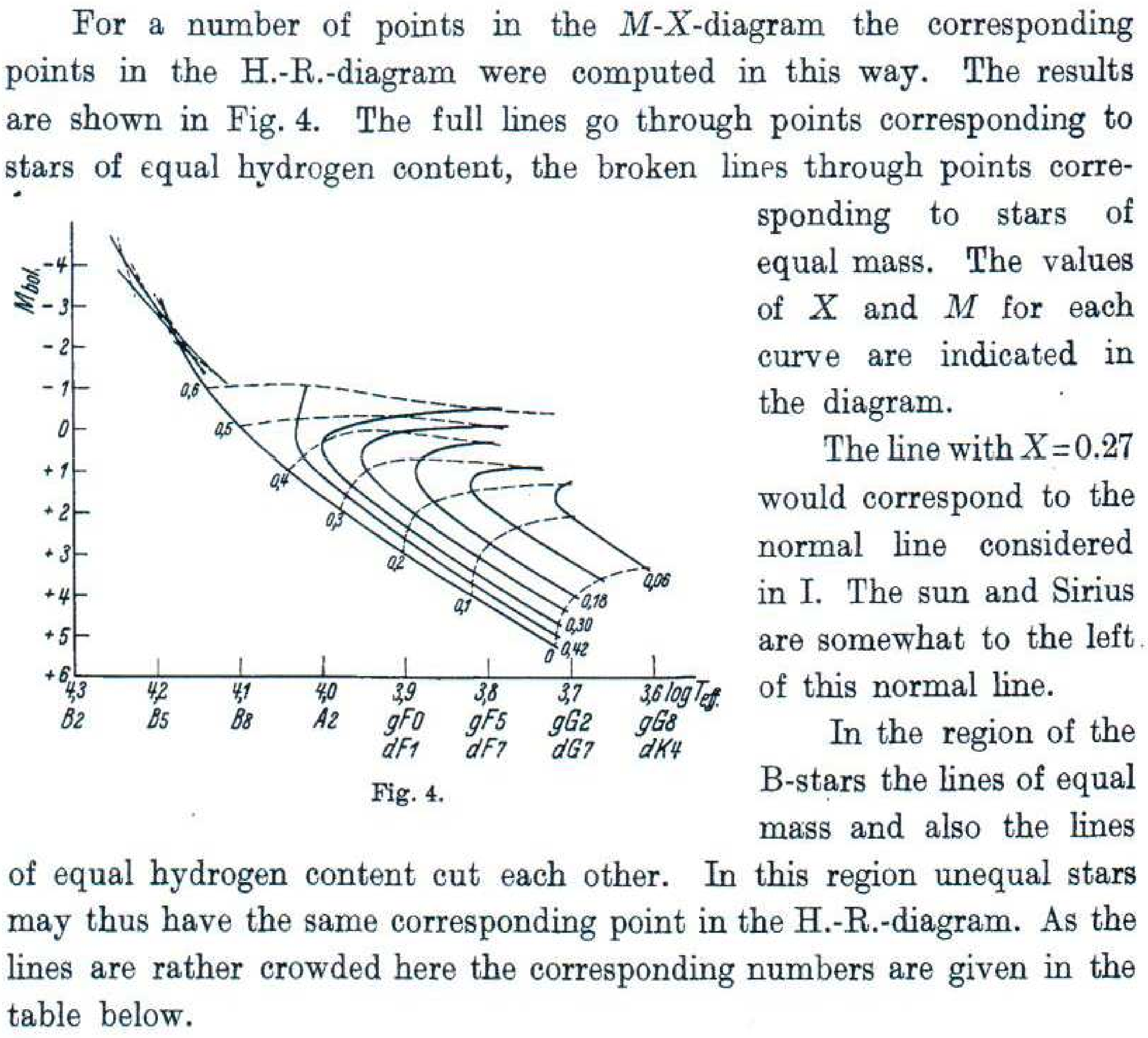
This expression was derived using the coulomb-field of the nucleus without taking account of the shielding by the electrons retained. The coulomb-field of the nucleus was also used for computing the absorption. When high densities are not involved the approximation is sufficiently accurate.

With the aid of this expression it is an easy matter to compute the total number of electrons retained by the various elements of the Russell-mixture (cfr. I, p. 139), and thus the number of free particles per H gram of this mixture, for different values of T and ψ/kT.

Only K-, L- and M-electrons were considered. When the number of electrons bound in shells with $n \overset{=}{>} 4$ begins to influence the number of free particles per H gram appreciably, it will be necessary to consider

H - R diagram

O interpretaci H-R diagramu r. 1933, odchod hvězd z hlavní posloupnosti, úhel sklonu závisí na obsahu vodíku



For a number of points in the *M*-*X*-diagram the corresponding points in the H.-R.-diagram were computed in this way. The results are shown in Fig. 4. The full lines go through points corresponding to stars of equal hydrogen content, the broken lines through points corresponding to stars of equal mass. The values of *X* and *M* for each curve are indicated in the diagram.

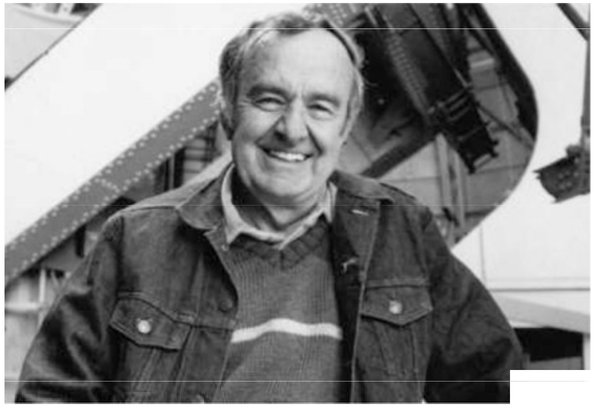
The line with $X = 0.27$ would correspond to the normal line considered in I. The sun and Sirius are somewhat to the left of this normal line.

In the region of the B-stars the lines of equal mass and also the lines of equal hydrogen content cut each other. In this region unequal stars may thus have the same corresponding point in the H.-R.-diagram. As the lines are rather crowded here the corresponding numbers are given in the table below.

Fig. 4.

H - R diagram

americký astrofyzik **Allan Sandage 1926 - 2010,** ***Observační přiblížení vývoje z pozorování vývojových stop pro M 67 a M 3, r. 1957***



OBSERVATIONAL APPROACH TO EVOLUTION. III. SEMIEMPIRICAL EVOLUTION TRACKS FOR M67 AND M3

Allan Sandage
Mount Wilson and Palomar Observatories
Carnegie Institution of Washington, California Institute of Technology

Received March 25, 1957

ABSTRACT

A method is presented for obtaining the tracks of evolution for individual stars in the subgiant and giant regions of color-magnitude diagrams of star clusters. The method utilizes the evolutionary information contained in the observed luminosity functions and color-magnitude diagrams of clusters. It is applied to the galactic cluster M67 and to the globular cluster M3. The evolutionary tracks, the time scale for evolution along these tracks, and the fraction of the total mass exhausted of hydrogen have been computed for both clusters, and the results are tabulated in Tables 3–10. The computed fraction of the mass exhausted of hydrogen for stars in M3 is compared with the theoretical predictions of the Hoyle-Schwarzschild (H-S) models. Fair agreement is obtained It is shown that the H-S models are capable of predicting nearly the correct luminosity function for M3 except at the very top of the giant sequence. The time taken for stars to evolve along the horizontal branch in M3 from $B - V = 0.50$ to $B - V = -0.10$ is found to be $2\ 3 \times 10^8$ years. The lifetime of the RR Lyrae phase of the evolution is 8×10^7 years. The expected rate of change of the period of the RR Lyrae stars due to this evolution is $\Delta t/t = 2.4 \times 10^{-11}$, or 0.1 second per century, which is about a factor of 5 below the limit of detectability with the available data. The observed period changes for RR Lyrae stars in M3 average twenty times this value and are believed to be due to causes other than evolution.

H - R diagram

Observační přiblížení vývoje z pozorování vývojových stop pro M 67 - otevřenou hvězdokupu M 67 a kulovou hvězdokupu M 3***, r. 1957,***

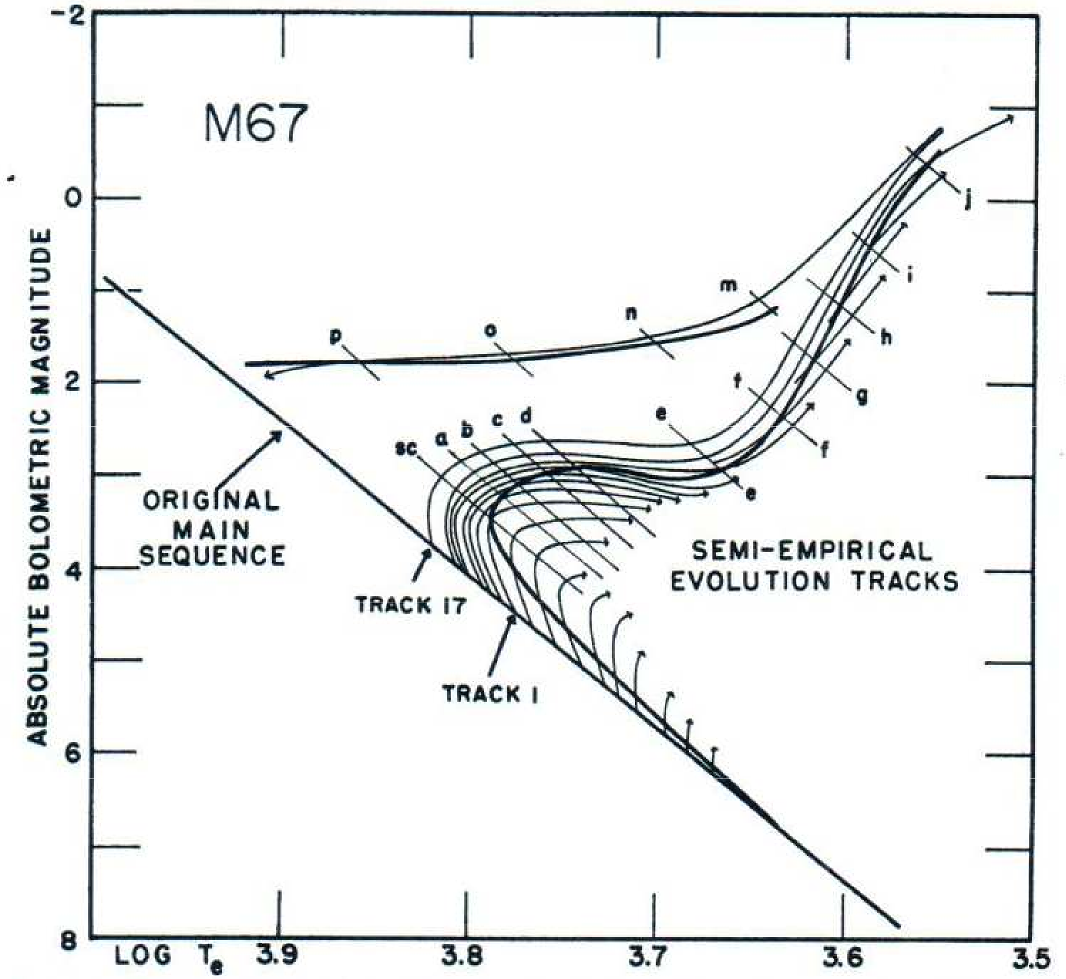


FIG. 2.—Semiempirical tracks of evolution for stars in M67. The various mapping points are shown as lines labeled $SC, a, \ldots, p$. The observed C-M diagram for M67 transformed to the M_{bol}, log T_e plane is shown as the heavy line cutting across the evolutionary tracks.

H - R diagram

Observační přiblížení vývoje z pozorování vývojových stop pro M 67 a M 3, r. 1957

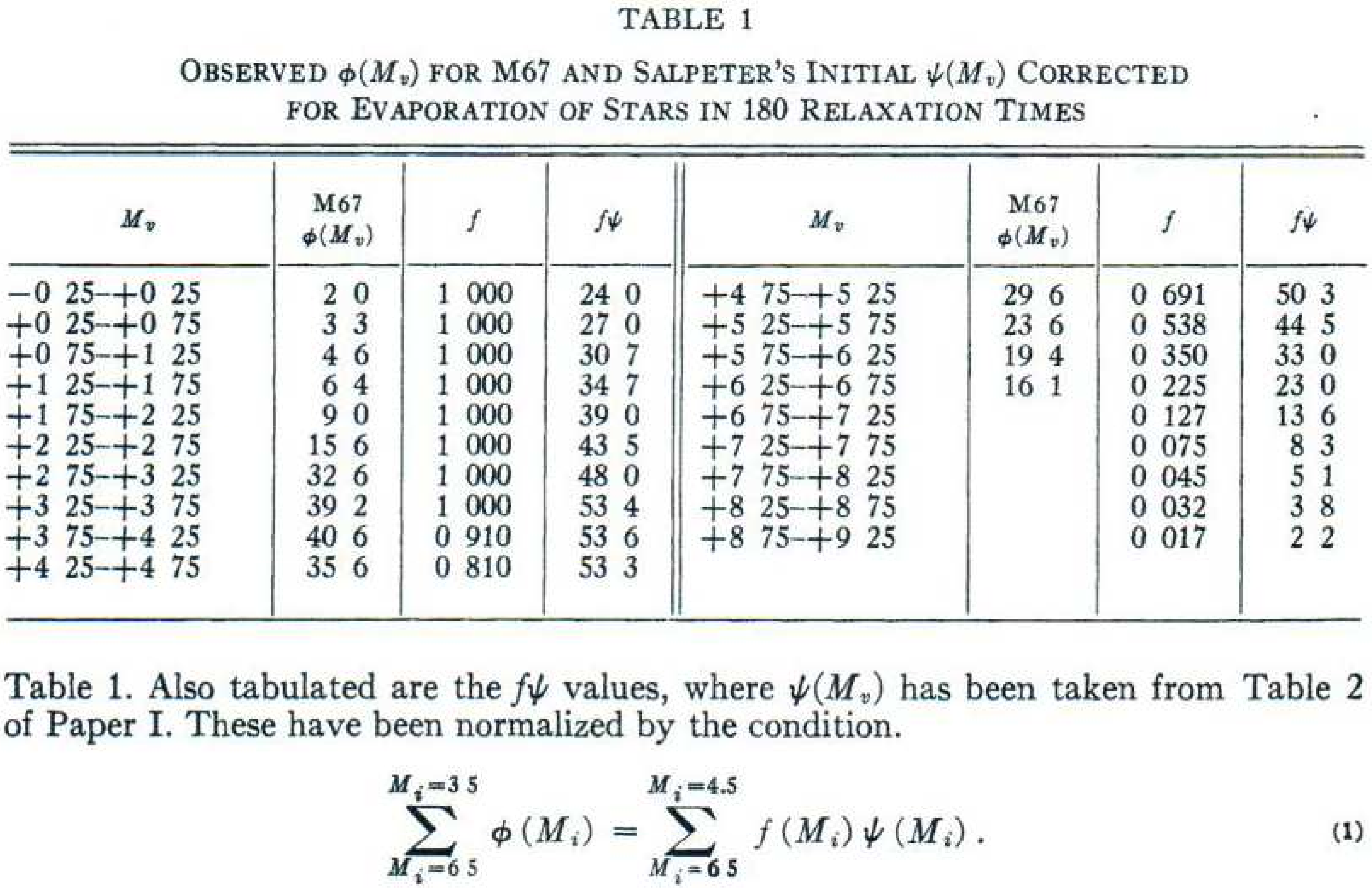


TABLE 1

OBSERVED $\phi(M_v)$ FOR M67 AND SALPETER'S INITIAL $\psi(M_v)$ CORRECTED FOR EVAPORATION OF STARS IN 180 RELAXATION TIMES

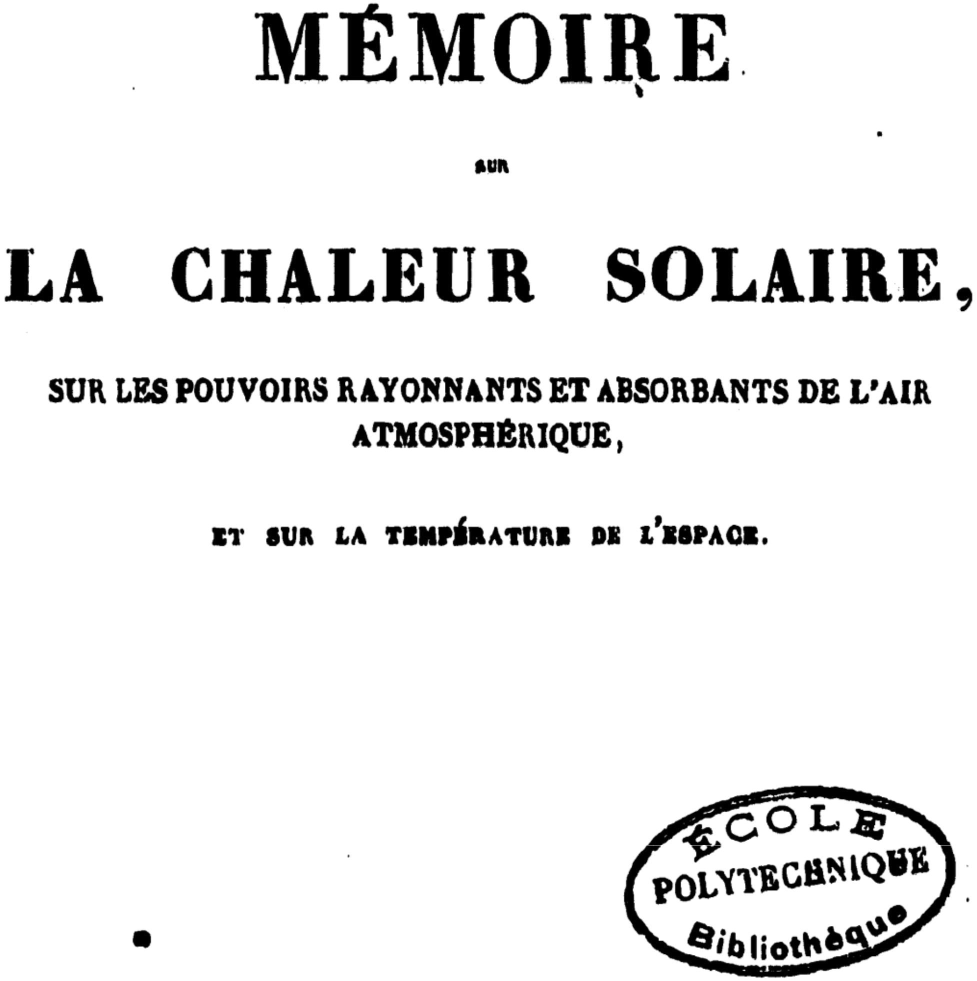
M_v	M67 $\phi(M_v)$	f	$f\psi$	M_v	M67 $\phi(M_v)$	f	$f\psi$
−0 25–+0 25	2 0	1 000	24 0	+4 75–+5 25	29 6	0 691	50 3
+0 25–+0 75	3 3	1 000	27 0	+5 25–+5 75	23 6	0 538	44 5
+0 75–+1 25	4 6	1 000	30 7	+5 75–+6 25	19 4	0 350	33 0
+1 25–+1 75	6 4	1 000	34 7	+6 25–+6 75	16 1	0 225	23 0
+1 75–+2 25	9 0	1 000	39 0	+6 75–+7 25		0 127	13 6
+2 25–+2 75	15 6	1 000	43 5	+7 25–+7 75		0 075	8 3
+2 75–+3 25	32 6	1 000	48 0	+7 75–+8 25		0 045	5 1
+3 25–+3 75	39 2	1 000	53 4	+8 25–+8 75		0 032	3 8
+3 75–+4 25	40 6	0 910	53 6	+8 75–+9 25		0 017	2 2
+4 25–+4 75	35 6	0 810	53 3				

Table 1. Also tabulated are the $f\psi$ values, where $\psi(M_v)$ has been taken from Table 2 of Paper I. These have been normalized by the condition.

$$\sum_{M_i=6\,5}^{M_i=3\,5} \phi(M_i) = \sum_{M_i=6\,5}^{M_i=4.5} f(M_i)\,\psi(M_i)\,. \tag{1}$$

Slunce

francouzský fyzik, **Claudie Servis Mathias Pouillet 1790 - 1868**
kontrukce pyrheliometru, měření dopadající energie/čas , 1,76 cal.cm



MÉMOIRE

SUR

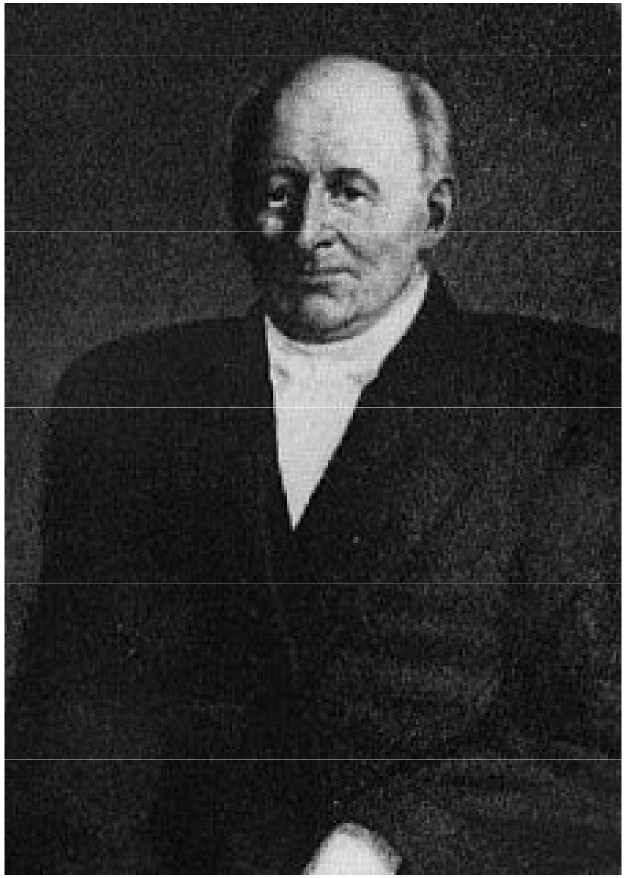
LA CHALEUR SOLAIRE,

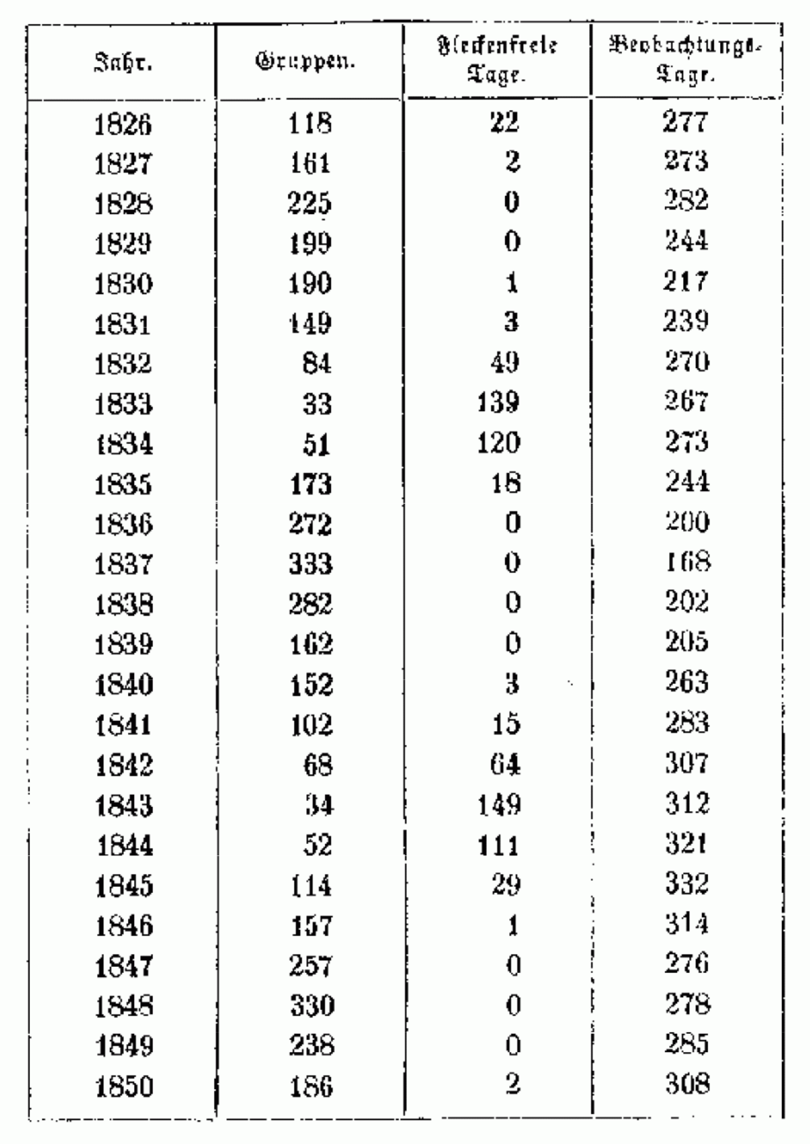
SUR LES POUVOIRS RAYONNANTS ET ABSORBANTS DE L'AIR
ATMOSPHÉRIQUE,

ET SUR LA TEMPÉRATURE DE L'ESPACE.

Slunce

německý astronom, farmaceut, **Heinrich Samuel Schwabe 1789 - 1875,** ***studium maxim a minim sluneční činnosti, stanovení periodicity na 10 roků***

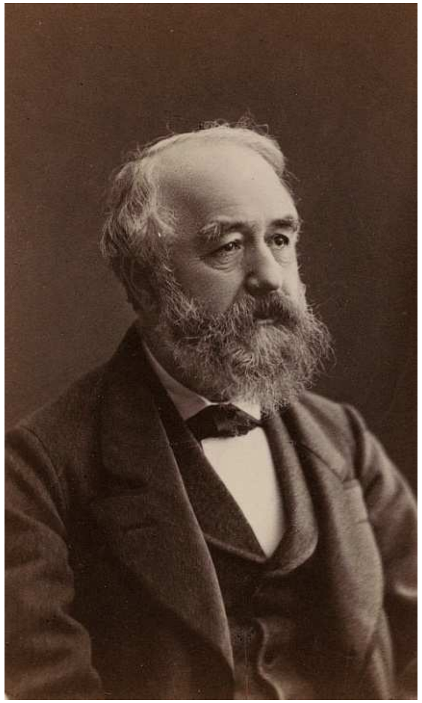


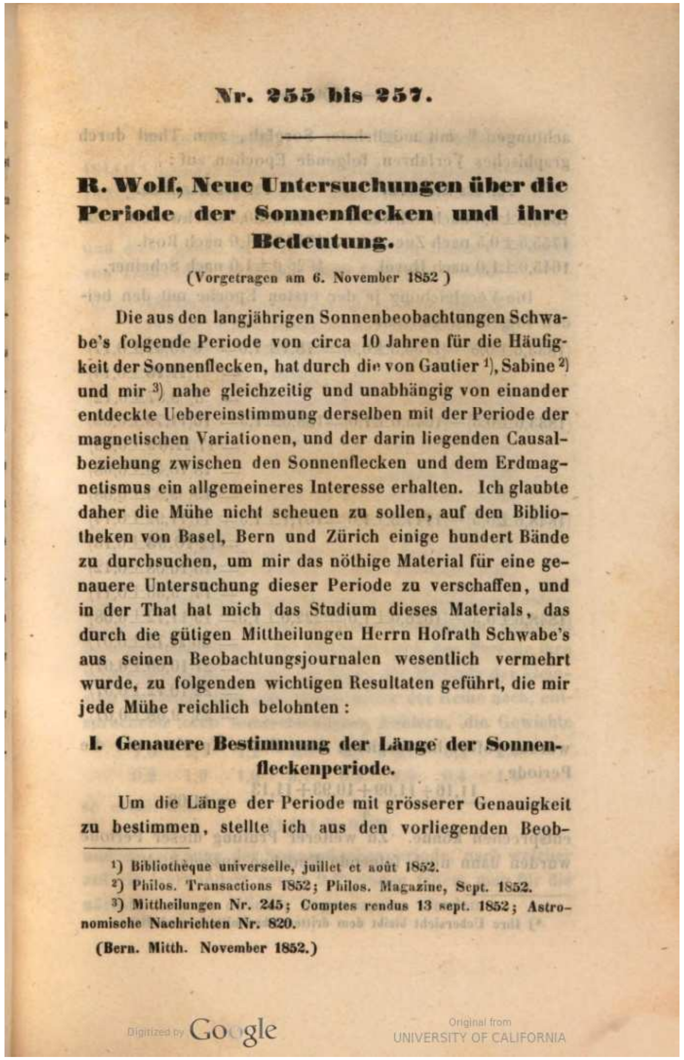


Jahr.	Gruppen.	Fleckenfreie Tage.	Beobachtungs-Tage.
1826	118	22	277
1827	161	2	273
1828	225	0	282
1829	199	0	244
1830	190	1	217
1831	149	3	239
1832	84	49	270
1833	33	139	267
1834	51	120	273
1835	173	18	244
1836	272	0	200
1837	333	0	168
1838	282	0	202
1839	162	0	205
1840	152	3	263
1841	102	15	283
1842	68	64	307
1843	34	149	312
1844	52	111	321
1845	114	29	332
1846	157	1	314
1847	257	0	276
1848	330	0	278
1849	238	0	285
1850	186	2	308

Slunce

švýcarský astronom **Rudolf Wolf 1816 - 1893,** ***Nové vyšetřování periody slunečních skvrn a její význam, r. 1852,*** upřesnění periody sluneční aktivity





Nr. 255 bis 257.

R. Wolf, Neue Untersuchungen über die Periode der Sonnenflecken und ihre Bedeutung.

(Vorgetragen am 6. November 1852)

Die aus den langjährigen Sonnenbeobachtungen Schwabe's folgende Periode von circa 10 Jahren für die Häufigkeit der Sonnenflecken, hat durch die von Gautier [1], Sabine [2] und mir [3] nahe gleichzeitig und unabhängig von einander entdeckte Uebereinstimmung derselben mit der Periode der magnetischen Variationen, und der darin liegenden Causalbeziehung zwischen den Sonnenflecken und dem Erdmagnetismus ein allgemeineres Interesse erhalten. Ich glaubte daher die Mühe nicht scheuen zu sollen, auf den Bibliotheken von Basel, Bern und Zürich einige hundert Bände zu durchsuchen, um mir das nöthige Material für eine genauere Untersuchung dieser Periode zu verschaffen, und in der That hat mich das Studium dieses Materials, das durch die gütigen Mittheilungen Herrn Hofrath Schwabe's aus seinen Beobachtungsjournalen wesentlich vermehrt wurde, zu folgenden wichtigen Resultaten geführt, die mir jede Mühe reichlich belohnten:

I. Genauere Bestimmung der Länge der Sonnenfleckenperiode.

Um die Länge der Periode mit grösserer Genauigkeit zu bestimmen, stellte ich aus den vorliegenden Beob-

---

[1] Bibliothèque universelle, juillet et août 1852.

[2] Philos. Transactions 1852; Philos. Magazine, Sept. 1852.

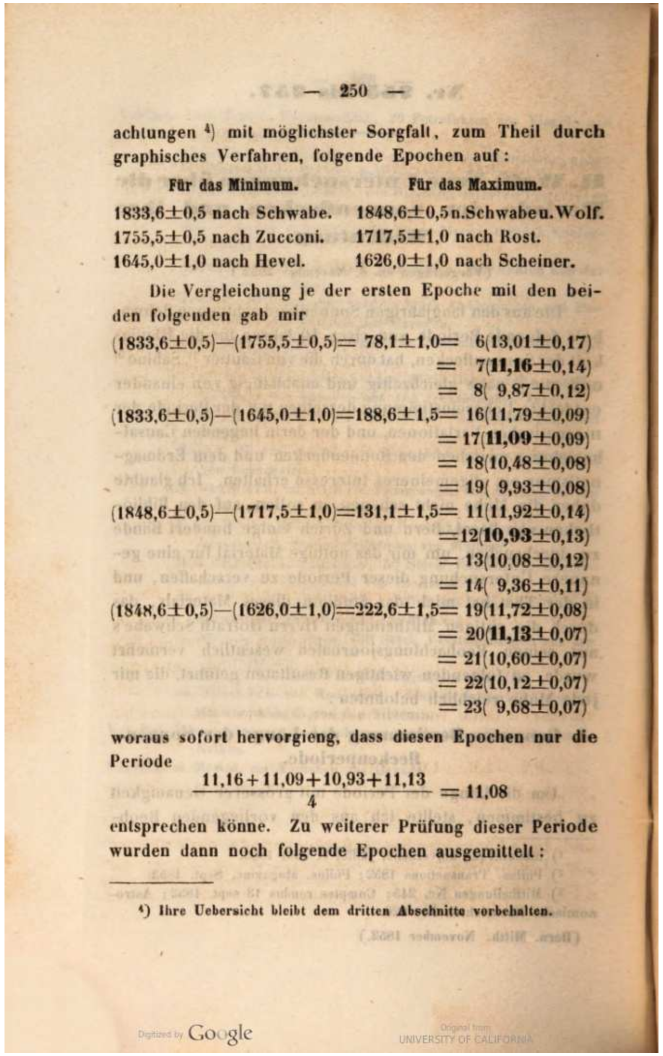
[3] Mittheilungen Nr. 245; Comptes rendus 13 sept. 1852; Astronomische Nachrichten Nr. 820.

(Bern. Mitth. November 1852.)

Slunce

švýcarský astronom **Rudolf Wolf 1816 - 1893,** zavedl Wolfovo – Curyšské číslo počtu slunečních skvrn r. 1848, ***R* = *k* (10*G* + *N*),**

k…korekční faktor, ***G***…počet skupin, ***N***…celkový počet skvrn



achtungen 4) mit möglichster Sorgfalt, zum Theil durch graphisches Verfahren, folgende Epochen auf:

Für das Minimum.	Für das Maximum.
1833,6±0,5 nach Schwabe.	1848,6±0,5 n. Schwabe u. Wolf.
1755,5±0,5 nach Zucconi.	1717,5±1,0 nach Rost.
1645,0±1,0 nach Hevel.	1626,0±1,0 nach Scheiner.

Die Vergleichung je der ersten Epoche mit den beiden folgenden gab mir

$$
\begin{aligned}
(1833{,}6\pm0{,}5)-(1755{,}5\pm0{,}5) = 78{,}1\pm1{,}0 &= 6(13{,}01\pm0{,}17)\\
&= 7(\mathbf{11{,}16}\pm0{,}14)\\
&= 8(\ 9{,}87\pm0{,}12)\\
(1833{,}6\pm0{,}5)-(1645{,}0\pm1{,}0) = 188{,}6\pm1{,}5 &= 16(11{,}79\pm0{,}09)\\
&= 17(\mathbf{11{,}09}\pm0{,}09)\\
&= 18(10{,}48\pm0{,}08)\\
&= 19(\ 9{,}93\pm0{,}08)\\
(1848{,}6\pm0{,}5)-(1717{,}5\pm1{,}0) = 131{,}1\pm1{,}5 &= 11(11{,}92\pm0{,}14)\\
&= 12(\mathbf{10{,}93}\pm0{,}13)\\
&= 13(10{,}08\pm0{,}12)\\
&= 14(\ 9{,}36\pm0{,}11)\\
(1848{,}6\pm0{,}5)-(1626{,}0\pm1{,}0) = 222{,}6\pm1{,}5 &= 19(11{,}72\pm0{,}08)\\
&= 20(\mathbf{11{,}13}\pm0{,}07)\\
&= 21(10{,}60\pm0{,}07)\\
&= 22(10{,}12\pm0{,}07)\\
&= 23(\ 9{,}68\pm0{,}07)
\end{aligned}
$$

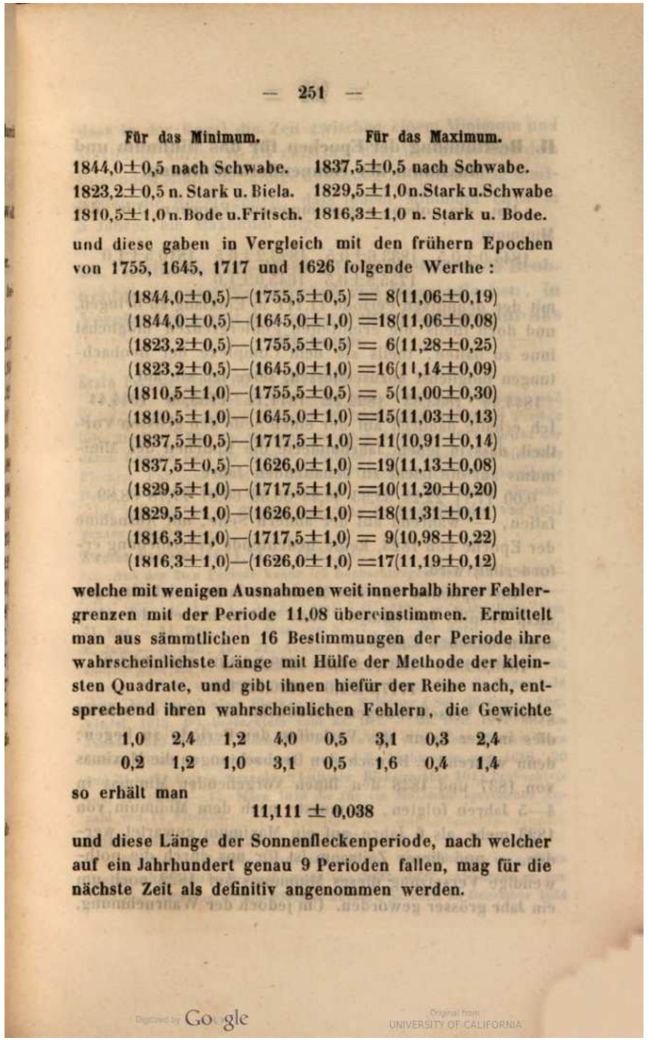
woraus sofort hervorgieng, dass diesen Epochen nur die Periode

$$\frac{11{,}16+11{,}09+10{,}93+11{,}13}{4} = 11{,}08$$

entsprechen könne. Zu weiterer Prüfung dieser Periode wurden dann noch folgende Epochen ausgemittelt:

---

4) Ihre Uebersicht bleibt dem dritten Abschnitte vorbehalten.



Für das Minimum.	Für das Maximum.
1844,0±0,5 nach Schwabe.	1837,5±0,5 nach Schwabe.
1823,2±0,5 n. Stark u. Biela.	1829,5±1,0 n. Stark u. Schwabe
1810,5±1,0 n. Bode u. Fritsch.	1816,3±1,0 n. Stark u. Bode.

und diese gaben in Vergleich mit den frühern Epochen von 1755, 1645, 1717 und 1626 folgende Werthe:

$$
\begin{aligned}
(1844{,}0\pm0{,}5)-(1755{,}5\pm0{,}5) &= 8(11{,}06\pm0{,}19)\\
(1844{,}0\pm0{,}5)-(1645{,}0\pm1{,}0) &= 18(11{,}06\pm0{,}08)\\
(1823{,}2\pm0{,}5)-(1755{,}5\pm0{,}5) &= 6(11{,}28\pm0{,}25)\\
(1823{,}2\pm0{,}5)-(1645{,}0\pm1{,}0) &= 16(11{,}14\pm0{,}09)\\
(1810{,}5\pm1{,}0)-(1755{,}5\pm0{,}5) &= 5(11{,}00\pm0{,}30)\\
(1810{,}5\pm1{,}0)-(1645{,}0\pm1{,}0) &= 15(11{,}03\pm0{,}13)\\
(1837{,}5\pm0{,}5)-(1717{,}5\pm1{,}0) &= 11(10{,}91\pm0{,}14)\\
(1837{,}5\pm0{,}5)-(1626{,}0\pm1{,}0) &= 19(11{,}13\pm0{,}08)\\
(1829{,}5\pm1{,}0)-(1717{,}5\pm1{,}0) &= 10(11{,}20\pm0{,}20)\\
(1829{,}5\pm1{,}0)-(1626{,}0\pm1{,}0) &= 18(11{,}31\pm0{,}11)\\
(1816{,}3\pm1{,}0)-(1717{,}5\pm1{,}0) &= 9(10{,}98\pm0{,}22)\\
(1816{,}3\pm1{,}0)-(1626{,}0\pm1{,}0) &= 17(11{,}19\pm0{,}12)
\end{aligned}
$$

welche mit wenigen Ausnahmen weit innerhalb ihrer Fehlergrenzen mit der Periode 11,08 übereinstimmen. Ermittelt man aus sämmtlichen 16 Bestimmungen der Periode ihre wahrscheinlichste Länge mit Hülfe der Methode der kleinsten Quadrate, und gibt ihnen hiefür der Reihe nach, entsprechend ihren wahrscheinlichen Fehlern, die Gewichte

1,0	2,4	1,2	4,0	0,5	3,1	0,3	2,4
0,2	1,2	1,0	3,1	0,5	1,6	0,4	1,4

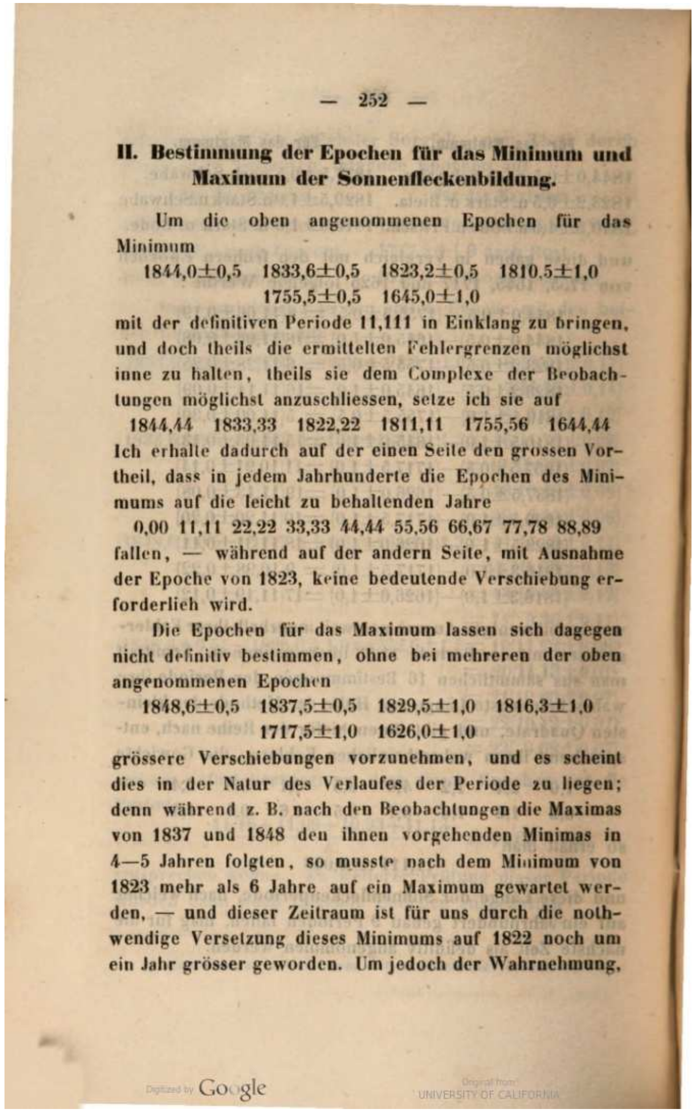
so erhält man

$$11{,}111 \pm 0{,}038$$

und diese Länge der Sonnenfleckenperiode, nach welcher auf ein Jahrhundert genau 9 Perioden fallen, mag für die nächste Zeit als definitiv angenommen werden.

Slunce

švýcarský astronom **Rudolf Wolf** , ***r. 1852 upřesnění periody sluneční aktivity***



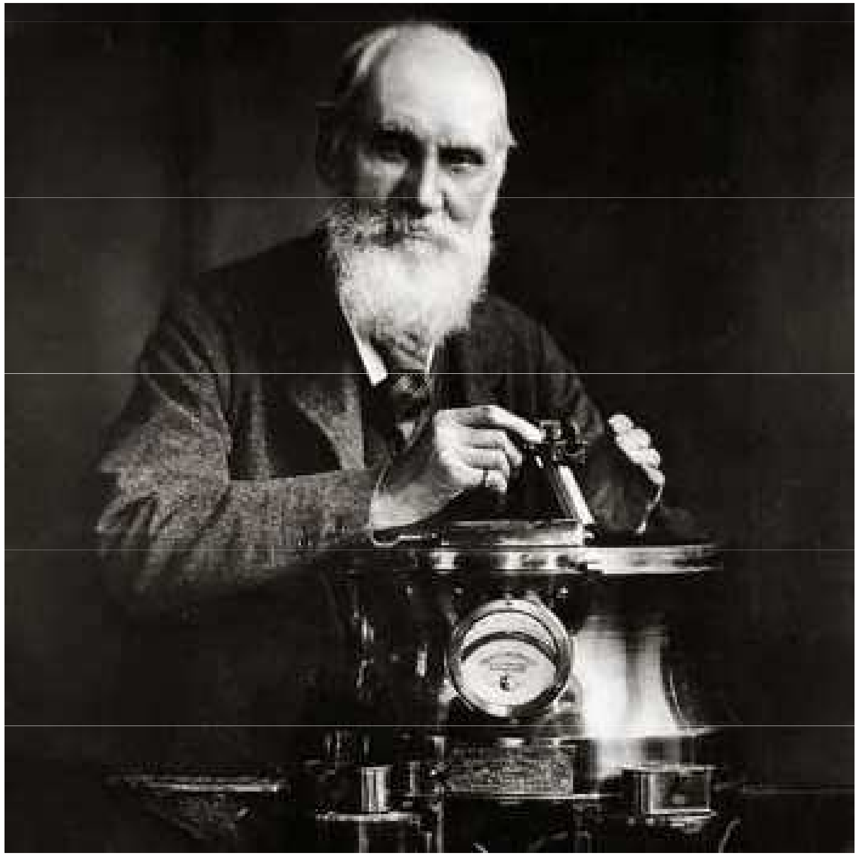
II. Bestimmung der Epochen für das Minimum und Maximum der Sonnenfleckenbildung.

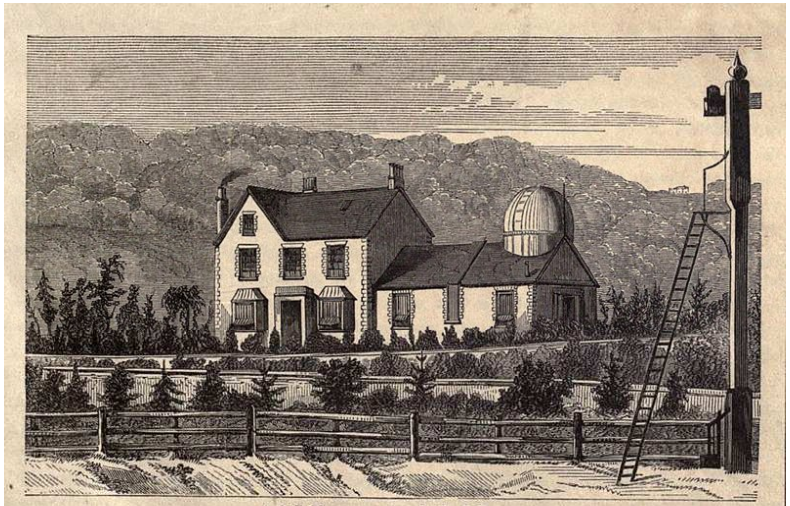
Um die oben angenommenen Epochen für das Minimum
1844,0±0,5   1833,6±0,5   1823,2±0,5   1810.5±1,0
1755,5±0,5   1645,0±1,0
mit der definitiven Periode 11,111 in Einklang zu bringen, und doch theils die ermittelten Fehlergrenzen möglichst inne zu halten, theils sie dem Complexe der Beobachtungen möglichst anzuschliessen, setze ich sie auf
1844,44   1833,33   1822,22   1811,11   1755,56   1644,44
Ich erhalte dadurch auf der einen Seite den grossen Vortheil, dass in jedem Jahrhunderte die Epochen des Minimums auf die leicht zu behaltenden Jahre
0,00 11,11 22,22 33,33 44,44 55,56 66,67 77,78 88,89
fallen, — während auf der andern Seite, mit Ausnahme der Epoche von 1823, keine bedeutende Verschiebung erforderlich wird.

Die Epochen für das Maximum lassen sich dagegen nicht definitiv bestimmen, ohne bei mehreren der oben angenommenen Epochen
1848,6±0,5   1837,5±0,5   1829,5±1,0   1816,3±1,0
1717,5±1,0   1626,0±1,0
grössere Verschiebungen vorzunehmen, und es scheint dies in der Natur des Verlaufes der Periode zu liegen; denn während z. B. nach den Beobachtungen die Maximas von 1837 und 1848 den ihnen vorgehenden Minimas in 4—5 Jahren folgten, so musste nach dem Minimum von 1823 mehr als 6 Jahre auf ein Maximum gewartet werden, — und dieser Zeitraum ist für uns durch die nothwendige Versetzung dieses Minimums auf 1822 noch um ein Jahr grösser geworden. Um jedoch der Wahrnehmung,

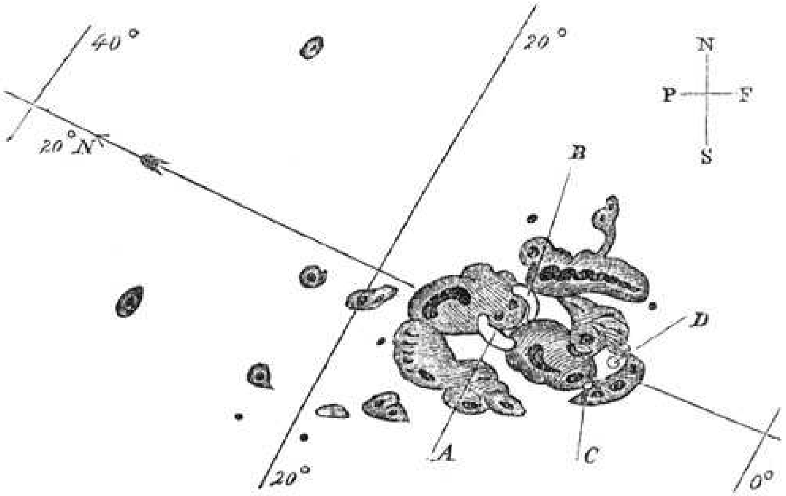
Slunce

anglický astronom, **Richard Christopher Carrington 1826 - 1875**
perioda rotace Slunce narůstá s větší heliocentrickou šířkou,
pozorování skvrn, motýlkový diagram,



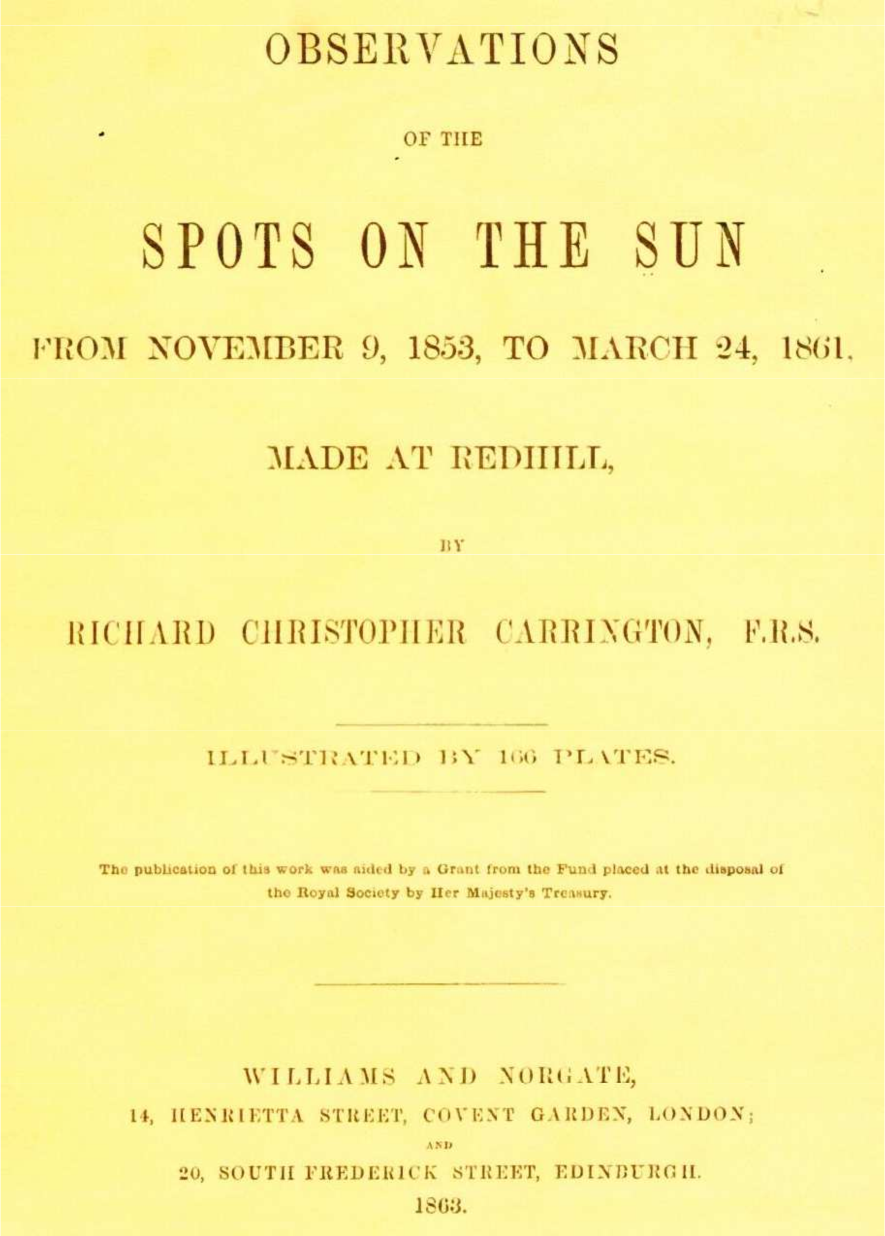


Redhill Observatory from the South-East.



Slunce

anglický astronom, **Richard Christopher Carrington 1826 - 1875**
Skvrny na Slunci, r. 1863, skvrny bližší slunečnímu rovníku rotují rychleji, **Galileo**



OBSERVATIONS

OF THE

SPOTS ON THE SUN

FROM NOVEMBER 9, 1853, TO MARCH 24, 1861,

MADE AT REDHILL,

BY

RICHARD CHRISTOPHER CARRINGTON, F.R.S.

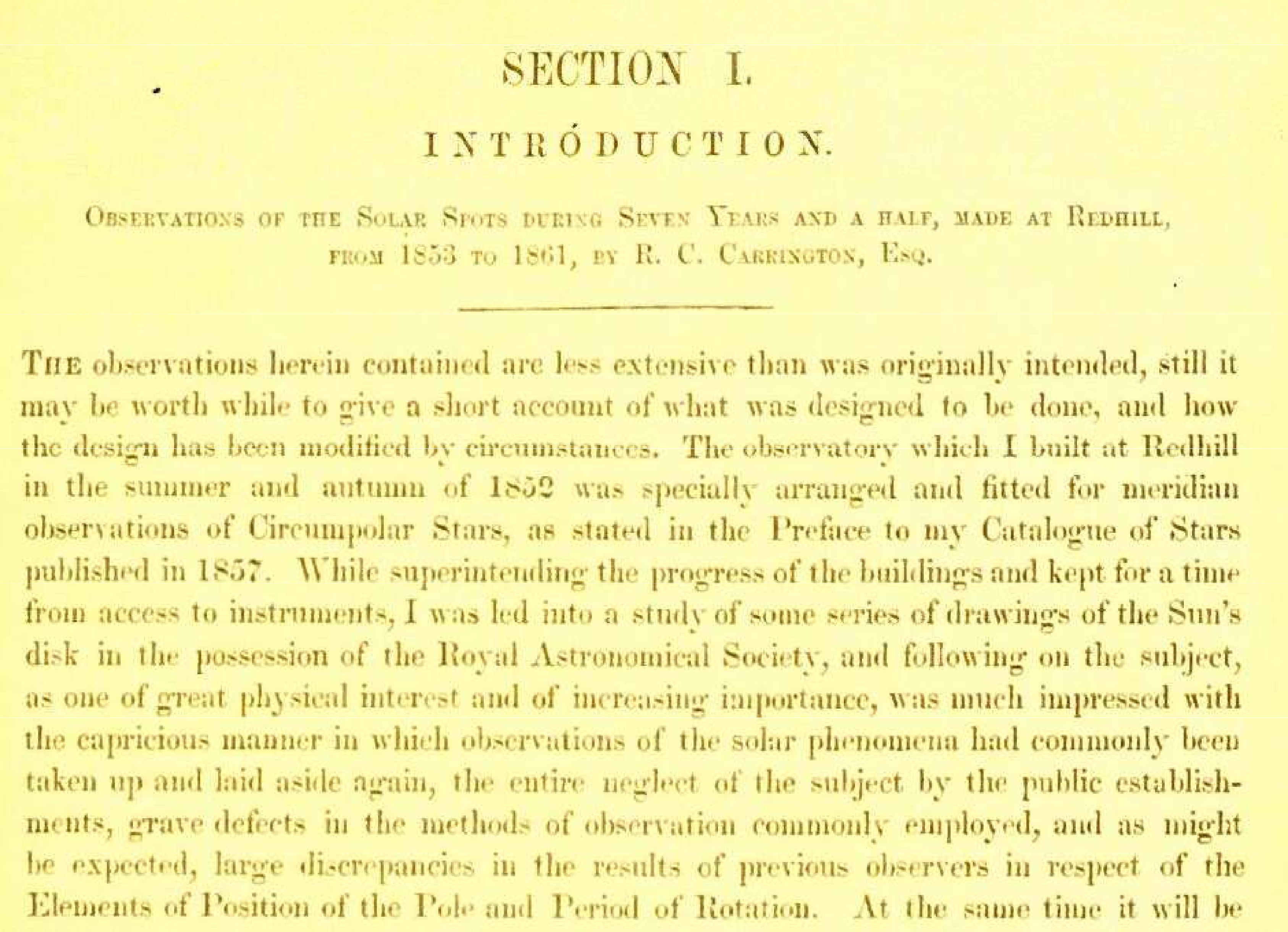
ILLUSTRATED BY 166 PLATES.

The publication of this work was aided by a Grant from the Fund placed at the disposal of
the Royal Society by Her Majesty's Treasury.

WILLIAMS AND NORGATE,
14, HENRIETTA STREET, COVENT GARDEN, LONDON;
AND
20, SOUTH FREDERICK STREET, EDINBURGH.
1863.

Slunce

Skvrny na Slunci, r. 1863



SECTION I.

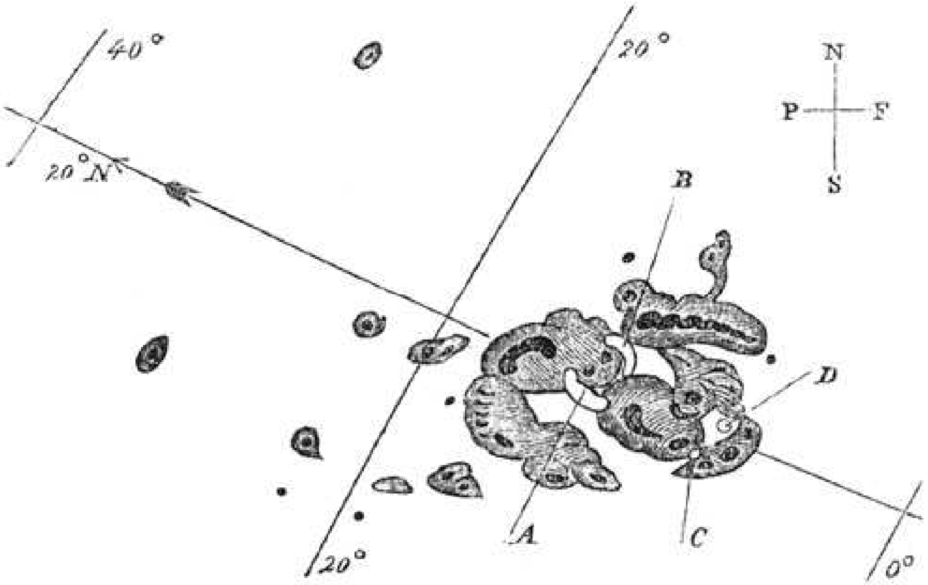
INTRODUCTION.

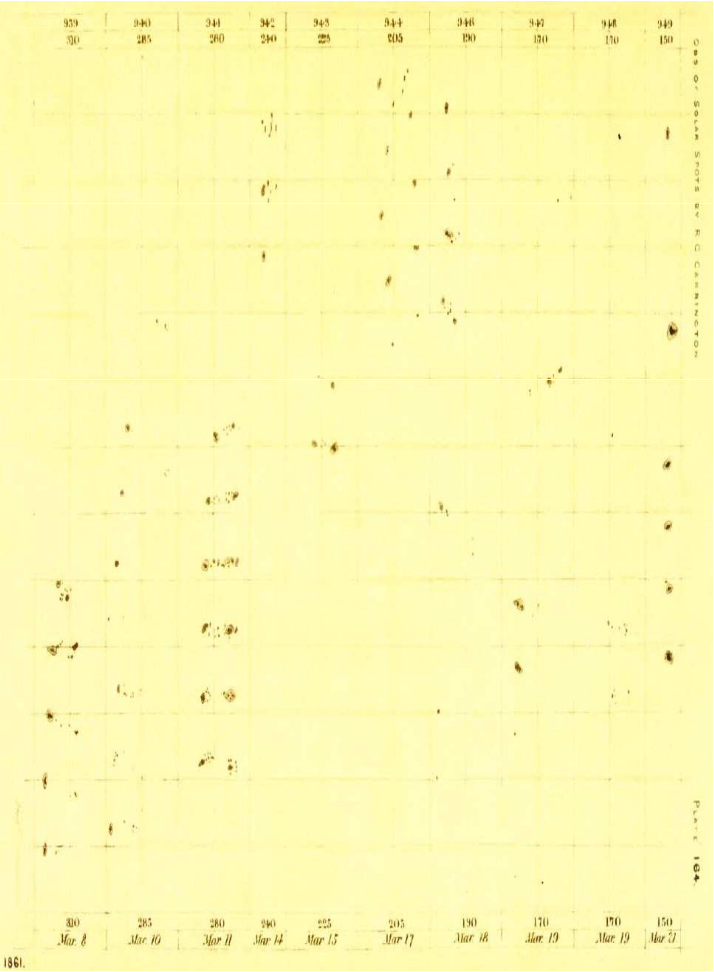
Observations of the Solar Spots during Seven Years and a half, made at Redhill, from 1853 to 1861, by R. C. Carrington, Esq.

THE observations herein contained are less extensive than was originally intended, still it may be worth while to give a short account of what was designed to be done, and how the design has been modified by circumstances. The observatory which I built at Redhill in the summer and autumn of 1852 was specially arranged and fitted for meridian observations of Circumpolar Stars, as stated in the Preface to my Catalogue of Stars published in 1857. While superintending the progress of the buildings and kept for a time from access to instruments, I was led into a study of some series of drawings of the Sun's disk in the possession of the Royal Astronomical Society, and following on the subject, as one of great physical interest and of increasing importance, was much impressed with the capricious manner in which observations of the solar phenomena had commonly been taken up and laid aside again, the entire neglect of the subject by the public establishments, grave defects in the methods of observation commonly employed, and as might be expected, large discrepancies in the results of previous observers in respect of the Elements of Position of the Pole and Period of Rotation. At the same time it will be

Slunce

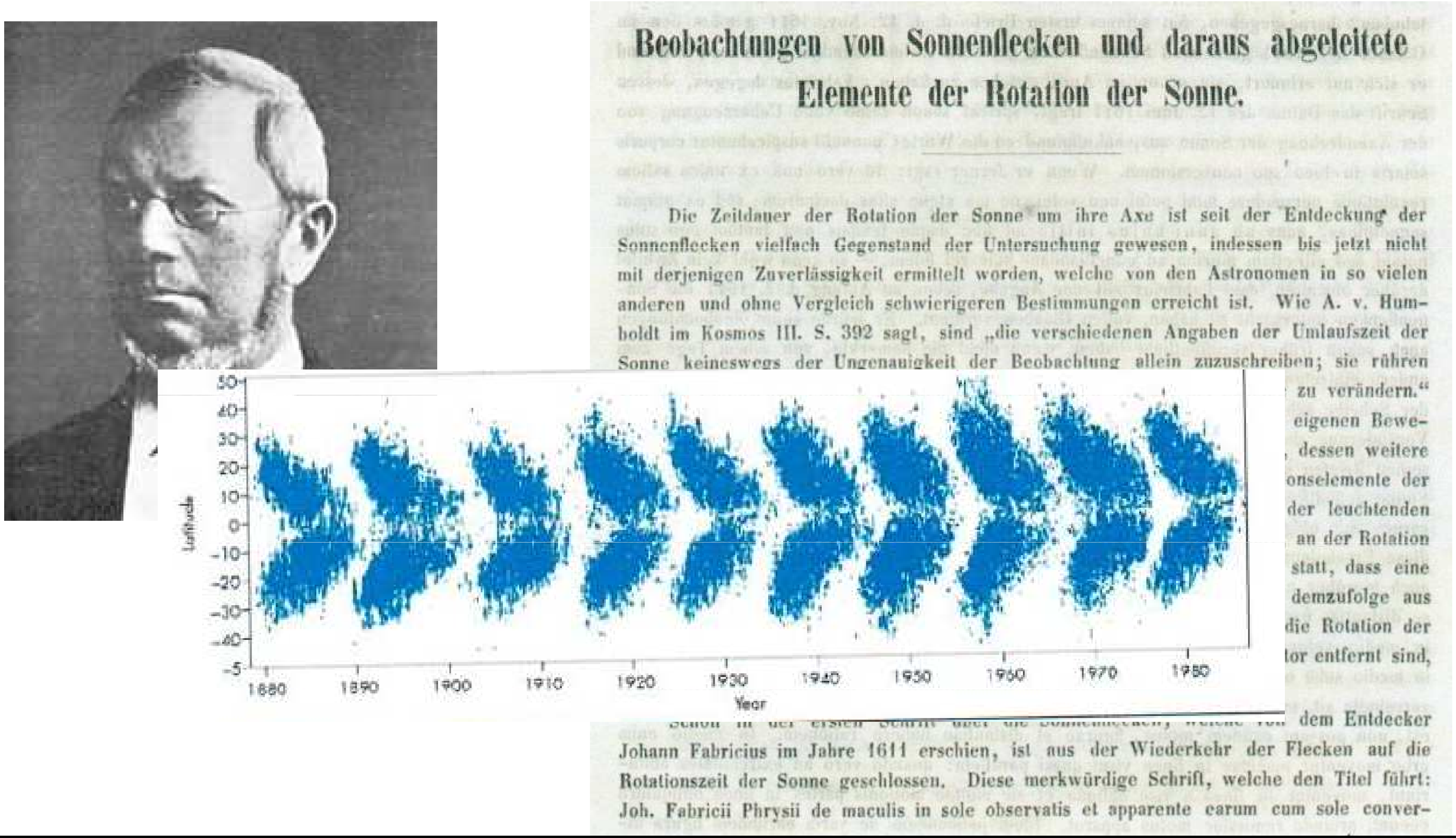
Skvrny na Slunci*, *r. 1863*, *Observations of Spots on the Sun from 9 November 1853 to 24 March 1861, Made at Redhill - Lalandova cena francouzské akademie věd na r. 1864, nákres pozorování slunečních skvrn 1. září 1859





Slunce

německý astronom Friedrich Wilhelm **Gustav Spörer 1822 -1874,** ***cykly skvrn se na začátku cyklu vyskytují ve vyšších heliografických šířkách, později níže a na konci poblíž rovníku, motýlkový diagram, perioda nízké aktivity slunečních skvrn v období 1645 - 1715, Maunderovo minimum, Pozorování slunečních skvrn r. 1861***



Beobachtungen von Sonnenflecken und daraus abgeleitete Elemente der Rotation der Sonne.

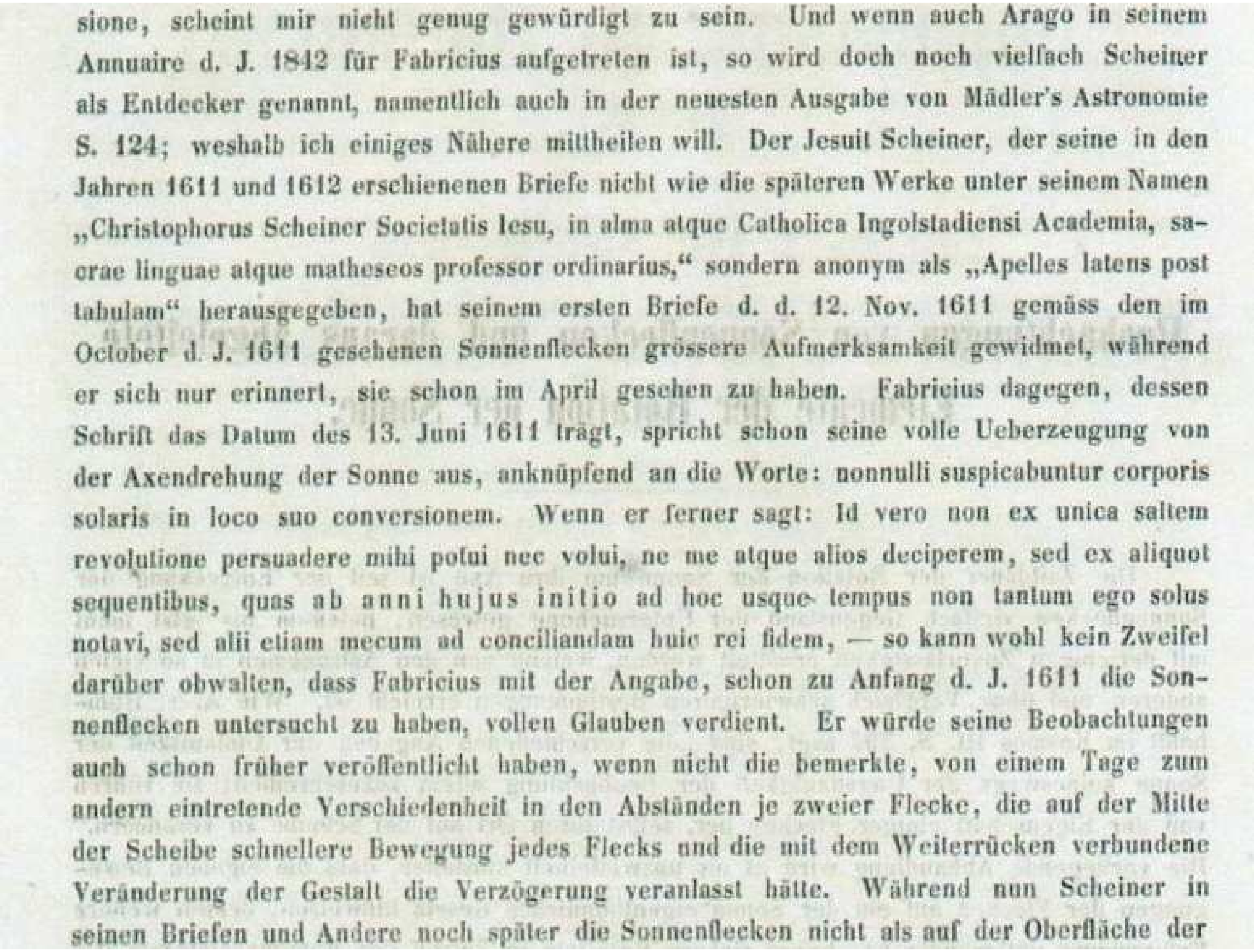
Die Zeitdauer der Rotation der Sonne um ihre Axe ist seit der Entdeckung der Sonnenflecken vielfach Gegenstand der Untersuchung gewesen, indessen bis jetzt nicht mit derjenigen Zuverlässigkeit ermittelt worden, welche von den Astronomen in so vielen anderen und ohne Vergleich schwierigeren Bestimmungen erreicht ist. Wie A. v. Humboldt im Kosmos III. S. 392 sagt, sind „die verschiedenen Angaben der Umlaufszeit der Sonne keineswegs der Ungenauigkeit der Beobachtung allein zuzuschreiben; sie rühren

zu verändern.“
eigenen Bewe-
, dessen weitere
onselemente der
der leuchtenden
an der Rotation
statt, dass eine
demzufolge aus
die Rotation der
tor entfernt sind,

dem Entdecker Johann Fabricius im Jahre 1611 erschien, ist aus der Wiederkehr der Flecken auf die Rotationszeit der Sonne geschlossen. Diese merkwürdige Schrift, welche den Titel führt: Joh. Fabricii Phrysii de maculis in sole observatis et apparente earum cum sole conver-

Slunce

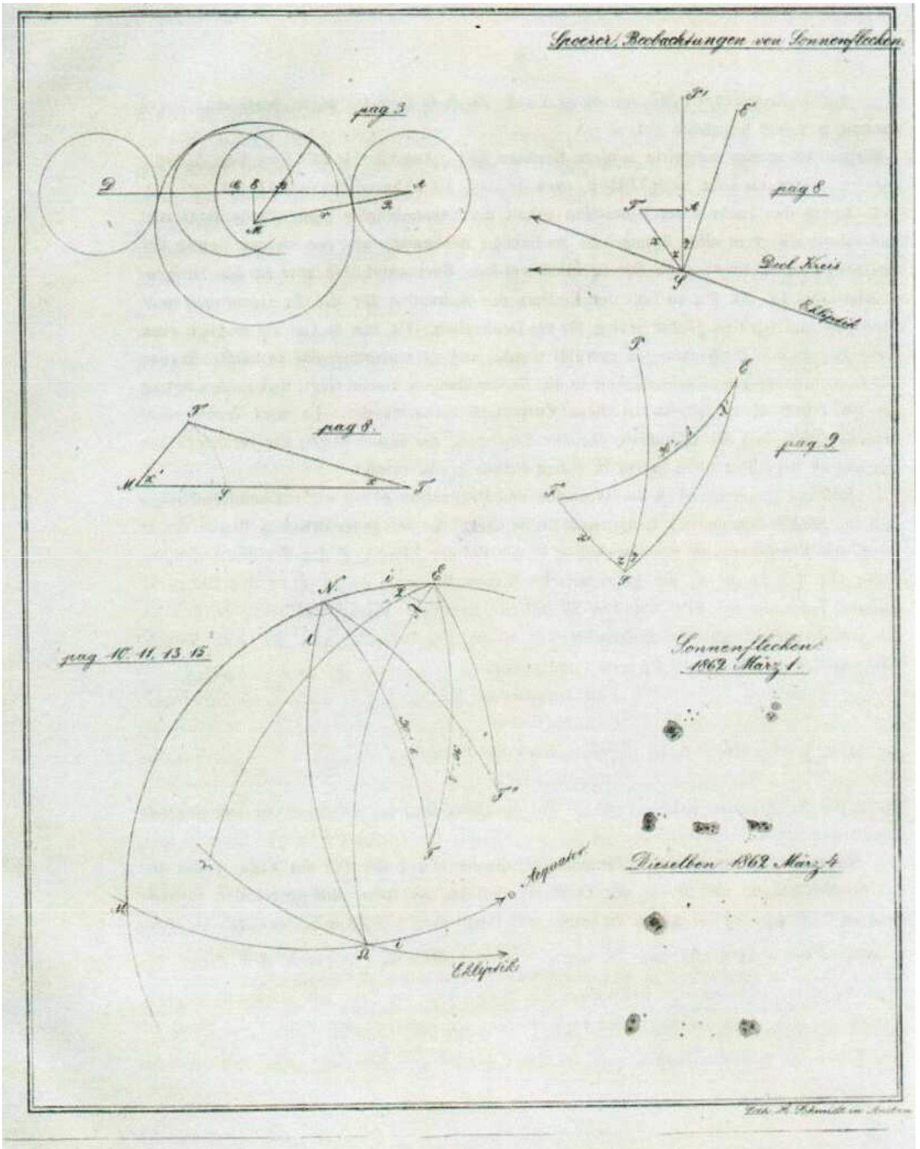
Pozorování slunečních skvrn r. 1861



sione, scheint mir nicht genug gewürdigt zu sein. Und wenn auch Arago in seinem Annuaire d. J. 1842 für Fabricius aufgetreten ist, so wird doch noch vielfach Scheiner als Entdecker genannt, namentlich auch in der neuesten Ausgabe von Mädler's Astronomie S. 124; weshalb ich einiges Nähere mittheilen will. Der Jesuit Scheiner, der seine in den Jahren 1611 und 1612 erschienenen Briefe nicht wie die späteren Werke unter seinem Namen „Christophorus Scheiner Societatis Iesu, in alma atque Catholica Ingolstadiensi Academia, sacrae linguae atque matheseos professor ordinarius,“ sondern anonym als „Apelles latens post tabulam“ herausgegeben, hat seinem ersten Briefe d. d. 12. Nov. 1611 gemäss den im October d. J. 1611 gesehenen Sonnenflecken grössere Aufmerksamkeit gewidmet, während er sich nur erinnert, sie schon im April gesehen zu haben. Fabricius dagegen, dessen Schrift das Datum des 13. Juni 1611 trägt, spricht schon seine volle Ueberzeugung von der Axendrehung der Sonne aus, anknüpfend an die Worte: nonnulli suspicabuntur corporis solaris in loco suo conversionem. Wenn er ferner sagt: Id vero non ex unica saltem revolutione persuadere mihi potui nec volui, ne me atque alios deciperem, sed ex aliquot sequentibus, quas ab anni hujus initio ad hoc usque tempus non tantum ego solus notavi, sed alii etiam mecum ad conciliandam huic rei fidem, — so kann wohl kein Zweifel darüber obwalten, dass Fabricius mit der Angabe, schon zu Anfang d. J. 1611 die Sonnenflecken untersucht zu haben, vollen Glauben verdient. Er würde seine Beobachtungen auch schon früher veröffentlicht haben, wenn nicht die bemerkte, von einem Tage zum andern eintretende Verschiedenheit in den Abständen je zweier Flecke, die auf der Mitte der Scheibe schnellere Bewegung jedes Flecks und die mit dem Weiterrücken verbundene Veränderung der Gestalt die Verzögerung veranlasst hätte. Während nun Scheiner in seinen Briefen und Andere noch später die Sonnenflecken nicht als auf der Oberfläche der

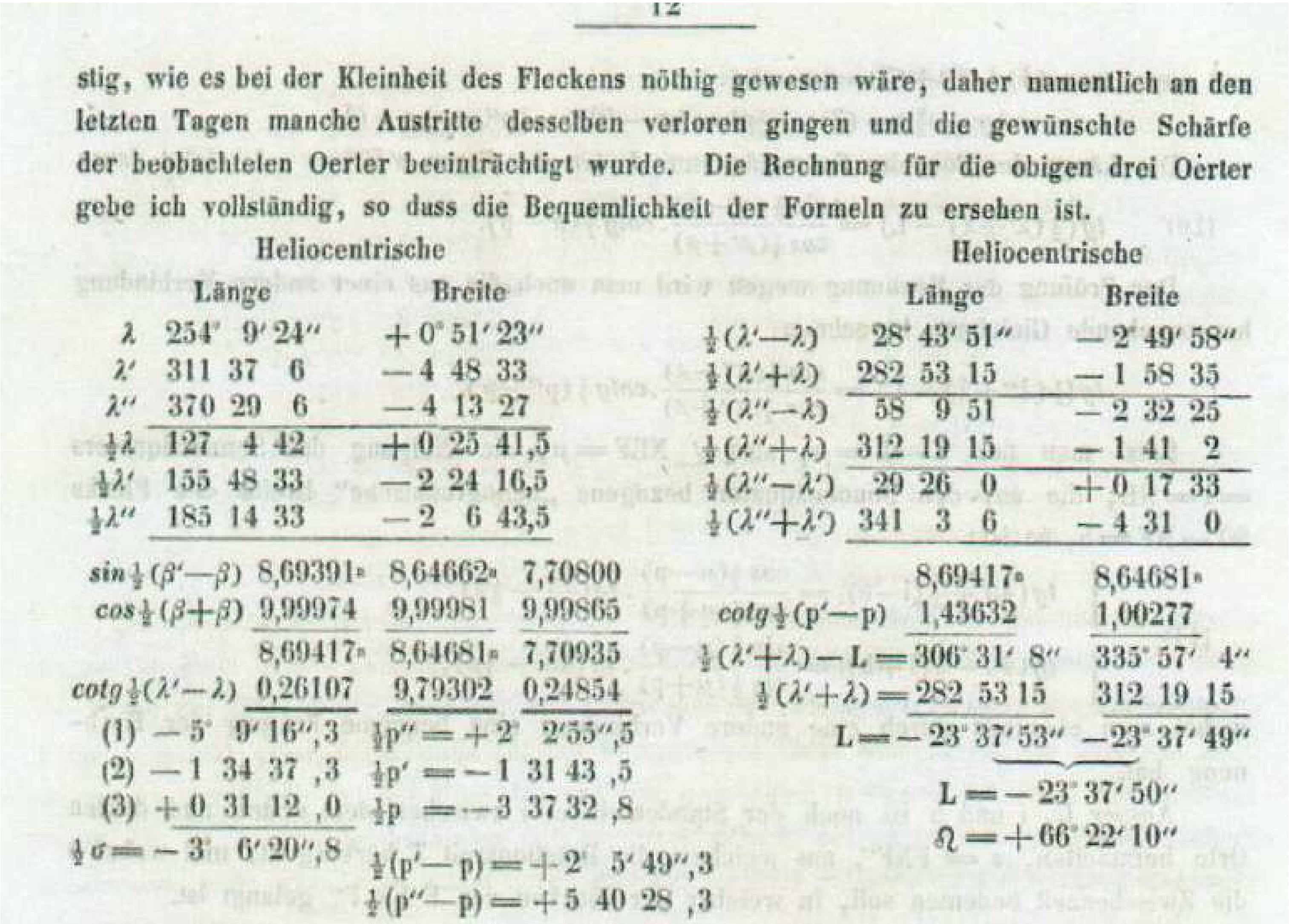
Slunce

Pozorování slunečních skvrn r. 1861



Slunce

Pozorování slunečních skvrn r. 1861



12

stig, wie es bei der Kleinheit des Fleckens nöthig gewesen wäre, daher namentlich an den letzten Tagen manche Austritte desselben verloren gingen und die gewünschte Schärfe der beobachteten Oerter beeinträchtigt wurde. Die Rechnung für die obigen drei Oerter gebe ich vollständig, so dass die Bequemlichkeit der Formeln zu ersehen ist.

	Heliocentrische Länge	Heliocentrische Breite
λ	254° 9′ 24″	+ 0° 51′ 23″
λ'	311 37 6	− 4 48 33
λ''	370 29 6	− 4 13 27
$\frac{1}{2}\lambda$	127 4 42	+ 0 25 41,5
$\frac{1}{2}\lambda'$	155 48 33	− 2 24 16,5
$\frac{1}{2}\lambda''$	185 14 33	− 2 6 43,5

	Heliocentrische Länge	Heliocentrische Breite
$\frac{1}{2}(\lambda'-\lambda)$	28° 43′ 51″	− 2° 49′ 58″
$\frac{1}{2}(\lambda'+\lambda)$	282 53 15	− 1 58 35
$\frac{1}{2}(\lambda''-\lambda)$	58 9 51	− 2 32 25
$\frac{1}{2}(\lambda''+\lambda)$	312 19 15	− 1 41 2
$\frac{1}{2}(\lambda''-\lambda')$	29 26 0	+ 0 17 33
$\frac{1}{2}(\lambda''+\lambda')$	341 3 6	− 4 31 0

$\sin\frac{1}{2}(\beta'-\beta)$	$8{,}69391_n$	$8{,}64662_n$	7,70800
$\cos\frac{1}{2}(\beta+\beta)$	9,99974	9,99981	9,99865
	$8{,}69417_n$	$8{,}64681_n$	7,70935
$\operatorname{cotg}\frac{1}{2}(\lambda'-\lambda)$	0,26107	9,79302	0,24854

(1) − 5° 9′ 16″,3 ½p″ = + 2° 2′ 55″,5

(2) − 1 34 37 ,3 ½p′ = − 1 31 43 ,5

(3) + 0 31 12 ,0 ½p = − 3 37 32 ,8

½σ = − 3° 6′ 20″,8

½(p′ − p) = + 2° 5′ 49″,3

½(p″ − p) = + 5 40 28 ,3

	$8{,}69417_n$	$8{,}64681_n$
$\operatorname{cotg}\frac{1}{2}(p'-p)$	1,43632	1,00277
$\frac{1}{2}(\lambda'+\lambda) - L =$	306° 31′ 8″	335° 57′ 4″
$\frac{1}{2}(\lambda'+\lambda) =$	282 53 15	312 19 15
$L =$	− 23° 37′ 53″	− 23° 37′ 49″

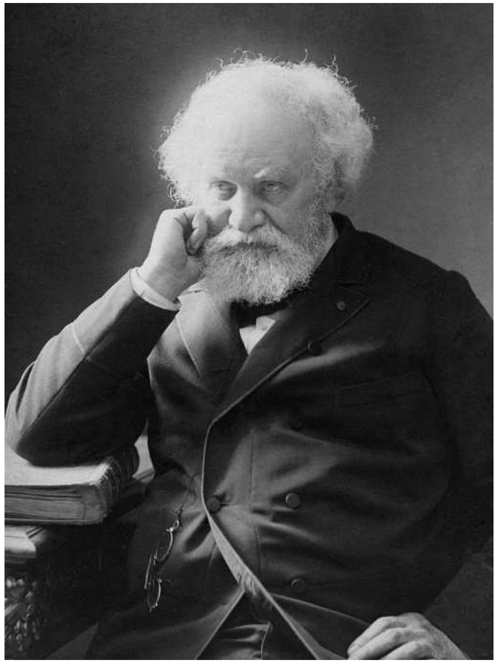
$$\underbrace{\qquad\qquad}$$

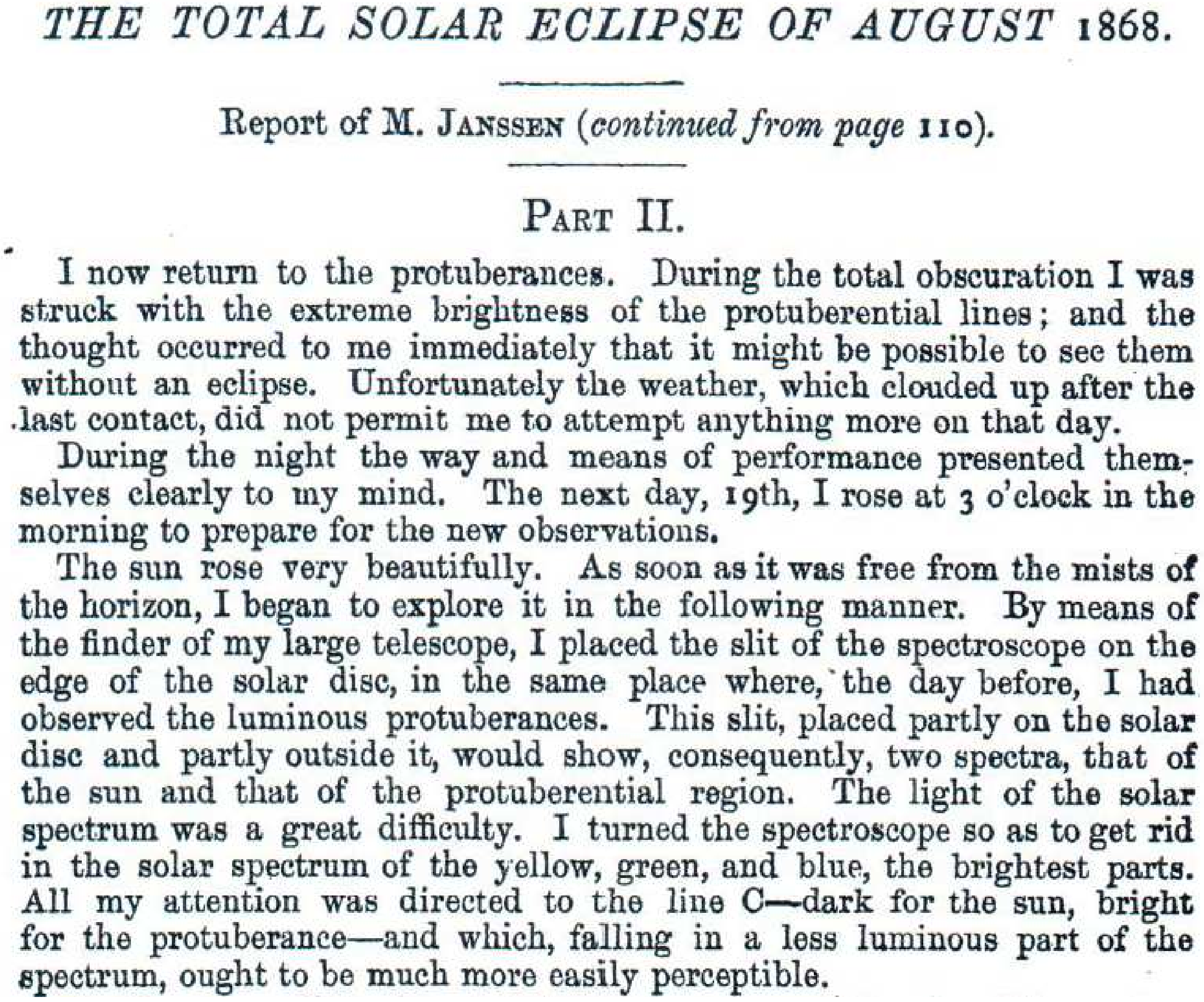
L = − 23° 37′ 50″

☊ = + 66° 22′ 10″

Slunce

francouzský astronom **Pierre Jules César Janssen 1824 – 1907**, v průběhu slunečního zatmění 18. srpna 1868 identifikoval čáru λ = 587,49 nm – helium,





THE TOTAL SOLAR ECLIPSE OF AUGUST 1868.

Report of M. Janssen (*continued from page* 110).

PART II.

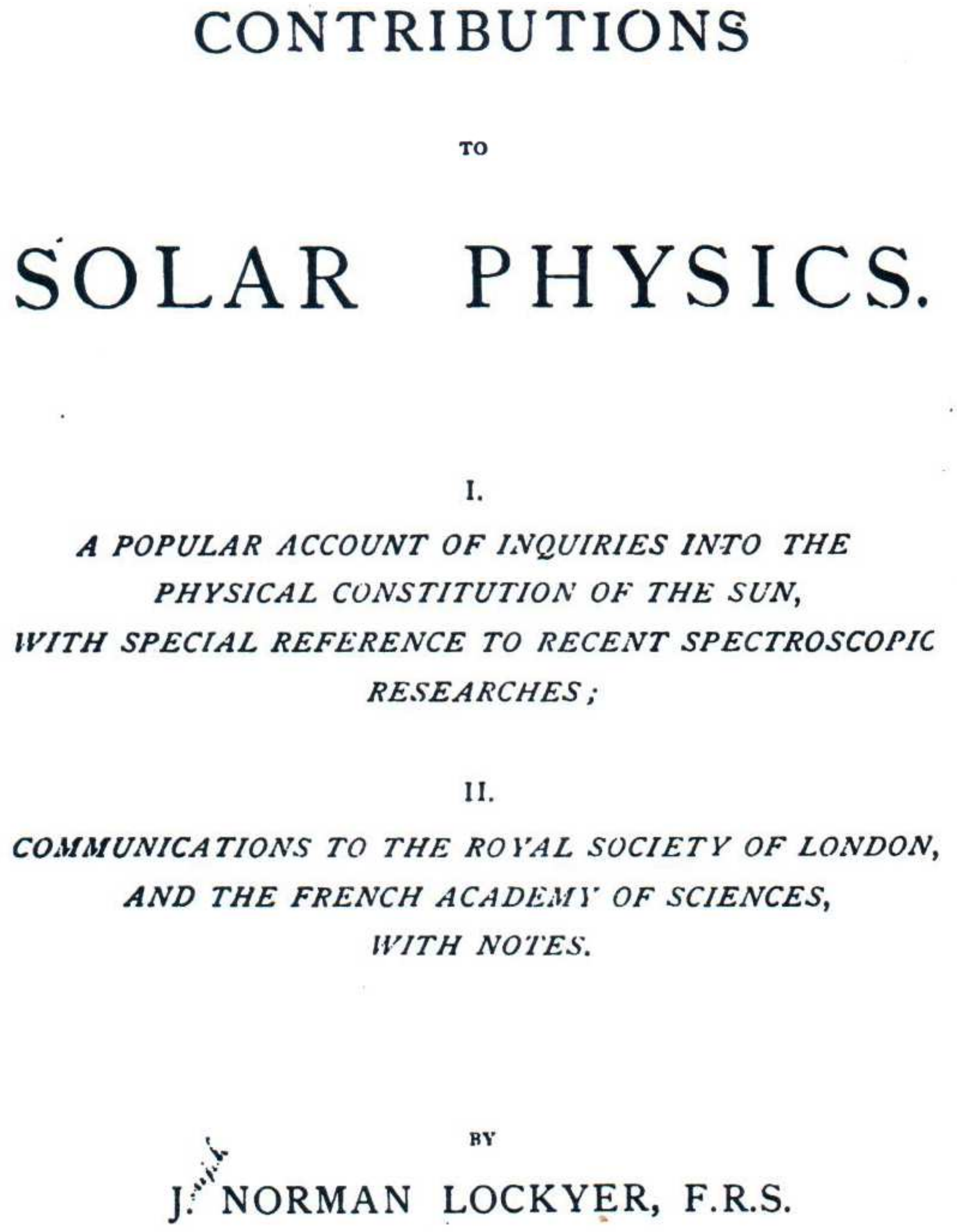
I now return to the protuberances. During the total obscuration I was struck with the extreme brightness of the protuberential lines; and the thought occurred to me immediately that it might be possible to see them without an eclipse. Unfortunately the weather, which clouded up after the last contact, did not permit me to attempt anything more on that day.

During the night the way and means of performance presented themselves clearly to my mind. The next day, 19th, I rose at 3 o'clock in the morning to prepare for the new observations.

The sun rose very beautifully. As soon as it was free from the mists of the horizon, I began to explore it in the following manner. By means of the finder of my large telescope, I placed the slit of the spectroscope on the edge of the solar disc, in the same place where, the day before, I had observed the luminous protuberances. This slit, placed partly on the solar disc and partly outside it, would show, consequently, two spectra, that of the sun and that of the protuberential region. The light of the solar spectrum was a great difficulty. I turned the spectroscope so as to get rid in the solar spectrum of the yellow, green, and blue, the brightest parts. All my attention was directed to the line C—dark for the sun, bright for the protuberance—and which, falling in a less luminous part of the spectrum, ought to be much more easily perceptible.

Slunce

anglický astronom, **Joseph Norman Lockyer 1836 - 1920,** ***Skvrny na Slunci r. 1874***



CONTRIBUTIONS

TO

SOLAR PHYSICS.

I.

A POPULAR ACCOUNT OF INQUIRIES INTO THE
PHYSICAL CONSTITUTION OF THE SUN,
WITH SPECIAL REFERENCE TO RECENT SPECTROSCOPIC
RESEARCHES;

II.

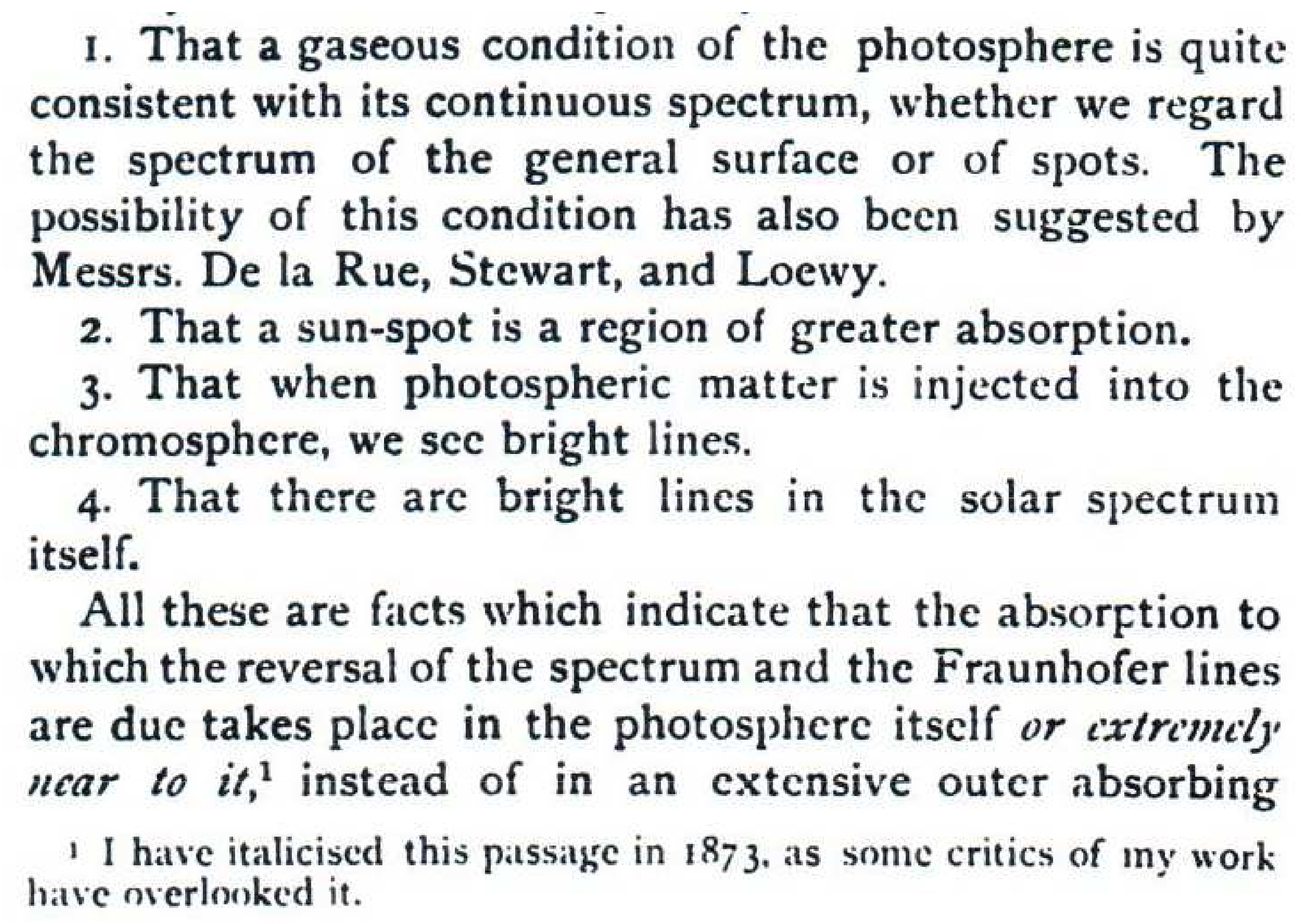
COMMUNICATIONS TO THE ROYAL SOCIETY OF LONDON,
AND THE FRENCH ACADEMY OF SCIENCES,
WITH NOTES.

BY

J. NORMAN LOCKYER, F.R.S.

Slunce

Skvrny na Slunci r. 1874



1. That a gaseous condition of the photosphere is quite consistent with its continuous spectrum, whether we regard the spectrum of the general surface or of spots. The possibility of this condition has also been suggested by Messrs. De la Rue, Stewart, and Loewy.

2. That a sun-spot is a region of greater absorption.

3. That when photospheric matter is injected into the chromosphere, we see bright lines.

4. That there are bright lines in the solar spectrum itself.

All these are facts which indicate that the absorption to which the reversal of the spectrum and the Fraunhofer lines are due takes place in the photosphere itself *or extremely near to it,*[1] instead of in an extensive outer absorbing

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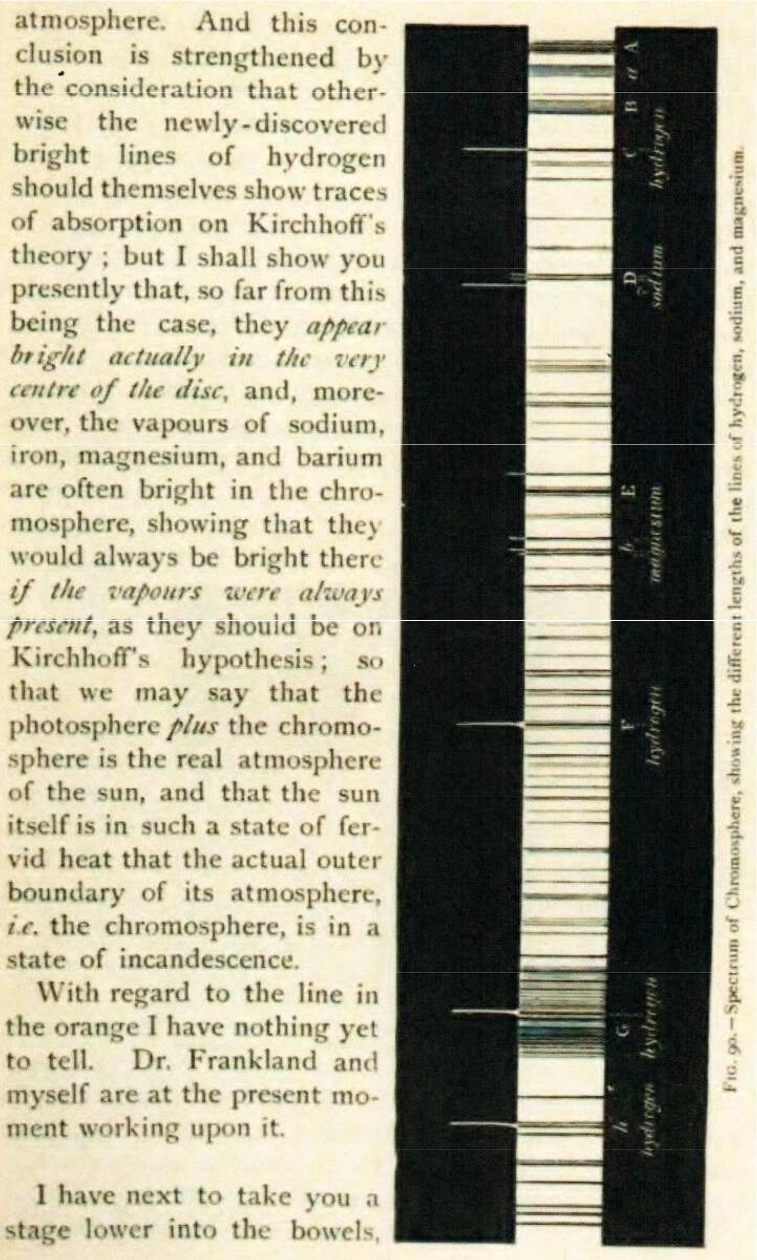
[1] I have italicised this passage in 1873, as some critics of my work have overlooked it.

Slunce

Skvrny na Slunci r. 1874, spektrum chromosféry



atmosphere. And this conclusion is strengthened by the consideration that otherwise the newly-discovered bright lines of hydrogen should themselves show traces of absorption on Kirchhoff's theory; but I shall show you presently that, so far from this being the case, they *appear bright actually in the very centre of the disc*, and, moreover, the vapours of sodium, iron, magnesium, and barium are often bright in the chromosphere, showing that they would always be bright there *if the vapours were always present*, as they should be on Kirchhoff's hypothesis; so that we may say that the photosphere *plus* the chromosphere is the real atmosphere of the sun, and that the sun itself is in such a state of fervid heat that the actual outer boundary of its atmosphere, *i.e.* the chromosphere, is in a state of incandescence.

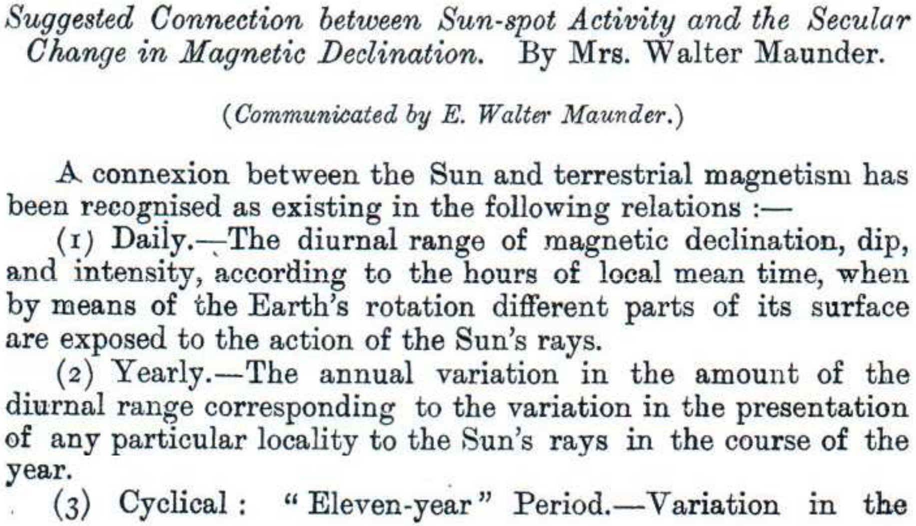
With regard to the line in the orange I have nothing yet to tell. Dr. Frankland and myself are at the present moment working upon it.

I have next to take you a stage lower into the bowels,

FIG. 90.—Spectrum of Chromosphere, showing the different lengths of the lines of hydrogen, sodium, and magnesium.

Slunce

anglický astronom **Edward Walter Maunder 1851 - 1928** ***Navrhované souvislosti mezi aktivitou slunečních skvrn a sekulárními změnami v magnetické deklinaci, r. 1904***, migrace slunečních skvrn v šířce průběhu slunečního cyklu, ***minimum sluneční činnosti 1645 -1715***, **Maunderovo minimum**, ***odhaleno r. 1893***, věděl o něm Spörer



Suggested Connection between Sun-spot Activity and the Secular Change in Magnetic Declination. By Mrs. Walter Maunder.

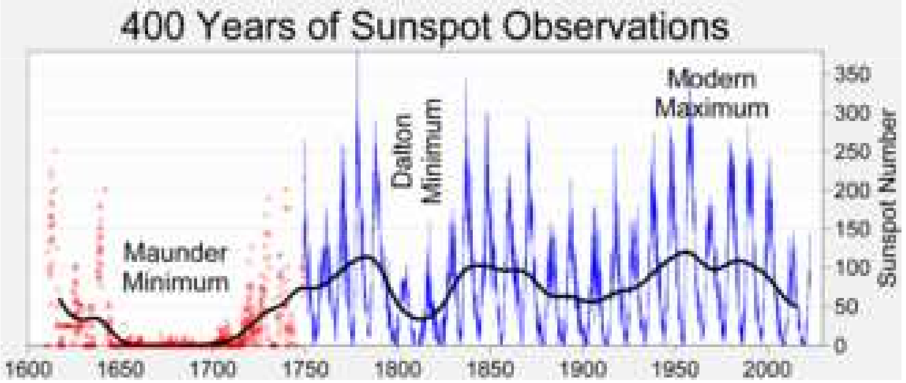
(*Communicated by E. Walter Maunder.*)

A connexion between the Sun and terrestrial magnetism has been recognised as existing in the following relations :—

(1) Daily.—The diurnal range of magnetic declination, dip, and intensity, according to the hours of local mean time, when by means of the Earth's rotation different parts of its surface are exposed to the action of the Sun's rays.

(2) Yearly.—The annual variation in the amount of the diurnal range corresponding to the variation in the presentation of any particular locality to the Sun's rays in the course of the year.

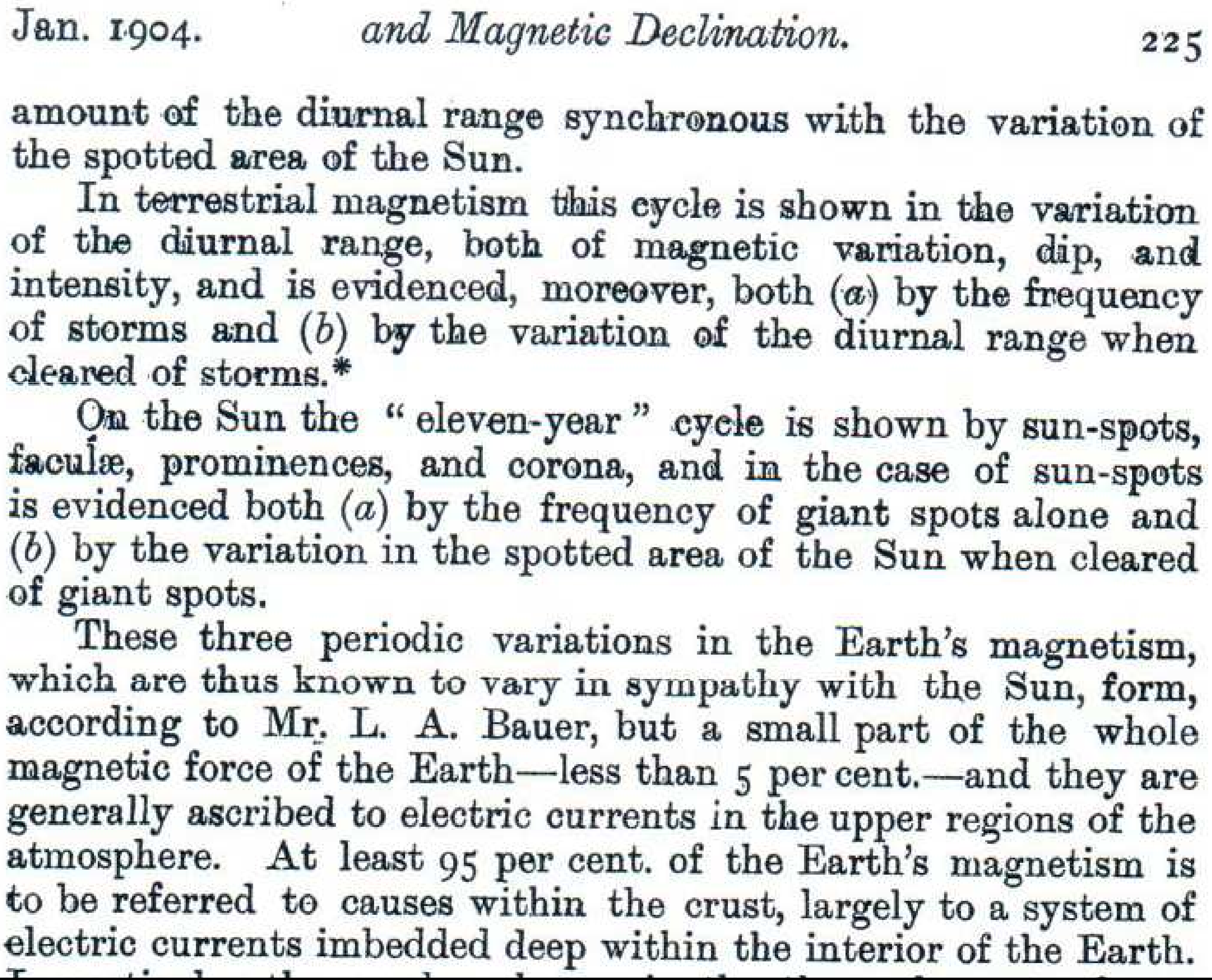
(3) Cyclical : "Eleven-year" Period.—Variation in the



Slunce

manželka Annie Russell Maunder 1868 - 1947…

Navrhované souvislosti mezi aktivitou slunečních skvrn a sekulárními změnami v magnetické deklinaci, r. 1904



Jan. 1904. *and Magnetic Declination.* 225

amount of the diurnal range synchronous with the variation of the spotted area of the Sun.

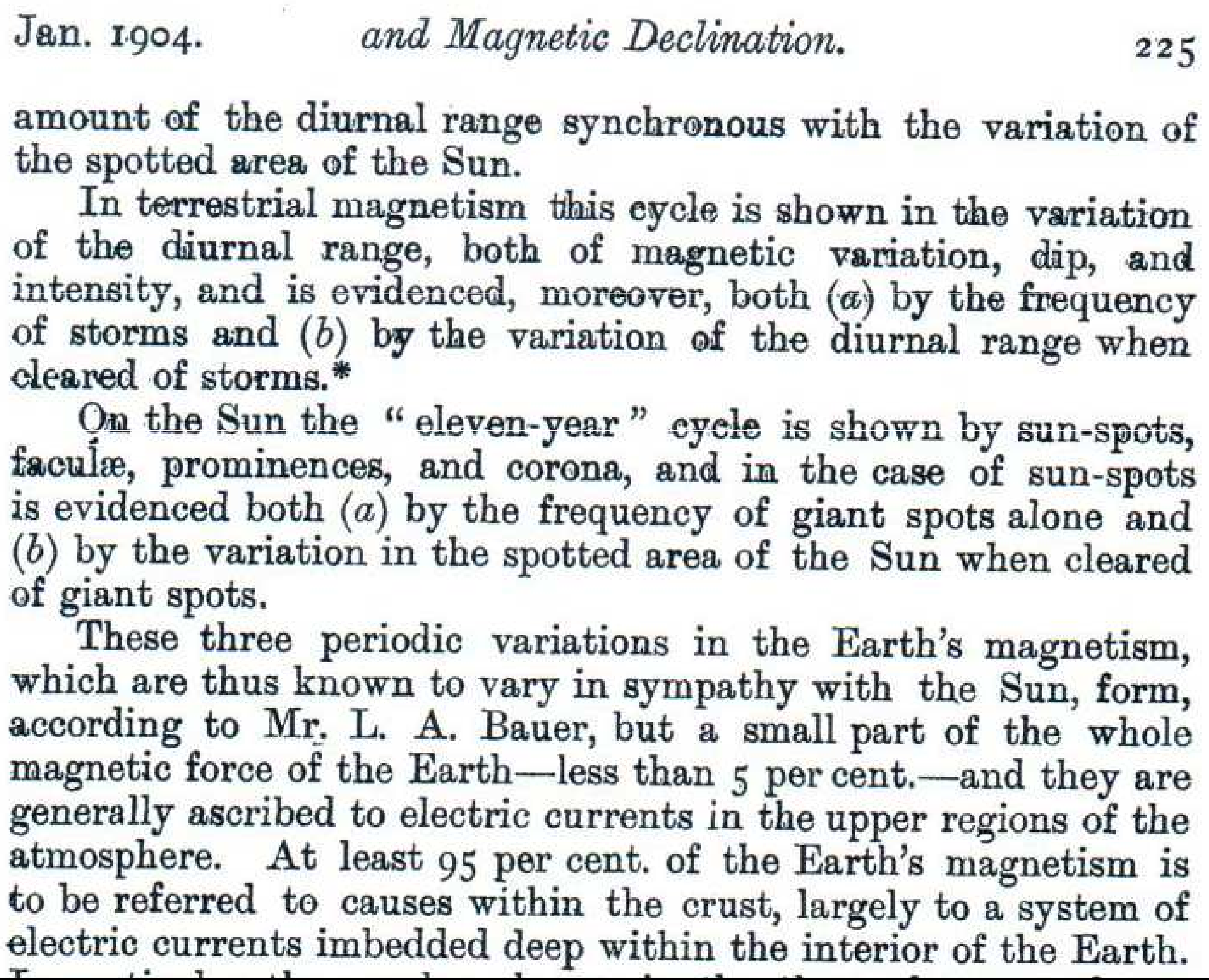
In terrestrial magnetism this cycle is shown in the variation of the diurnal range, both of magnetic variation, dip, and intensity, and is evidenced, moreover, both (*a*) by the frequency of storms and (*b*) by the variation of the diurnal range when cleared of storms.*

On the Sun the "eleven-year" cycle is shown by sun-spots, faculæ, prominences, and corona, and in the case of sun-spots is evidenced both (*a*) by the frequency of giant spots alone and (*b*) by the variation in the spotted area of the Sun when cleared of giant spots.

These three periodic variations in the Earth's magnetism, which are thus known to vary in sympathy with the Sun, form, according to Mr. L. A. Bauer, but a small part of the whole magnetic force of the Earth—less than 5 per cent.—and they are generally ascribed to electric currents in the upper regions of the atmosphere. At least 95 per cent. of the Earth's magnetism is to be referred to causes within the crust, largely to a system of electric currents imbedded deep within the interior of the Earth.

Slunce

Navrhované souvislosti mezi aktivitou slunečních skvrn a sekulárními změnami v magnetické deklinaci, r. 1904



amount of the diurnal range synchronous with the variation of the spotted area of the Sun.

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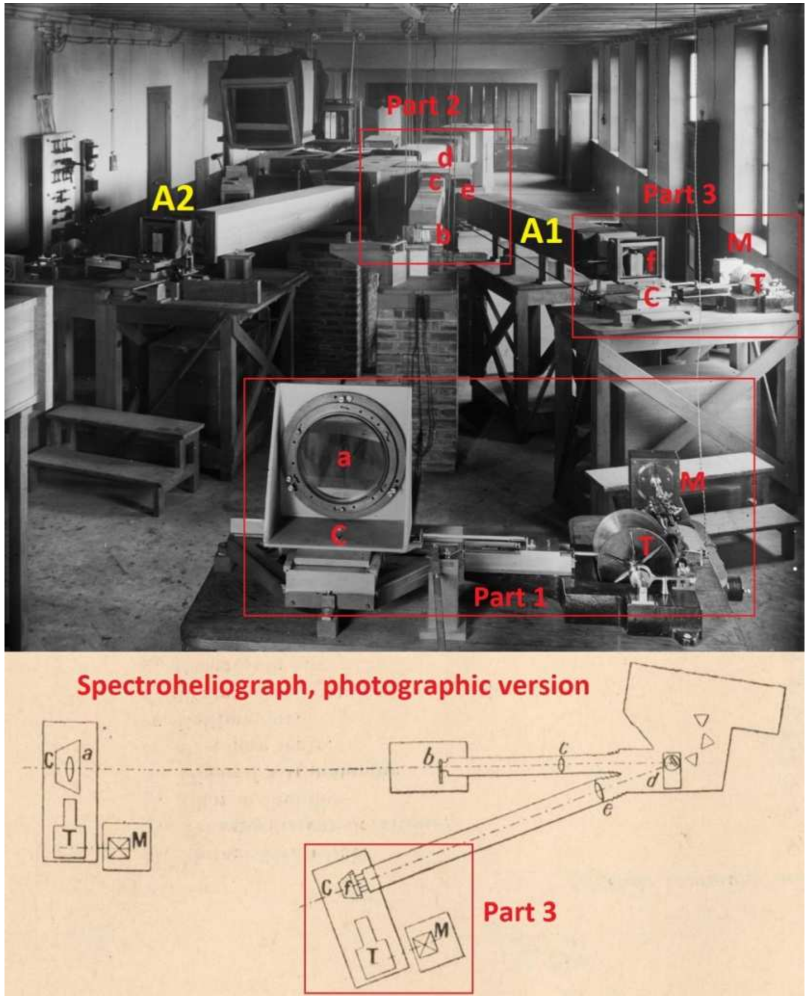
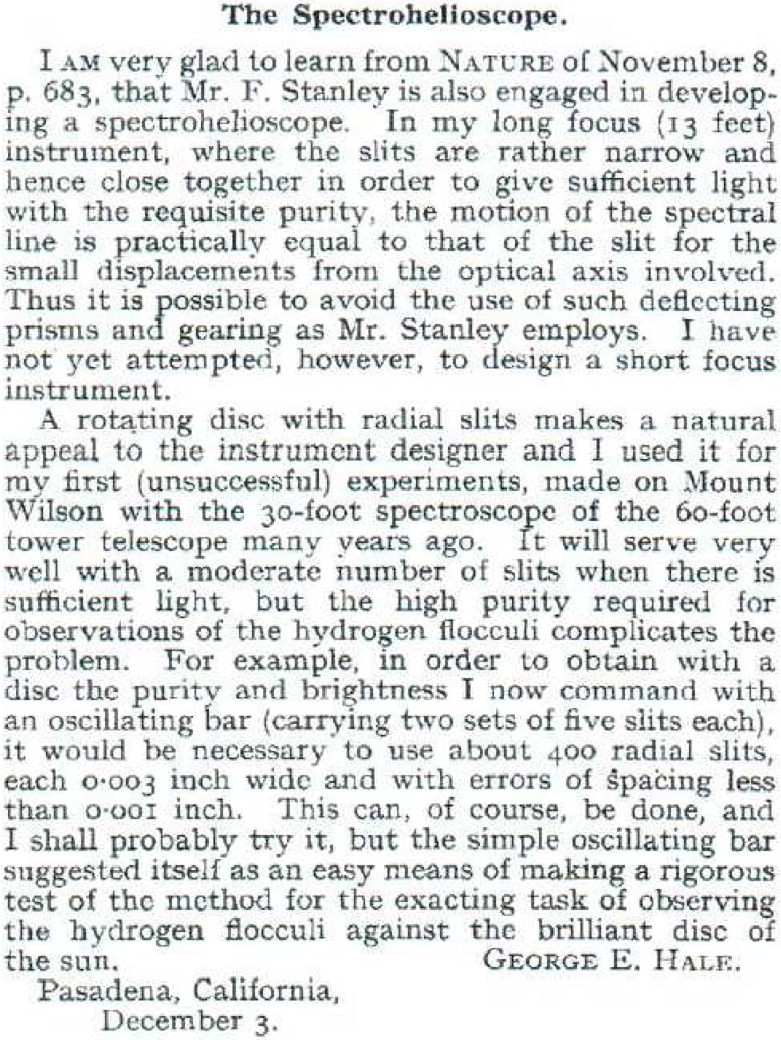
Slunce

francouzský astronom **Henri Alexandre Deslandres 1853 - 1948, Meudon** ***přístroj pro pozorování protuberancí Slunce - spektrohelioskop, spektroheliograf, 1893***



Slunce

americký astronom, **George Ellery Hale 1868 – 1938,** ***přístroj spektrohelioskop r. 1924 -*** sluneční spektroskop, nejčastěji pro čáru Hα**,** pozorování chromosféry, studium erupcí, protuberancí



The Spectrohelioscope.

I AM very glad to learn from NATURE of November 8, p. 683, that Mr. F. Stanley is also engaged in developing a spectrohelioscope. In my long focus (13 feet) instrument, where the slits are rather narrow and hence close together in order to give sufficient light with the requisite purity, the motion of the spectral line is practically equal to that of the slit for the small displacements from the optical axis involved. Thus it is possible to avoid the use of such deflecting prisms and gearing as Mr. Stanley employs. I have not yet attempted, however, to design a short focus instrument.

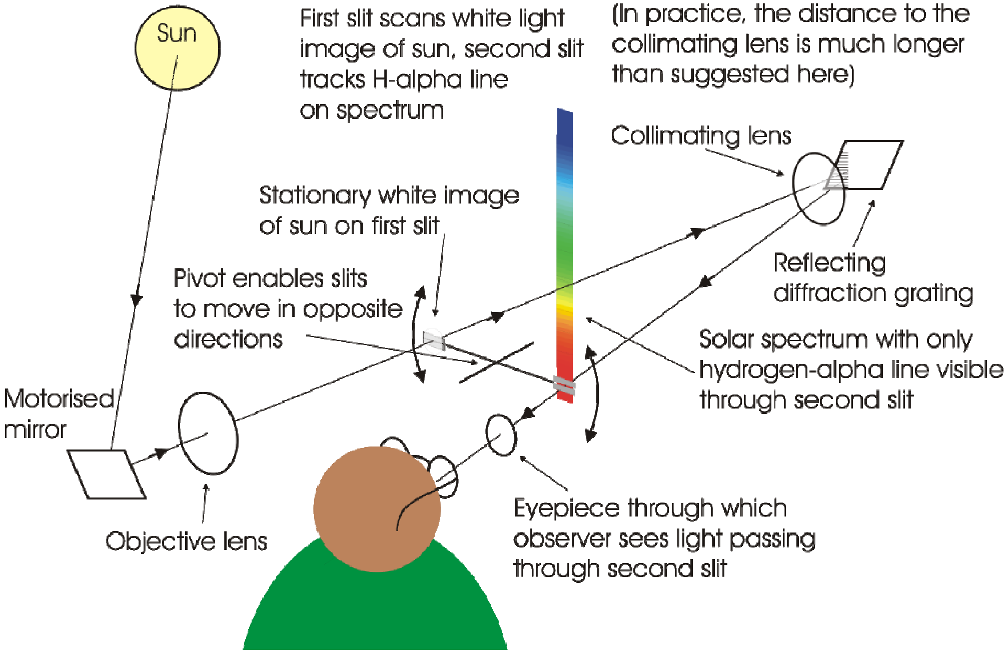
A rotating disc with radial slits makes a natural appeal to the instrument designer and I used it for my first (unsuccessful) experiments, made on Mount Wilson with the 30-foot spectroscope of the 60-foot tower telescope many years ago. It will serve very well with a moderate number of slits when there is sufficient light, but the high purity required for observations of the hydrogen flocculi complicates the problem. For example, in order to obtain with a disc the purity and brightness I now command with an oscillating bar (carrying two sets of five slits each), it would be necessary to use about 400 radial slits, each 0·003 inch wide and with errors of spacing less than 0·001 inch. This can, of course, be done, and I shall probably try it, but the simple oscillating bar suggested itself as an easy means of making a rigorous test of the method for the exacting task of observing the hydrogen flocculi against the brilliant disc of the sun.

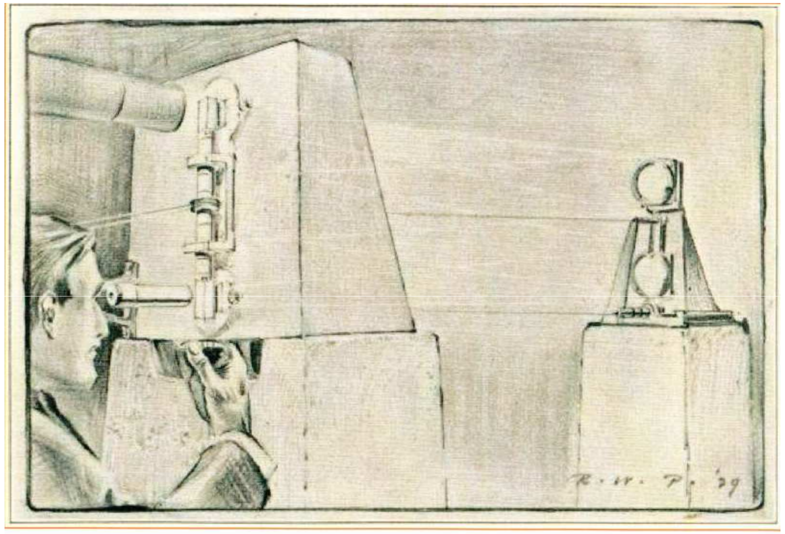
GEORGE E. HALE.

Pasadena, California,
December 3.

Slunce

schema slunečního spektroskopu

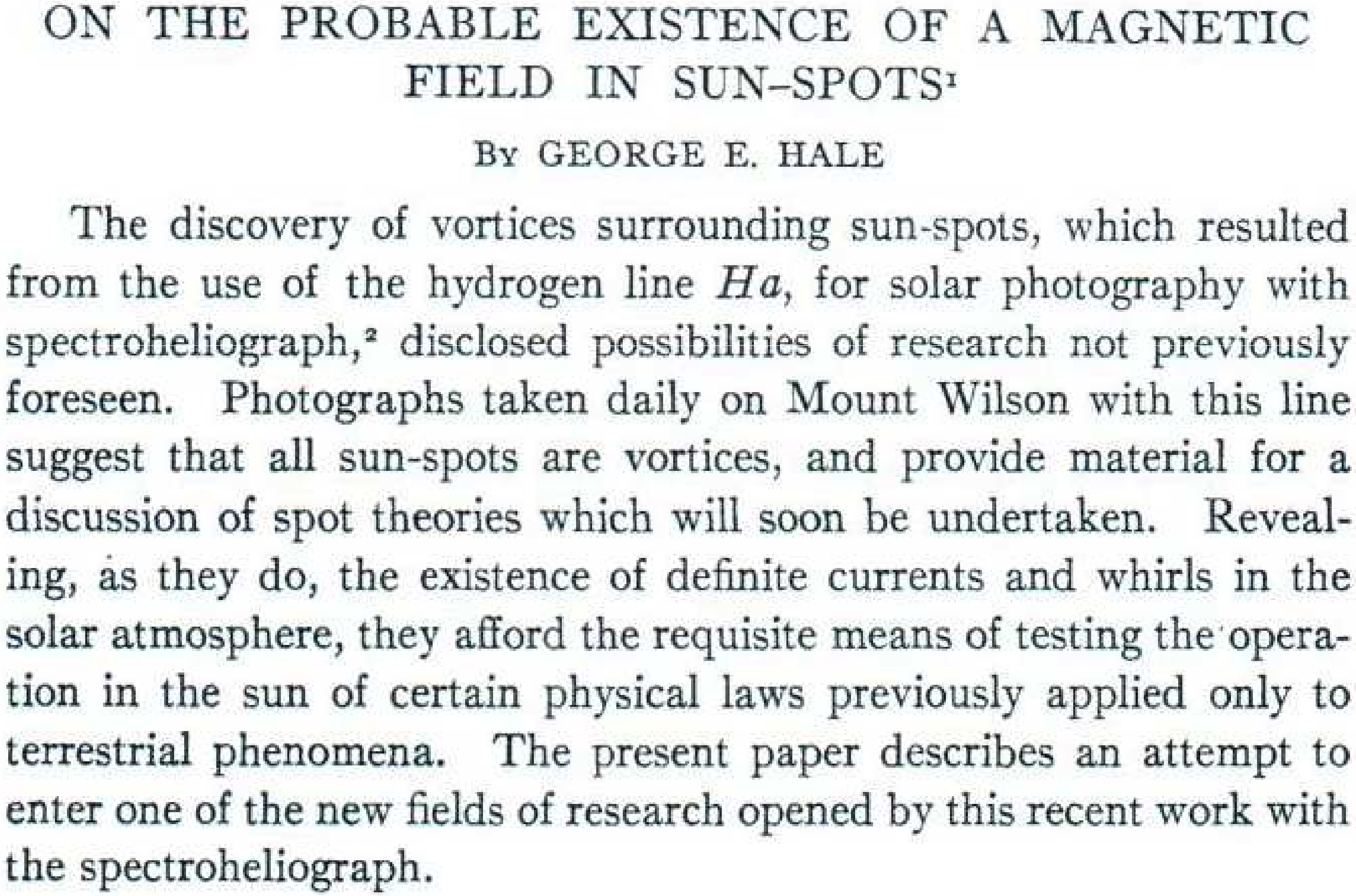




THE HALE
SPECTROHELIOSCOPE

Slunce

Pravděpodobná existence magnetického pole slunečních skvrn r. 1908, objev magnetického pole Slunce ve skvrnách, Zeemanův jev, šířka rozštěpení čar úměrná intenzitě magnetického pole,



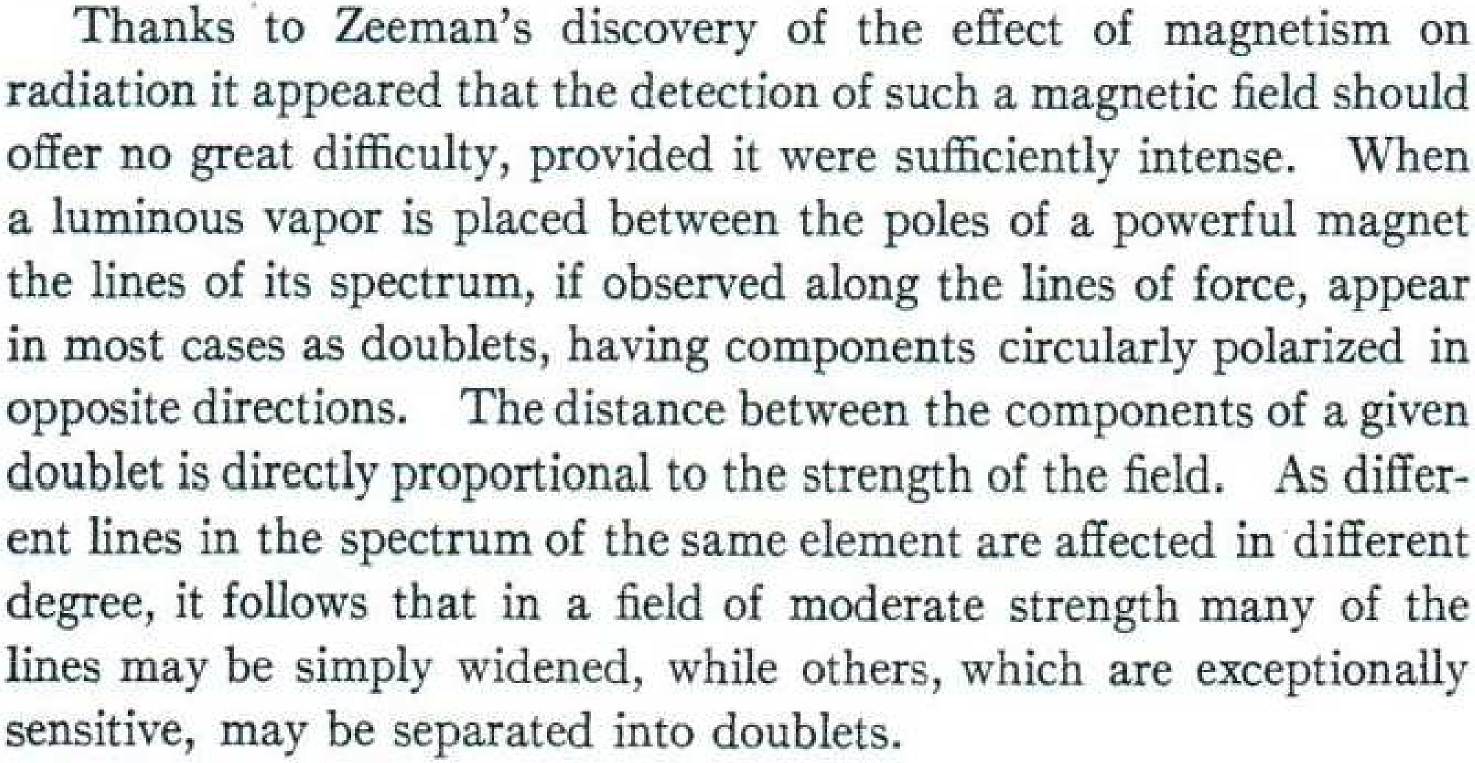
ON THE PROBABLE EXISTENCE OF A MAGNETIC FIELD IN SUN-SPOTS[1]

BY GEORGE E. HALE

The discovery of vortices surrounding sun-spots, which resulted from the use of the hydrogen line $H\alpha$, for solar photography with spectroheliograph,[2] disclosed possibilities of research not previously foreseen. Photographs taken daily on Mount Wilson with this line suggest that all sun-spots are vortices, and provide material for a discussion of spot theories which will soon be undertaken. Revealing, as they do, the existence of definite currents and whirls in the solar atmosphere, they afford the requisite means of testing the operation in the sun of certain physical laws previously applied only to terrestrial phenomena. The present paper describes an attempt to enter one of the new fields of research opened by this recent work with the spectroheliograph.

Slunce

Pravděpodobná existence magnetického pole slunečních skvrn r. 1908, objev magnetického pole Slunce ve skvrnách, Zeemanův jev, šířka rozštěpení čar úměrná intenzitě magnetického pole,



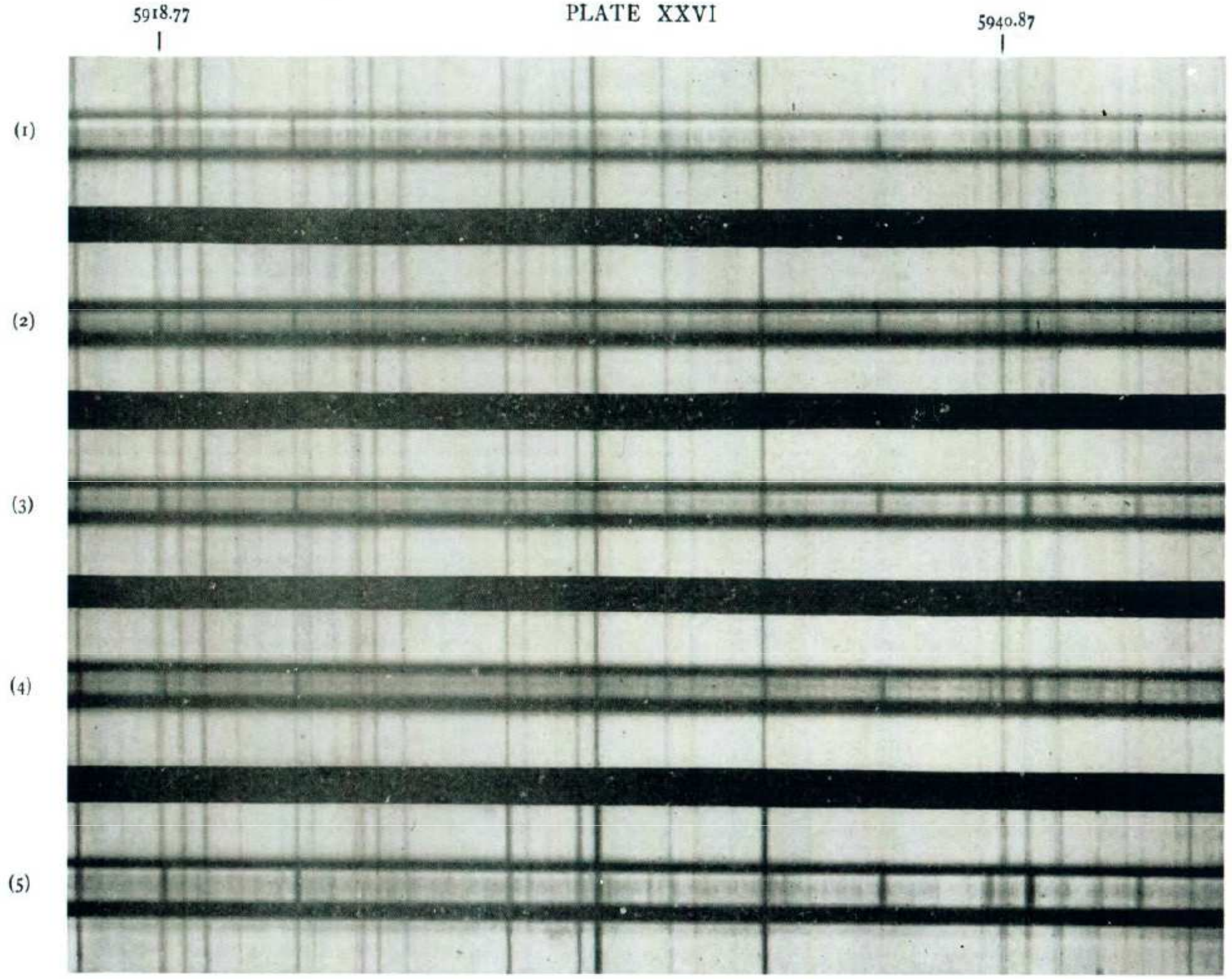
Thanks to Zeeman's discovery of the effect of magnetism on radiation it appeared that the detection of such a magnetic field should offer no great difficulty, provided it were sufficiently intense. When a luminous vapor is placed between the poles of a powerful magnet the lines of its spectrum, if observed along the lines of force, appear in most cases as doublets, having components circularly polarized in opposite directions. The distance between the components of a given doublet is directly proportional to the strength of the field. As different lines in the spectrum of the same element are affected in different degree, it follows that in a field of moderate strength many of the lines may be simply widened, while others, which are exceptionally sensitive, may be separated into doublets.

Slunce

Pravděpodobná existence magnetického pole slunečních skvrn r. 1908,

Zeemanův jev, šířka rozštěpení čar úměrná intenzitě magnetického pole,

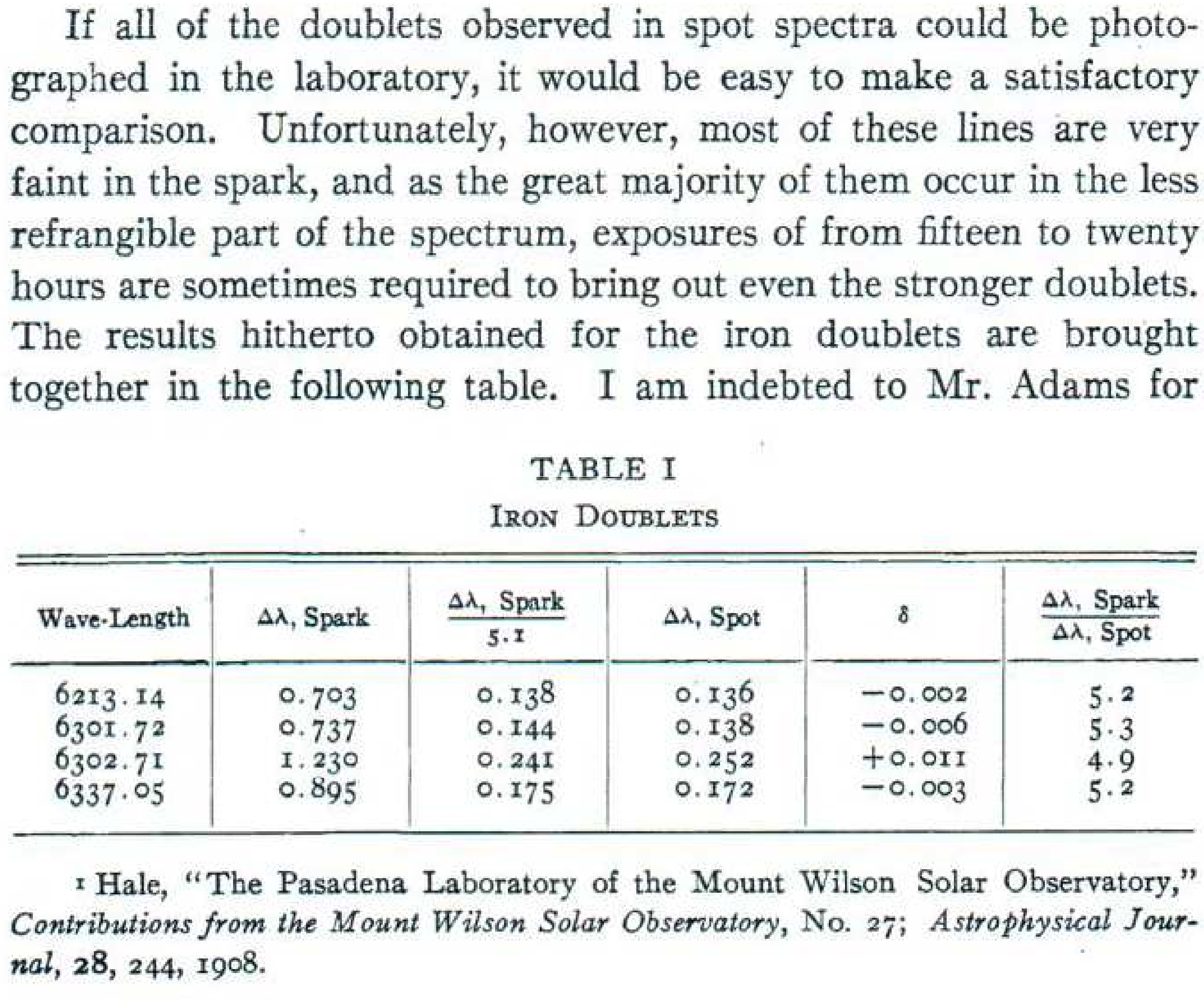
PLATE XXVI



(1) Southern spot, showing red components of doublets. Nicol 29° W. (2) One umbra of northern spot, showing violet components of doublets. Nicol 29° W. (3) Other umbra of northern spot, showing violet components of doublets. Nicol 29° W. (4) Some umbra of northern spot, showing red components of doublets. Nicol 61° E. (5) Spot spectrum without rhomb or Nicol, showing both components of doublets. Scale: 1 Ångström=6 mm.

Slunce

Pravděpodobná existence magnetického pole slunečních skvrn r. 1908,



If all of the doublets observed in spot spectra could be photographed in the laboratory, it would be easy to make a satisfactory comparison. Unfortunately, however, most of these lines are very faint in the spark, and as the great majority of them occur in the less refrangible part of the spectrum, exposures of from fifteen to twenty hours are sometimes required to bring out even the stronger doublets. The results hitherto obtained for the iron doublets are brought together in the following table. I am indebted to Mr. Adams for

TABLE I

IRON DOUBLETS

Wave-Length	Δλ, Spark	$\frac{\Delta\lambda,\ \text{Spark}}{5.1}$	Δλ, Spot	δ	$\frac{\Delta\lambda,\ \text{Spark}}{\Delta\lambda,\ \text{Spot}}$
6213.14	0.703	0.138	0.136	−0.002	5.2
6301.72	0.737	0.144	0.138	−0.006	5.3
6302.71	1.230	0.241	0.252	+0.011	4.9
6337.05	0.895	0.175	0.172	−0.003	5.2

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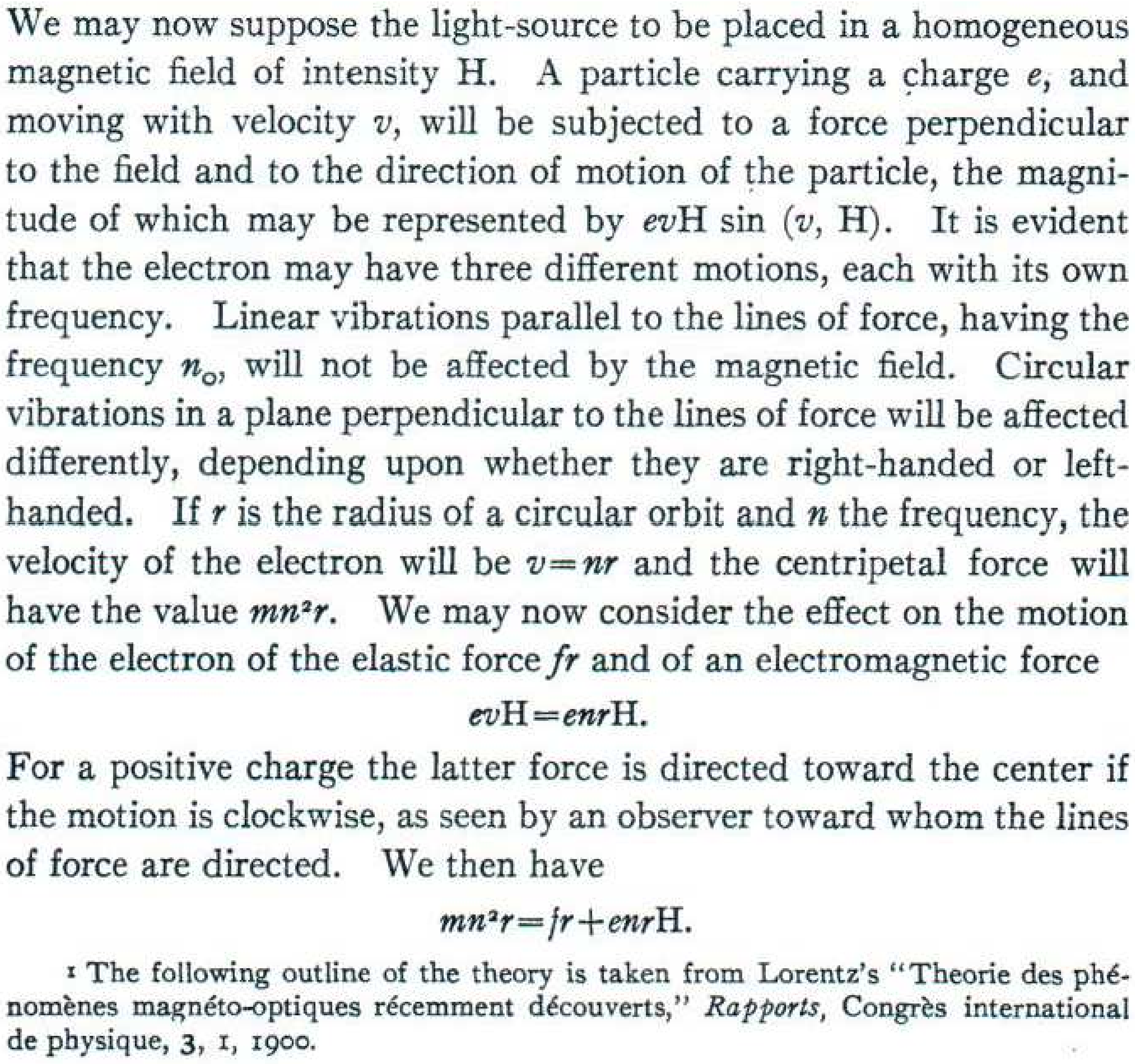
[1] Hale, "The Pasadena Laboratory of the Mount Wilson Solar Observatory," *Contributions from the Mount Wilson Solar Observatory*, No. 27; *Astrophysical Journal*, **28**, 244, 1908.

Slunce

Pravděpodobná existence magnetického pole slunečních skvrn r. 1908,



We may now suppose the light-source to be placed in a homogeneous magnetic field of intensity H. A particle carrying a charge e, and moving with velocity v, will be subjected to a force perpendicular to the field and to the direction of motion of the particle, the magnitude of which may be represented by $ev\mathrm{H} \sin (v, \mathrm{H})$. It is evident that the electron may have three different motions, each with its own frequency. Linear vibrations parallel to the lines of force, having the frequency n_0, will not be affected by the magnetic field. Circular vibrations in a plane perpendicular to the lines of force will be affected differently, depending upon whether they are right-handed or left-handed. If r is the radius of a circular orbit and n the frequency, the velocity of the electron will be $v=nr$ and the centripetal force will have the value mn^2r. We may now consider the effect on the motion of the electron of the elastic force fr and of an electromagnetic force

$$ev\mathrm{H}=enr\mathrm{H}.$$

For a positive charge the latter force is directed toward the center if the motion is clockwise, as seen by an observer toward whom the lines of force are directed. We then have

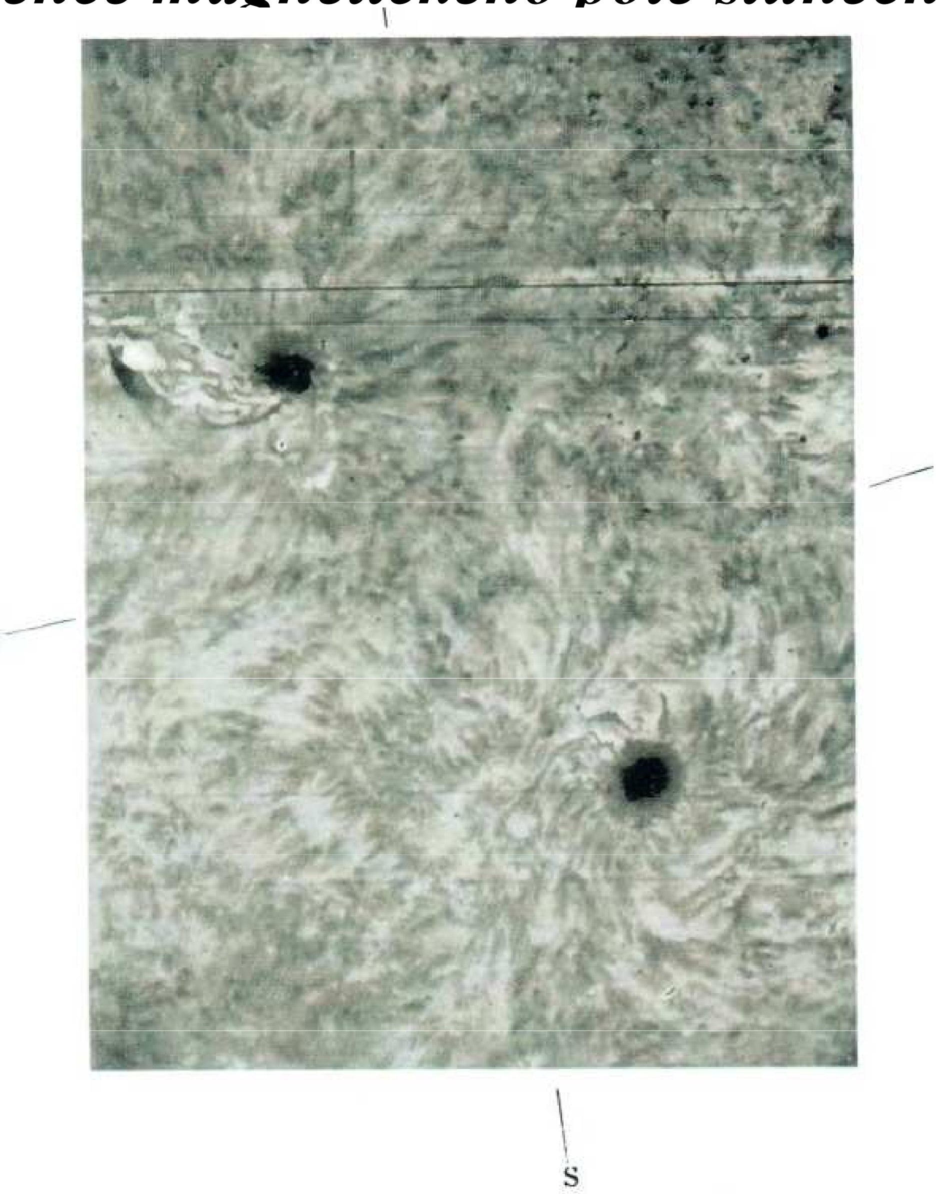
$$mn^2r=fr+enr\mathrm{H}.$$

---

[1] The following outline of the theory is taken from Lorentz's "Theorie des phénomènes magnéto-optiques récemment découverts," *Rapports*, Congrès international de physique, 3, 1, 1900.

Slunce

Pravděpodobná existence magnetického pole slunečních skvrn r. 1908,

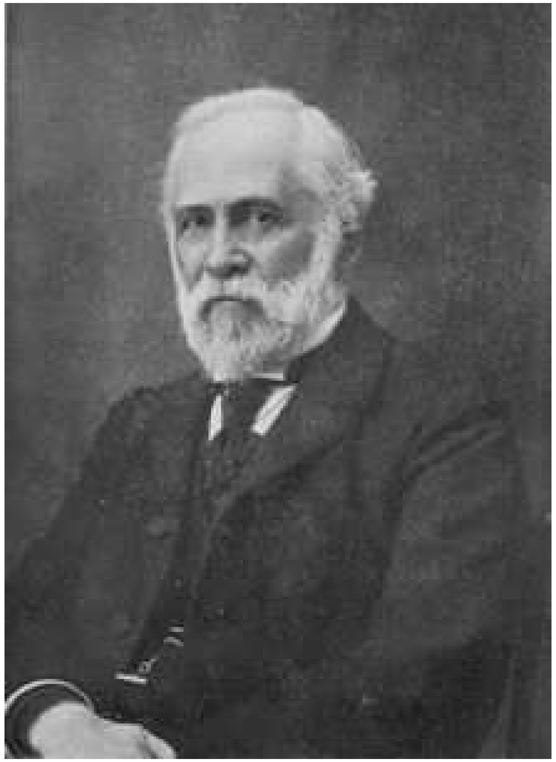


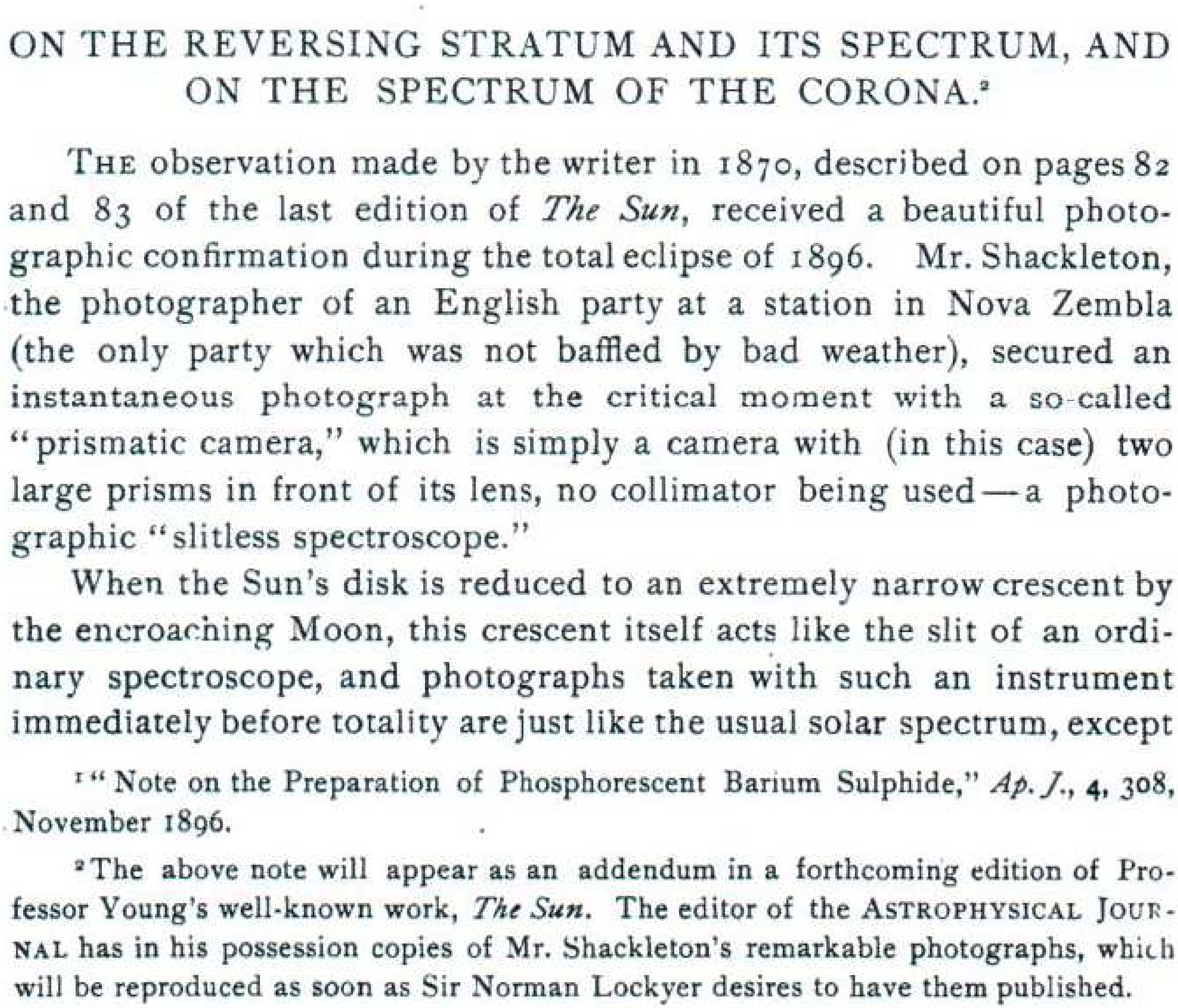
Sun-Spots and Hydrogen Flocculi, Showing Right- and Left-Handed Vortices

1908, September 9, 6^h 20^m A. M. Scale: Sun's Diameter=0.3 Meter

Slunce

americký astronom **Charles August Young 1834 - 1908,** ***Převracející vrstva a její spektrum a spektrum koróny r. 1897,*** zavedl **pojem převracející vrstva –** vznik absorpčních čar, koróna částí atmosféry Slunce





ON THE REVERSING STRATUM AND ITS SPECTRUM, AND ON THE SPECTRUM OF THE CORONA.[2]

THE observation made by the writer in 1870, described on pages 82 and 83 of the last edition of *The Sun*, received a beautiful photographic confirmation during the total eclipse of 1896. Mr. Shackleton, the photographer of an English party at a station in Nova Zembla (the only party which was not baffled by bad weather), secured an instantaneous photograph at the critical moment with a so-called "prismatic camera," which is simply a camera with (in this case) two large prisms in front of its lens, no collimator being used — a photographic "slitless spectroscope."

When the Sun's disk is reduced to an extremely narrow crescent by the encroaching Moon, this crescent itself acts like the slit of an ordinary spectroscope, and photographs taken with such an instrument immediately before totality are just like the usual solar spectrum, except

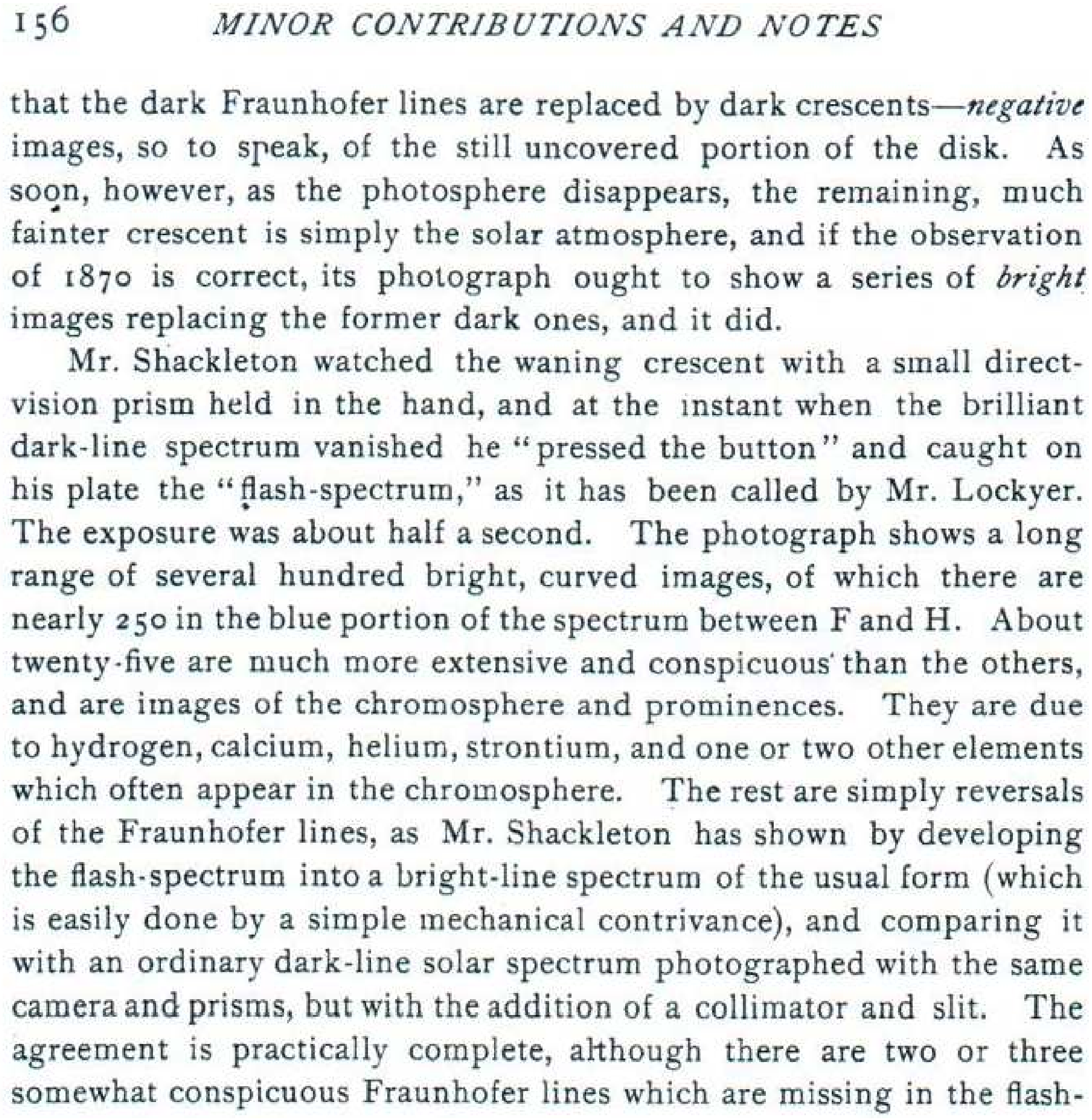
---

[1] "Note on the Preparation of Phosphorescent Barium Sulphide," *Ap. J.*, 4, 308, November 1896.

[2] The above note will appear as an addendum in a forthcoming edition of Professor Young's well-known work, *The Sun*. The editor of the ASTROPHYSICAL JOURNAL has in his possession copies of Mr. Shackleton's remarkable photographs, which will be reproduced as soon as Sir Norman Lockyer desires to have them published.

Slunce

Převracející vrstva a její spektrum a spektrum koróny r. 1897



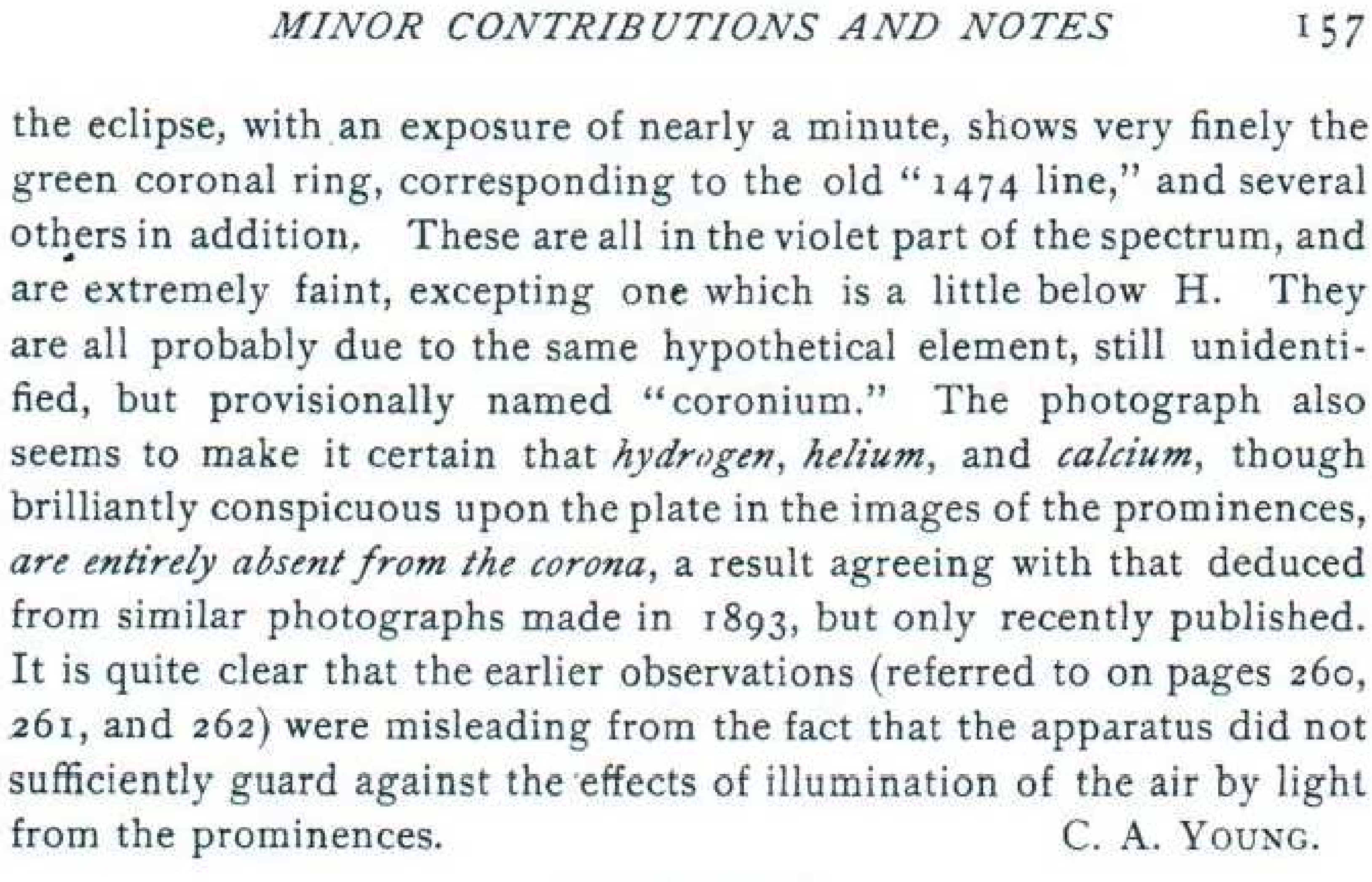
156 *MINOR CONTRIBUTIONS AND NOTES*

that the dark Fraunhofer lines are replaced by dark crescents—*negative* images, so to speak, of the still uncovered portion of the disk. As soon, however, as the photosphere disappears, the remaining, much fainter crescent is simply the solar atmosphere, and if the observation of 1870 is correct, its photograph ought to show a series of *bright* images replacing the former dark ones, and it did.

Mr. Shackleton watched the waning crescent with a small direct-vision prism held in the hand, and at the instant when the brilliant dark-line spectrum vanished he "pressed the button" and caught on his plate the "flash-spectrum," as it has been called by Mr. Lockyer. The exposure was about half a second. The photograph shows a long range of several hundred bright, curved images, of which there are nearly 250 in the blue portion of the spectrum between F and H. About twenty-five are much more extensive and conspicuous than the others, and are images of the chromosphere and prominences. They are due to hydrogen, calcium, helium, strontium, and one or two other elements which often appear in the chromosphere. The rest are simply reversals of the Fraunhofer lines, as Mr. Shackleton has shown by developing the flash-spectrum into a bright-line spectrum of the usual form (which is easily done by a simple mechanical contrivance), and comparing it with an ordinary dark-line solar spectrum photographed with the same camera and prisms, but with the addition of a collimator and slit. The agreement is practically complete, although there are two or three somewhat conspicuous Fraunhofer lines which are missing in the flash-

Slunce

Převracející vrstva a její spektrum a spektrum koróny r. 1897



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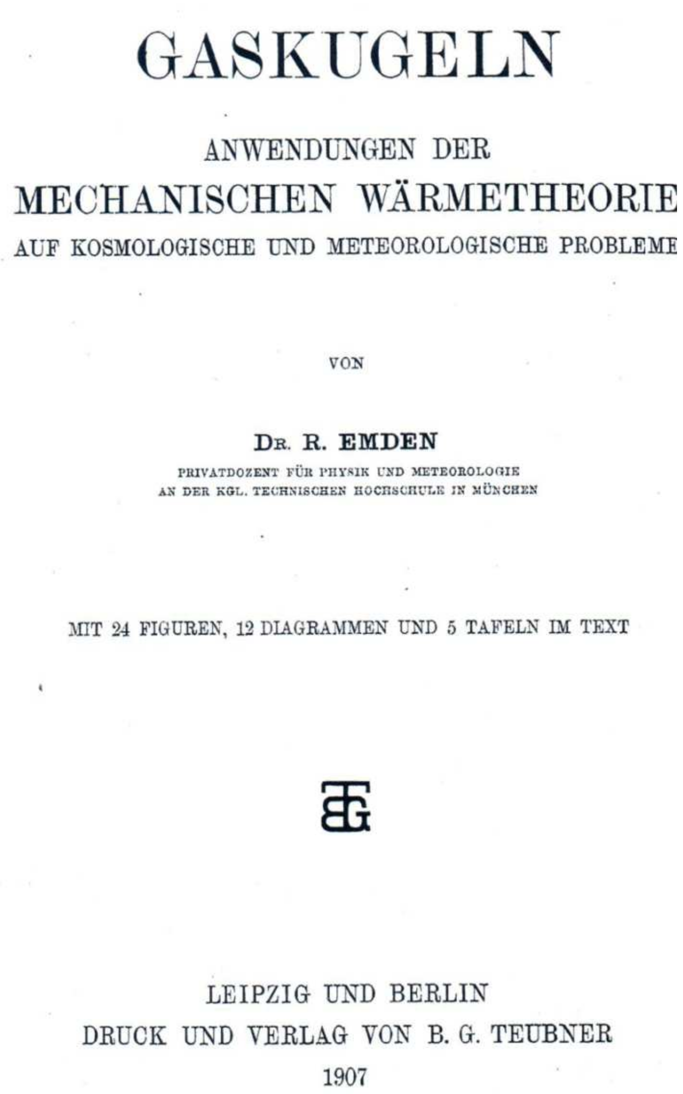
the eclipse, with an exposure of nearly a minute, shows very finely the green coronal ring, corresponding to the old "1474 line," and several others in addition. These are all in the violet part of the spectrum, and are extremely faint, excepting one which is a little below H. They are all probably due to the same hypothetical element, still unidentified, but provisionally named "coronium." The photograph also seems to make it certain that *hydrogen*, *helium*, and *calcium*, though brilliantly conspicuous upon the plate in the images of the prominences, *are entirely absent from the corona*, a result agreeing with that deduced from similar photographs made in 1893, but only recently published. It is quite clear that the earlier observations (referred to on pages 260, 261, and 262) were misleading from the fact that the apparatus did not sufficiently guard against the effects of illumination of the air by light from the prominences.

C. A. YOUNG.

Hvězdy

švýcarský astrofyzik, meteorolog **Robert Emden 1862 - 1940,** ***Plynné koule, r. 1907,*** Laneova-Emdenova rovnice,



GASKUGELN

ANWENDUNGEN DER
MECHANISCHEN WÄRMETHEORIE
AUF KOSMOLOGISCHE UND METEOROLOGISCHE PROBLEME

VON

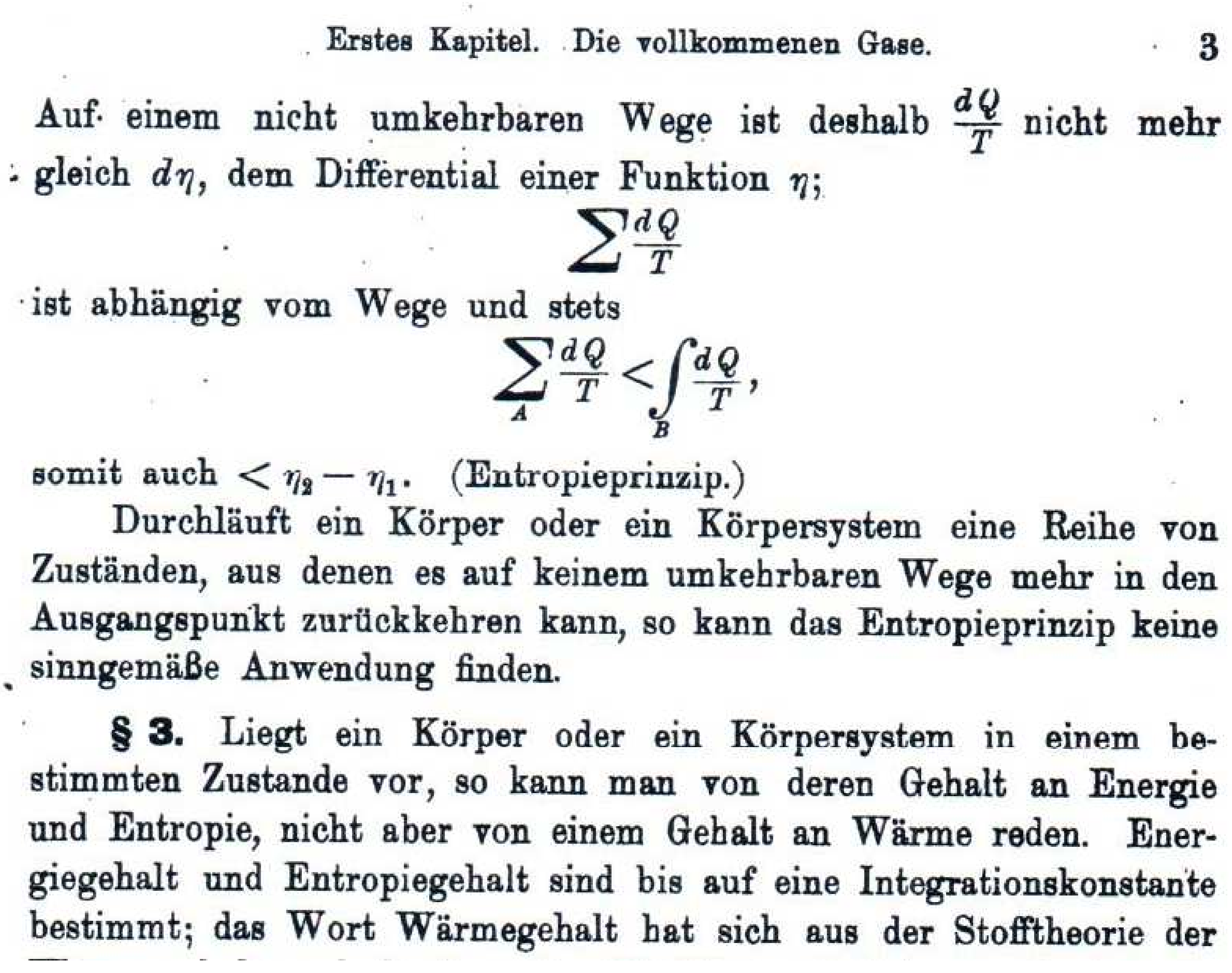
DR. R. EMDEN
PRIVATDOZENT FÜR PHYSIK UND METEOROLOGIE
AN DER KGL. TECHNISCHEN HOCHSCHULE IN MÜNCHEN

MIT 24 FIGUREN, 12 DIAGRAMMEN UND 5 TAFELN IM TEXT

LEIPZIG UND BERLIN
DRUCK UND VERLAG VON B. G. TEUBNER
1907

Hvězdy

Emden - Plynné koule, r. 1907,



Erstes Kapitel. Die vollkommenen Gase. 3

Auf einem nicht umkehrbaren Wege ist deshalb $\frac{dQ}{T}$ nicht mehr gleich $d\eta$, dem Differential einer Funktion η;

$$\sum \frac{dQ}{T}$$

ist abhängig vom Wege und stets

$$\sum_{A} \frac{dQ}{T} < \int_{B} \frac{dQ}{T},$$

somit auch $< \eta_2 - \eta_1$. (Entropieprinzip.)

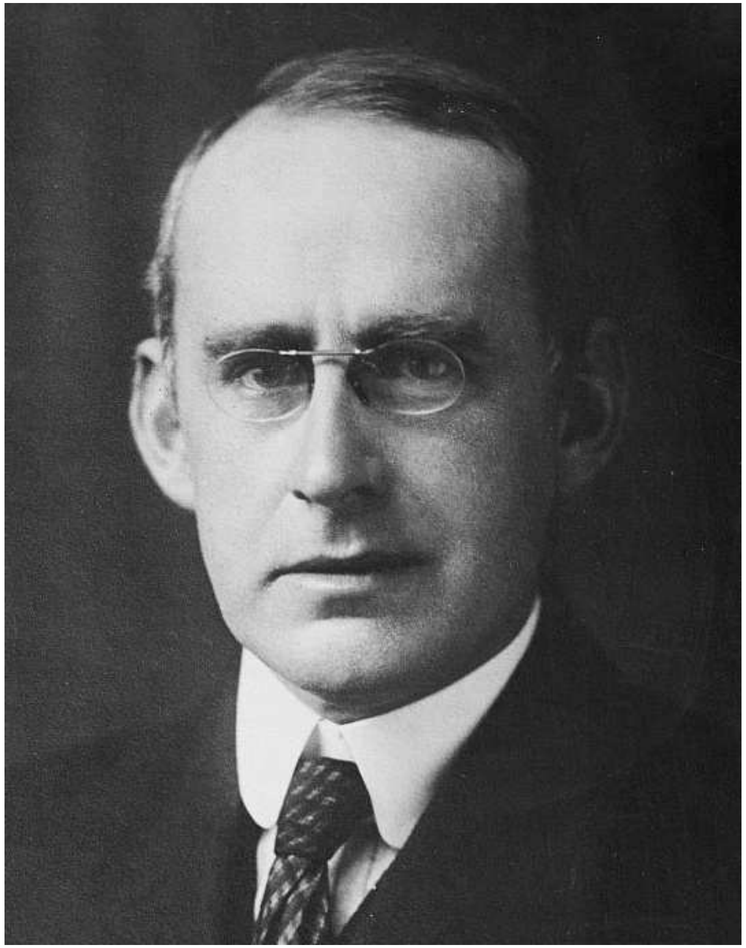
Durchläuft ein Körper oder ein Körpersystem eine Reihe von Zuständen, aus denen es auf keinem umkehrbaren Wege mehr in den Ausgangspunkt zurückkehren kann, so kann das Entropieprinzip keine sinngemäße Anwendung finden.

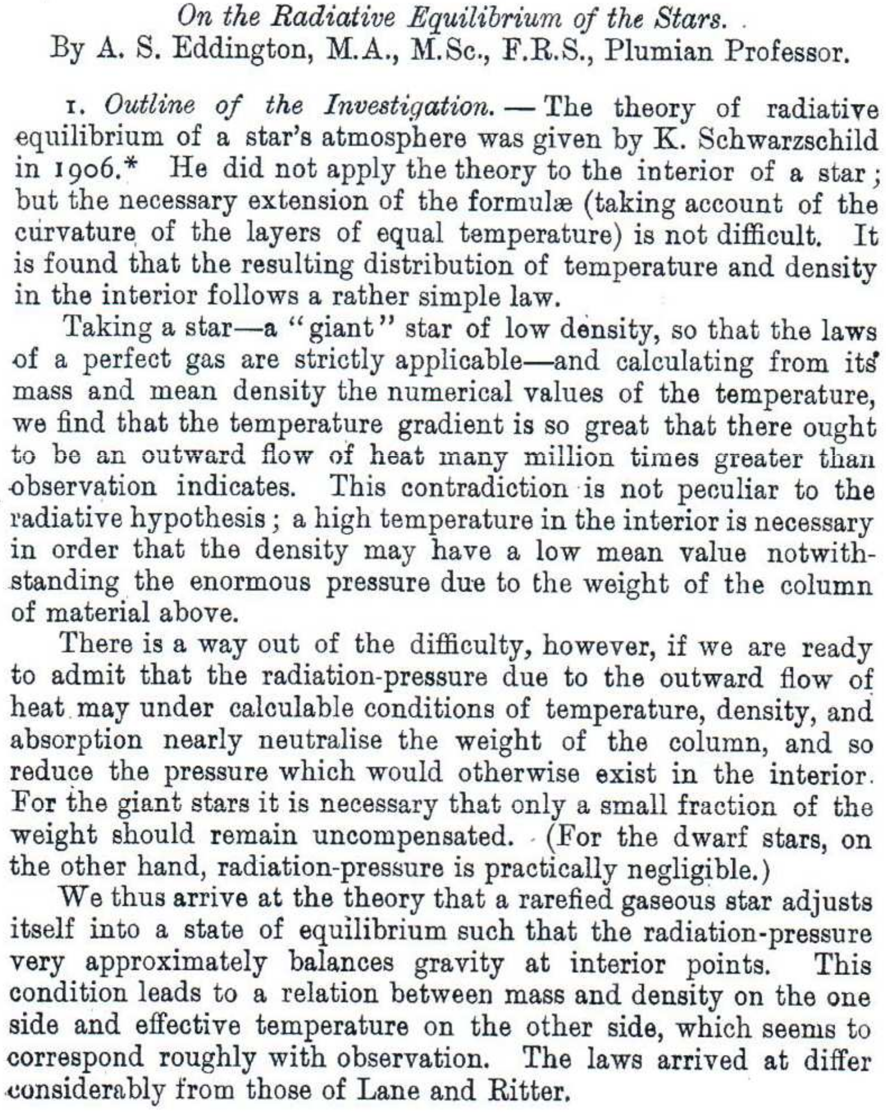
§ 3. Liegt ein Körper oder ein Körpersystem in einem bestimmten Zustande vor, so kann man von deren Gehalt an Energie und Entropie, nicht aber von einem Gehalt an Wärme reden. Energiegehalt und Entropiegehalt sind bis auf eine Integrationskonstante bestimmt; das Wort Wärmegehalt hat sich aus der Stofftheorie der

Hvězdy

Artur Stanley Eddington 1882 - 1944

anglický astrofyzik, ***Zářivá rovnováha ve hvězdách, r. 1916***





On the Radiative Equilibrium of the Stars.

By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor.

1. *Outline of the Investigation.* — The theory of radiative equilibrium of a star's atmosphere was given by K. Schwarzschild in 1906.* He did not apply the theory to the interior of a star; but the necessary extension of the formulæ (taking account of the curvature of the layers of equal temperature) is not difficult. It is found that the resulting distribution of temperature and density in the interior follows a rather simple law.

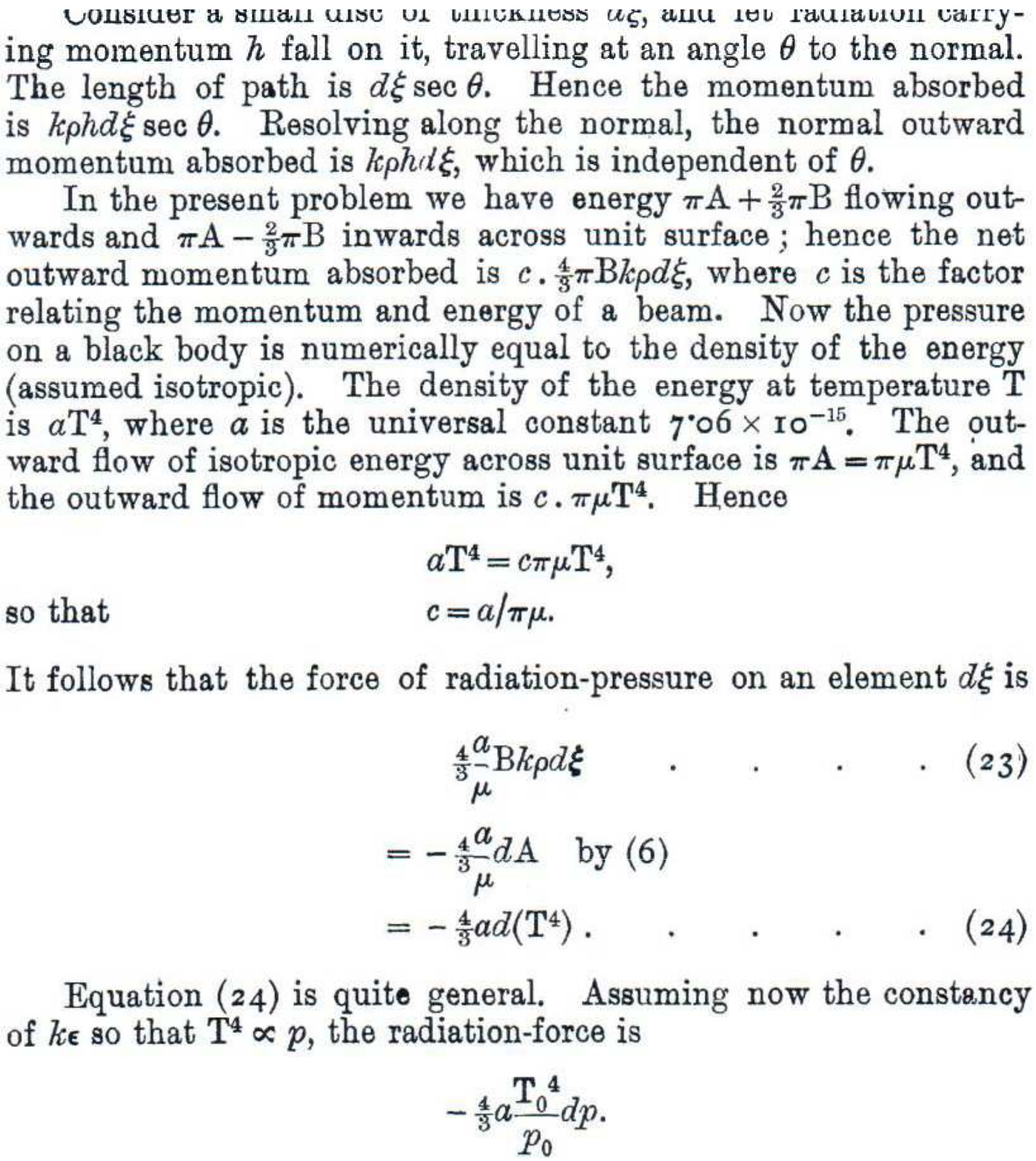
Taking a star—a "giant" star of low density, so that the laws of a perfect gas are strictly applicable—and calculating from its mass and mean density the numerical values of the temperature, we find that the temperature gradient is so great that there ought to be an outward flow of heat many million times greater than observation indicates. This contradiction is not peculiar to the radiative hypothesis; a high temperature in the interior is necessary in order that the density may have a low mean value notwithstanding the enormous pressure due to the weight of the column of material above.

There is a way out of the difficulty, however, if we are ready to admit that the radiation-pressure due to the outward flow of heat may under calculable conditions of temperature, density, and absorption nearly neutralise the weight of the column, and so reduce the pressure which would otherwise exist in the interior. For the giant stars it is necessary that only a small fraction of the weight should remain uncompensated. (For the dwarf stars, on the other hand, radiation-pressure is practically negligible.)

We thus arrive at the theory that a rarefied gaseous star adjusts itself into a state of equilibrium such that the radiation-pressure very approximately balances gravity at interior points. This condition leads to a relation between mass and density on the one side and effective temperature on the other side, which seems to correspond roughly with observation. The laws arrived at differ considerably from those of Lane and Ritter.

Hvězdy - Artur Stanley Eddington

anglický astrofyzik, ***Zářivá rovnováha ve hvězdách, r. 1916***



Consider a small disc of thickness $d\xi$, and let radiation carrying momentum h fall on it, travelling at an angle θ to the normal. The length of path is $d\xi \sec\theta$. Hence the momentum absorbed is $k\rho h d\xi \sec\theta$. Resolving along the normal, the normal outward momentum absorbed is $k\rho h d\xi$, which is independent of θ.

In the present problem we have energy $\pi A + \frac{2}{3}\pi B$ flowing outwards and $\pi A - \frac{2}{3}\pi B$ inwards across unit surface; hence the net outward momentum absorbed is $c \,.\, \frac{4}{3}\pi B k\rho d\xi$, where c is the factor relating the momentum and energy of a beam. Now the pressure on a black body is numerically equal to the density of the energy (assumed isotropic). The density of the energy at temperature T is aT^4, where a is the universal constant $7{\cdot}06 \times 10^{-15}$. The outward flow of isotropic energy across unit surface is $\pi A = \pi\mu T^4$, and the outward flow of momentum is $c \,.\, \pi\mu T^4$. Hence

$$aT^4 = c\pi\mu T^4,$$

so that

$$c = a/\pi\mu.$$

It follows that the force of radiation-pressure on an element $d\xi$ is

$$\frac{4}{3}\frac{a}{\mu} B k\rho d\xi \quad . \quad . \quad . \quad . \quad (23)$$

$$= -\frac{4}{3}\frac{a}{\mu} dA \quad \text{by } (6)$$

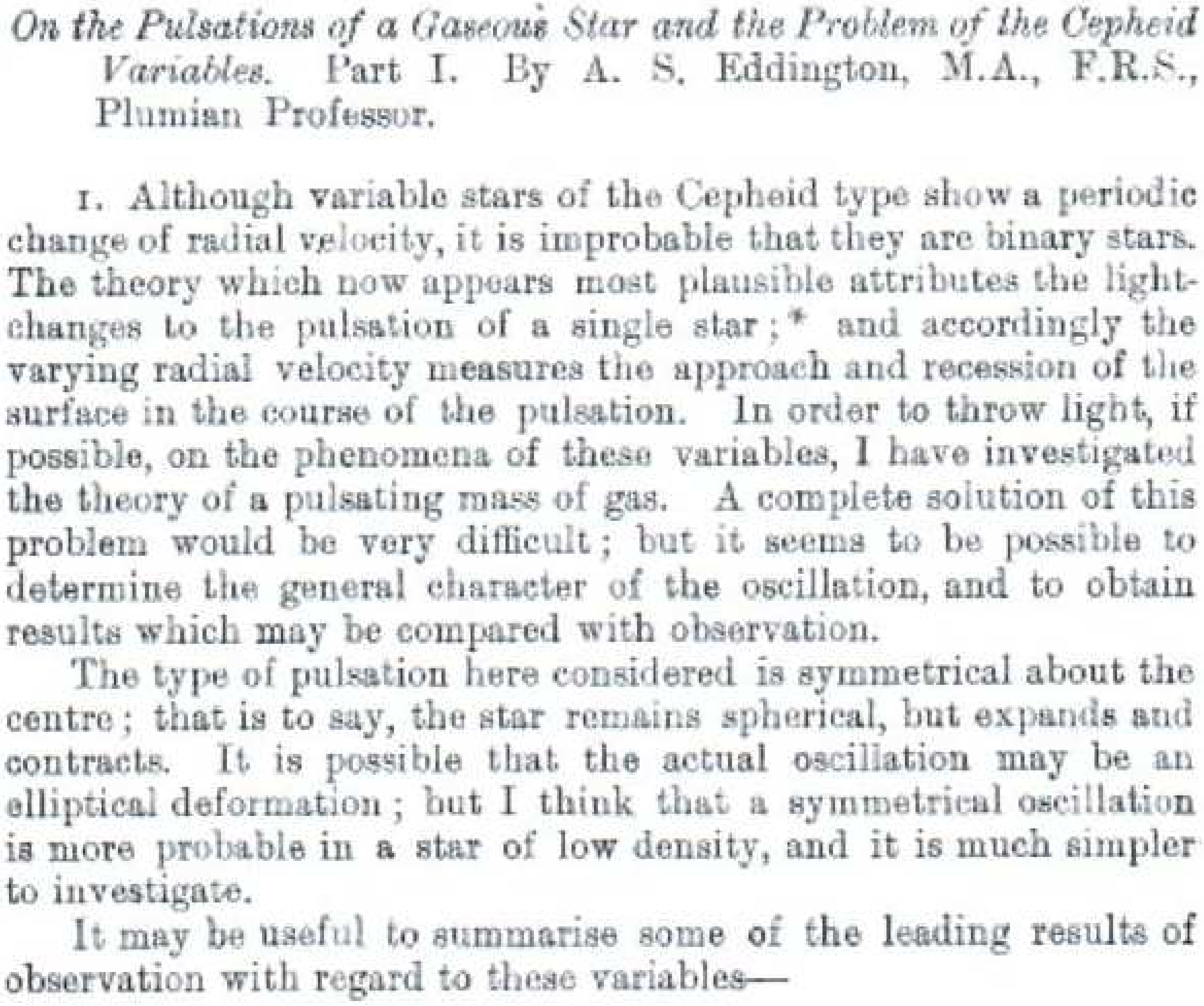
$$= -\frac{4}{3} a d(T^4) \,. \quad . \quad . \quad . \quad . \quad (24)$$

Equation (24) is quite general. Assuming now the constancy of $k\epsilon$ so that $T^4 \propto p$, the radiation-force is

$$-\frac{4}{3} a \frac{T_0{}^4}{p_0} dp.$$

Hvězdy
Artur Stanley Eddington

anglický astrofyzik, ***r. 1918/19 - O pulsacích plynných hvězd a problému cefeid***



On the Pulsations of a Gaseous Star and the Problem of the Cepheid Variables. Part I. By A. S. Eddington, M.A., F.R.S., Plumian Professor.

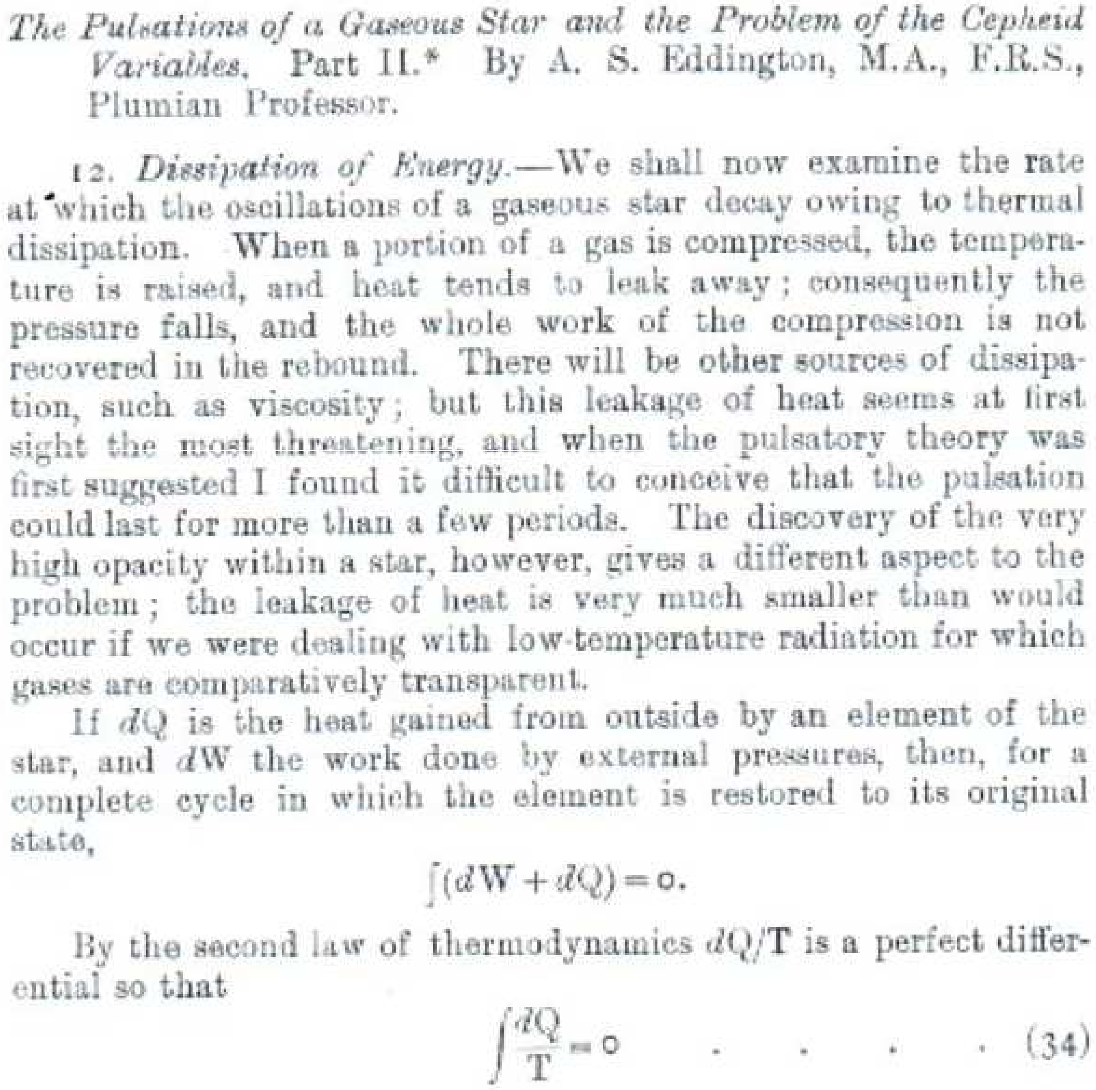
1. Although variable stars of the Cepheid type show a periodic change of radial velocity, it is improbable that they are binary stars. The theory which now appears most plausible attributes the light-changes to the pulsation of a single star;* and accordingly the varying radial velocity measures the approach and recession of the surface in the course of the pulsation. In order to throw light, if possible, on the phenomena of these variables, I have investigated the theory of a pulsating mass of gas. A complete solution of this problem would be very difficult; but it seems to be possible to determine the general character of the oscillation, and to obtain results which may be compared with observation.

The type of pulsation here considered is symmetrical about the centre; that is to say, the star remains spherical, but expands and contracts. It is possible that the actual oscillation may be an elliptical deformation; but I think that a symmetrical oscillation is more probable in a star of low density, and it is much simpler to investigate.

It may be useful to summarise some of the leading results of observation with regard to these variables—

Hvězdy
Artur Stanley Eddington

anglický astrofyzik, ***r. 1918/19 – O pulsacích plynných hvězd a problému cefeid***



The Pulsations of a Gaseous Star and the Problem of the Cepheid Variables. Part II.* By A. S. Eddington, M.A., F.R.S., Plumian Professor.

12. *Dissipation of Energy.*—We shall now examine the rate at which the oscillations of a gaseous star decay owing to thermal dissipation. When a portion of a gas is compressed, the temperature is raised, and heat tends to leak away; consequently the pressure falls, and the whole work of the compression is not recovered in the rebound. There will be other sources of dissipation, such as viscosity; but this leakage of heat seems at first sight the most threatening, and when the pulsatory theory was first suggested I found it difficult to conceive that the pulsation could last for more than a few periods. The discovery of the very high opacity within a star, however, gives a different aspect to the problem; the leakage of heat is very much smaller than would occur if we were dealing with low-temperature radiation for which gases are comparatively transparent.

If dQ is the heat gained from outside by an element of the star, and dW the work done by external pressures, then, for a complete cycle in which the element is restored to its original state,

$$\int(dW + dQ) = 0.$$

By the second law of thermodynamics dQ/T is a perfect differential so that

$$\int\frac{dQ}{T} = 0 \quad . \quad . \quad . \quad . \quad (34)$$

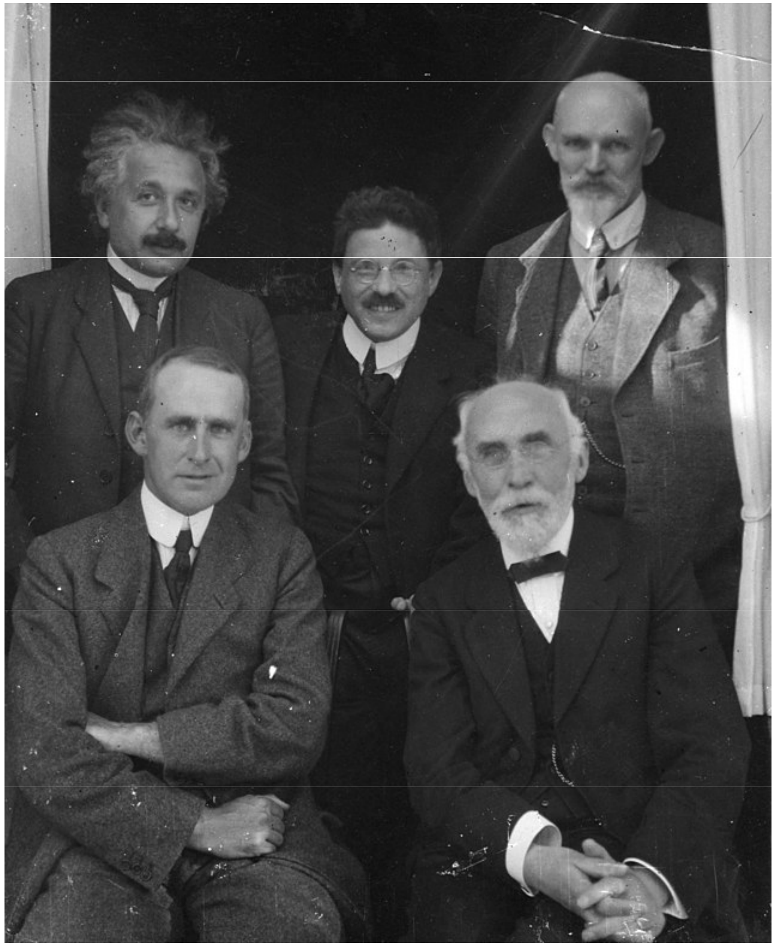
Hvězdy
Artur Stanley Eddington

anglický astrofyzik, ***Vnitřní stavba hvězd r. 1920***

základní úvaha o energiích hvězd

„Jestliže jen 5% hmoty hvězdy původně tvořené vodíkem se přemění na složitější prvky, pak celková uvolněné teplo bude více než stačit našim požadavkům a už nemusíme hledat jiný zdroj energie hvězd.“

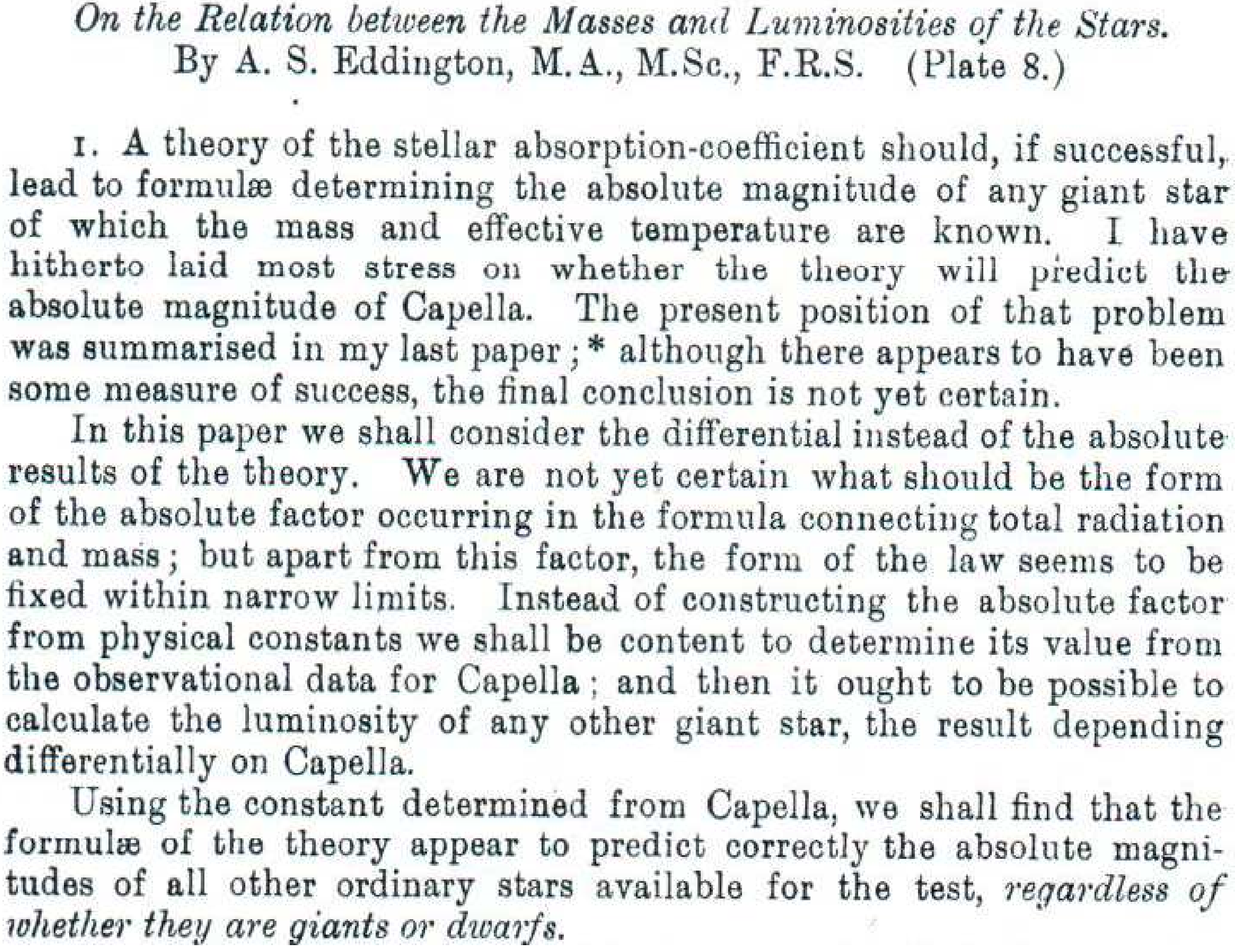
Observatory vol. XLIII, 557, 1920



Hvězdy

Artur Stanley Eddington

anglický astrofyzik***, O vztahu mezi hmotností a zářivým výkonem hvězd r. 1926***



On the Relation between the Masses and Luminosities of the Stars.

By A. S. Eddington, M.A., M.Sc., F.R.S. (Plate 8.)

1. A theory of the stellar absorption-coefficient should, if successful, lead to formulæ determining the absolute magnitude of any giant star of which the mass and effective temperature are known. I have hitherto laid most stress on whether the theory will predict the absolute magnitude of Capella. The present position of that problem was summarised in my last paper;* although there appears to have been some measure of success, the final conclusion is not yet certain.

In this paper we shall consider the differential instead of the absolute results of the theory. We are not yet certain what should be the form of the absolute factor occurring in the formula connecting total radiation and mass; but apart from this factor, the form of the law seems to be fixed within narrow limits. Instead of constructing the absolute factor from physical constants we shall be content to determine its value from the observational data for Capella; and then it ought to be possible to calculate the luminosity of any other giant star, the result depending differentially on Capella.

Using the constant determined from Capella, we shall find that the formulæ of the theory appear to predict correctly the absolute magnitudes of all other ordinary stars available for the test, *regardless of whether they are giants or dwarfs.*

Hvězdy
Artur Stanley Eddington

O vztahu mezi hmotností a zářivým výkonem hvězd r. 1926

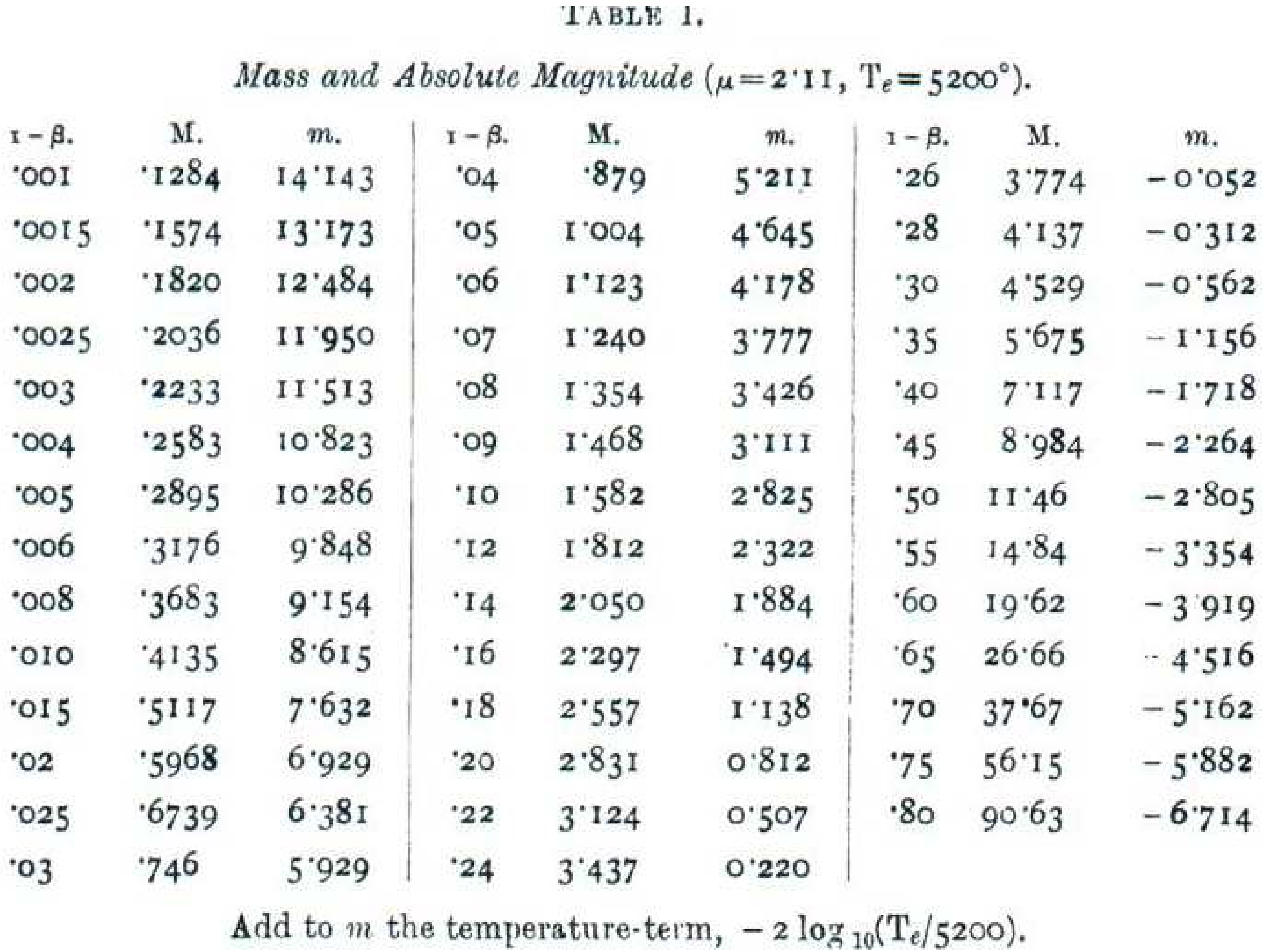


TABLE I.

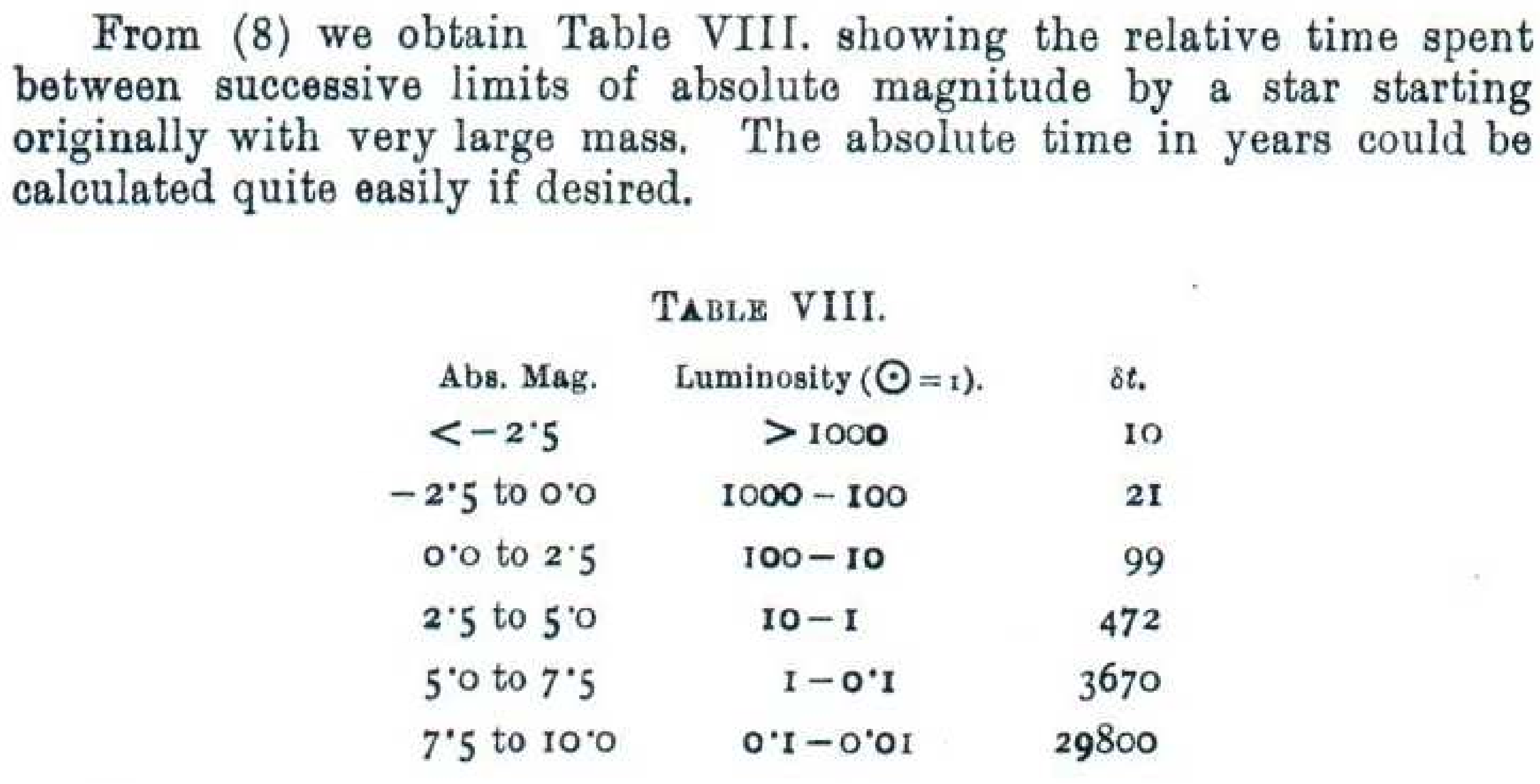
Mass and Absolute Magnitude ($\mu = 2·11$, $T_e = 5200°$).

$1-\beta$.	M.	*m*.	$1-\beta$.	M.	*m*.	$1-\beta$.	M.	*m*.
·001	·1284	14·143	·04	·879	5·211	·26	3·774	−0·052
·0015	·1574	13·173	·05	1·004	4·645	·28	4·137	−0·312
·002	·1820	12·484	·06	1·123	4·178	·30	4·529	−0·562
·0025	·2036	11·950	·07	1·240	3·777	·35	5·675	−1·156
·003	·2233	11·513	·08	1·354	3·426	·40	7·117	−1·718
·004	·2583	10·823	·09	1·468	3·111	·45	8·984	−2·264
·005	·2895	10·286	·10	1·582	2·825	·50	11·46	−2·805
·006	·3176	9·848	·12	1·812	2·322	·55	14·84	−3·354
·008	·3683	9·154	·14	2·050	1·884	·60	19·62	−3·919
·010	·4135	8·615	·16	2·297	1·494	·65	26·66	−4·516
·015	·5117	7·632	·18	2·557	1·138	·70	37·67	−5·162
·02	·5968	6·929	·20	2·831	0·812	·75	56·15	−5·882
·025	·6739	6·381	·22	3·124	0·507	·80	90·63	−6·714
·03	·746	5·929	·24	3·437	0·220			

Add to *m* the temperature-term, $-2\log_{10}(T_e/5200)$.

Hvězdy
Artur Stanley Eddington

O vztahu mezi hmotností a zářivým výkonem hvězd r. 1926



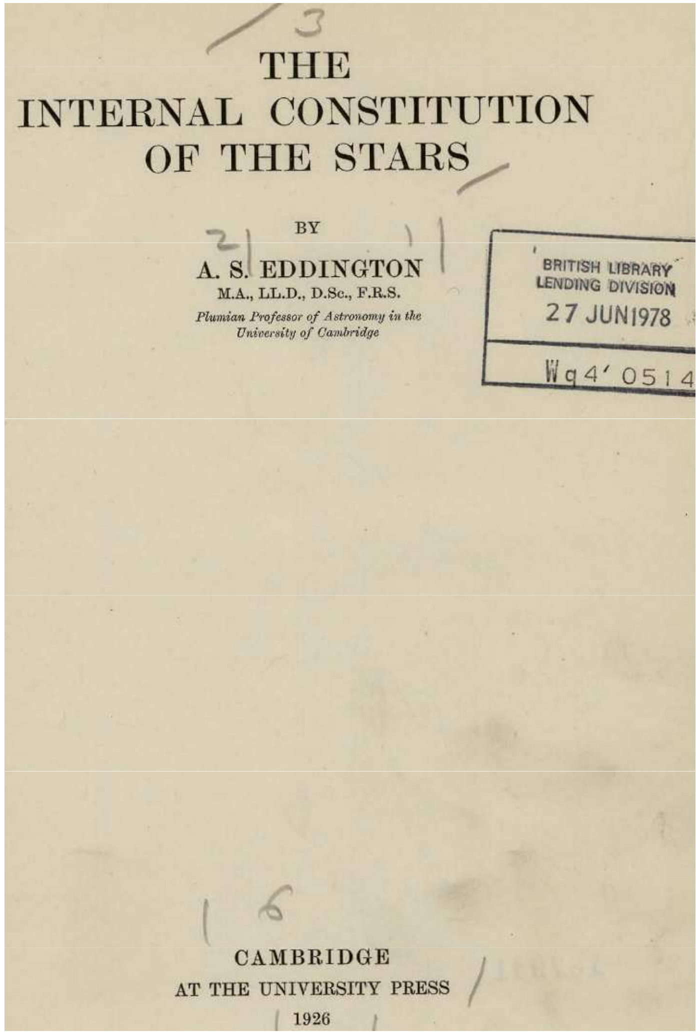
From (8) we obtain Table VIII. showing the relative time spent between successive limits of absolute magnitude by a star starting originally with very large mass. The absolute time in years could be calculated quite easily if desired.

TABLE VIII.

Abs. Mag.	Luminosity (⊙ = 1).	δt.
< −2·5	> 1000	10
−2·5 to 0·0	1000 − 100	21
0·0 to 2·5	100 − 10	99
2·5 to 5·0	10 − 1	472
5·0 to 7·5	1 − 0·1	3670
7·5 to 10·0	0·1 − 0·01	29800

Hvězdy
Artur Stanley Eddington

Stavba nitra hvězd r. 1926, teorie nitra hvězd



3

THE
INTERNAL CONSTITUTION
OF THE STARS

BY

2

A. S. EDDINGTON
M.A., LL.D., D.Sc., F.R.S.
Plumian Professor of Astronomy in the
University of Cambridge

6

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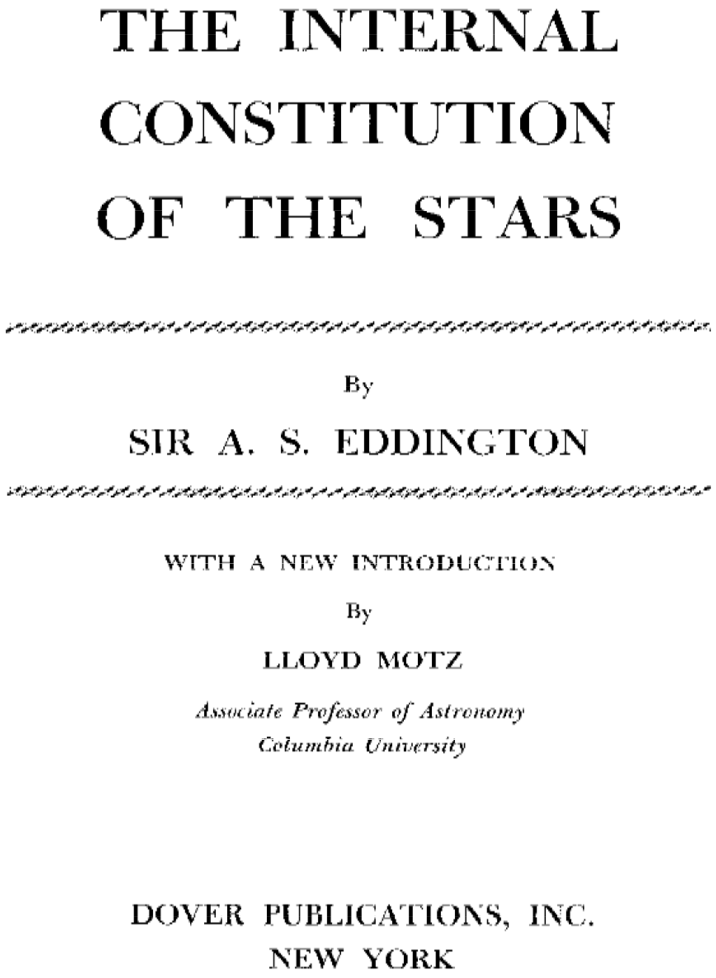
AT THE UNIVERSITY PRESS
1926



THE INTERNAL
CONSTITUTION
OF THE STARS

By

SIR A. S. EDDINGTON

WITH A NEW INTRODUCTION

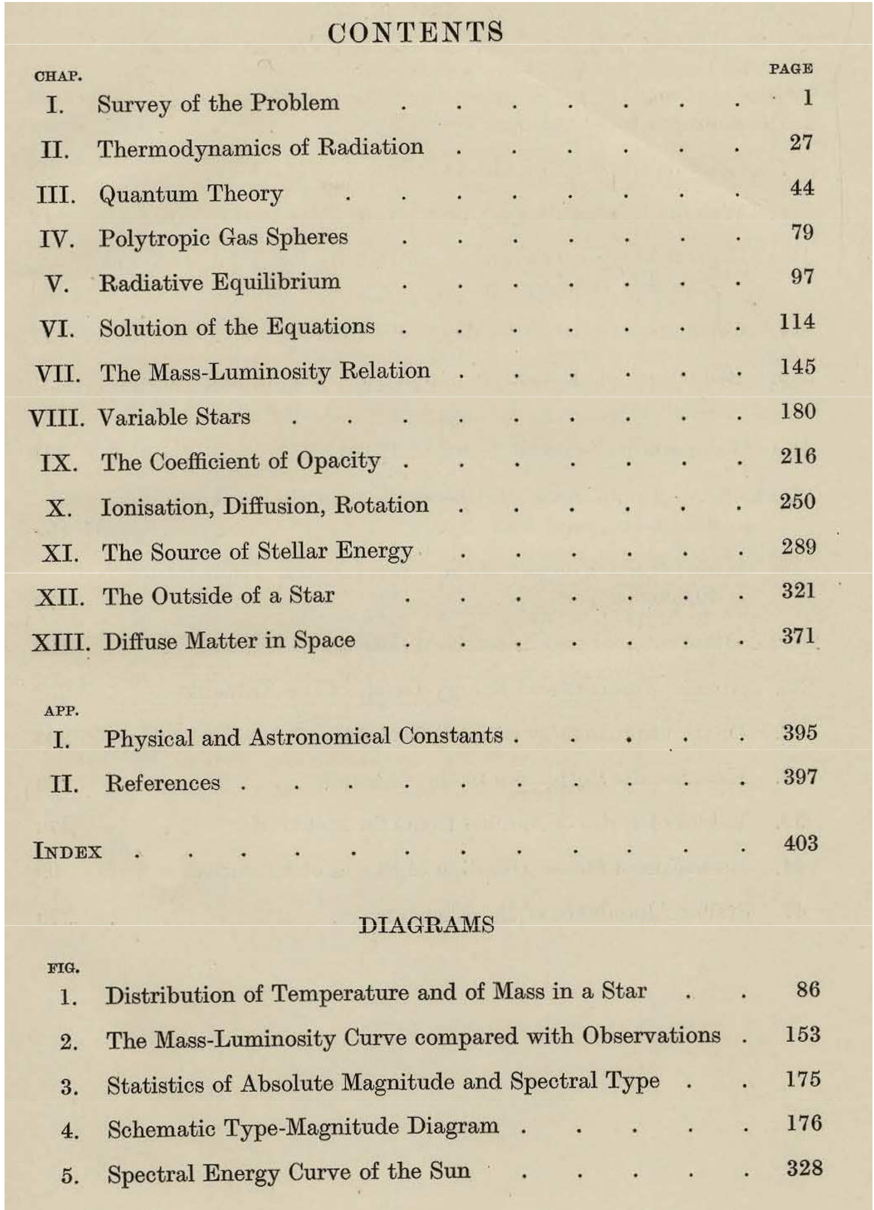
By

LLOYD MOTZ
Associate Professor of Astronomy
Columbia University

DOVER PUBLICATIONS, INC.
NEW YORK

Hvězdy - Artur Stanley Eddington

Stavba nitra hvězd r. 1926



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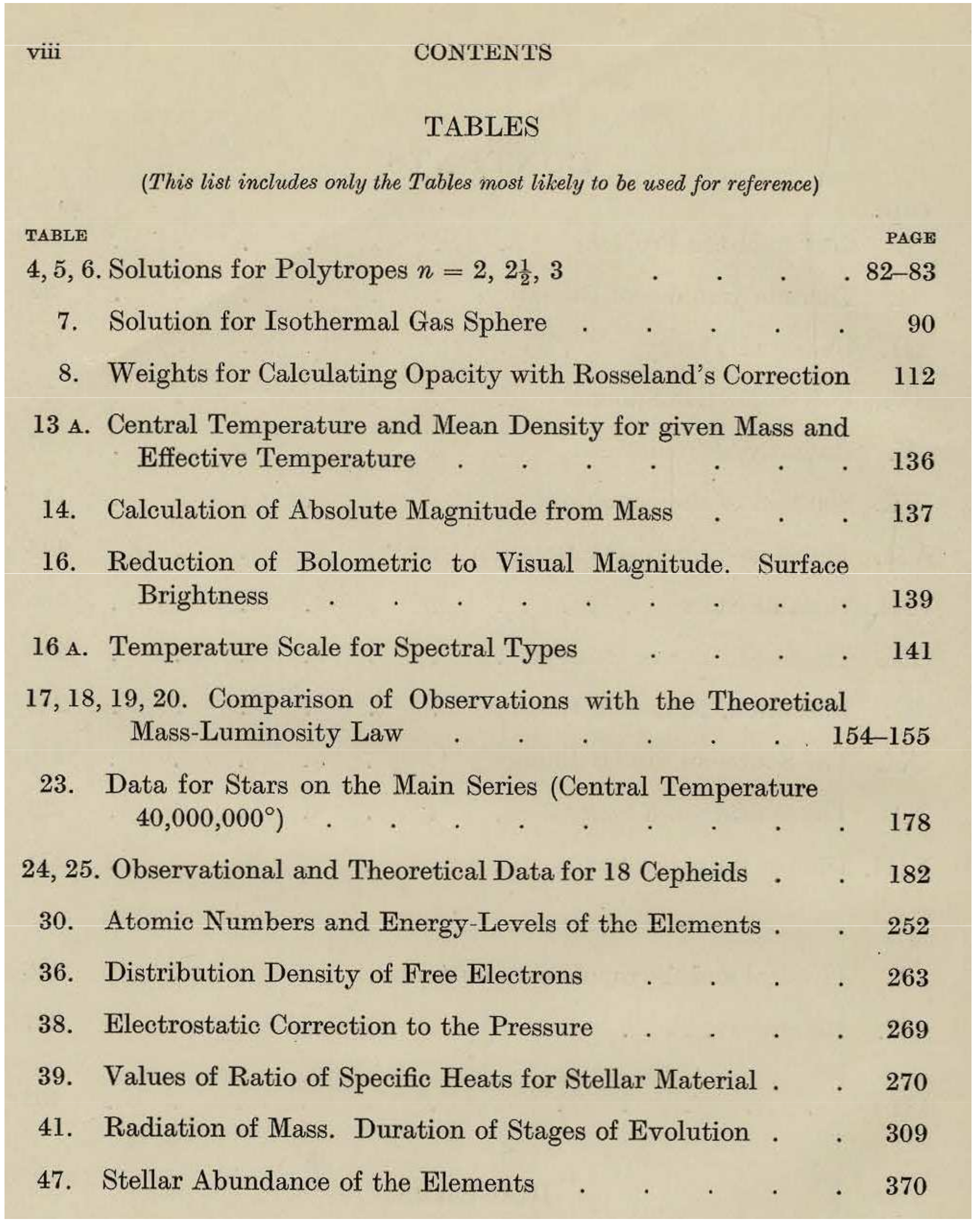
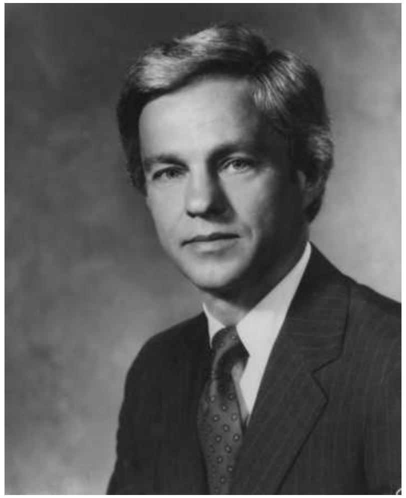


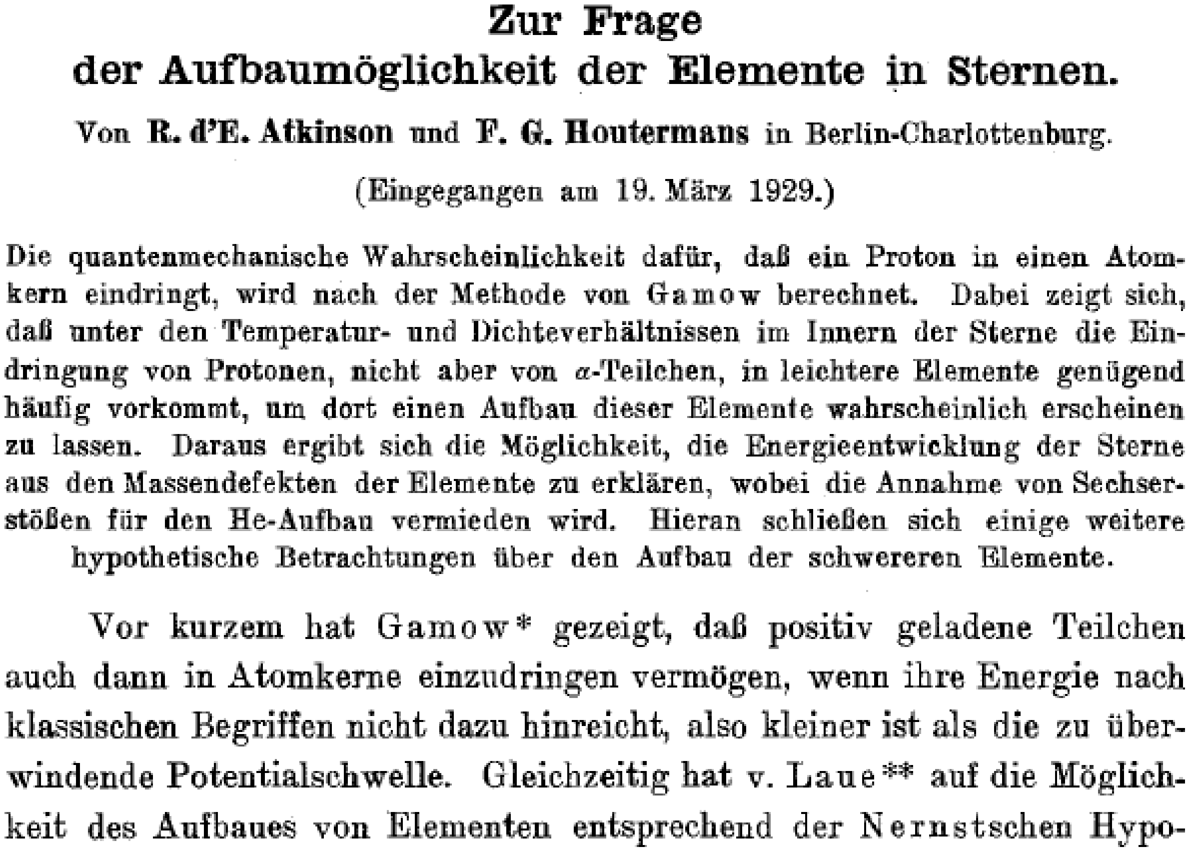
TABLES

(*This list includes only the Tables most likely to be used for reference*)

Hvězdy
Robert d´Escourt Atkinson 1898 - 1982
Friedrich George Hourtemans* 1903 - 1966

* holandsko-rakousko-německý atomový fyzik, **existence tunelového jevu při jaderných reakcích v nitru hvězd**, ***K otázce možného vzniku prvků ve hvězdách r.1929***





Zur Frage der Aufbaumöglichkeit der Elemente in Sternen.

Von **R. d'E. Atkinson** und **F. G. Houtermans** in Berlin-Charlottenburg.

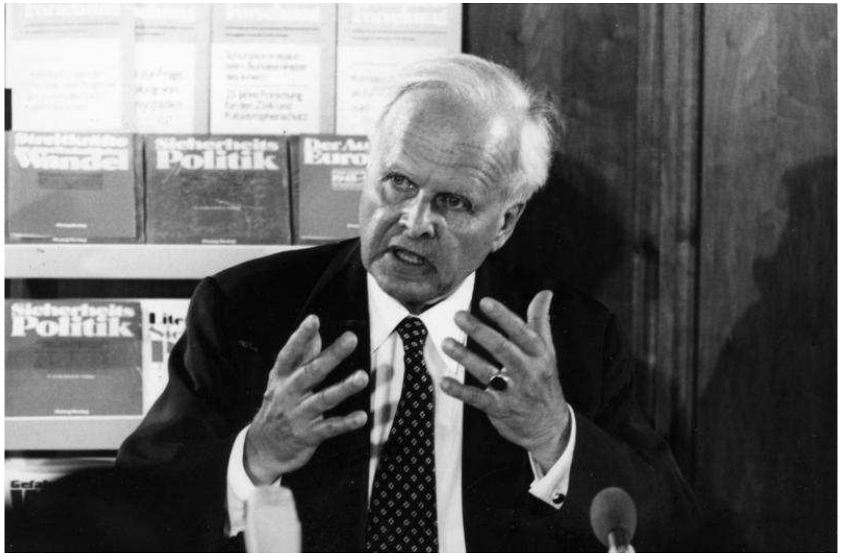
(Eingegangen am 19. März 1929.)

Die quantenmechanische Wahrscheinlichkeit dafür, daß ein Proton in einen Atomkern eindringt, wird nach der Methode von Gamow berechnet. Dabei zeigt sich, daß unter den Temperatur- und Dichteverhältnissen im Innern der Sterne die Eindringung von Protonen, nicht aber von α-Teilchen, in leichtere Elemente genügend häufig vorkommt, um dort einen Aufbau dieser Elemente wahrscheinlich erscheinen zu lassen. Daraus ergibt sich die Möglichkeit, die Energieentwicklung der Sterne aus den Massendefekten der Elemente zu erklären, wobei die Annahme von Sechserstößen für den He-Aufbau vermieden wird. Hieran schließen sich einige weitere hypothetische Betrachtungen über den Aufbau der schwereren Elemente.

Vor kurzem hat Gamow* gezeigt, daß positiv geladene Teilchen auch dann in Atomkerne einzudringen vermögen, wenn ihre Energie nach klassischen Begriffen nicht dazu hinreicht, also kleiner ist als die zu überwindende Potentialschwelle. Gleichzeitig hat v. Laue** auf die Möglichkeit des Aufbaues von Elementen entsprechend der Nernstschen Hypo-

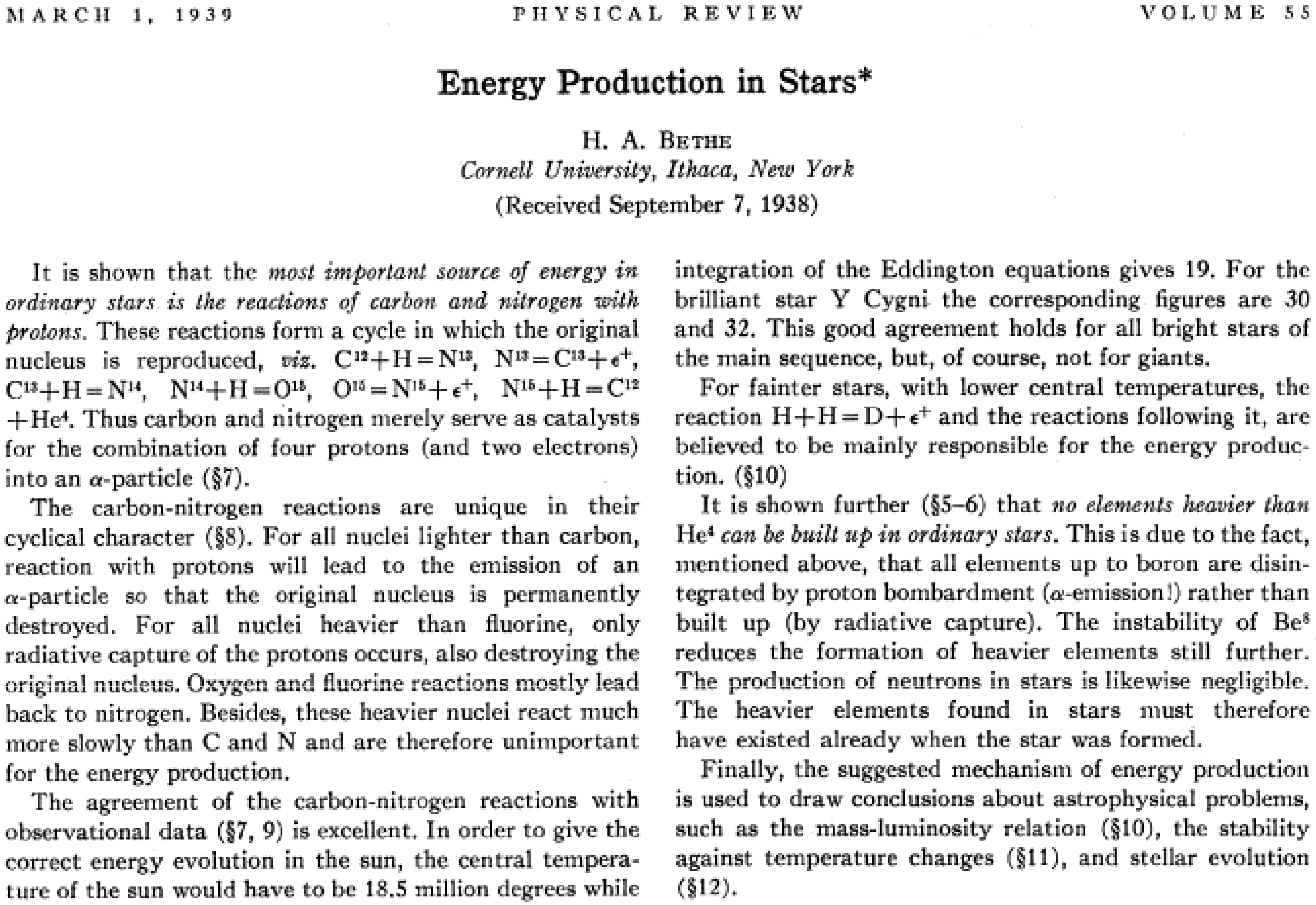
Hvězdy - zdroje jejich energie

americký fyzik německého původu **Hans Albert Bethe 1906 – 2005** , ***Nobelova cena za fyziku r.1967***, ***rozvoj teorie jaderných reakcí, zejména za objevy produkce energie v hvězdách,*** 1933 práce na fotodisintegraci deutéria, ***r. 1937 spolupráce s Carlem Friedrichem von Weizsackerem (1912 - 2007***), přechod na výzkum zdrojů energie hvězd, účast na ***projektu Manhattan***, vývoj atomové bomby, později vodíkové, r. 1947 vyložil Lambův posuv ve spektru vodíku, astrofyzika - supernovy, neutronové hvězdy, gravitační kolaps - černé díry, interpretace neutrinového nedostatku



Hvězdy - Hans Albert Bethe

Produkce energie ve hvězdách, r. 1939



MARCH 1, 1939 PHYSICAL REVIEW VOLUME 55

Energy Production in Stars*

H. A. Bethe
Cornell University, Ithaca, New York

(Received September 7, 1938)

It is shown that the *most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons.* These reactions form a cycle in which the original nucleus is reproduced, *viz.* $C^{12}+H=N^{13}$, $N^{13}=C^{13}+\epsilon^+$, $C^{13}+H=N^{14}$, $N^{14}+H=O^{15}$, $O^{15}=N^{15}+\epsilon^+$, $N^{15}+H=C^{12}+He^4$. Thus carbon and nitrogen merely serve as catalysts for the combination of four protons (and two electrons) into an α-particle (§7).

The carbon-nitrogen reactions are unique in their cyclical character (§8). For all nuclei lighter than carbon, reaction with protons will lead to the emission of an α-particle so that the original nucleus is permanently destroyed. For all nuclei heavier than fluorine, only radiative capture of the protons occurs, also destroying the original nucleus. Oxygen and fluorine reactions mostly lead back to nitrogen. Besides, these heavier nuclei react much more slowly than C and N and are therefore unimportant for the energy production.

The agreement of the carbon-nitrogen reactions with observational data (§7, 9) is excellent. In order to give the correct energy evolution in the sun, the central temperature of the sun would have to be 18.5 million degrees while integration of the Eddington equations gives 19. For the brilliant star Y Cygni the corresponding figures are 30 and 32. This good agreement holds for all bright stars of the main sequence, but, of course, not for giants.

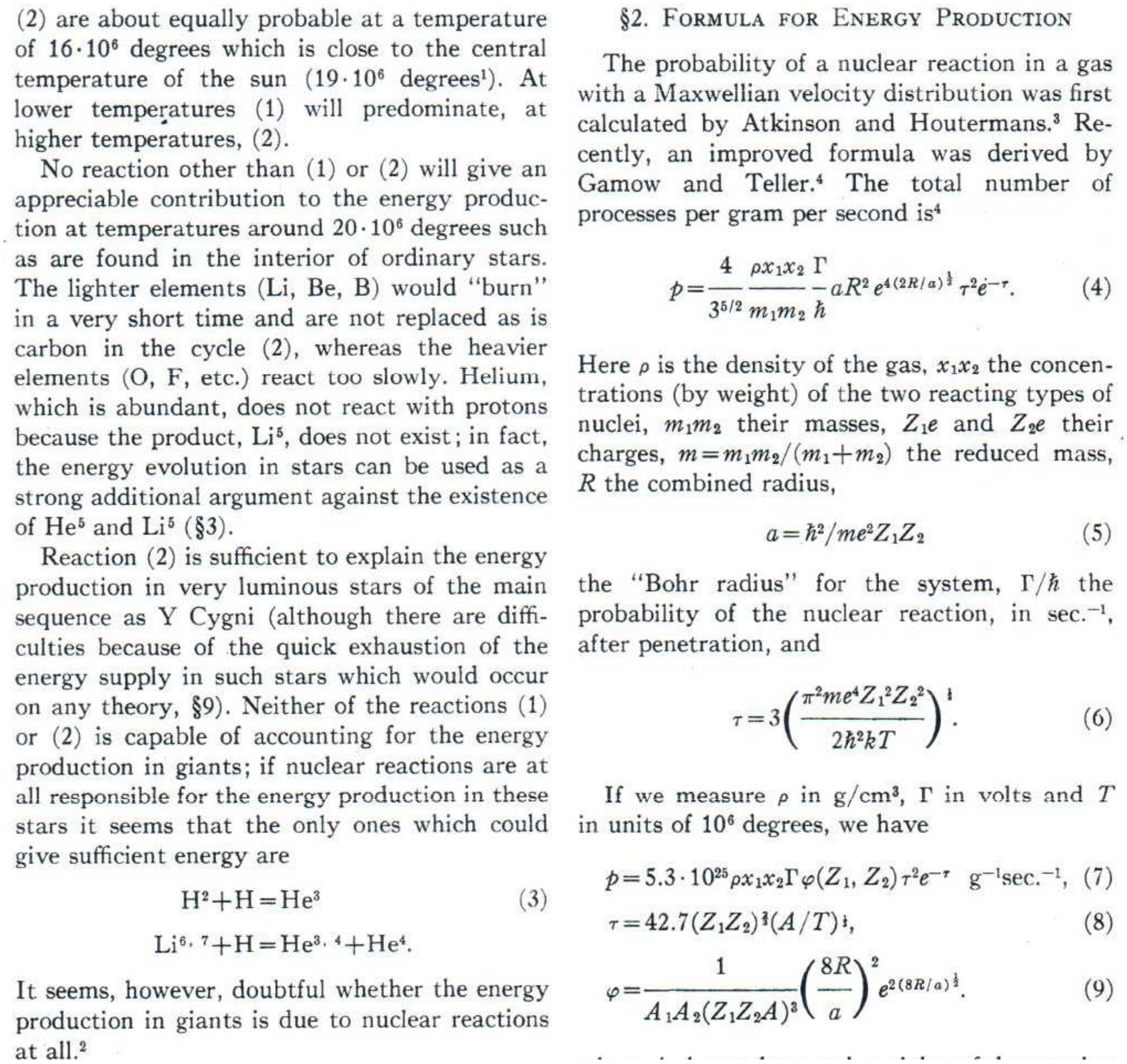
For fainter stars, with lower central temperatures, the reaction $H+H=D+\epsilon^+$ and the reactions following it, are believed to be mainly responsible for the energy production. (§10)

It is shown further (§5–6) that *no elements heavier than* He^4 *can be built up in ordinary stars.* This is due to the fact, mentioned above, that all elements up to boron are disintegrated by proton bombardment (α-emission!) rather than built up (by radiative capture). The instability of Be^8 reduces the formation of heavier elements still further. The production of neutrons in stars is likewise negligible. The heavier elements found in stars must therefore have existed already when the star was formed.

Finally, the suggested mechanism of energy production is used to draw conclusions about astrophysical problems, such as the mass-luminosity relation (§10), the stability against temperature changes (§11), and stellar evolution (§12).

Hvězdy - Hans Albert Bethe

Produkce energie ve hvězdách, r. 1939



(2) are about equally probable at a temperature of $16\cdot10^6$ degrees which is close to the central temperature of the sun ($19\cdot10^6$ degrees[1]). At lower temperatures (1) will predominate, at higher temperatures, (2).

No reaction other than (1) or (2) will give an appreciable contribution to the energy production at temperatures around $20\cdot10^6$ degrees such as are found in the interior of ordinary stars. The lighter elements (Li, Be, B) would "burn" in a very short time and are not replaced as is carbon in the cycle (2), whereas the heavier elements (O, F, etc.) react too slowly. Helium, which is abundant, does not react with protons because the product, Li^5, does not exist; in fact, the energy evolution in stars can be used as a strong additional argument against the existence of He^5 and Li^5 (§3).

Reaction (2) is sufficient to explain the energy production in very luminous stars of the main sequence as Y Cygni (although there are difficulties because of the quick exhaustion of the energy supply in such stars which would occur on any theory, §9). Neither of the reactions (1) or (2) is capable of accounting for the energy production in giants; if nuclear reactions are at all responsible for the energy production in these stars it seems that the only ones which could give sufficient energy are

$$H^2+H=He^3 \qquad (3)$$

$$Li^{6,\,7}+H=He^{3,\,4}+He^4.$$

It seems, however, doubtful whether the energy production in giants is due to nuclear reactions at all.[2]

§2. FORMULA FOR ENERGY PRODUCTION

The probability of a nuclear reaction in a gas with a Maxwellian velocity distribution was first calculated by Atkinson and Houtermans.[3] Recently, an improved formula was derived by Gamow and Teller.[4] The total number of processes per gram per second is[4]

$$p=\frac{4}{3^{5/2}}\frac{\rho x_1x_2}{m_1m_2}\frac{\Gamma}{\hbar}aR^2e^{4(2R/a)^{\frac{1}{2}}}\tau^2e^{-\tau}. \qquad (4)$$

Here ρ is the density of the gas, x_1x_2 the concentrations (by weight) of the two reacting types of nuclei, m_1m_2 their masses, Z_1e and Z_2e their charges, $m=m_1m_2/(m_1+m_2)$ the reduced mass, R the combined radius,

$$a=\hbar^2/me^2Z_1Z_2 \qquad (5)$$

the "Bohr radius" for the system, $\Gamma/\hbar$ the probability of the nuclear reaction, in sec.$^{-1}$, after penetration, and

$$\tau=3\left(\frac{\pi^2me^4Z_1^2Z_2^2}{2\hbar^2kT}\right)^{\frac{1}{3}}. \qquad (6)$$

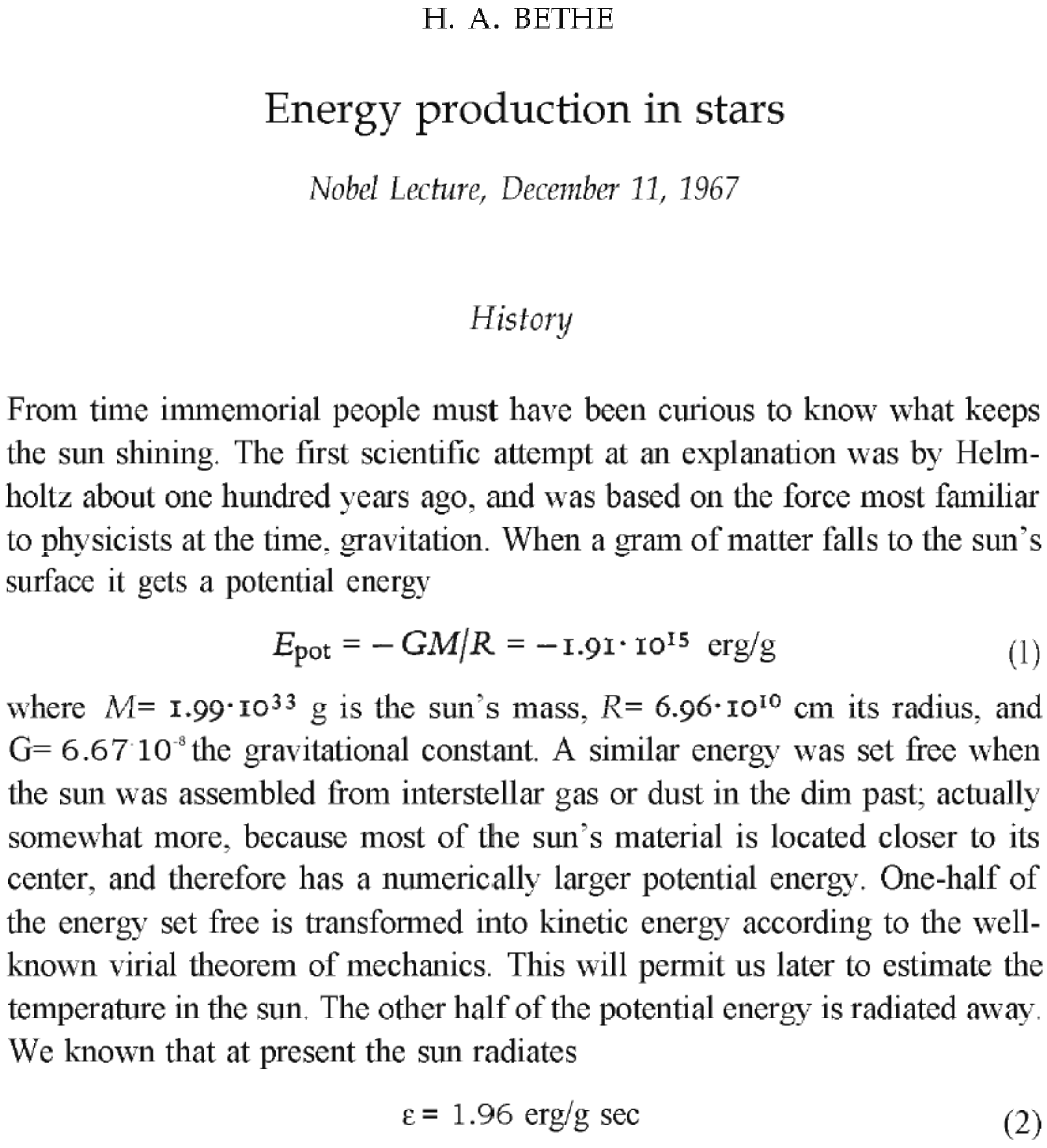
If we measure ρ in g/cm^3, Γ in volts and T in units of 10^6 degrees, we have

$$p=5.3\cdot10^{25}\rho x_1x_2\Gamma\varphi(Z_1,Z_2)\tau^2e^{-\tau}\quad \mathrm{g^{-1}sec.^{-1}}, \qquad (7)$$

$$\tau=42.7(Z_1Z_2)^{\frac{2}{3}}(A/T)^{\frac{1}{3}}, \qquad (8)$$

$$\varphi=\frac{1}{A_1A_2(Z_1Z_2A)^3}\left(\frac{8R}{a}\right)^2e^{2(8R/a)^{\frac{1}{2}}}. \qquad (9)$$

Hvězdy - Hans Albert Bethe
Nobelova cena r. 1967



H. A. BETHE

Energy production in stars

Nobel Lecture, December 11, 1967

History

From time immemorial people must have been curious to know what keeps the sun shining. The first scientific attempt at an explanation was by Helmholtz about one hundred years ago, and was based on the force most familiar to physicists at the time, gravitation. When a gram of matter falls to the sun's surface it gets a potential energy

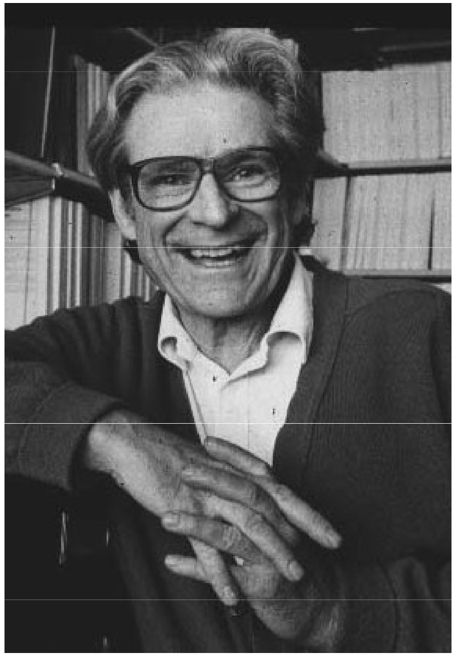
$$E_{\mathrm{pot}} = -GM/R = -1.91 \cdot 10^{15} \text{ erg/g} \qquad (1)$$

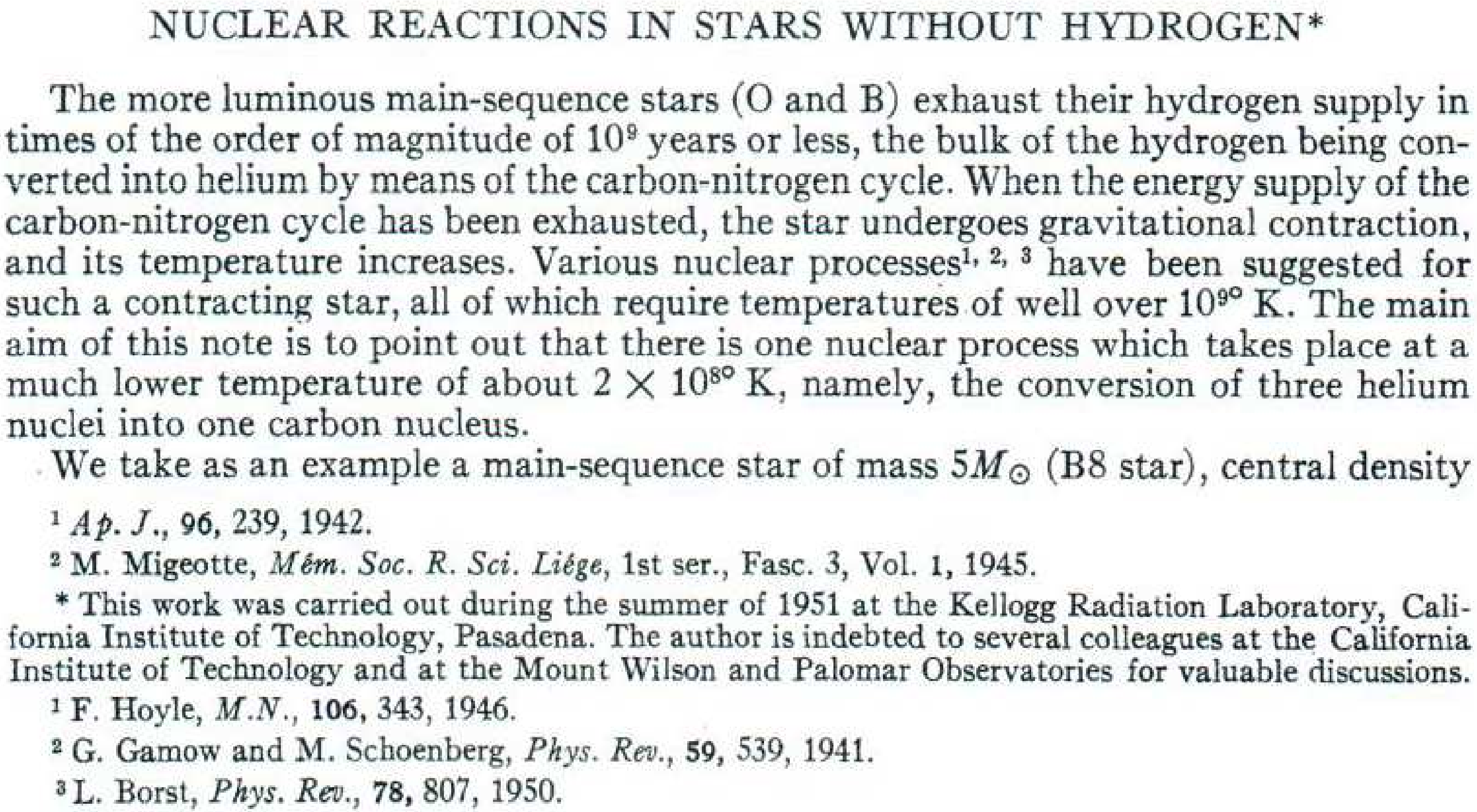
where $M = 1.99 \cdot 10^{33}$ g is the sun's mass, $R = 6.96 \cdot 10^{10}$ cm its radius, and $G = 6.67 \cdot 10^{-8}$ the gravitational constant. A similar energy was set free when the sun was assembled from interstellar gas or dust in the dim past; actually somewhat more, because most of the sun's material is located closer to its center, and therefore has a numerically larger potential energy. One-half of the energy set free is transformed into kinetic energy according to the well-known virial theorem of mechanics. This will permit us later to estimate the temperature in the sun. The other half of the potential energy is radiated away. We known that at present the sun radiates

$$\varepsilon = 1.96 \text{ erg/g sec} \qquad (2)$$

Hvězdy - Edwin Ernest Salpeter 1924 - 2008

americký fyzik rakouského původu, profesor Cornellovy univerzity, ***Nukleární reakce ve hvězdách bez vodíku, r. 1951***, přeměna helia na uhlík, **3 α proces**, akreční disky černých děr, aktivita galaktických jader





NUCLEAR REACTIONS IN STARS WITHOUT HYDROGEN*

The more luminous main-sequence stars (O and B) exhaust their hydrogen supply in times of the order of magnitude of 10^9 years or less, the bulk of the hydrogen being converted into helium by means of the carbon-nitrogen cycle. When the energy supply of the carbon-nitrogen cycle has been exhausted, the star undergoes gravitational contraction, and its temperature increases. Various nuclear processes[1, 2, 3] have been suggested for such a contracting star, all of which require temperatures of well over $10^{9\circ}$ K. The main aim of this note is to point out that there is one nuclear process which takes place at a much lower temperature of about $2 \times 10^{8\circ}$ K, namely, the conversion of three helium nuclei into one carbon nucleus.

We take as an example a main-sequence star of mass $5M_\odot$ (B8 star), central density

[1] *Ap. J.*, **96**, 239, 1942.

[2] M. Migeotte, *Mém. Soc. R. Sci. Liége*, 1st ser., Fasc. 3, Vol. **1**, 1945.

* This work was carried out during the summer of 1951 at the Kellogg Radiation Laboratory, California Institute of Technology, Pasadena. The author is indebted to several colleagues at the California Institute of Technology and at the Mount Wilson and Palomar Observatories for valuable discussions.

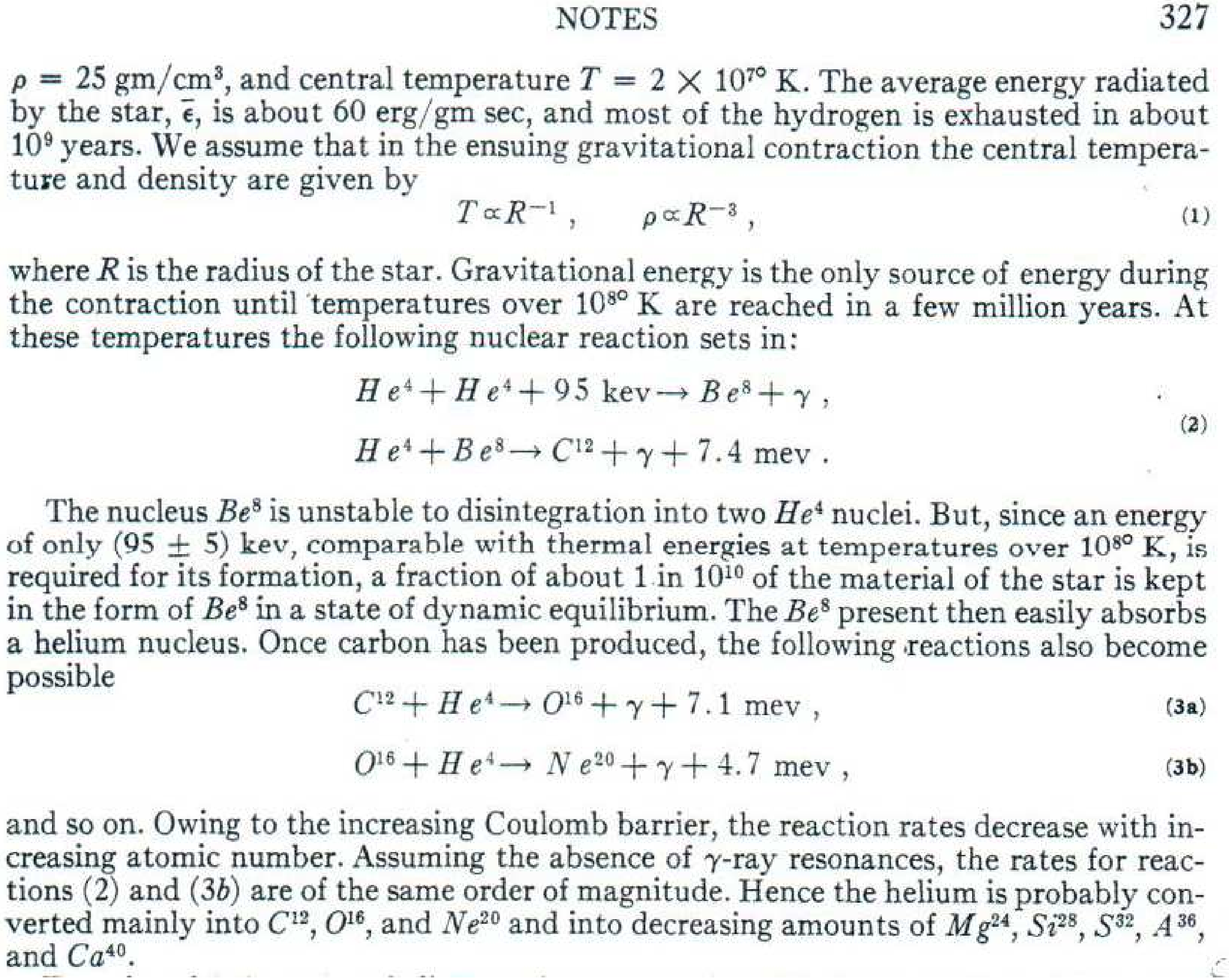
[1] F. Hoyle, *M.N.*, **106**, 343, 1946.

[2] G. Gamow and M. Schoenberg, *Phys. Rev.*, **59**, 539, 1941.

[3] L. Borst, *Phys. Rev.*, **78**, 807, 1950.

Hvězdy
Edwin Ernest Salpeter

Nukleární reakce ve hvězdách bez vodíku, r. 1951



$\rho = 25$ gm/cm³, and central temperature $T = 2 \times 10^{7\circ}$ K. The average energy radiated by the star, $\bar{\epsilon}$, is about 60 erg/gm sec, and most of the hydrogen is exhausted in about 10^9 years. We assume that in the ensuing gravitational contraction the central temperature and density are given by

$$T \propto R^{-1}, \qquad \rho \propto R^{-3}, \tag{1}$$

where R is the radius of the star. Gravitational energy is the only source of energy during the contraction until temperatures over $10^{8\circ}$ K are reached in a few million years. At these temperatures the following nuclear reaction sets in:

$$\begin{aligned} He^4 + He^4 + 95 \text{ kev} &\rightarrow Be^8 + \gamma\,, \\ He^4 + Be^8 &\rightarrow C^{12} + \gamma + 7.4 \text{ mev}\,. \end{aligned} \tag{2}$$

The nucleus Be^8 is unstable to disintegration into two He^4 nuclei. But, since an energy of only (95 ± 5) kev, comparable with thermal energies at temperatures over $10^{8\circ}$ K, is required for its formation, a fraction of about 1 in 10^{10} of the material of the star is kept in the form of Be^8 in a state of dynamic equilibrium. The Be^8 present then easily absorbs a helium nucleus. Once carbon has been produced, the following reactions also become possible

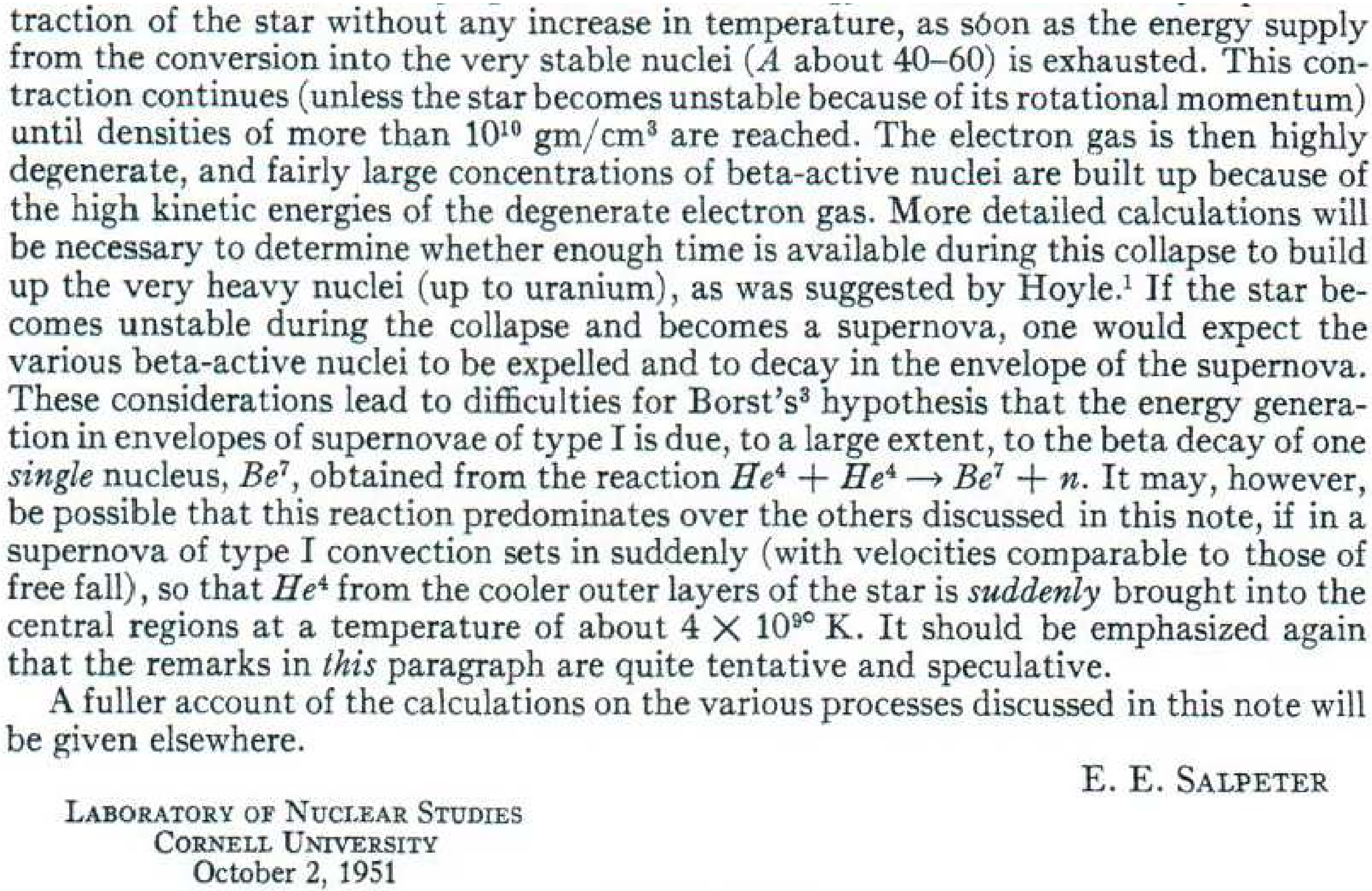
$$C^{12} + He^4 \rightarrow O^{16} + \gamma + 7.1 \text{ mev}\,, \tag{3a}$$

$$O^{16} + He^4 \rightarrow Ne^{20} + \gamma + 4.7 \text{ mev}\,, \tag{3b}$$

and so on. Owing to the increasing Coulomb barrier, the reaction rates decrease with increasing atomic number. Assuming the absence of γ-ray resonances, the rates for reactions (2) and (3b) are of the same order of magnitude. Hence the helium is probably converted mainly into C^{12}, O^{16}, and Ne^{20} and into decreasing amounts of Mg^{24}, Si^{28}, S^{32}, A^{36}, and Ca^{40}.

Hvězdy
Edwin Ernest Salpeter

Nukleární reakce ve hvězdách bez vodíku, r. 1951



traction of the star without any increase in temperature, as soon as the energy supply from the conversion into the very stable nuclei (*A* about 40–60) is exhausted. This contraction continues (unless the star becomes unstable because of its rotational momentum) until densities of more than 10^{10} gm/cm^3 are reached. The electron gas is then highly degenerate, and fairly large concentrations of beta-active nuclei are built up because of the high kinetic energies of the degenerate electron gas. More detailed calculations will be necessary to determine whether enough time is available during this collapse to build up the very heavy nuclei (up to uranium), as was suggested by Hoyle.[1] If the star becomes unstable during the collapse and becomes a supernova, one would expect the various beta-active nuclei to be expelled and to decay in the envelope of the supernova. These considerations lead to difficulties for Borst's[3] hypothesis that the energy generation in envelopes of supernovae of type I is due, to a large extent, to the beta decay of one *single* nucleus, Be^7, obtained from the reaction $He^4 + He^4 \rightarrow Be^7 + n$. It may, however, be possible that this reaction predominates over the others discussed in this note, if in a supernova of type I convection sets in suddenly (with velocities comparable to those of free fall), so that He^4 from the cooler outer layers of the star is *suddenly* brought into the central regions at a temperature of about $4 \times 10^{9\circ}$ K. It should be emphasized again that the remarks in *this* paragraph are quite tentative and speculative.

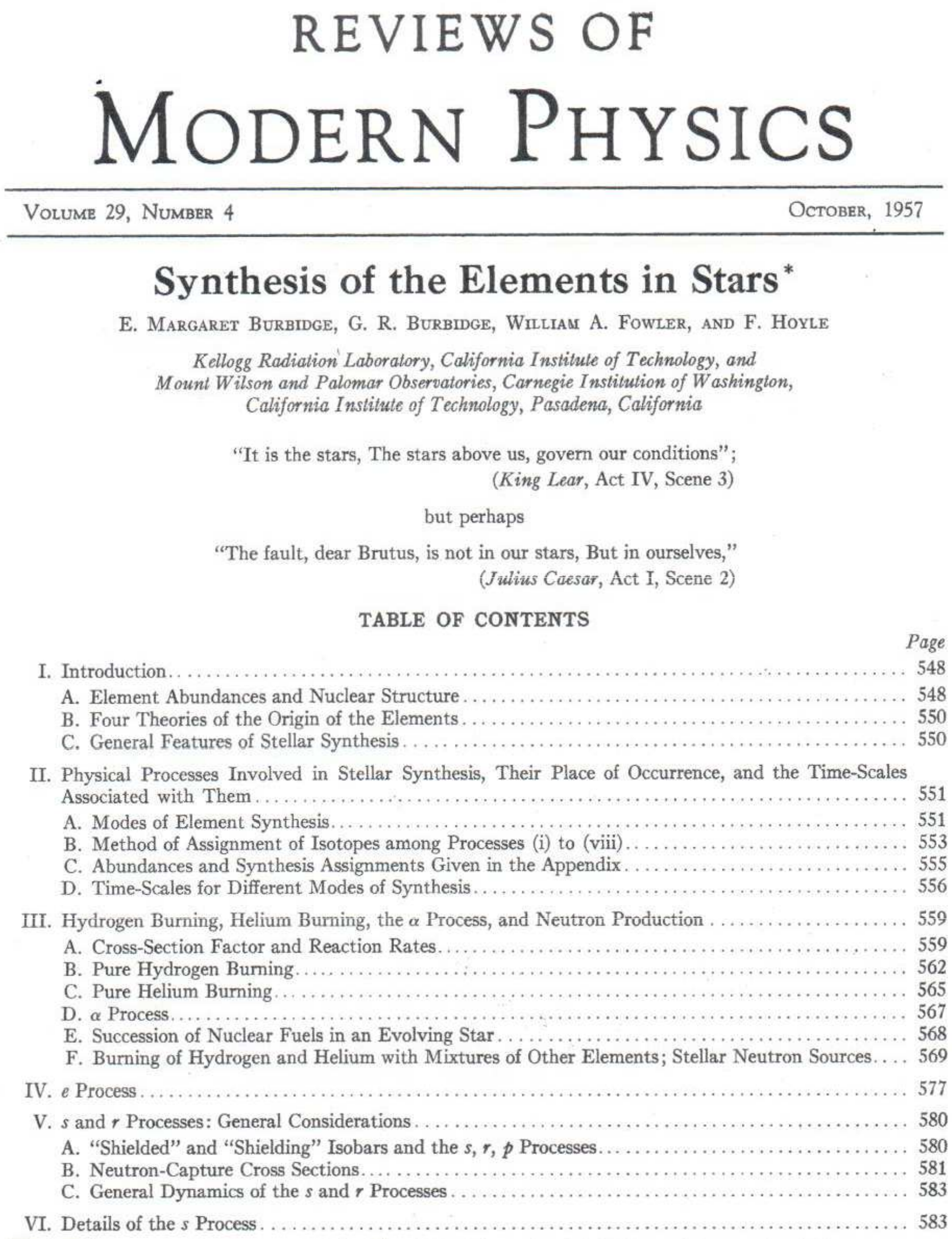
A fuller account of the calculations on the various processes discussed in this note will be given elsewhere.

E. E. SALPETER

LABORATORY OF NUCLEAR STUDIES
CORNELL UNIVERSITY
October 2, 1951

Hvězdy - vznik prvků

W. A. Fowler, F. Hoyle E. M. Burbidge, G. R. Burbidge, - FHBB



REVIEWS OF
MODERN PHYSICS

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Synthesis of the Elements in Stars*

E. MARGARET BURBIDGE, G. R. BURBIDGE, WILLIAM A. FOWLER, AND F. HOYLE

Kellogg Radiation Laboratory, California Institute of Technology, and
Mount Wilson and Palomar Observatories, Carnegie Institution of Washington,
California Institute of Technology, Pasadena, California

> "It is the stars, The stars above us, govern our conditions";
> (*King Lear*, Act IV, Scene 3)
>
> but perhaps
>
> "The fault, dear Brutus, is not in our stars, But in ourselves,"
> (*Julius Caesar*, Act I, Scene 2)

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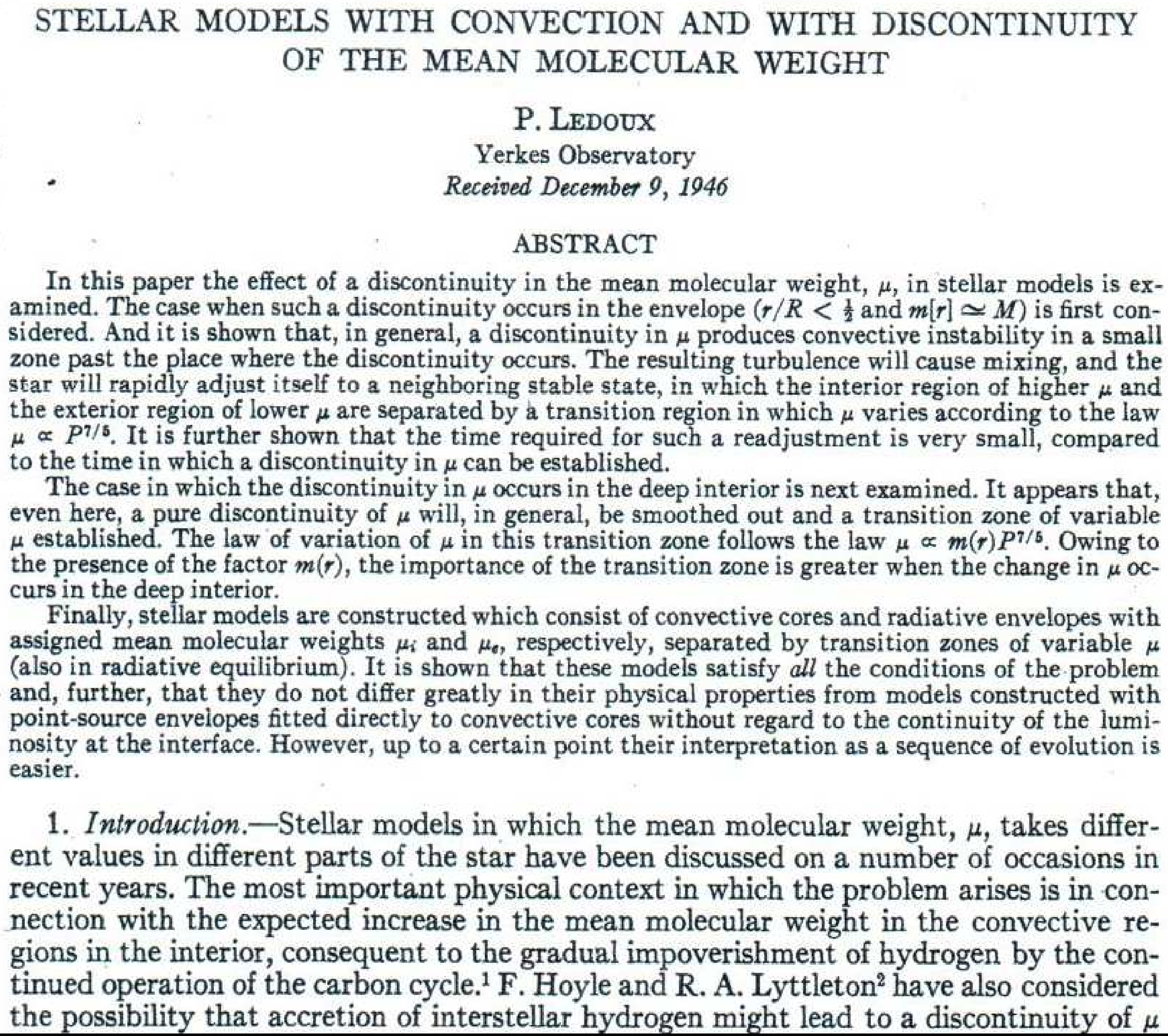
Paul Ledoux 1914 - 1988

belgický astrofyzik, ***Hvězdné modely s konvekcí a diskontinuitou střední molekulové hmotnosti r. 1947***, ***kritérium nestability***



STELLAR MODELS WITH CONVECTION AND WITH DISCONTINUITY OF THE MEAN MOLECULAR WEIGHT

P. Ledoux
Yerkes Observatory

Received December 9, 1946

ABSTRACT

In this paper the effect of a discontinuity in the mean molecular weight, μ, in stellar models is examined. The case when such a discontinuity occurs in the envelope ($r/R < \frac{1}{2}$ and $m[r] \simeq M$) is first considered. And it is shown that, in general, a discontinuity in μ produces convective instability in a small zone past the place where the discontinuity occurs. The resulting turbulence will cause mixing, and the star will rapidly adjust itself to a neighboring stable state, in which the interior region of higher μ and the exterior region of lower μ are separated by a transition region in which μ varies according to the law $\mu \propto P^{7/5}$. It is further shown that the time required for such a readjustment is very small, compared to the time in which a discontinuity in μ can be established.

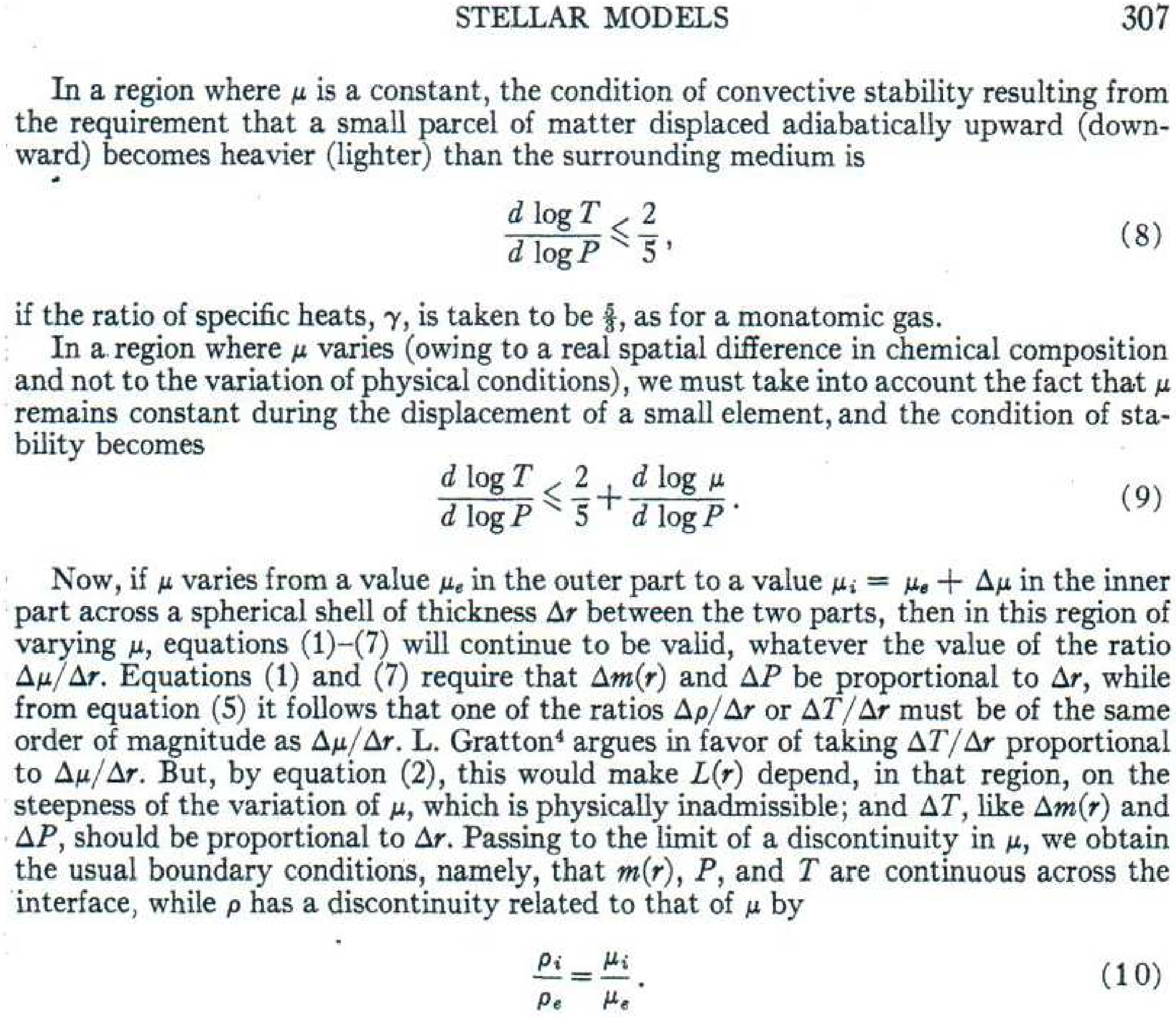
The case in which the discontinuity in μ occurs in the deep interior is next examined. It appears that, even here, a pure discontinuity of μ will, in general, be smoothed out and a transition zone of variable μ established. The law of variation of μ in this transition zone follows the law $\mu \propto m(r)P^{7/5}$. Owing to the presence of the factor $m(r)$, the importance of the transition zone is greater when the change in μ occurs in the deep interior.

Finally, stellar models are constructed which consist of convective cores and radiative envelopes with assigned mean molecular weights μ_i and μ_e, respectively, separated by transition zones of variable μ (also in radiative equilibrium). It is shown that these models satisfy *all* the conditions of the problem and, further, that they do not differ greatly in their physical properties from models constructed with point-source envelopes fitted directly to convective cores without regard to the continuity of the luminosity at the interface. However, up to a certain point their interpretation as a sequence of evolution is easier.

1. *Introduction.*—Stellar models in which the mean molecular weight, μ, takes different values in different parts of the star have been discussed on a number of occasions in recent years. The most important physical context in which the problem arises is in connection with the expected increase in the mean molecular weight in the convective regions in the interior, consequent to the gradual impoverishment of hydrogen by the continued operation of the carbon cycle.[1] F. Hoyle and R. A. Lyttleton[2] have also considered the possibility that accretion of interstellar hydrogen might lead to a discontinuity of μ

Hvězdy
Paul Ledoux

Hvězdné modely s konvekcí a diskontinuitou střední molekulové hmotnosti r. 1947



In a region where μ is a constant, the condition of convective stability resulting from the requirement that a small parcel of matter displaced adiabatically upward (downward) becomes heavier (lighter) than the surrounding medium is

$$\frac{d\log T}{d\log P} \leqslant \frac{2}{5}, \tag{8}$$

if the ratio of specific heats, γ, is taken to be $\frac{5}{3}$, as for a monatomic gas.

In a region where μ varies (owing to a real spatial difference in chemical composition and not to the variation of physical conditions), we must take into account the fact that μ remains constant during the displacement of a small element, and the condition of stability becomes

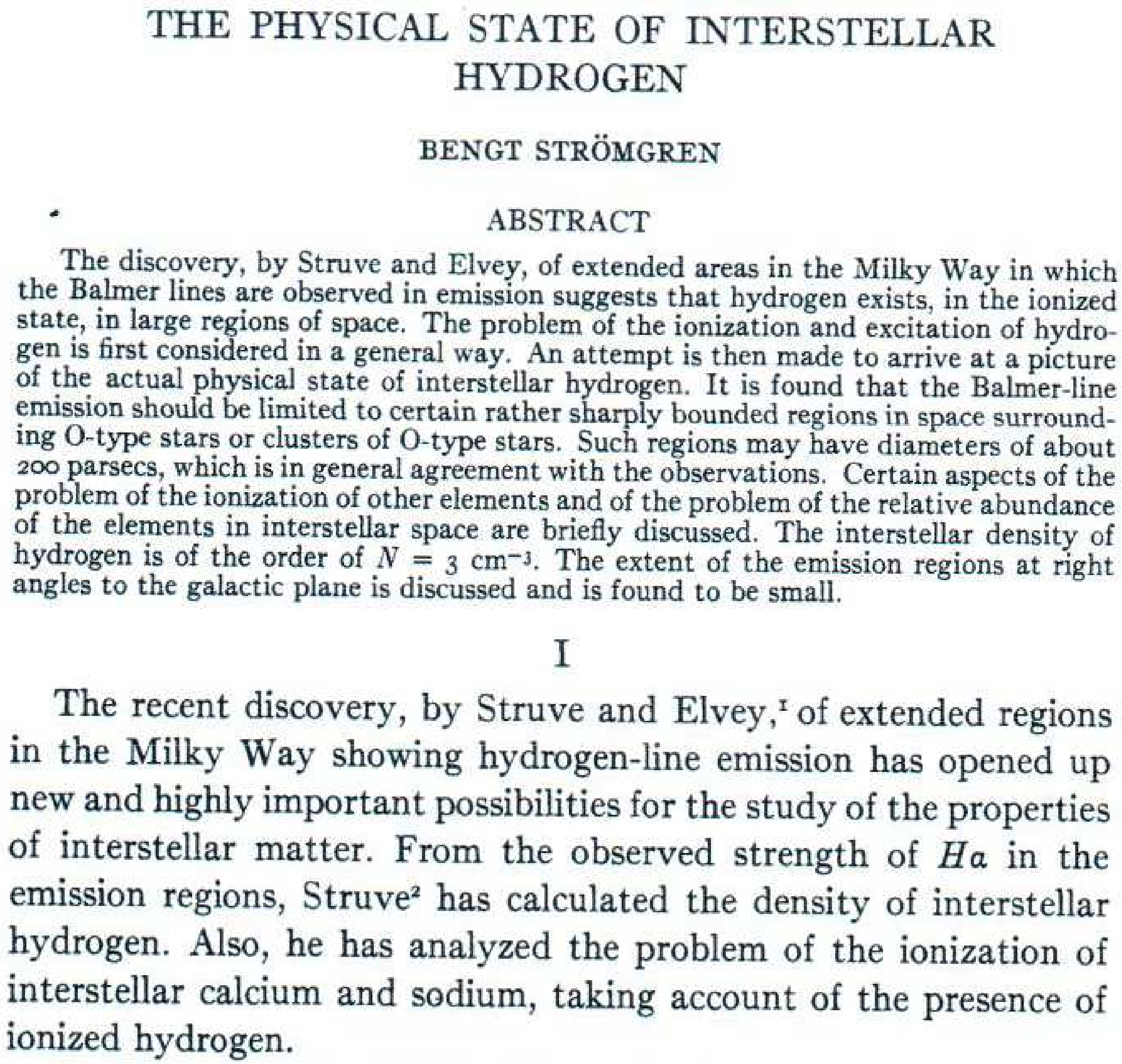
$$\frac{d\log T}{d\log P} \leqslant \frac{2}{5} + \frac{d\log \mu}{d\log P}. \tag{9}$$

Now, if μ varies from a value μ_e in the outer part to a value $\mu_i = \mu_e + \Delta\mu$ in the inner part across a spherical shell of thickness Δr between the two parts, then in this region of varying μ, equations (1)–(7) will continue to be valid, whatever the value of the ratio $\Delta\mu/\Delta r$. Equations (1) and (7) require that $\Delta m(r)$ and ΔP be proportional to Δr, while from equation (5) it follows that one of the ratios $\Delta\rho/\Delta r$ or $\Delta T/\Delta r$ must be of the same order of magnitude as $\Delta\mu/\Delta r$. L. Gratton[4] argues in favor of taking $\Delta T/\Delta r$ proportional to $\Delta\mu/\Delta r$. But, by equation (2), this would make $L(r)$ depend, in that region, on the steepness of the variation of μ, which is physically inadmissible; and ΔT, like $\Delta m(r)$ and ΔP, should be proportional to Δr. Passing to the limit of a discontinuity in μ, we obtain the usual boundary conditions, namely, that $m(r)$, P, and T are continuous across the interface, while ρ has a discontinuity related to that of μ by

$$\frac{\rho_i}{\rho_e} = \frac{\mu_i}{\mu_e}. \tag{10}$$

Hvězdy

dánský astrofyzik **Bengt Georg Daniel Strömgren 1908-1987,**
O fyzikálním stavu mezihvězdného vodíku r. 1939, existence ionizovaných oblastí kolem horkých hvězd,



THE PHYSICAL STATE OF INTERSTELLAR HYDROGEN

BENGT STRÖMGREN

ABSTRACT

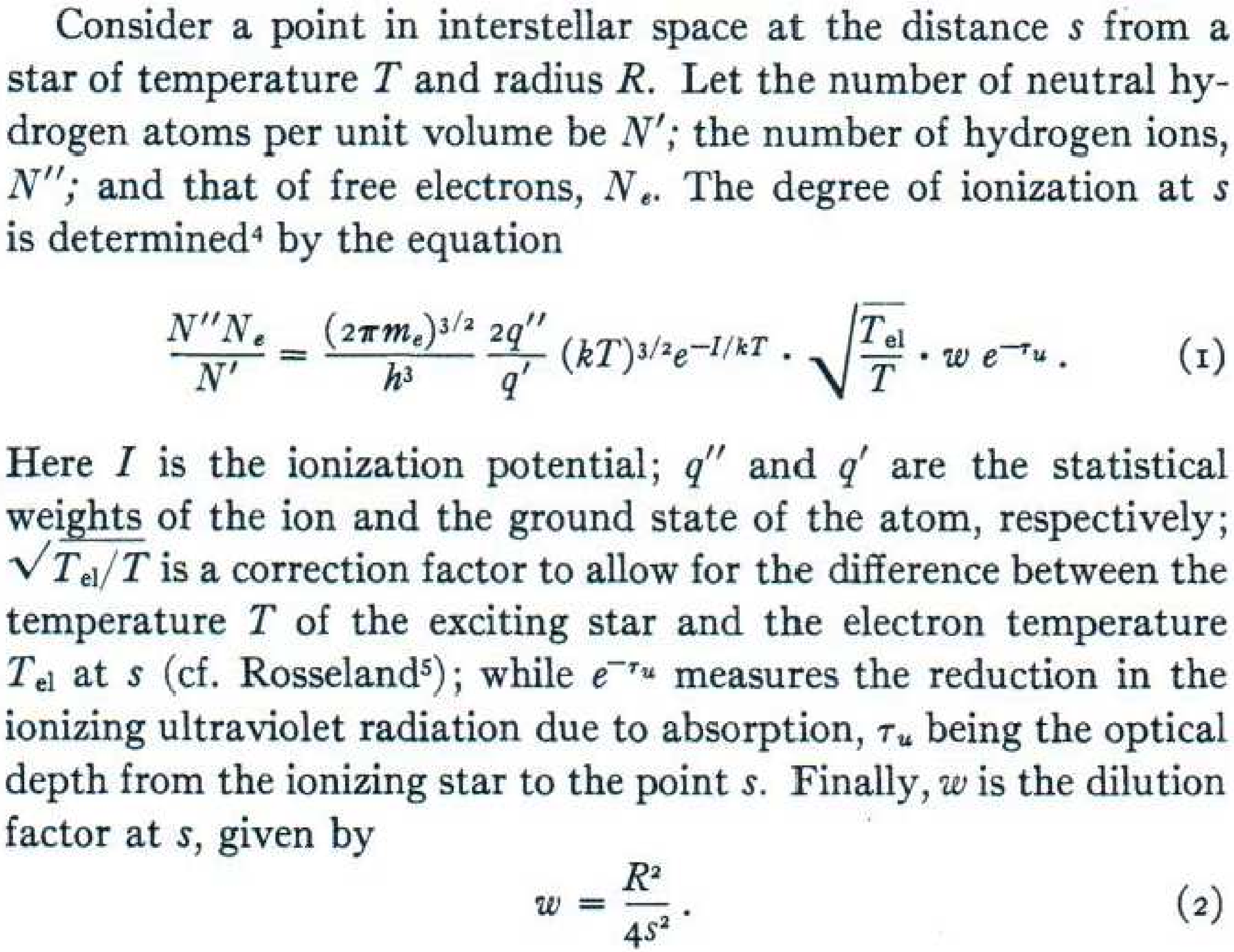
The discovery, by Struve and Elvey, of extended areas in the Milky Way in which the Balmer lines are observed in emission suggests that hydrogen exists, in the ionized state, in large regions of space. The problem of the ionization and excitation of hydrogen is first considered in a general way. An attempt is then made to arrive at a picture of the actual physical state of interstellar hydrogen. It is found that the Balmer-line emission should be limited to certain rather sharply bounded regions in space surrounding O-type stars or clusters of O-type stars. Such regions may have diameters of about 200 parsecs, which is in general agreement with the observations. Certain aspects of the problem of the ionization of other elements and of the problem of the relative abundance of the elements in interstellar space are briefly discussed. The interstellar density of hydrogen is of the order of $N = 3$ cm^{-3}. The extent of the emission regions at right angles to the galactic plane is discussed and is found to be small.

I

The recent discovery, by Struve and Elvey,[1] of extended regions in the Milky Way showing hydrogen-line emission has opened up new and highly important possibilities for the study of the properties of interstellar matter. From the observed strength of $H\alpha$ in the emission regions, Struve[2] has calculated the density of interstellar hydrogen. Also, he has analyzed the problem of the ionization of interstellar calcium and sodium, taking account of the presence of ionized hydrogen.

Hvězdy

Strömgren ***O fyzikálním stavu mezihvězdného vodíku r. 1939,*** existence ionizačních oblastí kolem horkých hvězd,



Consider a point in interstellar space at the distance s from a star of temperature T and radius R. Let the number of neutral hydrogen atoms per unit volume be N'; the number of hydrogen ions, N''; and that of free electrons, N_e. The degree of ionization at s is determined[4] by the equation

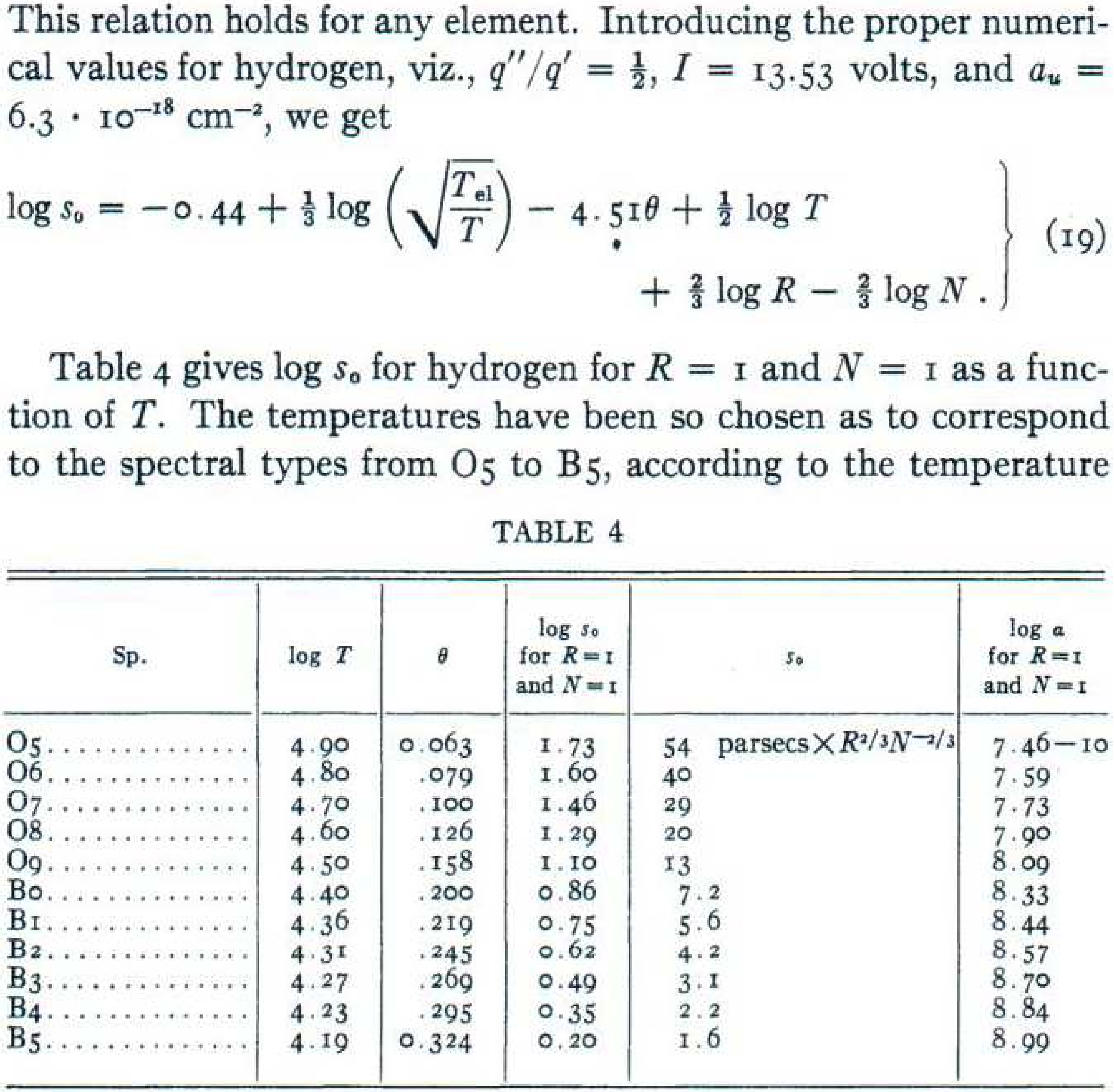
$$\frac{N''N_e}{N'} = \frac{(2\pi m_e)^{3/2}}{h^3}\,\frac{2q''}{q'}\,(kT)^{3/2}e^{-I/kT}\cdot\sqrt{\frac{T_{\rm el}}{T}}\cdot w\,e^{-\tau_u}\,. \tag{1}$$

Here I is the ionization potential; q'' and q' are the statistical weights of the ion and the ground state of the atom, respectively; $\sqrt{T_{\rm el}/T}$ is a correction factor to allow for the difference between the temperature T of the exciting star and the electron temperature $T_{\rm el}$ at s (cf. Rosseland[5]); while $e^{-\tau_u}$ measures the reduction in the ionizing ultraviolet radiation due to absorption, τ_u being the optical depth from the ionizing star to the point s. Finally, w is the dilution factor at s, given by

$$w = \frac{R^2}{4s^2}\,. \tag{2}$$

Hvězdy

O fyzikálním stavu mezihvězdného vodíku r. 1939, existence ionizačních oblastí kolem horkých hvězd,



This relation holds for any element. Introducing the proper numerical values for hydrogen, viz., $q''/q' = \frac{1}{2}$, $I = 13.53$ volts, and $a_u = 6.3 \cdot 10^{-18}$ cm^{-2}, we get

$$\left.\begin{aligned} \log s_0 = -0.44 + \tfrac{1}{3}\log\left(\sqrt{\frac{T_{el}}{T}}\right) - 4.51\theta + \tfrac{1}{2}\log T \\ + \tfrac{2}{3}\log R - \tfrac{2}{3}\log N\,. \end{aligned}\right\} \quad (19)$$

Table 4 gives $\log s_0$ for hydrogen for $R = 1$ and $N = 1$ as a function of T. The temperatures have been so chosen as to correspond to the spectral types from O5 to B5, according to the temperature

TABLE 4

Sp.	log T	θ	log s_0 for $R=1$ and $N=1$	s_0	log a for $R=1$ and $N=1$
O5	4.90	0.063	1.73	54 parsecs$\times R^{2/3}N^{-2/3}$	7.46−10
O6	4.80	.079	1.60	40	7.59
O7	4.70	.100	1.46	29	7.73
O8	4.60	.126	1.29	20	7.90
O9	4.50	.158	1.10	13	8.09
B0	4.40	.200	0.86	7.2	8.33
B1	4.36	.219	0.75	5.6	8.44
B2	4.31	.245	0.62	4.2	8.57
B3	4.27	.269	0.49	3.1	8.70
B4	4.23	.295	0.35	2.2	8.84
B5	4.19	0.324	0.20	1.6	8.99

Hvězdy

O fyzikálním stavu mezihvězdného vodíku r. 1939, existence ionizačních oblastí kolem horkých hvězd,

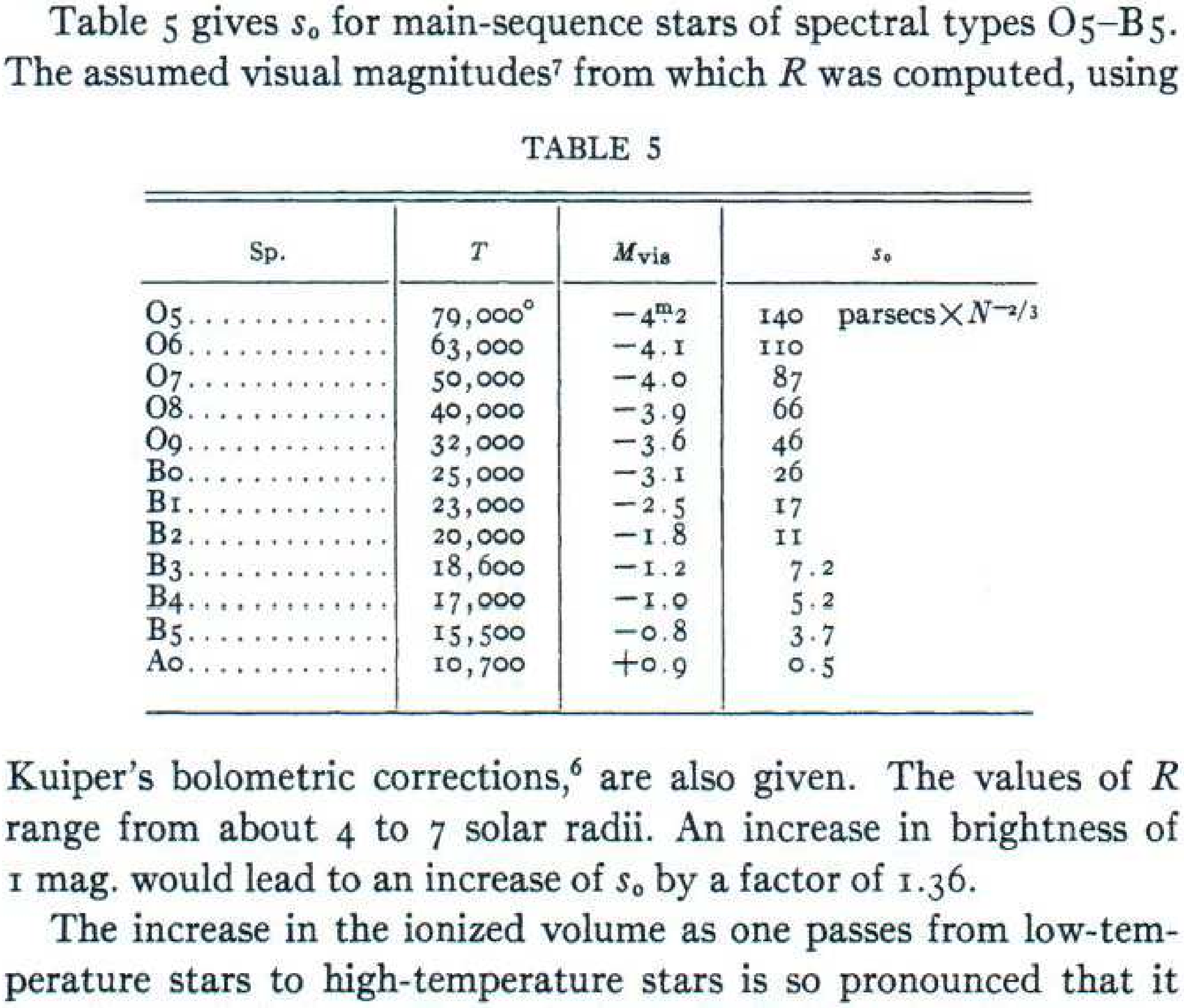


Table 5 gives s_0 for main-sequence stars of spectral types O5–B5. The assumed visual magnitudes[7] from which R was computed, using

TABLE 5

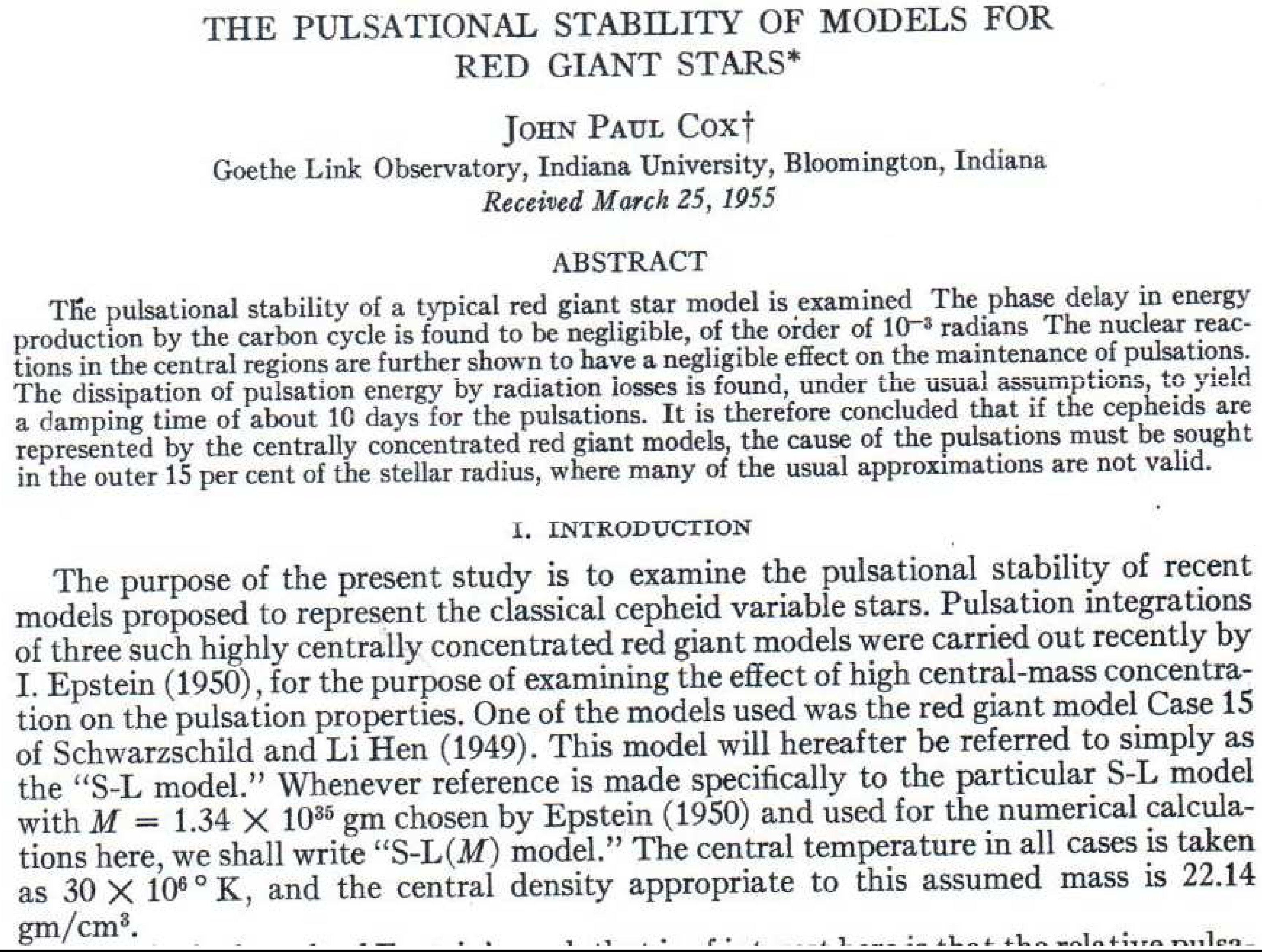
Sp.	T	M_{vis}	s_0
O5	79,000°	$-4^{\text{m}}.2$	140 parsecs × $N^{-2/3}$
O6	63,000	−4.1	110
O7	50,000	−4.0	87
O8	40,000	−3.9	66
O9	32,000	−3.6	46
B0	25,000	−3.1	26
B1	23,000	−2.5	17
B2	20,000	−1.8	11
B3	18,600	−1.2	7.2
B4	17,000	−1.0	5.2
B5	15,500	−0.8	3.7
A0	10,700	+0.9	0.5

Kuiper's bolometric corrections,[6] are also given. The values of R range from about 4 to 7 solar radii. An increase in brightness of 1 mag. would lead to an increase of s_0 by a factor of 1.36.

The increase in the ionized volume as one passes from low-temperature stars to high-temperature stars is so pronounced that it

Hvězdy
John Paul Cox 1926 - 1984

americký astrofyzik, ***Pulsační stabilita modelů rudých obrů, r. 1955***



THE PULSATIONAL STABILITY OF MODELS FOR RED GIANT STARS*

JOHN PAUL COX†

Goethe Link Observatory, Indiana University, Bloomington, Indiana

Received March 25, 1955

ABSTRACT

The pulsational stability of a typical red giant star model is examined The phase delay in energy production by the carbon cycle is found to be negligible, of the order of 10^{-3} radians The nuclear reactions in the central regions are further shown to have a negligible effect on the maintenance of pulsations. The dissipation of pulsation energy by radiation losses is found, under the usual assumptions, to yield a damping time of about 10 days for the pulsations. It is therefore concluded that if the cepheids are represented by the centrally concentrated red giant models, the cause of the pulsations must be sought in the outer 15 per cent of the stellar radius, where many of the usual approximations are not valid.

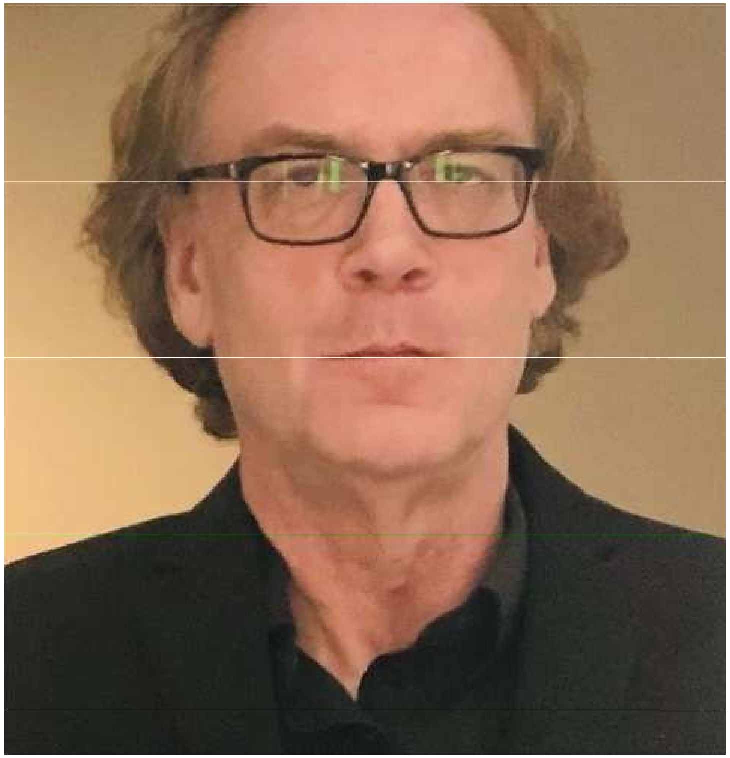
I. INTRODUCTION

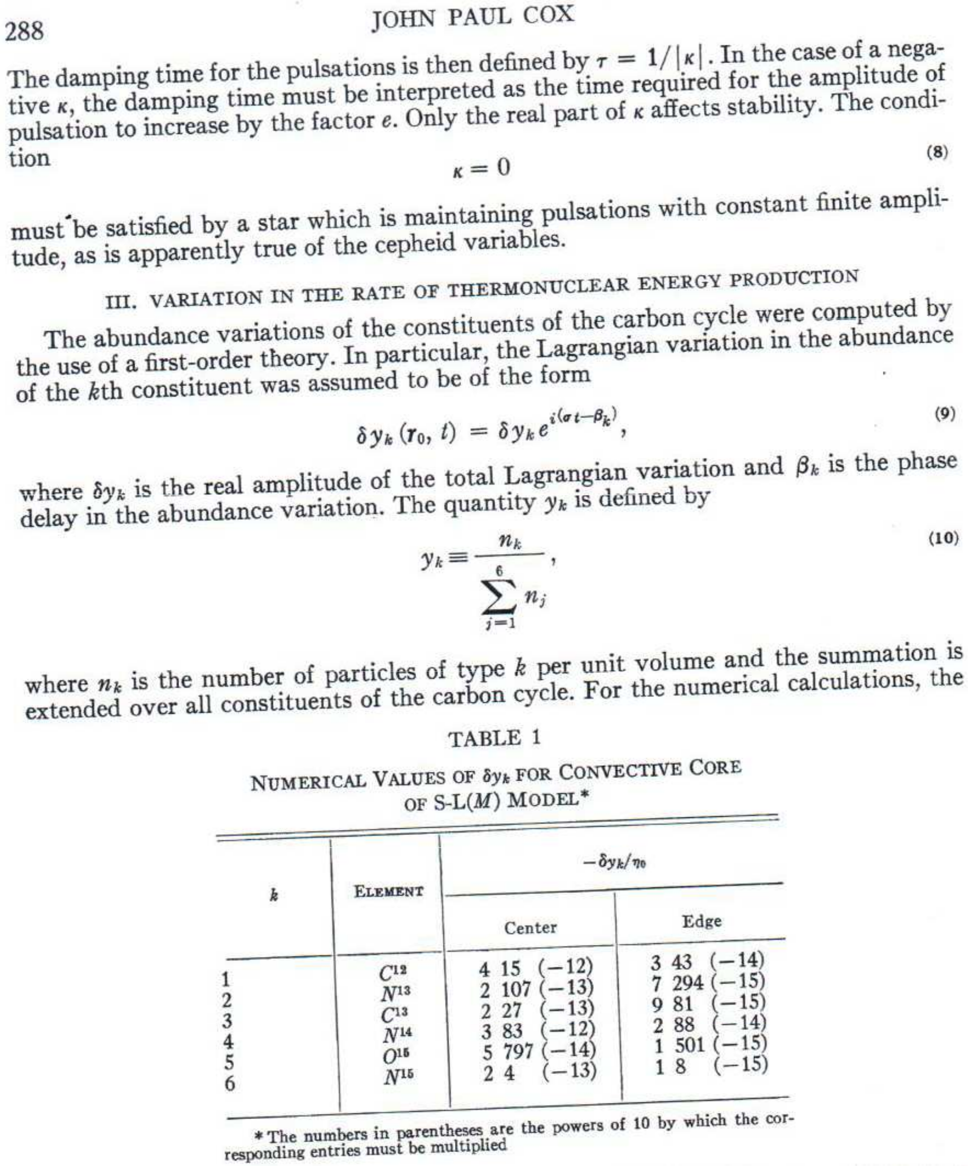
The purpose of the present study is to examine the pulsational stability of recent models proposed to represent the classical cepheid variable stars. Pulsation integrations of three such highly centrally concentrated red giant models were carried out recently by I. Epstein (1950), for the purpose of examining the effect of high central-mass concentration on the pulsation properties. One of the models used was the red giant model Case 15 of Schwarzschild and Li Hen (1949). This model will hereafter be referred to simply as the "S-L model." Whenever reference is made specifically to the particular S-L model with $M = 1.34 \times 10^{35}$ gm chosen by Epstein (1950) and used for the numerical calculations here, we shall write "S-L(M) model." The central temperature in all cases is taken as 30×10^{6} ° K, and the central density appropriate to this assumed mass is 22.14 gm/cm^{3}.

Hvězdy

John Paul Cox

americký astrofyzik, ***Pulsační stabilita modelů rudých obrů, r. 1955***





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The damping time for the pulsations is then defined by $\tau = 1/|\kappa|$. In the case of a negative κ, the damping time must be interpreted as the time required for the amplitude of pulsation to increase by the factor e. Only the real part of κ affects stability. The condition

$$\kappa = 0 \tag{8}$$

must be satisfied by a star which is maintaining pulsations with constant finite amplitude, as is apparently true of the cepheid variables.

III. VARIATION IN THE RATE OF THERMONUCLEAR ENERGY PRODUCTION

The abundance variations of the constituents of the carbon cycle were computed by the use of a first-order theory. In particular, the Lagrangian variation in the abundance of the kth constituent was assumed to be of the form

$$\delta y_k(r_0, t) = \delta y_k e^{i(\sigma t - \beta_k)}, \tag{9}$$

where δy_k is the real amplitude of the total Lagrangian variation and β_k is the phase delay in the abundance variation. The quantity y_k is defined by

$$y_k \equiv \frac{n_k}{\sum_{j=1}^{6} n_j}, \tag{10}$$

where n_k is the number of particles of type k per unit volume and the summation is extended over all constituents of the carbon cycle. For the numerical calculations, the

TABLE 1

NUMERICAL VALUES OF δy_k FOR CONVECTIVE CORE OF S-L(M) MODEL*

k	ELEMENT	$-\delta y_k/\eta_0$	
		Center	Edge
1	C^{12}	4 15 (−12)	3 43 (−14)
2	N^{13}	2 107 (−13)	7 294 (−15)
3	C^{13}	2 27 (−13)	9 81 (−15)
4	N^{14}	3 83 (−12)	2 88 (−14)
5	O^{15}	5 797 (−14)	1 501 (−15)
6	N^{15}	2 4 (−13)	1 8 (−15)

* The numbers in parentheses are the powers of 10 by which the corresponding entries must be multiplied

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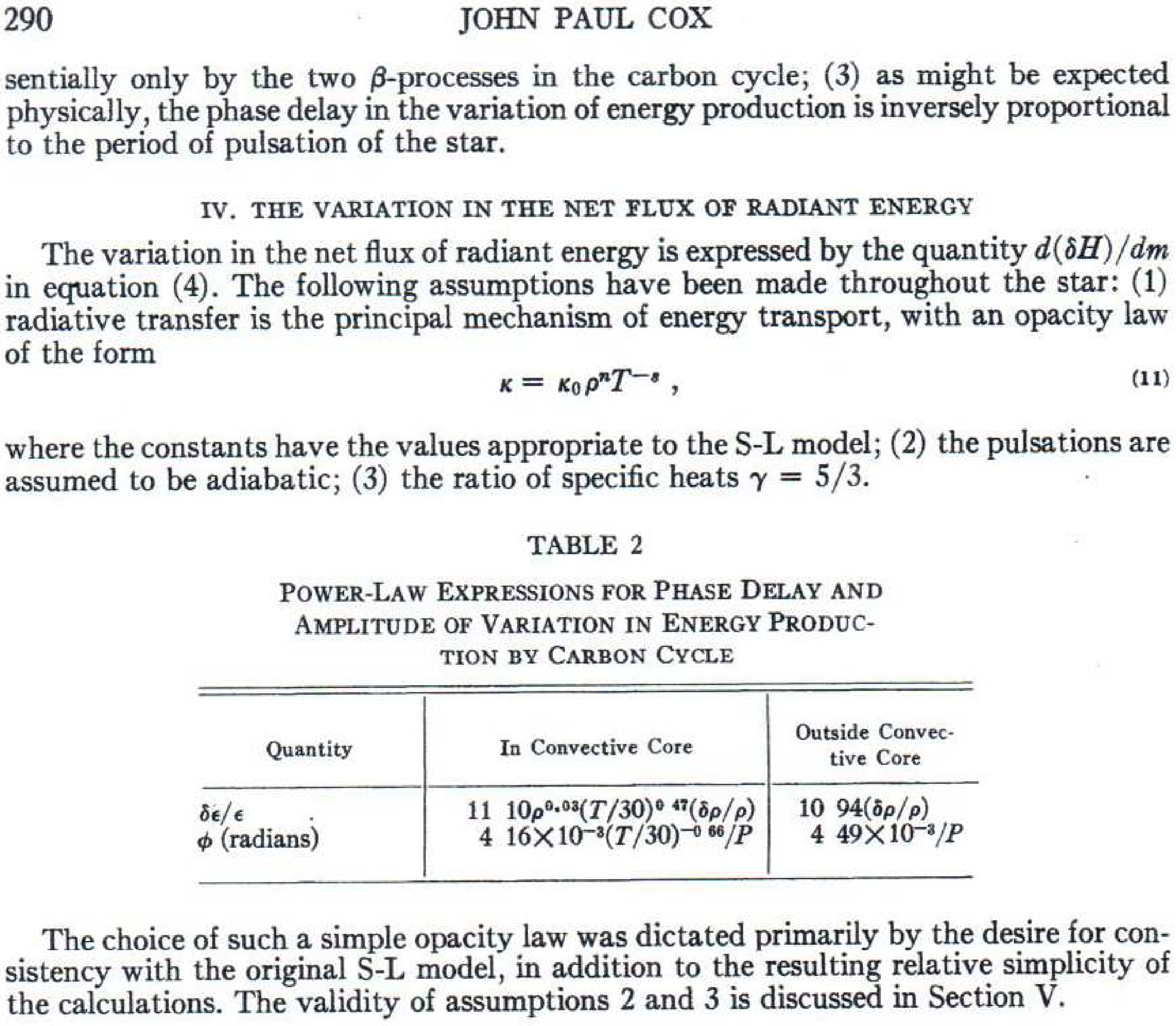
americký astrofyzik, ***Pulsační stabilita modelů rudých obrů, r. 1955***



sentially only by the two β-processes in the carbon cycle; (3) as might be expected physically, the phase delay in the variation of energy production is inversely proportional to the period of pulsation of the star.

IV. THE VARIATION IN THE NET FLUX OF RADIANT ENERGY

The variation in the net flux of radiant energy is expressed by the quantity $d(\delta H)/dm$ in equation (4). The following assumptions have been made throughout the star: (1) radiative transfer is the principal mechanism of energy transport, with an opacity law of the form

$$\kappa = \kappa_0 \rho^n T^{-s} , \tag{11}$$

where the constants have the values appropriate to the S-L model; (2) the pulsations are assumed to be adiabatic; (3) the ratio of specific heats $\gamma = 5/3$.

TABLE 2

POWER-LAW EXPRESSIONS FOR PHASE DELAY AND AMPLITUDE OF VARIATION IN ENERGY PRODUCTION BY CARBON CYCLE

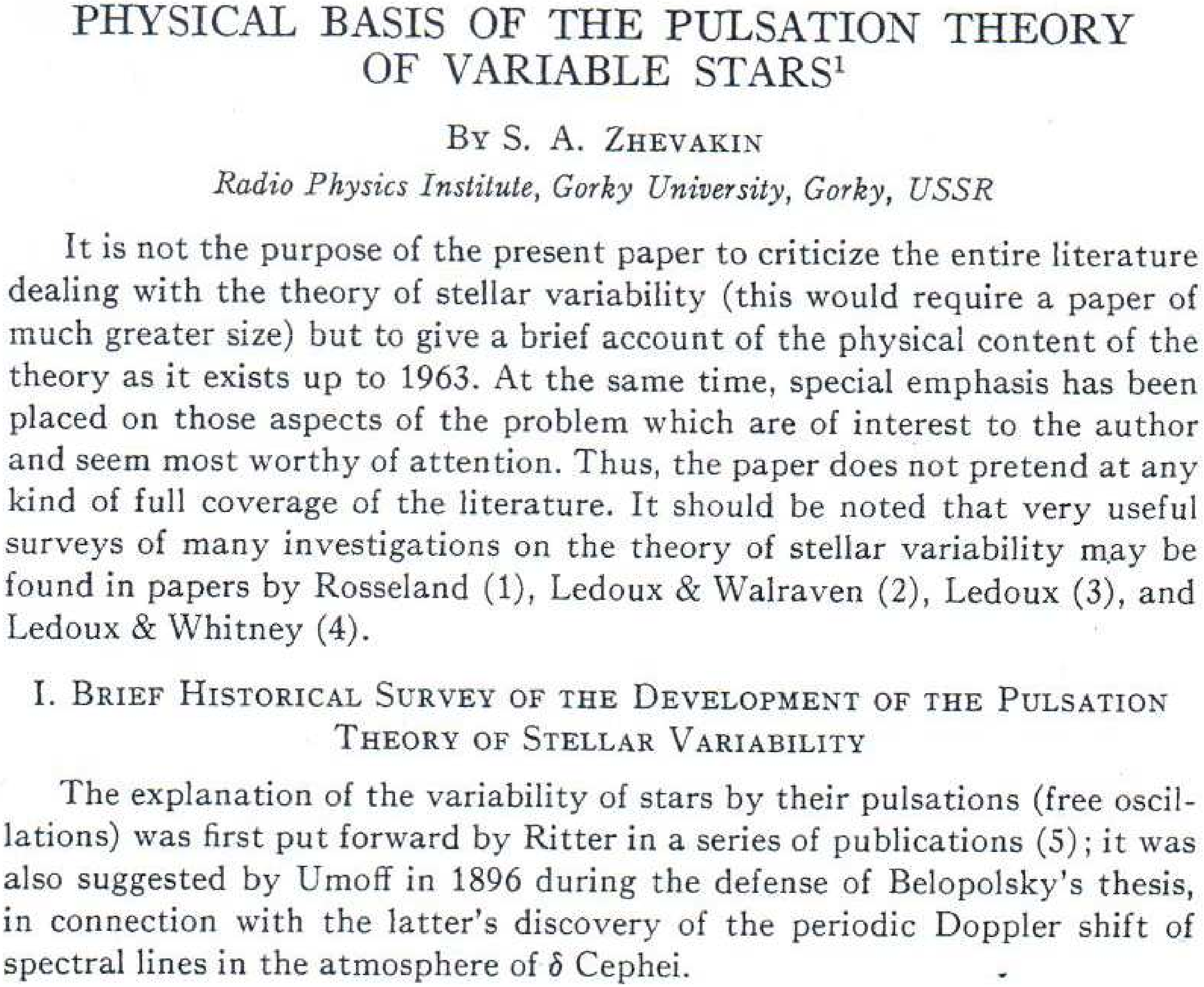
Quantity	In Convective Core	Outside Convective Core
$\delta\epsilon/\epsilon$	$11\ 10\rho^{0.03}(T/30)^{0\ 47}(\delta\rho/\rho)$	$10\ 94(\delta\rho/\rho)$
ϕ (radians)	$4\ 16\times 10^{-3}(T/30)^{-0\ 66}/P$	$4\ 49\times 10^{-3}/P$

The choice of such a simple opacity law was dictated primarily by the desire for consistency with the original S-L model, in addition to the resulting relative simplicity of the calculations. The validity of assumptions 2 and 3 is discussed in Section V.

Hvězdy - Sergej Aleksandrovič Ževakin
1916 - 2001

ruský astrofyzik, ***Fyzikální základy pulsační teorie proměnných hvězd r.1963,*** chemické složení pulsačních vrstev cefeid, úloha helia, r. 1953, 1963

Figure 6: Sergei Aleksandrovich Zhevakin 1916 – 2001



PHYSICAL BASIS OF THE PULSATION THEORY OF VARIABLE STARS[1]

By S. A. Zhevakin

Radio Physics Institute, Gorky University, Gorky, USSR

It is not the purpose of the present paper to criticize the entire literature dealing with the theory of stellar variability (this would require a paper of much greater size) but to give a brief account of the physical content of the theory as it exists up to 1963. At the same time, special emphasis has been placed on those aspects of the problem which are of interest to the author and seem most worthy of attention. Thus, the paper does not pretend at any kind of full coverage of the literature. It should be noted that very useful surveys of many investigations on the theory of stellar variability may be found in papers by Rosseland (1), Ledoux & Walraven (2), Ledoux (3), and Ledoux & Whitney (4).

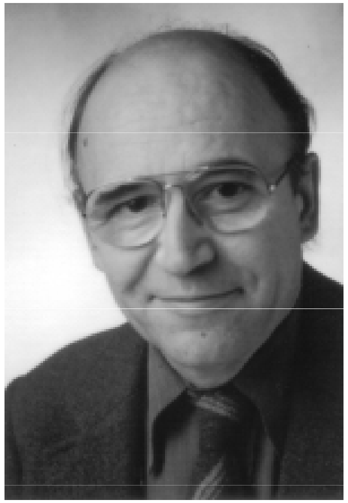
I. Brief Historical Survey of the Development of the Pulsation Theory of Stellar Variability

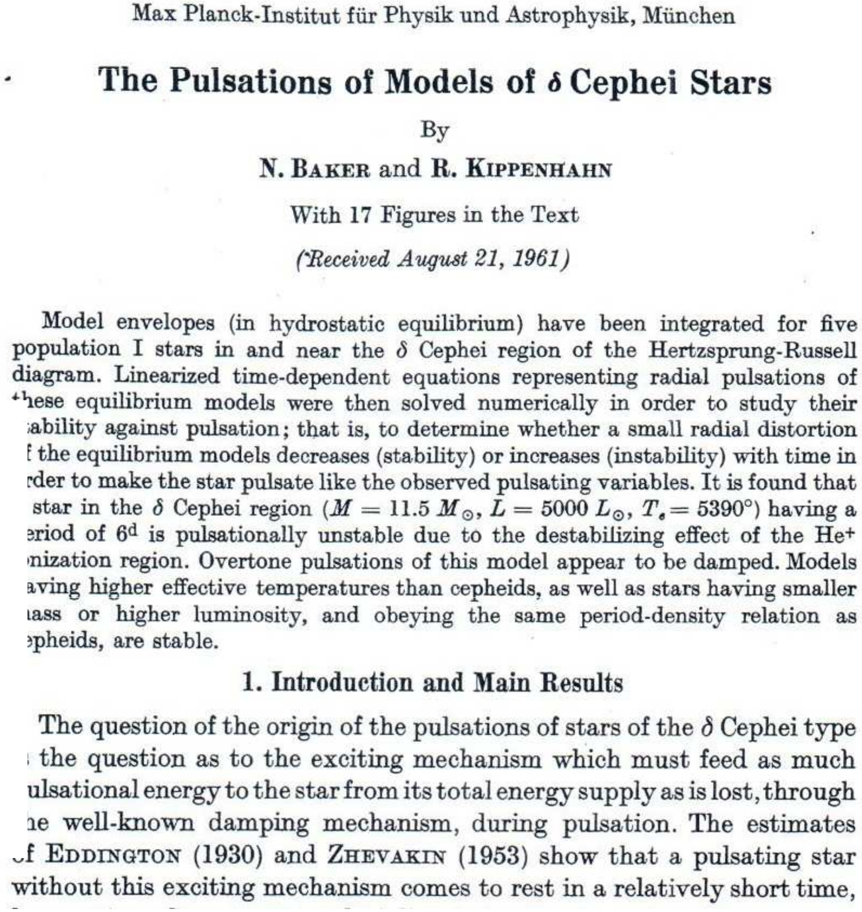
The explanation of the variability of stars by their pulsations (free oscillations) was first put forward by Ritter in a series of publications (5); it was also suggested by Umoff in 1896 during the defense of Belopolsky's thesis, in connection with the latter's discovery of the periodic Doppler shift of spectral lines in the atmosphere of δ Cephei.

Hvězdy
Norman Hodgson Baker 1931 - 2005, Rudolf Kippenhahn 1926 - 2020

americký astrofyzik, německý astrofyzik českého původu

Pulsační modely hvězd δ Cephei, r. 1962





Max Planck-Institut für Physik und Astrophysik, München

The Pulsations of Models of δ Cephei Stars

By

N. Baker and R. Kippenhahn

With 17 Figures in the Text

(Received August 21, 1961)

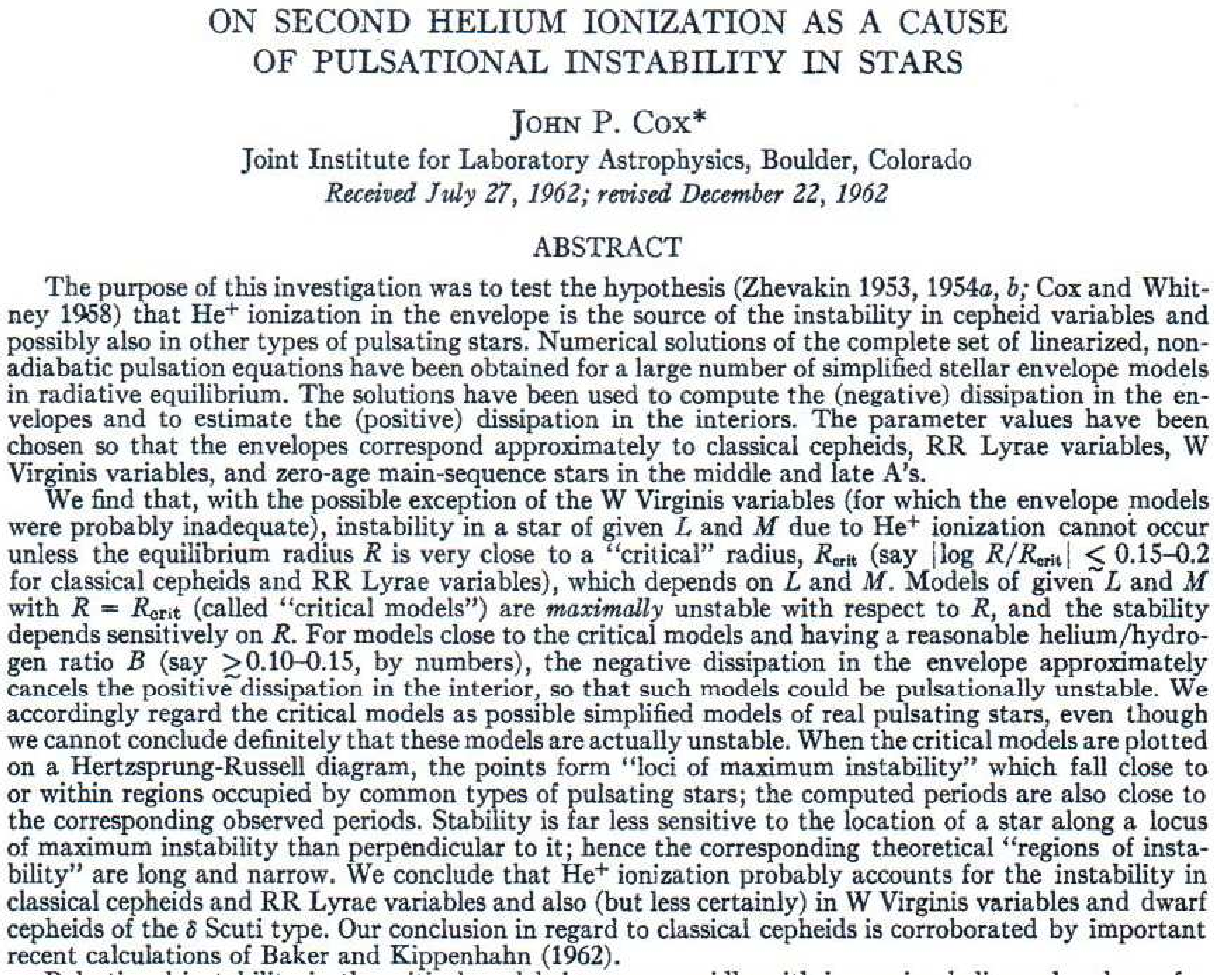
Model envelopes (in hydrostatic equilibrium) have been integrated for five population I stars in and near the δ Cephei region of the Hertzsprung-Russell diagram. Linearized time-dependent equations representing radial pulsations of hese equilibrium models were then solved numerically in order to study their ability against pulsation; that is, to determine whether a small radial distortion f the equilibrium models decreases (stability) or increases (instability) with time in rder to make the star pulsate like the observed pulsating variables. It is found that star in the δ Cephei region ($M = 11.5\,M_\odot$, $L = 5000\,L_\odot$, $T_e = 5390°$) having a eriod of 6^d is pulsationally unstable due to the destabilizing effect of the He^+ nization region. Overtone pulsations of this model appear to be damped. Models aving higher effective temperatures than cepheids, as well as stars having smaller ass or higher luminosity, and obeying the same period-density relation as pheids, are stable.

1. Introduction and Main Results

The question of the origin of the pulsations of stars of the δ Cephei type the question as to the exciting mechanism which must feed as much ulsational energy to the star from its total energy supply as is lost, through he well-known damping mechanism, during pulsation. The estimates f EDDINGTON (1930) and ZHEVAKIN (1953) show that a pulsating star without this exciting mechanism comes to rest in a relatively short time,

Hvězdy
John Paul Cox

americký astrofyzik, ***Druhá ionizace helia jako případ pulsační nestability, r. 1963***



ON SECOND HELIUM IONIZATION AS A CAUSE OF PULSATIONAL INSTABILITY IN STARS

JOHN P. COX*
Joint Institute for Laboratory Astrophysics, Boulder, Colorado

Received July 27, 1962; revised December 22, 1962

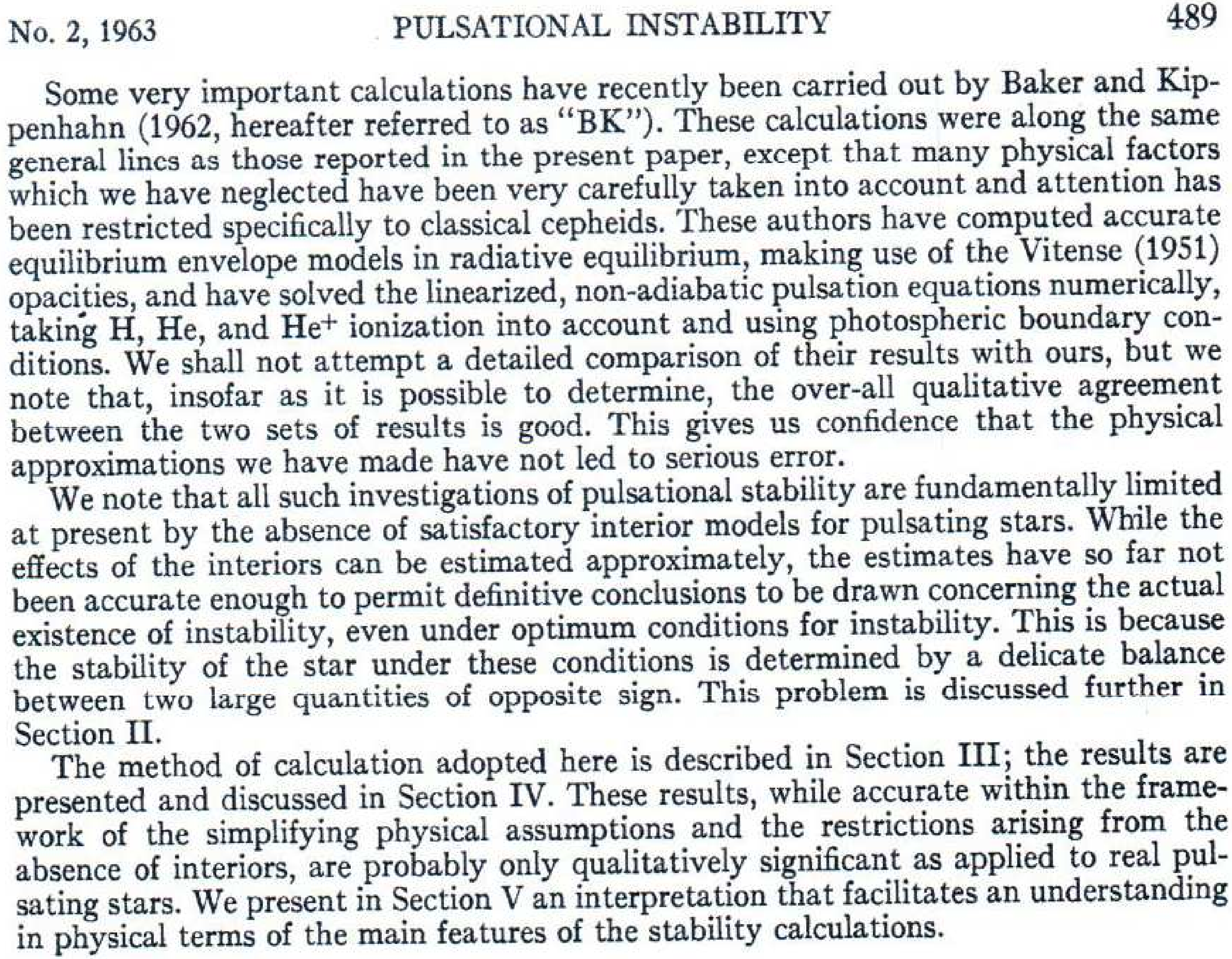
ABSTRACT

The purpose of this investigation was to test the hypothesis (Zhevakin 1953, 1954*a*, *b;* Cox and Whitney 1958) that He^+ ionization in the envelope is the source of the instability in cepheid variables and possibly also in other types of pulsating stars. Numerical solutions of the complete set of linearized, non-adiabatic pulsation equations have been obtained for a large number of simplified stellar envelope models in radiative equilibrium. The solutions have been used to compute the (negative) dissipation in the envelopes and to estimate the (positive) dissipation in the interiors. The parameter values have been chosen so that the envelopes correspond approximately to classical cepheids, RR Lyrae variables, W Virginis variables, and zero-age main-sequence stars in the middle and late A's.

We find that, with the possible exception of the W Virginis variables (for which the envelope models were probably inadequate), instability in a star of given L and M due to He^+ ionization cannot occur unless the equilibrium radius R is very close to a "critical" radius, $R_{\rm crit}$ (say $|\log R/R_{\rm crit}| \lesssim 0.15$–$0.2$ for classical cepheids and RR Lyrae variables), which depends on L and M. Models of given L and M with $R = R_{\rm crit}$ (called "critical models") are *maximally* unstable with respect to R, and the stability depends sensitively on R. For models close to the critical models and having a reasonable helium/hydrogen ratio B (say $\gtrsim 0.10$–0.15, by numbers), the negative dissipation in the envelope approximately cancels the positive dissipation in the interior, so that such models could be pulsationally unstable. We accordingly regard the critical models as possible simplified models of real pulsating stars, even though we cannot conclude definitely that these models are actually unstable. When the critical models are plotted on a Hertzsprung-Russell diagram, the points form "loci of maximum instability" which fall close to or within regions occupied by common types of pulsating stars; the computed periods are also close to the corresponding observed periods. Stability is far less sensitive to the location of a star along a locus of maximum instability than perpendicular to it; hence the corresponding theoretical "regions of instability" are long and narrow. We conclude that He^+ ionization probably accounts for the instability in classical cepheids and RR Lyrae variables and also (but less certainly) in W Virginis variables and dwarf cepheids of the δ Scuti type. Our conclusion in regard to classical cepheids is corroborated by important recent calculations of Baker and Kippenhahn (1962).

John Paul Cox 1926 - 1984

americký astrofyzik, ***Druhá ionizace helia jako případ pulsační nestability, r. 1963***



No. 2, 1963 PULSATIONAL INSTABILITY 489

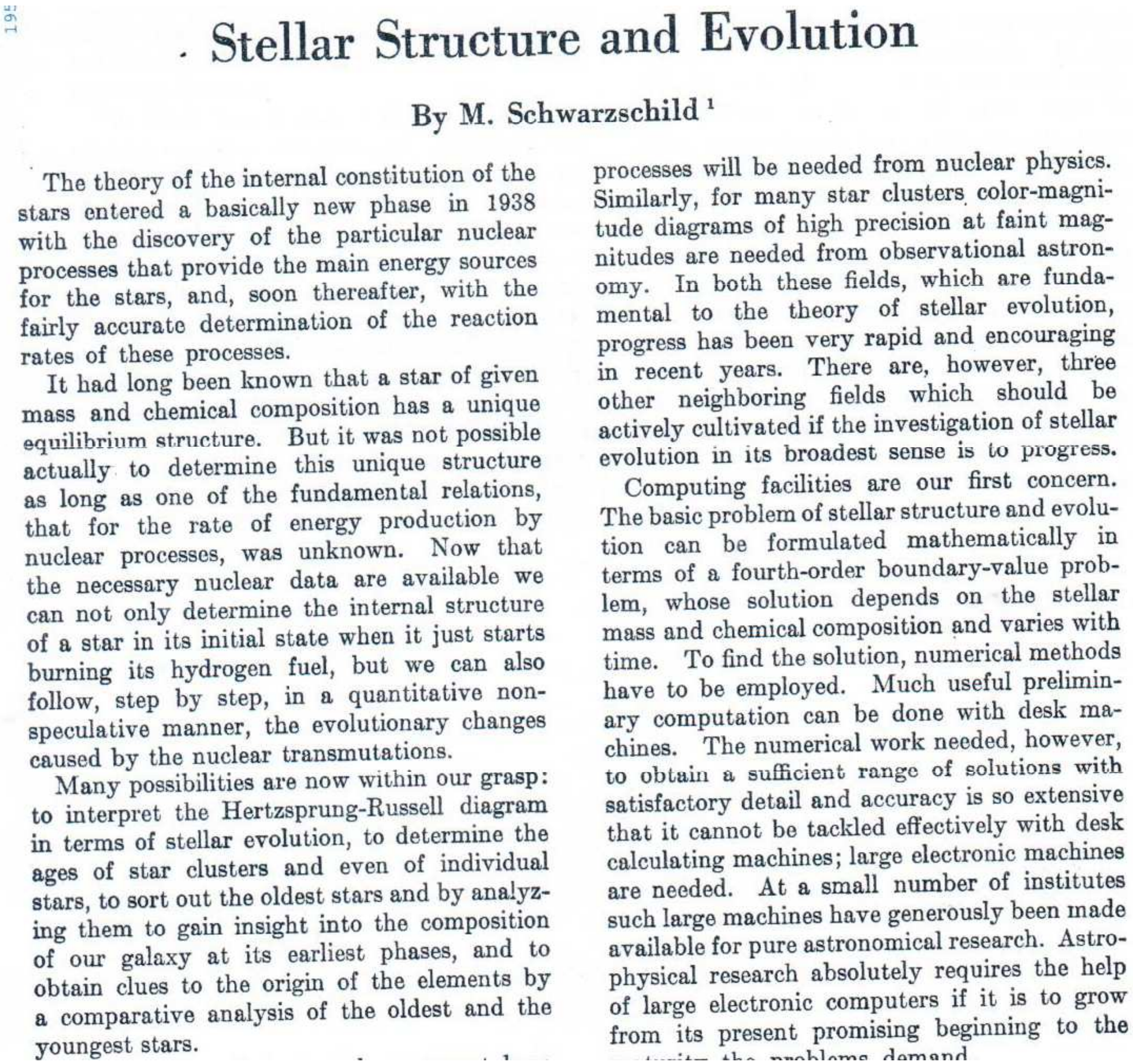
Some very important calculations have recently been carried out by Baker and Kippenhahn (1962, hereafter referred to as "BK"). These calculations were along the same general lines as those reported in the present paper, except that many physical factors which we have neglected have been very carefully taken into account and attention has been restricted specifically to classical cepheids. These authors have computed accurate equilibrium envelope models in radiative equilibrium, making use of the Vitense (1951) opacities, and have solved the linearized, non-adiabatic pulsation equations numerically, taking H, He, and He^+ ionization into account and using photospheric boundary conditions. We shall not attempt a detailed comparison of their results with ours, but we note that, insofar as it is possible to determine, the over-all qualitative agreement between the two sets of results is good. This gives us confidence that the physical approximations we have made have not led to serious error.

We note that all such investigations of pulsational stability are fundamentally limited at present by the absence of satisfactory interior models for pulsating stars. While the effects of the interiors can be estimated approximately, the estimates have so far not been accurate enough to permit definitive conclusions to be drawn concerning the actual existence of instability, even under optimum conditions for instability. This is because the stability of the star under these conditions is determined by a delicate balance between two large quantities of opposite sign. This problem is discussed further in Section II.

The method of calculation adopted here is described in Section III; the results are presented and discussed in Section IV. These results, while accurate within the framework of the simplifying physical assumptions and the restrictions arising from the absence of interiors, are probably only qualitatively significant as applied to real pulsating stars. We present in Section V an interpretation that facilitates an understanding in physical terms of the main features of the stability calculations.

Hvězdy
Martin Schwarzschild 1912 - 1997

americký astrofyzik, ***r. 1961 – vývoj hvězd po odchodu z hlavní posloupnosti***



Stellar Structure and Evolution

By M. Schwarzschild[1]

The theory of the internal constitution of the stars entered a basically new phase in 1938 with the discovery of the particular nuclear processes that provide the main energy sources for the stars, and, soon thereafter, with the fairly accurate determination of the reaction rates of these processes.

It had long been known that a star of given mass and chemical composition has a unique equilibrium structure. But it was not possible actually to determine this unique structure as long as one of the fundamental relations, that for the rate of energy production by nuclear processes, was unknown. Now that the necessary nuclear data are available we can not only determine the internal structure of a star in its initial state when it just starts burning its hydrogen fuel, but we can also follow, step by step, in a quantitative non-speculative manner, the evolutionary changes caused by the nuclear transmutations.

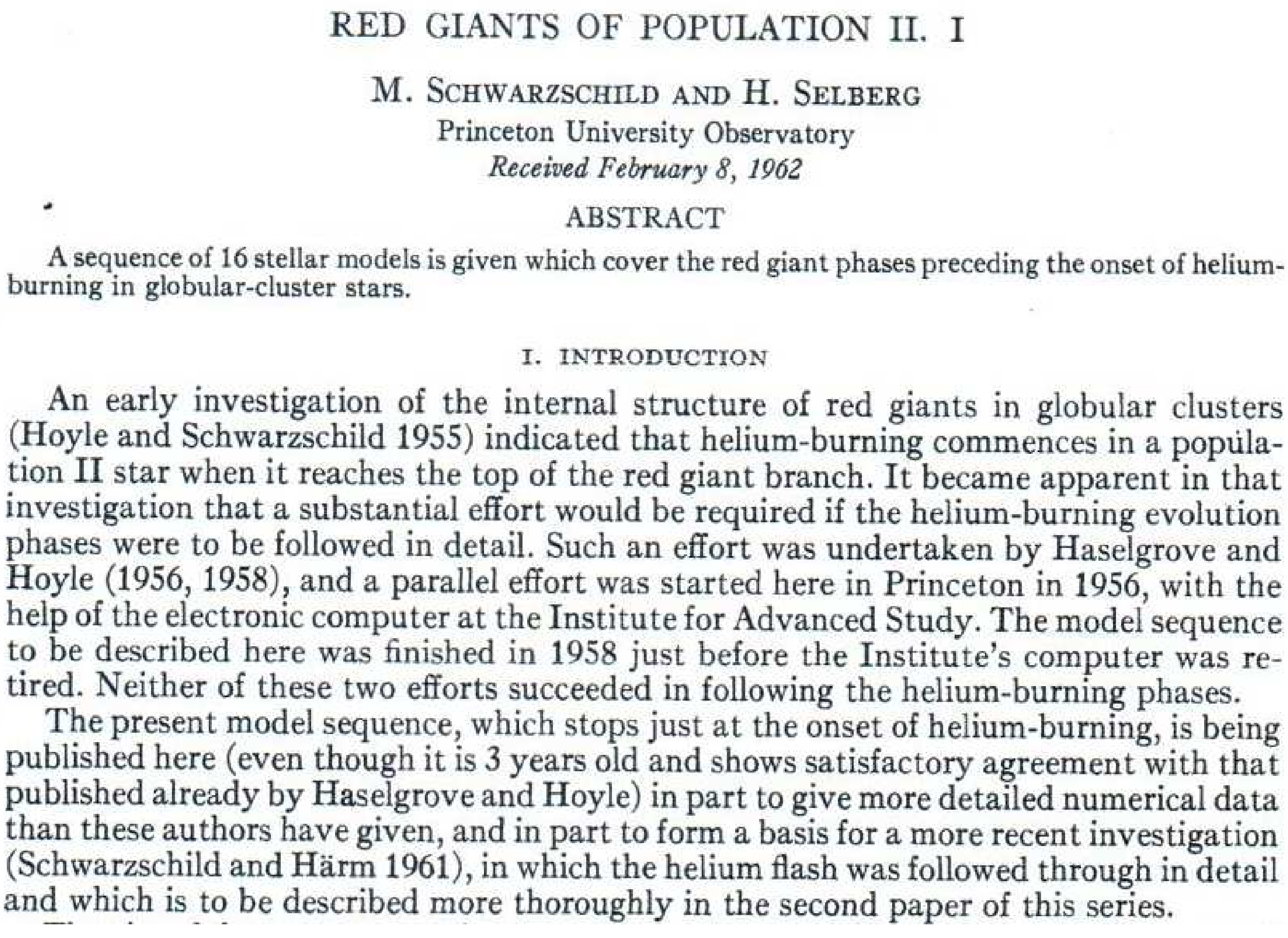
Many possibilities are now within our grasp: to interpret the Hertzsprung-Russell diagram in terms of stellar evolution, to determine the ages of star clusters and even of individual stars, to sort out the oldest stars and by analyzing them to gain insight into the composition of our galaxy at its earliest phases, and to obtain clues to the origin of the elements by a comparative analysis of the oldest and the youngest stars.

processes will be needed from nuclear physics. Similarly, for many star clusters color-magnitude diagrams of high precision at faint magnitudes are needed from observational astronomy. In both these fields, which are fundamental to the theory of stellar evolution, progress has been very rapid and encouraging in recent years. There are, however, three other neighboring fields which should be actively cultivated if the investigation of stellar evolution in its broadest sense is to progress.

Computing facilities are our first concern. The basic problem of stellar structure and evolution can be formulated mathematically in terms of a fourth-order boundary-value problem, whose solution depends on the stellar mass and chemical composition and varies with time. To find the solution, numerical methods have to be employed. Much useful preliminary computation can be done with desk machines. The numerical work needed, however, to obtain a sufficient range of solutions with satisfactory detail and accuracy is so extensive that it cannot be tackled effectively with desk calculating machines; large electronic machines are needed. At a small number of institutes such large machines have generously been made available for pure astronomical research. Astrophysical research absolutely requires the help of large electronic computers if it is to grow from its present promising beginning to the

Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962



RED GIANTS OF POPULATION II. I

M. SCHWARZSCHILD AND H. SELBERG
Princeton University Observatory

Received February 8, 1962

ABSTRACT

A sequence of 16 stellar models is given which cover the red giant phases preceding the onset of helium-burning in globular-cluster stars.

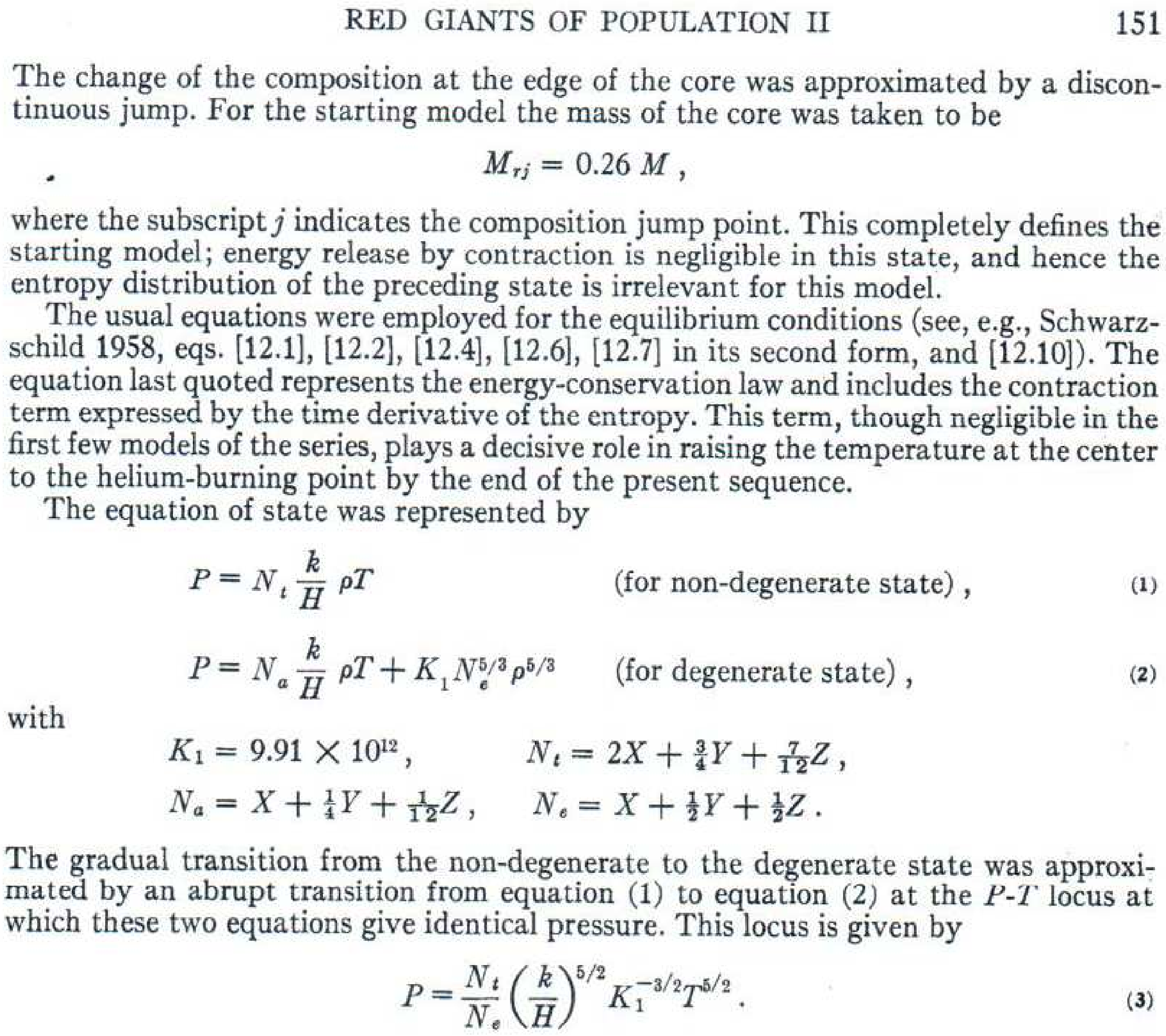
I. INTRODUCTION

An early investigation of the internal structure of red giants in globular clusters (Hoyle and Schwarzschild 1955) indicated that helium-burning commences in a population II star when it reaches the top of the red giant branch. It became apparent in that investigation that a substantial effort would be required if the helium-burning evolution phases were to be followed in detail. Such an effort was undertaken by Haselgrove and Hoyle (1956, 1958), and a parallel effort was started here in Princeton in 1956, with the help of the electronic computer at the Institute for Advanced Study. The model sequence to be described here was finished in 1958 just before the Institute's computer was retired. Neither of these two efforts succeeded in following the helium-burning phases.

The present model sequence, which stops just at the onset of helium-burning, is being published here (even though it is 3 years old and shows satisfactory agreement with that published already by Haselgrove and Hoyle) in part to give more detailed numerical data than these authors have given, and in part to form a basis for a more recent investigation (Schwarzschild and Härm 1961), in which the helium flash was followed through in detail and which is to be described more thoroughly in the second paper of this series.

Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962



The change of the composition at the edge of the core was approximated by a discontinuous jump. For the starting model the mass of the core was taken to be

$$M_{rj} = 0.26\,M\,,$$

where the subscript *j* indicates the composition jump point. This completely defines the starting model; energy release by contraction is negligible in this state, and hence the entropy distribution of the preceding state is irrelevant for this model.

The usual equations were employed for the equilibrium conditions (see, e.g., Schwarzschild 1958, eqs. [12.1], [12.2], [12.4], [12.6], [12.7] in its second form, and [12.10]). The equation last quoted represents the energy-conservation law and includes the contraction term expressed by the time derivative of the entropy. This term, though negligible in the first few models of the series, plays a decisive role in raising the temperature at the center to the helium-burning point by the end of the present sequence.

The equation of state was represented by

$$P = N_t \frac{k}{H} \rho T \qquad \text{(for non-degenerate state)}\,, \tag{1}$$

$$P = N_a \frac{k}{H} \rho T + K_1 N_e^{5/3} \rho^{5/3} \qquad \text{(for degenerate state)}\,, \tag{2}$$

with

$$K_1 = 9.91 \times 10^{12}\,, \qquad N_t = 2X + \tfrac{3}{4}Y + \tfrac{7}{12}Z\,,$$

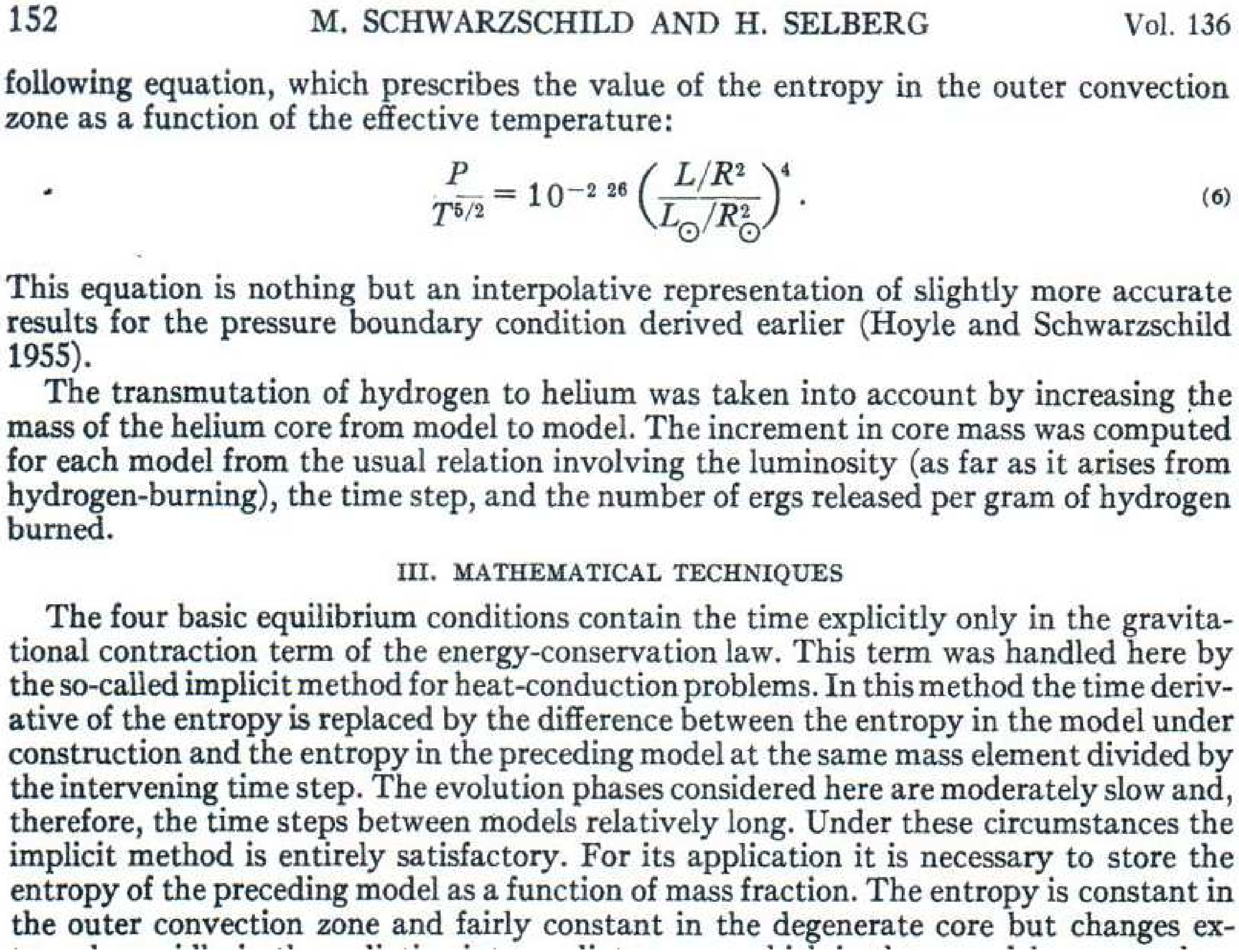
$$N_a = X + \tfrac{1}{4}Y + \tfrac{1}{12}Z\,, \qquad N_e = X + \tfrac{1}{2}Y + \tfrac{1}{2}Z\,.$$

The gradual transition from the non-degenerate to the degenerate state was approximated by an abrupt transition from equation (1) to equation (2) at the *P-T* locus at which these two equations give identical pressure. This locus is given by

$$P = \frac{N_t}{N_e}\left(\frac{k}{H}\right)^{5/2} K_1^{-3/2} T^{5/2}\,. \tag{3}$$

Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962



following equation, which prescribes the value of the entropy in the outer convection zone as a function of the effective temperature:

$$\frac{P}{T^{5/2}} = 10^{-2.26} \left(\frac{L/R^2}{L_{\odot}/R_{\odot}^2}\right)^4 . \qquad (6)$$

This equation is nothing but an interpolative representation of slightly more accurate results for the pressure boundary condition derived earlier (Hoyle and Schwarzschild 1955).

The transmutation of hydrogen to helium was taken into account by increasing the mass of the helium core from model to model. The increment in core mass was computed for each model from the usual relation involving the luminosity (as far as it arises from hydrogen-burning), the time step, and the number of ergs released per gram of hydrogen burned.

III. MATHEMATICAL TECHNIQUES

The four basic equilibrium conditions contain the time explicitly only in the gravitational contraction term of the energy-conservation law. This term was handled here by the so-called implicit method for heat-conduction problems. In this method the time derivative of the entropy is replaced by the difference between the entropy in the model under construction and the entropy in the preceding model at the same mass element divided by the intervening time step. The evolution phases considered here are moderately slow and, therefore, the time steps between models relatively long. Under these circumstances the implicit method is entirely satisfactory. For its application it is necessary to store the entropy of the preceding model as a function of mass fraction. The entropy is constant in the outer convection zone and fairly constant in the degenerate core but changes ex-

Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962

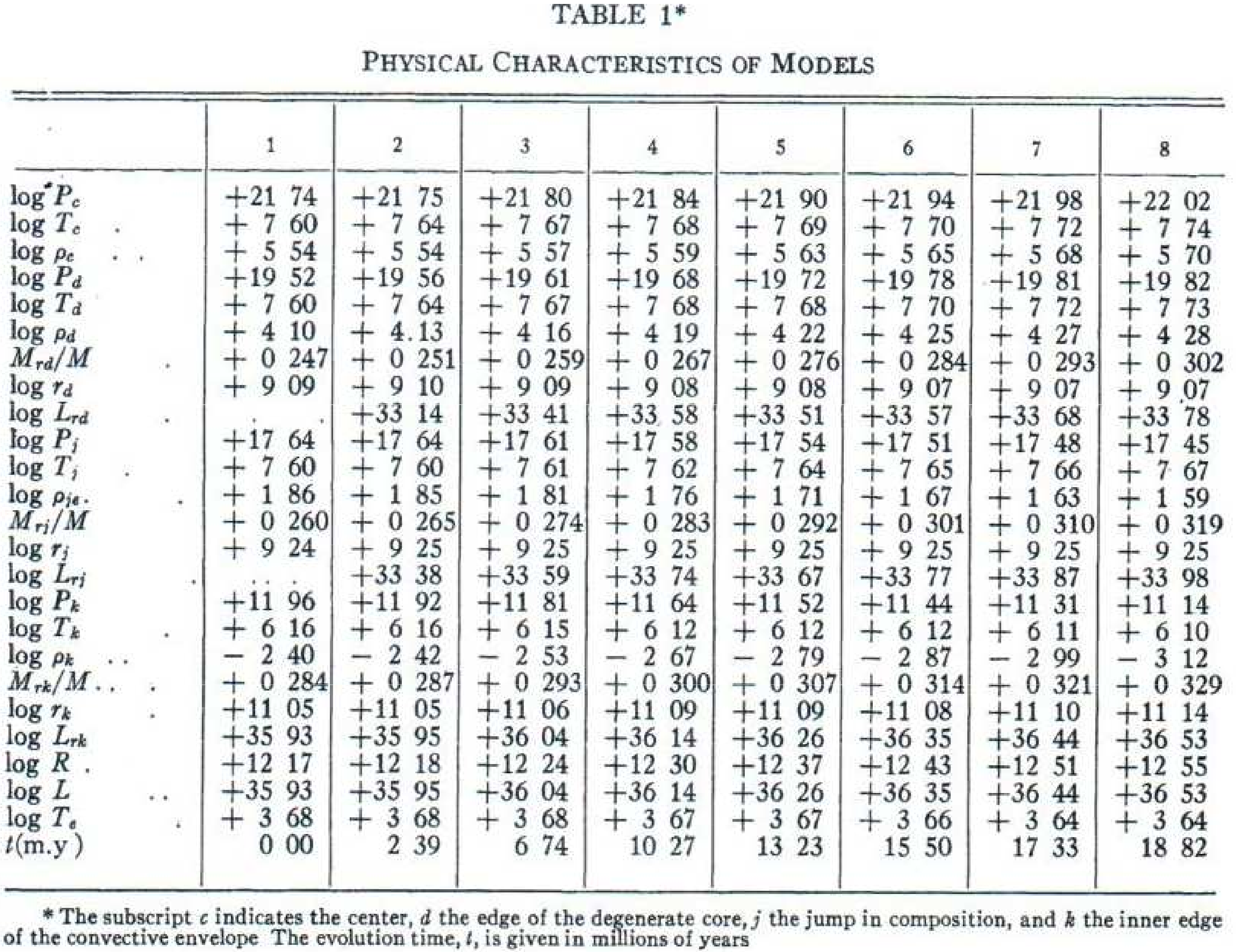


TABLE 1*

PHYSICAL CHARACTERISTICS OF MODELS

	1	2	3	4	5	6	7	8
$\log P_c$	+21 74	+21 75	+21 80	+21 84	+21 90	+21 94	+21 98	+22 02
$\log T_c$	+ 7 60	+ 7 64	+ 7 67	+ 7 68	+ 7 69	+ 7 70	+ 7 72	+ 7 74
$\log \rho_c$	+ 5 54	+ 5 54	+ 5 57	+ 5 59	+ 5 63	+ 5 65	+ 5 68	+ 5 70
$\log P_d$	+19 52	+19 56	+19 61	+19 68	+19 72	+19 78	+19 81	+19 82
$\log T_d$	+ 7 60	+ 7 64	+ 7 67	+ 7 68	+ 7 68	+ 7 70	+ 7 72	+ 7 73
$\log \rho_d$	+ 4 10	+ 4.13	+ 4 16	+ 4 19	+ 4 22	+ 4 25	+ 4 27	+ 4 28
M_{rd}/M	+ 0 247	+ 0 251	+ 0 259	+ 0 267	+ 0 276	+ 0 284	+ 0 293	+ 0 302
$\log r_d$	+ 9 09	+ 9 10	+ 9 09	+ 9 08	+ 9 08	+ 9 07	+ 9 07	+ 9 07
$\log L_{rd}$	. . .	+33 14	+33 41	+33 58	+33 51	+33 57	+33 68	+33 78
$\log P_j$	+17 64	+17 64	+17 61	+17 58	+17 54	+17 51	+17 48	+17 45
$\log T_j$	+ 7 60	+ 7 60	+ 7 61	+ 7 62	+ 7 64	+ 7 65	+ 7 66	+ 7 67
$\log \rho_{je}$	+ 1 86	+ 1 85	+ 1 81	+ 1 76	+ 1 71	+ 1 67	+ 1 63	+ 1 59
M_{rj}/M	+ 0 260	+ 0 265	+ 0 274	+ 0 283	+ 0 292	+ 0 301	+ 0 310	+ 0 319
$\log r_j$	+ 9 24	+ 9 25	+ 9 25	+ 9 25	+ 9 25	+ 9 25	+ 9 25	+ 9 25
$\log L_{rj}$	. . .	+33 38	+33 59	+33 74	+33 67	+33 77	+33 87	+33 98
$\log P_k$	+11 96	+11 92	+11 81	+11 64	+11 52	+11 44	+11 31	+11 14
$\log T_k$	+ 6 16	+ 6 16	+ 6 15	+ 6 12	+ 6 12	+ 6 12	+ 6 11	+ 6 10
$\log \rho_k$	− 2 40	− 2 42	− 2 53	− 2 67	− 2 79	− 2 87	− 2 99	− 3 12
M_{rk}/M	+ 0 284	+ 0 287	+ 0 293	+ 0 300	+ 0 307	+ 0 314	+ 0 321	+ 0 329
$\log r_k$	+11 05	+11 05	+11 06	+11 09	+11 09	+11 08	+11 10	+11 14
$\log L_{rk}$	+35 93	+35 95	+36 04	+36 14	+36 26	+36 35	+36 44	+36 53
$\log R$	+12 17	+12 18	+12 24	+12 30	+12 37	+12 43	+12 51	+12 55
$\log L$	+35 93	+35 95	+36 04	+36 14	+36 26	+36 35	+36 44	+36 53
$\log T_e$	+ 3 68	+ 3 68	+ 3 68	+ 3 67	+ 3 67	+ 3 66	+ 3 64	+ 3 64
t(m.y)	0 00	2 39	6 74	10 27	13 23	15 50	17 33	18 82

* The subscript c indicates the center, d the edge of the degenerate core, j the jump in composition, and k the inner edge of the convective envelope The evolution time, t, is given in millions of years

Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962

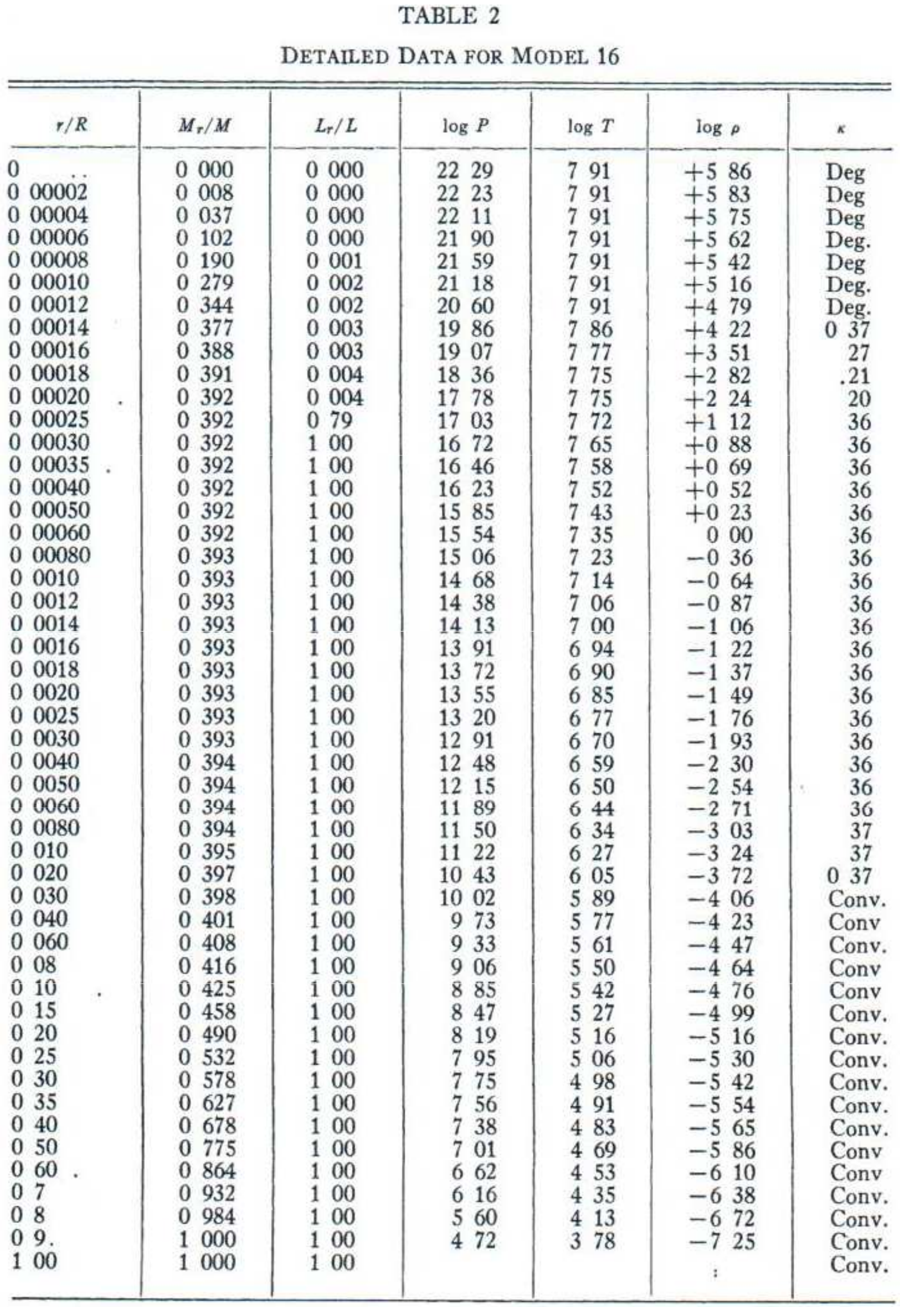


TABLE 2

DETAILED DATA FOR MODEL 16

r/R	M_r/M	L_r/L	log P	log T	log ρ	κ
0 . .	0 000	0 000	22 29	7 91	+5 86	Deg
0 00002	0 008	0 000	22 23	7 91	+5 83	Deg
0 00004	0 037	0 000	22 11	7 91	+5 75	Deg
0 00006	0 102	0 000	21 90	7 91	+5 62	Deg.
0 00008	0 190	0 001	21 59	7 91	+5 42	Deg
0 00010	0 279	0 002	21 18	7 91	+5 16	Deg.
0 00012	0 344	0 002	20 60	7 91	+4 79	Deg.
0 00014	0 377	0 003	19 86	7 86	+4 22	0 37
0 00016	0 388	0 003	19 07	7 77	+3 51	27
0 00018	0 391	0 004	18 36	7 75	+2 82	.21
0 00020	0 392	0 004	17 78	7 75	+2 24	20
0 00025	0 392	0 79	17 03	7 72	+1 12	36
0 00030	0 392	1 00	16 72	7 65	+0 88	36
0 00035	0 392	1 00	16 46	7 58	+0 69	36
0 00040	0 392	1 00	16 23	7 52	+0 52	36
0 00050	0 392	1 00	15 85	7 43	+0 23	36
0 00060	0 392	1 00	15 54	7 35	0 00	36
0 00080	0 393	1 00	15 06	7 23	−0 36	36
0 0010	0 393	1 00	14 68	7 14	−0 64	36
0 0012	0 393	1 00	14 38	7 06	−0 87	36
0 0014	0 393	1 00	14 13	7 00	−1 06	36
0 0016	0 393	1 00	13 91	6 94	−1 22	36
0 0018	0 393	1 00	13 72	6 90	−1 37	36
0 0020	0 393	1 00	13 55	6 85	−1 49	36
0 0025	0 393	1 00	13 20	6 77	−1 76	36
0 0030	0 393	1 00	12 91	6 70	−1 93	36
0 0040	0 394	1 00	12 48	6 59	−2 30	36
0 0050	0 394	1 00	12 15	6 50	−2 54	36
0 0060	0 394	1 00	11 89	6 44	−2 71	36
0 0080	0 394	1 00	11 50	6 34	−3 03	37
0 010	0 395	1 00	11 22	6 27	−3 24	37
0 020	0 397	1 00	10 43	6 05	−3 72	0 37
0 030	0 398	1 00	10 02	5 89	−4 06	Conv.
0 040	0 401	1 00	9 73	5 77	−4 23	Conv
0 060	0 408	1 00	9 33	5 61	−4 47	Conv.
0 08	0 416	1 00	9 06	5 50	−4 64	Conv
0 10	0 425	1 00	8 85	5 42	−4 76	Conv
0 15	0 458	1 00	8 47	5 27	−4 99	Conv.
0 20	0 490	1 00	8 19	5 16	−5 16	Conv.
0 25	0 532	1 00	7 95	5 06	−5 30	Conv.
0 30	0 578	1 00	7 75	4 98	−5 42	Conv.
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0 40	0 678	1 00	7 38	4 83	−5 65	Conv.
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0 60	0 864	1 00	6 62	4 53	−6 10	Conv
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Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962

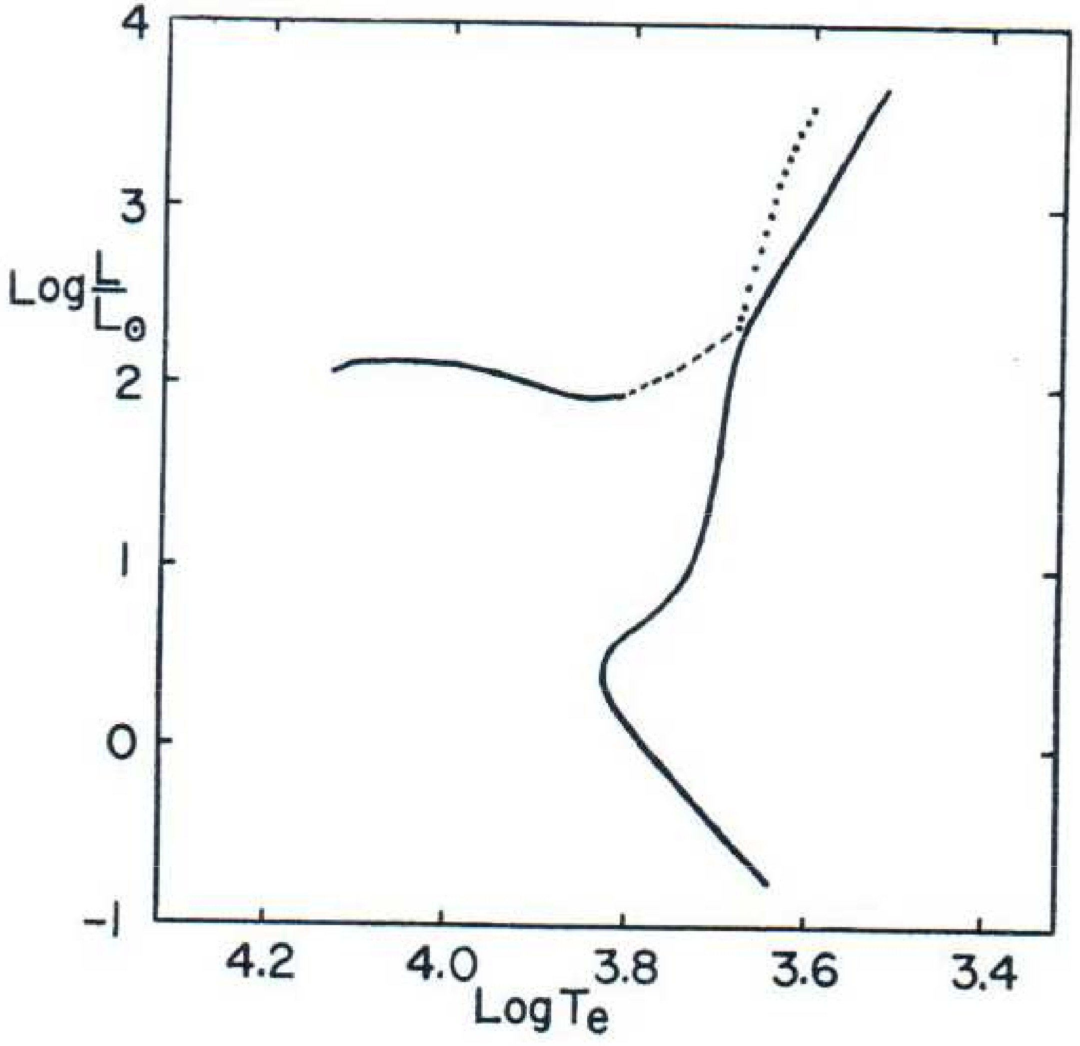


FIG. 1.—Hertzsprung-Russell diagram of the present models (*dots*) compared with a globular cluster (*curve*).

Hvězdy
Martin Schwarzschild

Rudí obři populace II. r. 1962

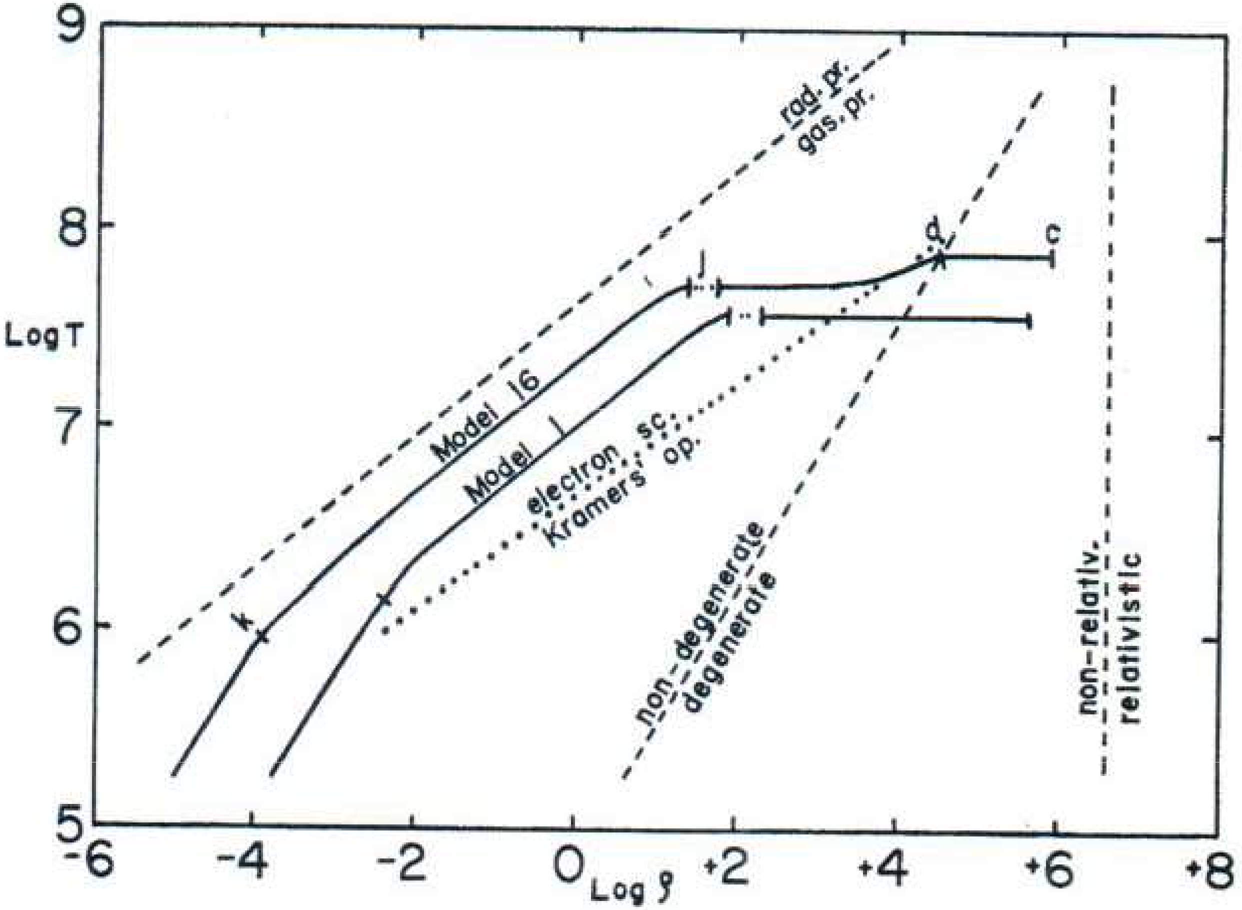
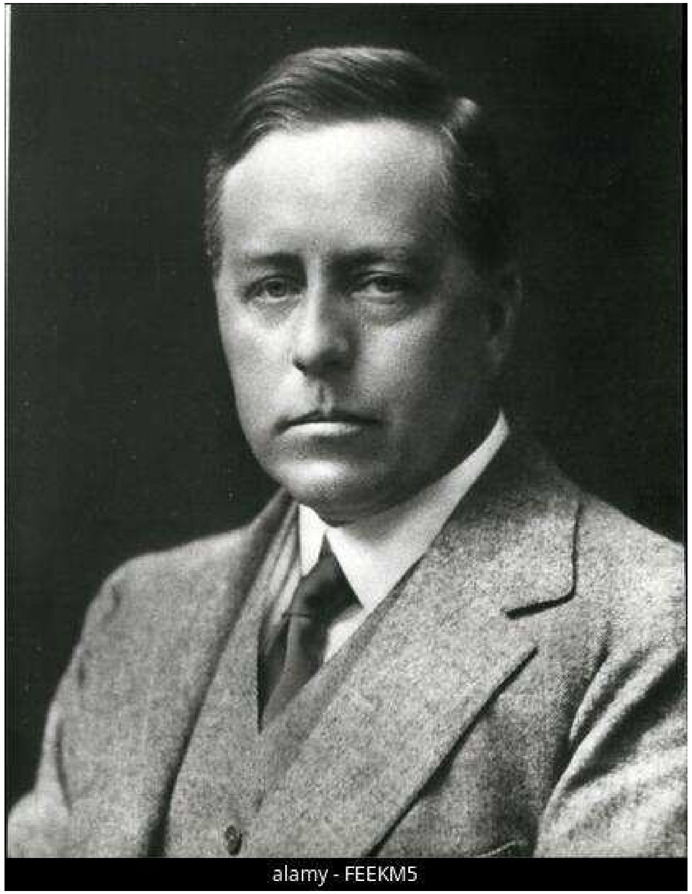
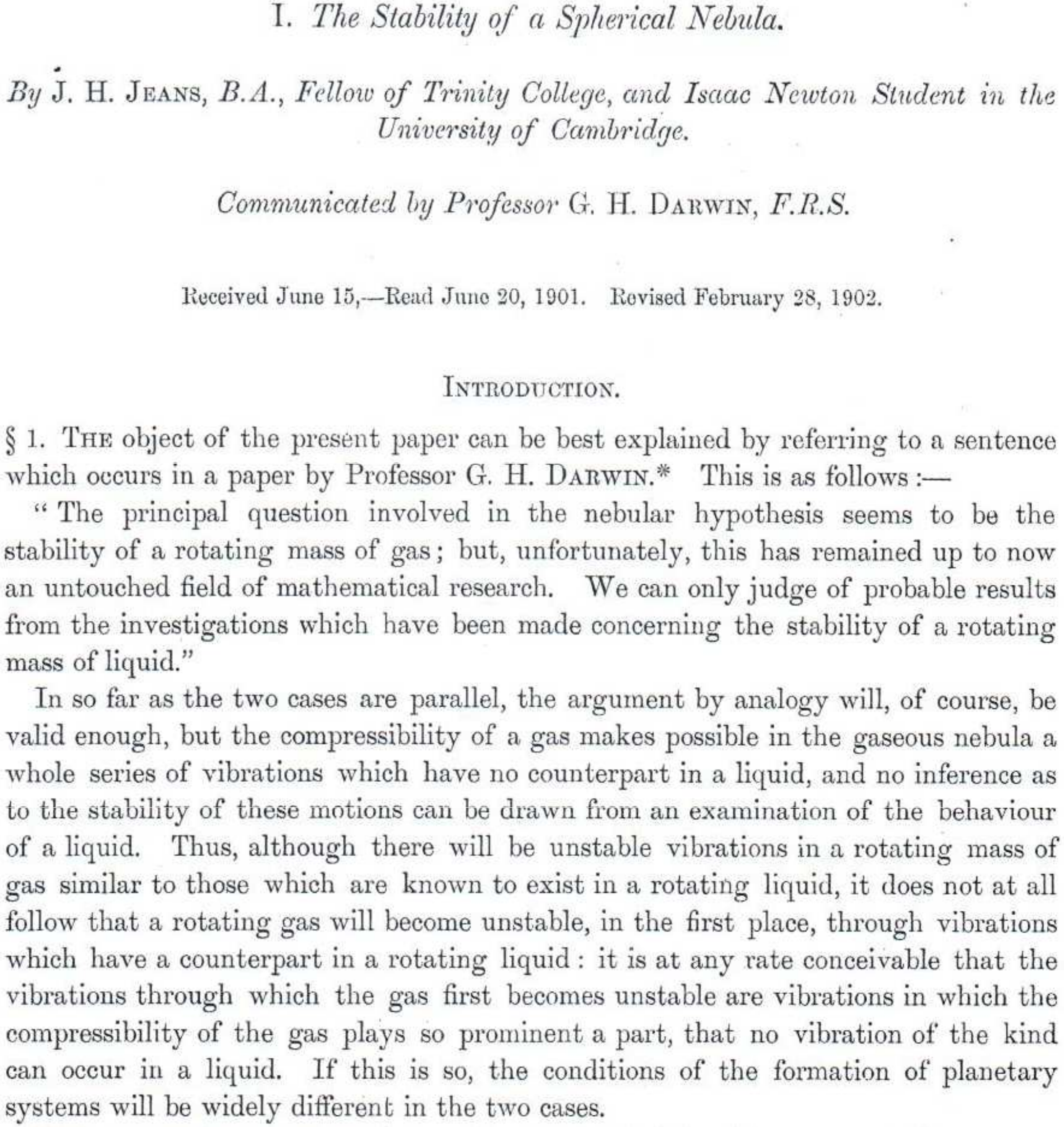


FIG. 2.—Temperature-density diagram for model 1 and model 16. The letter *c* indicates the center, *d* the edge of the degenerate core, *j* the jump in composition and density, and *k* the inner edge of the convective envelope.

Hvězdy
podmínky vzniku hvězd

James Hopwood Jeans 1877 - 1946, *Stabilita sférické mlhoviny r. 1901*





I. *The Stability of a Spherical Nebula.*

By J. H. Jeans, *B.A., Fellow of Trinity College, and Isaac Newton Student in the University of Cambridge.*

Communicated by Professor G. H. Darwin, *F.R.S.*

Received June 15,—Read June 20, 1901. Revised February 28, 1902.

Introduction.

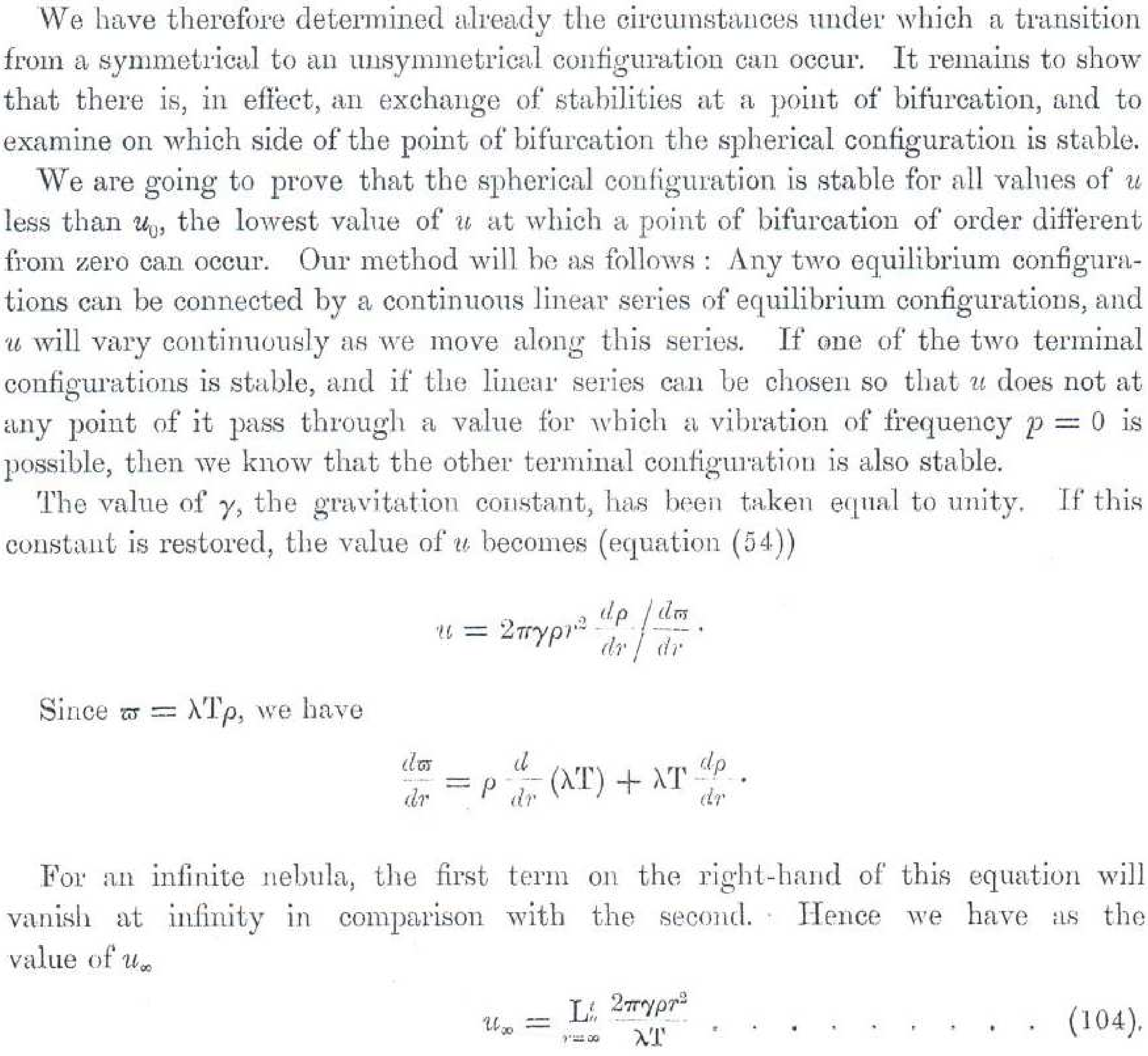
§ 1. The object of the present paper can be best explained by referring to a sentence which occurs in a paper by Professor G. H. Darwin.* This is as follows :—

"The principal question involved in the nebular hypothesis seems to be the stability of a rotating mass of gas ; but, unfortunately, this has remained up to now an untouched field of mathematical research. We can only judge of probable results from the investigations which have been made concerning the stability of a rotating mass of liquid."

In so far as the two cases are parallel, the argument by analogy will, of course, be valid enough, but the compressibility of a gas makes possible in the gaseous nebula a whole series of vibrations which have no counterpart in a liquid, and no inference as to the stability of these motions can be drawn from an examination of the behaviour of a liquid. Thus, although there will be unstable vibrations in a rotating mass of gas similar to those which are known to exist in a rotating liquid, it does not at all follow that a rotating gas will become unstable, in the first place, through vibrations which have a counterpart in a rotating liquid : it is at any rate conceivable that the vibrations through which the gas first becomes unstable are vibrations in which the compressibility of the gas plays so prominent a part, that no vibration of the kind can occur in a liquid. If this is so, the conditions of the formation of planetary systems will be widely different in the two cases.

Hvězdy - podmínky vzniku hvězd

Stabilita sférické mlhoviny r. 1901



We have therefore determined already the circumstances under which a transition from a symmetrical to an unsymmetrical configuration can occur. It remains to show that there is, in effect, an exchange of stabilities at a point of bifurcation, and to examine on which side of the point of bifurcation the spherical configuration is stable.

We are going to prove that the spherical configuration is stable for all values of u less than u_0, the lowest value of u at which a point of bifurcation of order different from zero can occur. Our method will be as follows: Any two equilibrium configurations can be connected by a continuous linear series of equilibrium configurations, and u will vary continuously as we move along this series. If one of the two terminal configurations is stable, and if the linear series can be chosen so that u does not at any point of it pass through a value for which a vibration of frequency $p = 0$ is possible, then we know that the other terminal configuration is also stable.

The value of γ, the gravitation constant, has been taken equal to unity. If this constant is restored, the value of u becomes (equation (54))

$$u = 2\pi\gamma\rho r^2 \frac{d\rho}{dr} \Big/ \frac{d\varpi}{dr}.$$

Since $\varpi = \lambda T \rho$, we have

$$\frac{d\varpi}{dr} = \rho \frac{d}{dr}(\lambda T) + \lambda T \frac{d\rho}{dr}.$$

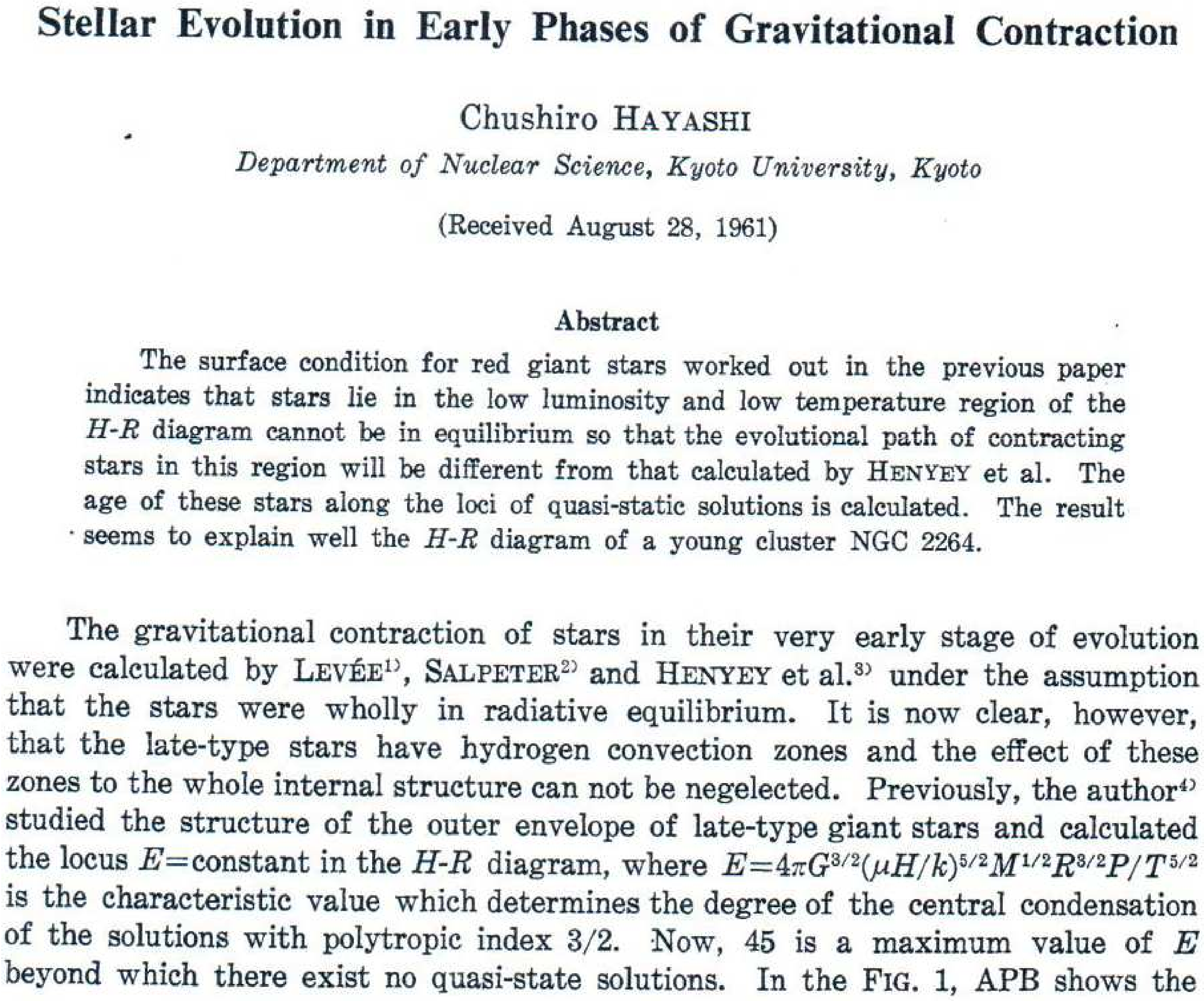
For an infinite nebula, the first term on the right-hand of this equation will vanish at infinity in comparison with the second. Hence we have as the value of u_∞

$$u_\infty = \mathop{\mathrm{Lt}}_{r=\infty} \frac{2\pi\gamma\rho r^2}{\lambda T} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (104).$$

Hvězdy - příchod na hlavní posloupnost

japonský astrofyzik, **Chushiro Hayashi 1920 – 2010,**

Vývoj hvězd při gravitační kontrakci, r. 1961



Stellar Evolution in Early Phases of Gravitational Contraction

Chushiro HAYASHI

Department of Nuclear Science, Kyoto University, Kyoto

(Received August 28, 1961)

Abstract

The surface condition for red giant stars worked out in the previous paper indicates that stars lie in the low luminosity and low temperature region of the *H-R* diagram cannot be in equilibrium so that the evolutional path of contracting stars in this region will be different from that calculated by HENYEY et al. The age of these stars along the loci of quasi-static solutions is calculated. The result seems to explain well the *H-R* diagram of a young cluster NGC 2264.

The gravitational contraction of stars in their very early stage of evolution were calculated by LEVÉE1), SALPETER2) and HENYEY et al.3) under the assumption that the stars were wholly in radiative equilibrium. It is now clear, however, that the late-type stars have hydrogen convection zones and the effect of these zones to the whole internal structure can not be negelected. Previously, the author4) studied the structure of the outer envelope of late-type giant stars and calculated the locus $E=$constant in the *H-R* diagram, where $E=4\pi G^{3/2}(\mu H/k)^{5/2}M^{1/2}R^{3/2}P/T^{5/2}$ is the characteristic value which determines the degree of the central condensation of the solutions with polytropic index 3/2. Now, 45 is a maximum value of E beyond which there exist no quasi-state solutions. In the FIG. 1, APB shows the

Hvězdy - příchod na hlavní posloupnost

Vývoj hvězd při gravitační kontrakci, r. 1961

The life time of the contraction is calculated for Population I stars using the expressions

$$\frac{dE}{dt}=-L, \qquad E=-\frac{3\gamma-4}{3(\gamma-1)}\,\frac{3}{5-n}\,\frac{GM^2}{R}\,. \tag{1}$$

For the sake of simplicity we take $\gamma=5/3$ and $n=2$. For the curve APB the previous results[4] for $E=40$ are used, and for the line CPD $L\sim M^{4.5}R^{-0.5}$ is assumed and the normalization constant is taken from the results by HENYEY et. al.[3] If $L\sim R^{-\alpha}$ along the path, the age of a star from the time when $R=\infty$ is given by

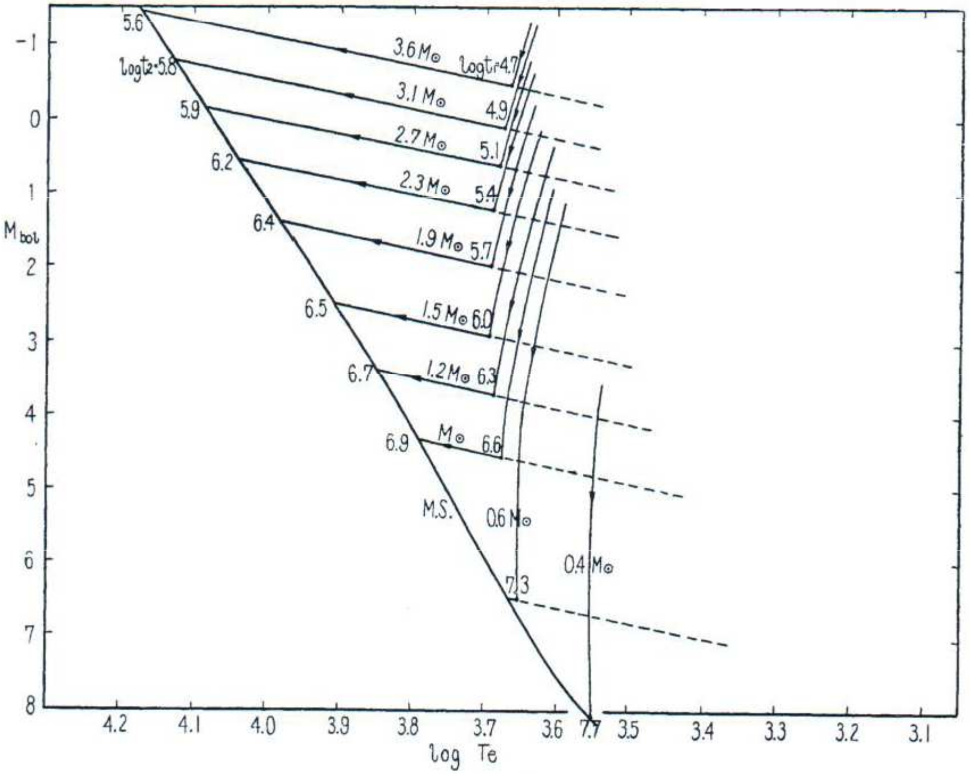
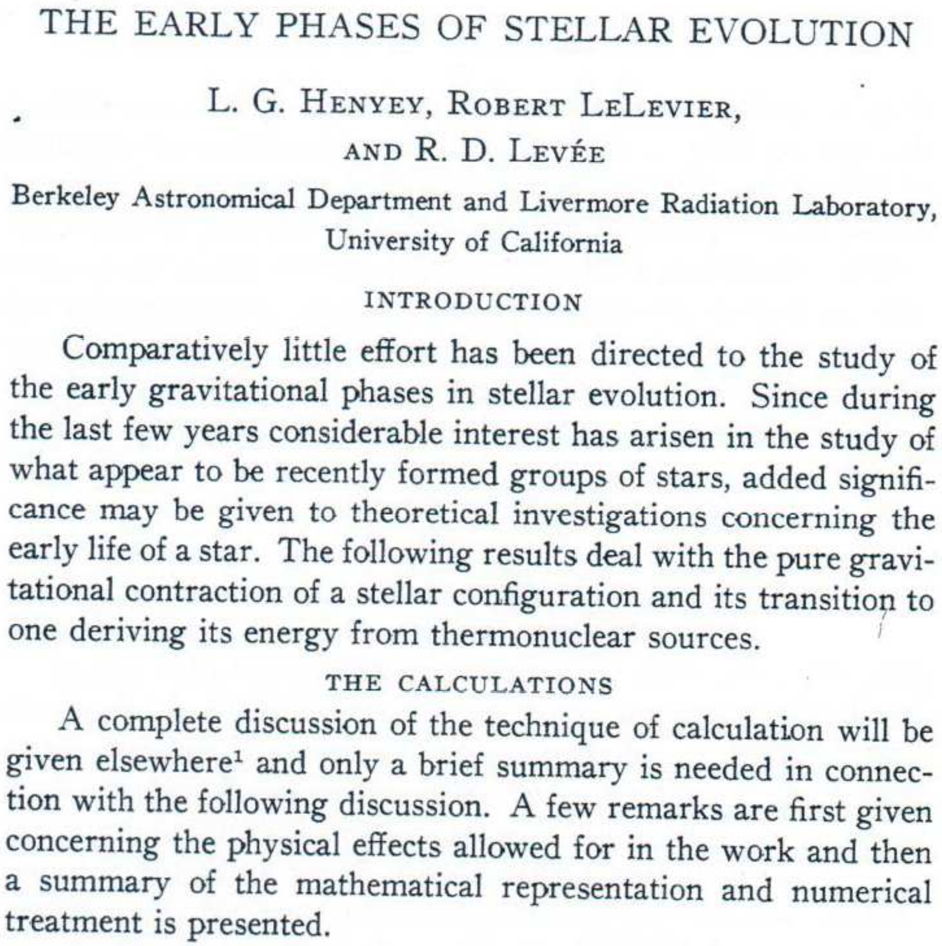


FIG. 2. Evolutional tracks and ages of stars with different masses in gravitational contraction. t_1 and t_2 denote the ages (in yeasr) at the turning point and on the main sequence, respectively.

Hvězdy - příchod na hlavní posloupnost
Luis George Henyey 1910 - 1970

americký astrofyzik maďarského původu, ***Počáteční fáze hvězdného vývoje, r. 1955***



THE EARLY PHASES OF STELLAR EVOLUTION

L. G. HENYEY, ROBERT LELEVIER,
AND R. D. LEVÉE

Berkeley Astronomical Department and Livermore Radiation Laboratory,
University of California

INTRODUCTION

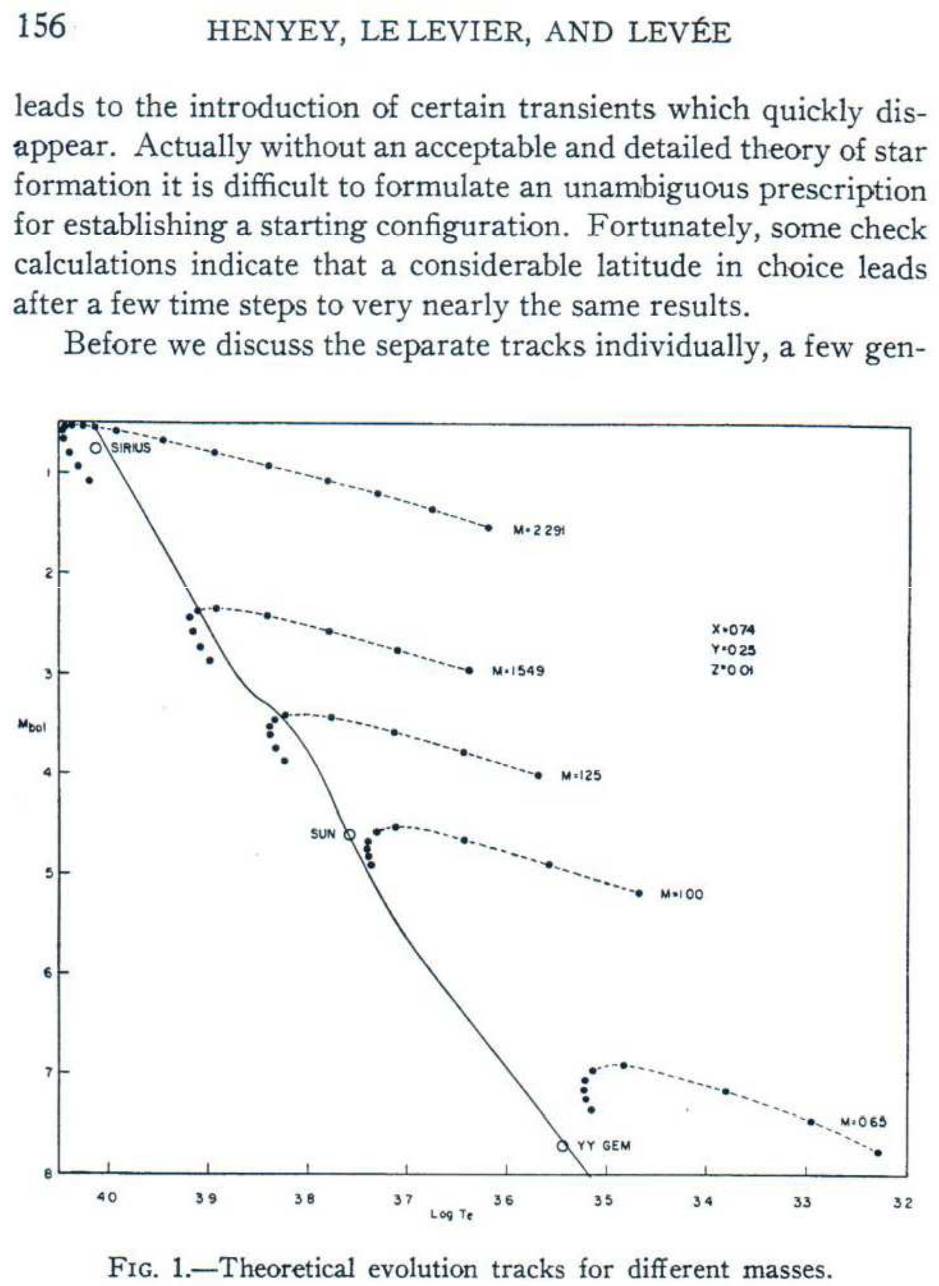
Comparatively little effort has been directed to the study of the early gravitational phases in stellar evolution. Since during the last few years considerable interest has arisen in the study of what appear to be recently formed groups of stars, added significance may be given to theoretical investigations concerning the early life of a star. The following results deal with the pure gravitational contraction of a stellar configuration and its transition to one deriving its energy from thermonuclear sources.

THE CALCULATIONS

A complete discussion of the technique of calculation will be given elsewhere[1] and only a brief summary is needed in connection with the following discussion. A few remarks are first given concerning the physical effects allowed for in the work and then a summary of the mathematical representation and numerical treatment is presented.

Hvězdy - příchod na hlavní posloupnost

Počáteční fáze hvězdného vývoje, r. 1955



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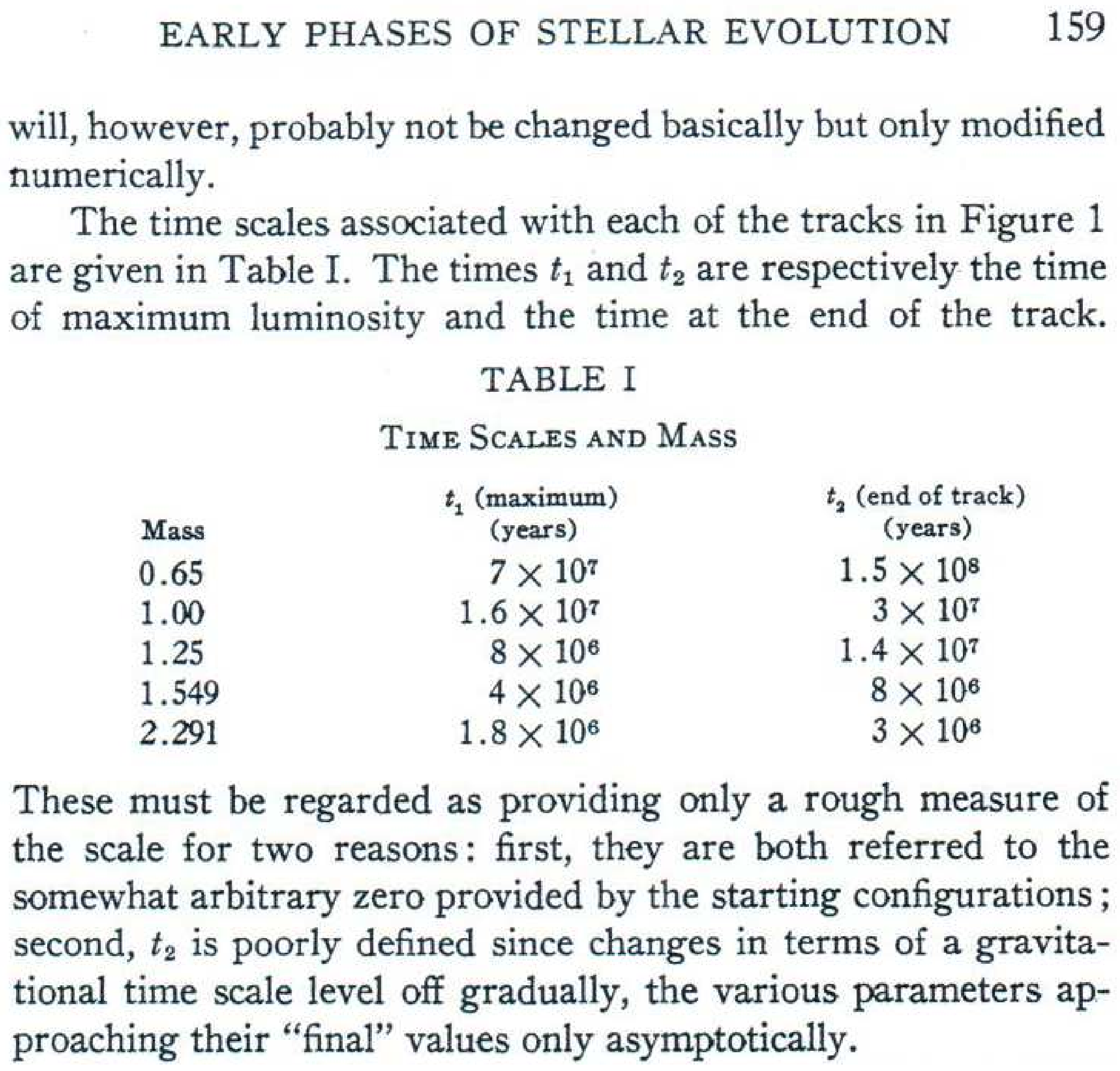
leads to the introduction of certain transients which quickly disappear. Actually without an acceptable and detailed theory of star formation it is difficult to formulate an unambiguous prescription for establishing a starting configuration. Fortunately, some check calculations indicate that a considerable latitude in choice leads after a few time steps to very nearly the same results.

Before we discuss the separate tracks individually, a few gen-

FIG. 1.—Theoretical evolution tracks for different masses.

Hvězdy - příchod na hlavní posloupnost

Počáteční fáze hvězdného vývoje, r. 1955



EARLY PHASES OF STELLAR EVOLUTION 159

will, however, probably not be changed basically but only modified numerically.

The time scales associated with each of the tracks in Figure 1 are given in Table I. The times t_1 and t_2 are respectively the time of maximum luminosity and the time at the end of the track.

TABLE I

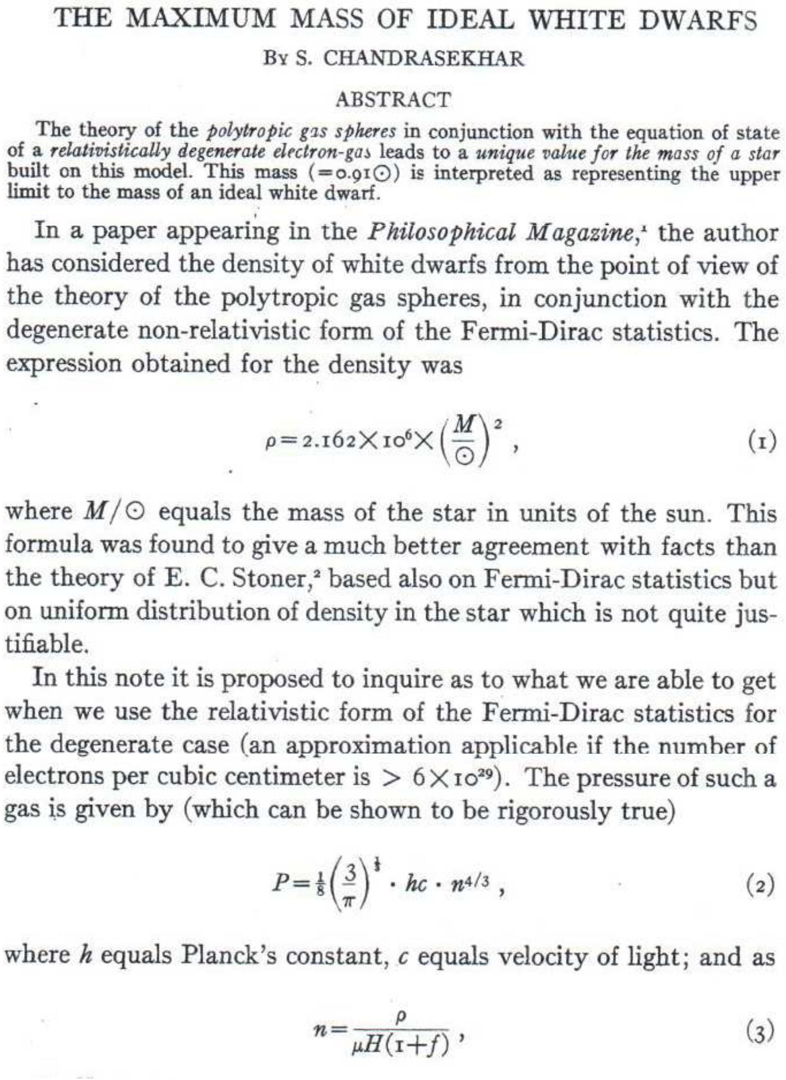
TIME SCALES AND MASS

Mass	t_1 (maximum) (years)	t_2 (end of track) (years)
0.65	7×10^7	1.5×10^8
1.00	1.6×10^7	3×10^7
1.25	8×10^6	1.4×10^7
1.549	4×10^6	8×10^6
2.291	1.8×10^6	3×10^6

These must be regarded as providing only a rough measure of the scale for two reasons: first, they are both referred to the somewhat arbitrary zero provided by the starting configurations; second, t_2 is poorly defined since changes in terms of a gravitational time scale level off gradually, the various parameters approaching their "final" values only asymptotically.

Hvězdy
Subrahmanyan Chandrasekhar 1910 - 1995

Maximální hmotnost ideálních bílých trpaslíků, r. 1931 - fyzikální podmínky v degenerovaných hvězdách, limitní hmotnost



THE MAXIMUM MASS OF IDEAL WHITE DWARFS

BY S. CHANDRASEKHAR

ABSTRACT

The theory of the *polytropic gas spheres* in conjunction with the equation of state of a *relativistically degenerate electron-gas* leads to a *unique value for the mass of a star built on this model*. This mass ($=0.91\odot$) is interpreted as representing the upper limit to the mass of an ideal white dwarf.

In a paper appearing in the *Philosophical Magazine*,[1] the author has considered the density of white dwarfs from the point of view of the theory of the polytropic gas spheres, in conjunction with the degenerate non-relativistic form of the Fermi-Dirac statistics. The expression obtained for the density was

$$\rho = 2.162 \times 10^6 \times \left(\frac{M}{\odot}\right)^2 , \qquad (1)$$

where $M/\odot$ equals the mass of the star in units of the sun. This formula was found to give a much better agreement with facts than the theory of E. C. Stoner,[2] based also on Fermi-Dirac statistics but on uniform distribution of density in the star which is not quite justifiable.

In this note it is proposed to inquire as to what we are able to get when we use the relativistic form of the Fermi-Dirac statistics for the degenerate case (an approximation applicable if the number of electrons per cubic centimeter is $> 6 \times 10^{29}$). The pressure of such a gas is given by (which can be shown to be rigorously true)

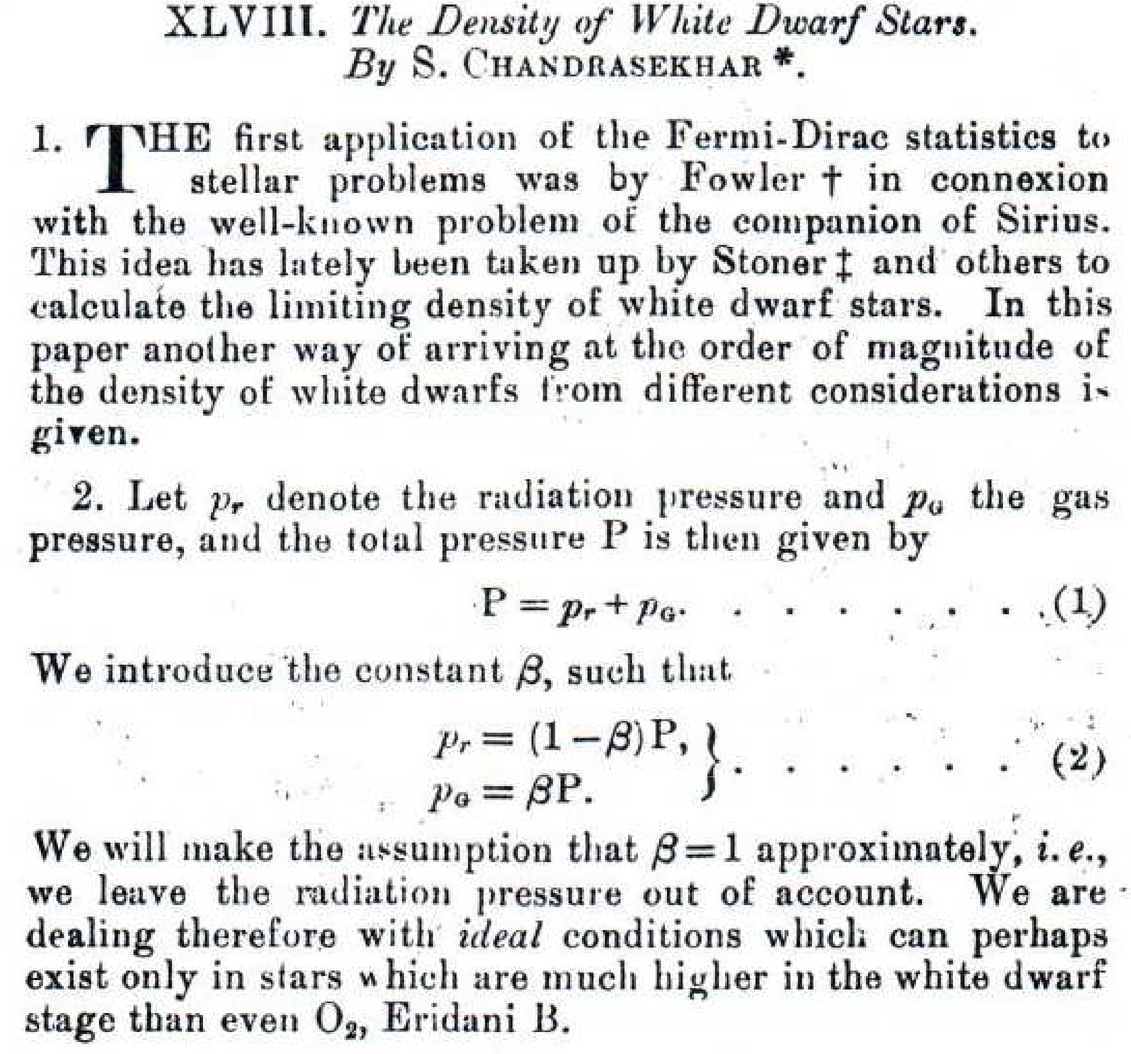
$$P = \tfrac{1}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} \cdot hc \cdot n^{4/3} , \qquad (2)$$

where h equals Planck's constant, c equals velocity of light; and as

$$n = \frac{\rho}{\mu H(1+f)} , \qquad (3)$$

Hvězdy
Subrahmanyan Chandrasekhar

Hustota bílých trpaslíků, r. 1931 - fyzikální podmínky v degenerovaných hvězdách



XLVIII. *The Density of White Dwarf Stars.*
By S. CHANDRASEKHAR *.

1. THE first application of the Fermi-Dirac statistics to stellar problems was by Fowler † in connexion with the well-known problem of the companion of Sirius. This idea has lately been taken up by Stoner ‡ and others to calculate the limiting density of white dwarf stars. In this paper another way of arriving at the order of magnitude of the density of white dwarfs from different considerations is given.

2. Let p_r denote the radiation pressure and p_G the gas pressure, and the total pressure P is then given by

$$P = p_r + p_G. \qquad (1)$$

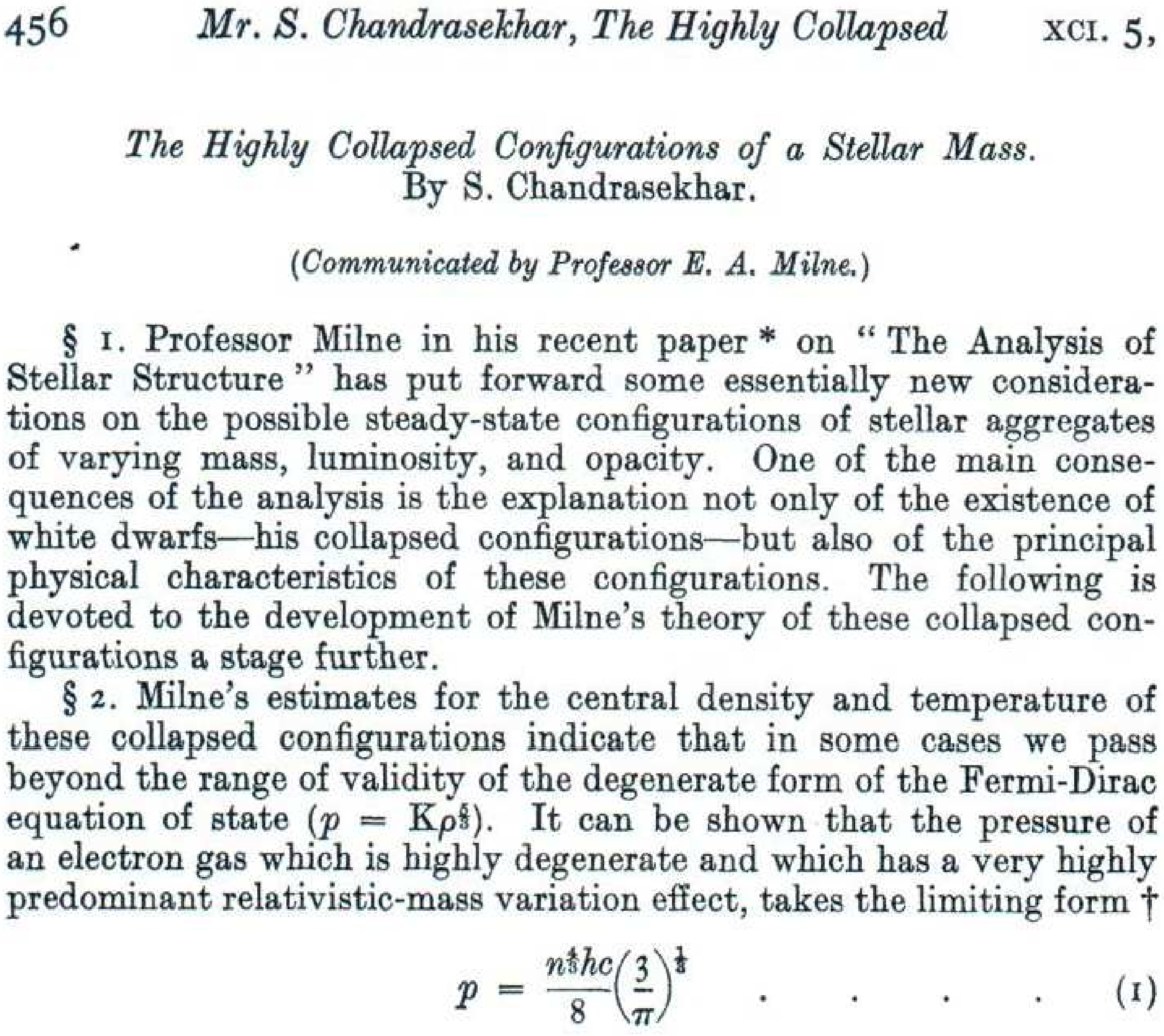
We introduce the constant β, such that

$$\left.\begin{aligned} p_r &= (1-\beta)P, \\ p_G &= \beta P. \end{aligned}\right\} \qquad (2)$$

We will make the assumption that $\beta = 1$ approximately, *i. e.*, we leave the radiation pressure out of account. We are dealing therefore with *ideal* conditions which can perhaps exist only in stars which are much higher in the white dwarf stage than even O_2, Eridani B.

Hvězdy - Subrahmanyan Chandrasekhar

Vysoce kolabující konfigurace hvězdných hmot, r. 1931 - fyzikální podmínky v degenerovaných hvězdách



456 *Mr. S. Chandrasekhar, The Highly Collapsed* XCI. 5,

The Highly Collapsed Configurations of a Stellar Mass.
By S. Chandrasekhar.

(*Communicated by Professor E. A. Milne.*)

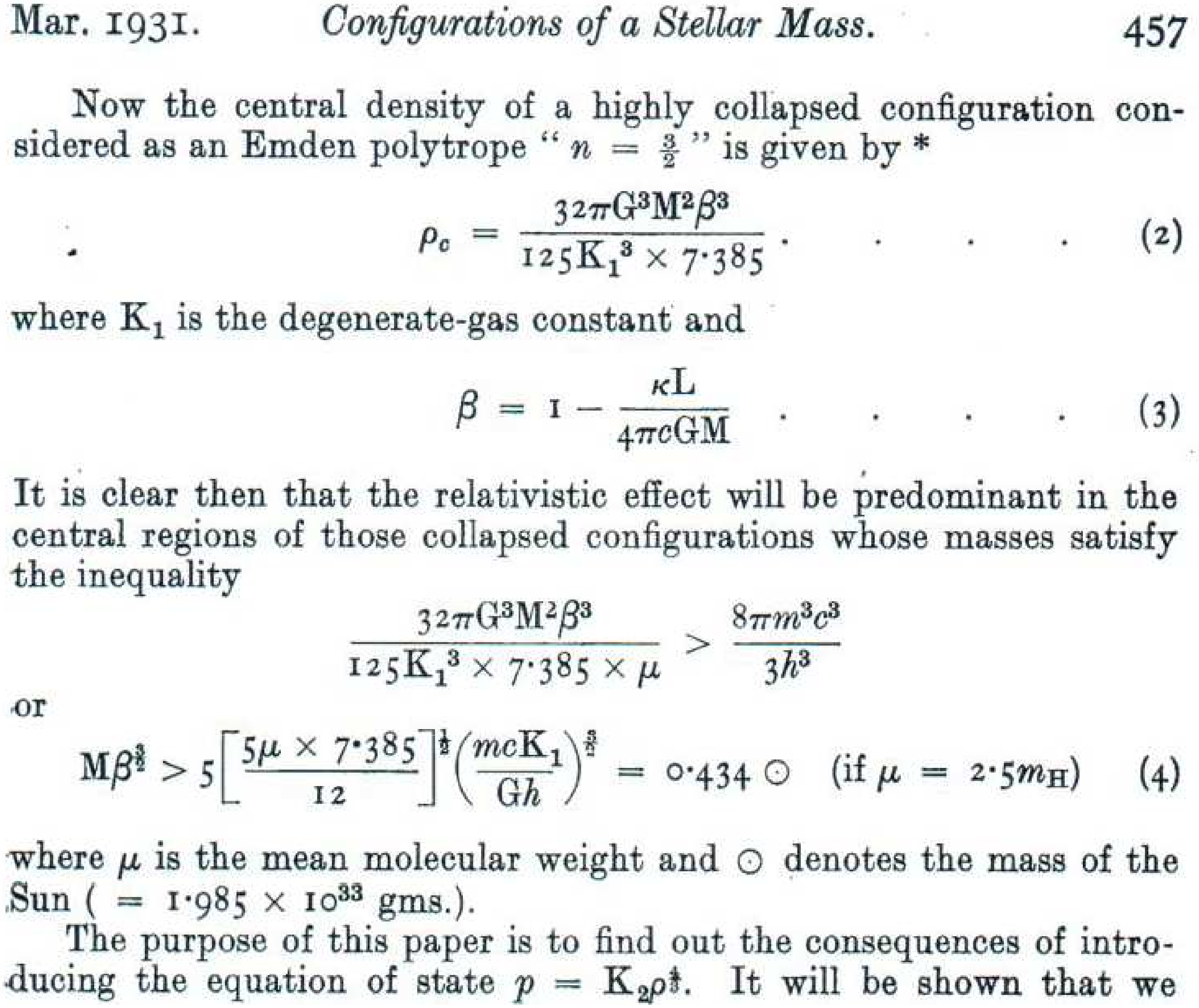
§ 1. Professor Milne in his recent paper[*] on "The Analysis of Stellar Structure" has put forward some essentially new considerations on the possible steady-state configurations of stellar aggregates of varying mass, luminosity, and opacity. One of the main consequences of the analysis is the explanation not only of the existence of white dwarfs—his collapsed configurations—but also of the principal physical characteristics of these configurations. The following is devoted to the development of Milne's theory of these collapsed configurations a stage further.

§ 2. Milne's estimates for the central density and temperature of these collapsed configurations indicate that in some cases we pass beyond the range of validity of the degenerate form of the Fermi-Dirac equation of state ($p = K\rho^{5/3}$). It can be shown that the pressure of an electron gas which is highly degenerate and which has a very highly predominant relativistic-mass variation effect, takes the limiting form[†]

$$p = \frac{n^{4/3}hc}{8}\left(\frac{3}{\pi}\right)^{1/3} \quad . \quad . \quad . \quad . \quad (1)$$

Hvězdy - Subrahmanyan Chandrasekhar

Vysoce kolabující konfigurace hvězdných hmot, r. 1931 - fyzikální podmínky v degenerovaných hvězdách



Mar. 1931. *Configurations of a Stellar Mass.* 457

Now the central density of a highly collapsed configuration considered as an Emden polytrope "$n = \frac{3}{2}$" is given by *

$$\rho_c = \frac{32\pi G^3 M^2 \beta^3}{125 K_1^3 \times 7\cdot385} \quad . \quad . \quad . \quad . \quad (2)$$

where K_1 is the degenerate-gas constant and

$$\beta = 1 - \frac{\kappa L}{4\pi c G M} \quad . \quad . \quad . \quad . \quad (3)$$

It is clear then that the relativistic effect will be predominant in the central regions of those collapsed configurations whose masses satisfy the inequality

$$\frac{32\pi G^3 M^2 \beta^3}{125 K_1^3 \times 7\cdot385 \times \mu} > \frac{8\pi m^3 c^3}{3h^3}$$

or

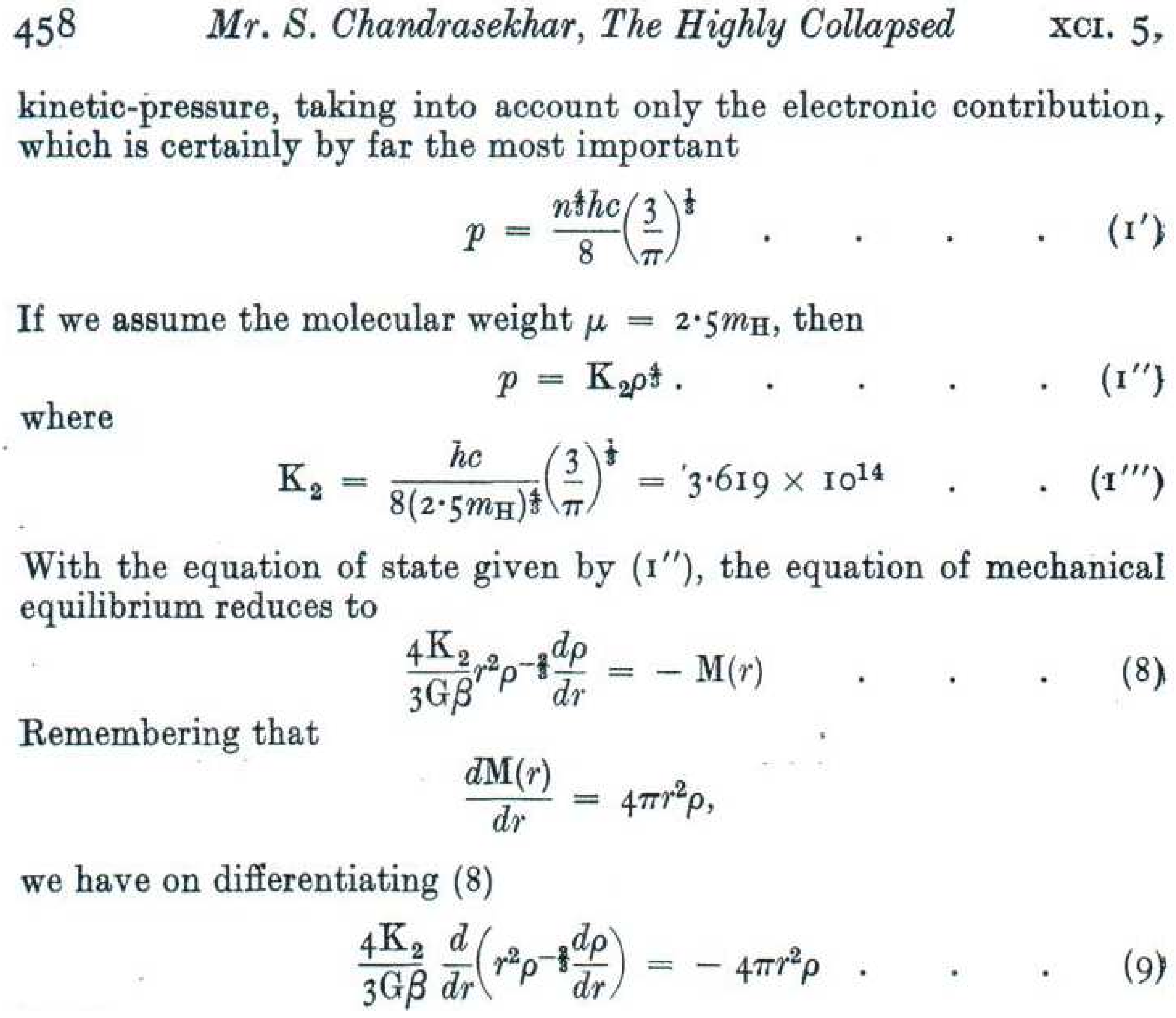
$$M\beta^{\frac{3}{2}} > 5\left[\frac{5\mu \times 7\cdot385}{12}\right]^{\frac{1}{2}}\left(\frac{mcK_1}{Gh}\right)^{\frac{3}{2}} = 0\cdot434 \odot \quad (\text{if } \mu = 2\cdot5 m_H) \quad (4)$$

where μ is the mean molecular weight and $\odot$ denotes the mass of the Sun ($= 1\cdot985 \times 10^{33}$ gms.).

The purpose of this paper is to find out the consequences of introducing the equation of state $p = K_2\rho^{\frac{4}{3}}$. It will be shown that we

Hvězdy - Subrahmanyan Chandrasekhar

Vysoce kolabující konfigurace hvězdných hmot, r. 1931 - fyzikální podmínky v degenerovaných hvězdách



kinetic-pressure, taking into account only the electronic contribution, which is certainly by far the most important

$$p = \frac{n^{\frac{4}{3}}hc}{8}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad (1')$$

If we assume the molecular weight $\mu = 2{\cdot}5m_{\mathrm{H}}$, then

$$p = \mathrm{K}_2\rho^{\frac{4}{3}} \,. \quad . \quad . \quad . \quad . \quad (1'')$$

where

$$\mathrm{K}_2 = \frac{hc}{8(2{\cdot}5m_{\mathrm{H}})^{\frac{4}{3}}}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} = 3{\cdot}619 \times 10^{14} \quad . \quad . \quad (1''')$$

With the equation of state given by (1″), the equation of mechanical equilibrium reduces to

$$\frac{4\mathrm{K}_2}{3\mathrm{G}\beta}r^2\rho^{-\frac{2}{3}}\frac{d\rho}{dr} = -\,\mathrm{M}(r) \quad . \quad . \quad . \quad (8)$$

Remembering that

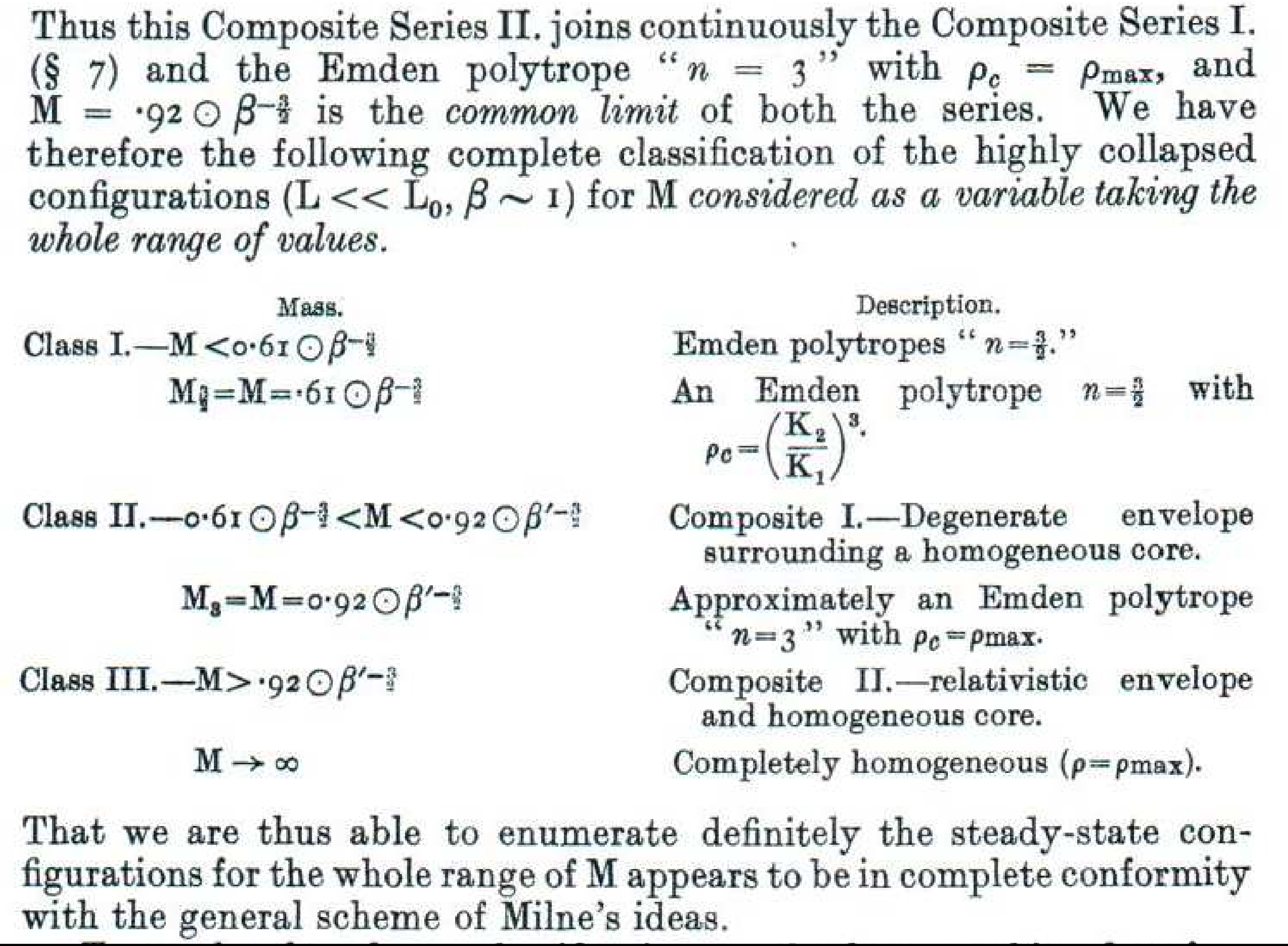
$$\frac{d\mathrm{M}(r)}{dr} = 4\pi r^2\rho,$$

we have on differentiating (8)

$$\frac{4\mathrm{K}_2}{3\mathrm{G}\beta}\,\frac{d}{dr}\left(r^2\rho^{-\frac{2}{3}}\frac{d\rho}{dr}\right) = -\,4\pi r^2\rho \quad . \quad . \quad . \quad (9)$$

Hvězdy - Subrahmanyan Chandrasekhar

Vysoce kolabující konfigurace hvězdných hmot, r. 1931 - fyzikální podmínky v degenerovaných hvězdách



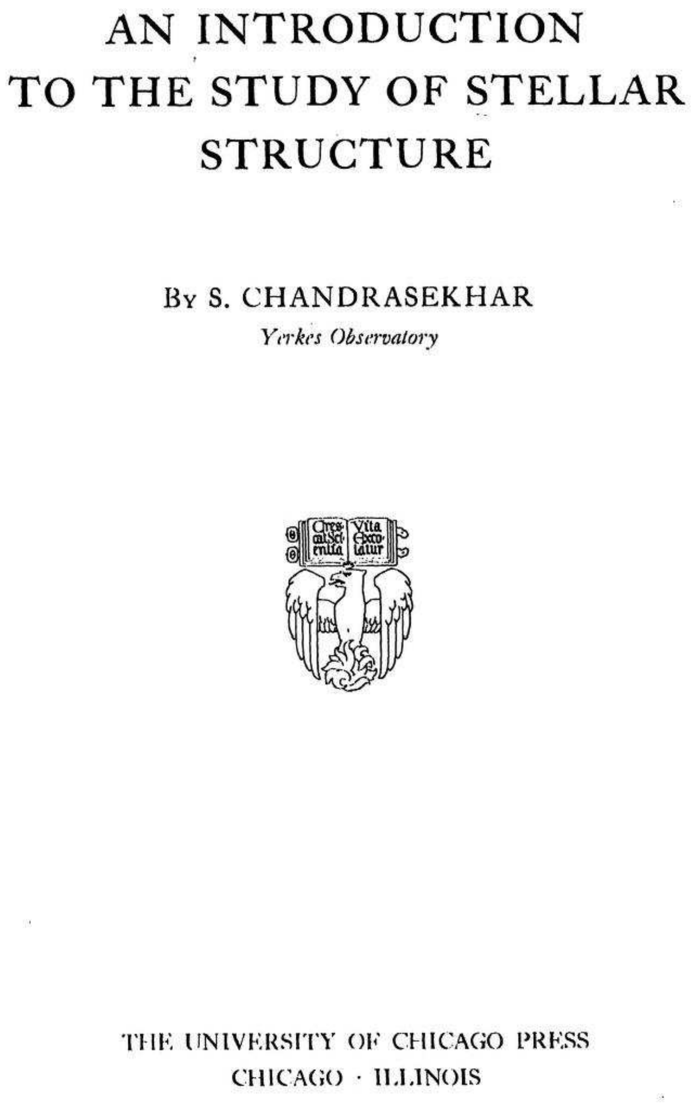
Thus this Composite Series II. joins continuously the Composite Series I. (§ 7) and the Emden polytrope "$n = 3$" with $\rho_c = \rho_{\max}$, and $M = \cdot 92 \odot \beta^{-\frac{3}{2}}$ is the *common limit* of both the series. We have therefore the following complete classification of the highly collapsed configurations ($L << L_0$, $\beta \sim 1$) for M *considered as a variable taking the whole range of values.*

Mass.	Description.
Class I.—$M < 0{\cdot}61 \odot \beta^{-\frac{3}{2}}$	Emden polytropes "$n = \frac{3}{2}$."
$M_{\frac{3}{2}} = M = {\cdot}61 \odot \beta^{-\frac{3}{2}}$	An Emden polytrope $n = \frac{3}{2}$ with $\rho_c = \left(\frac{K_2}{K_1}\right)^3$.
Class II.—$0{\cdot}61 \odot \beta^{-\frac{3}{2}} < M < 0{\cdot}92 \odot \beta'^{-\frac{3}{2}}$	Composite I.—Degenerate envelope surrounding a homogeneous core.
$M_3 = M = 0{\cdot}92 \odot \beta'^{-\frac{3}{2}}$	Approximately an Emden polytrope "$n = 3$" with $\rho_c = \rho_{\max}$.
Class III.—$M > {\cdot}92 \odot \beta'^{-\frac{3}{2}}$	Composite II.—relativistic envelope and homogeneous core.
$M \to \infty$	Completely homogeneous ($\rho = \rho_{\max}$).

That we are thus able to enumerate definitely the steady-state configurations for the whole range of M appears to be in complete conformity with the general scheme of Milne's ideas.

Hvězdy - Subrahmanyan Chandrasekhar

Úvod do studia stavby hvězd, r. 1939 – učebnice, fyzikální podmínky v nitru hvězd



AN INTRODUCTION
TO THE STUDY OF STELLAR
STRUCTURE

BY S. CHANDRASEKHAR
Yerkes Observatory

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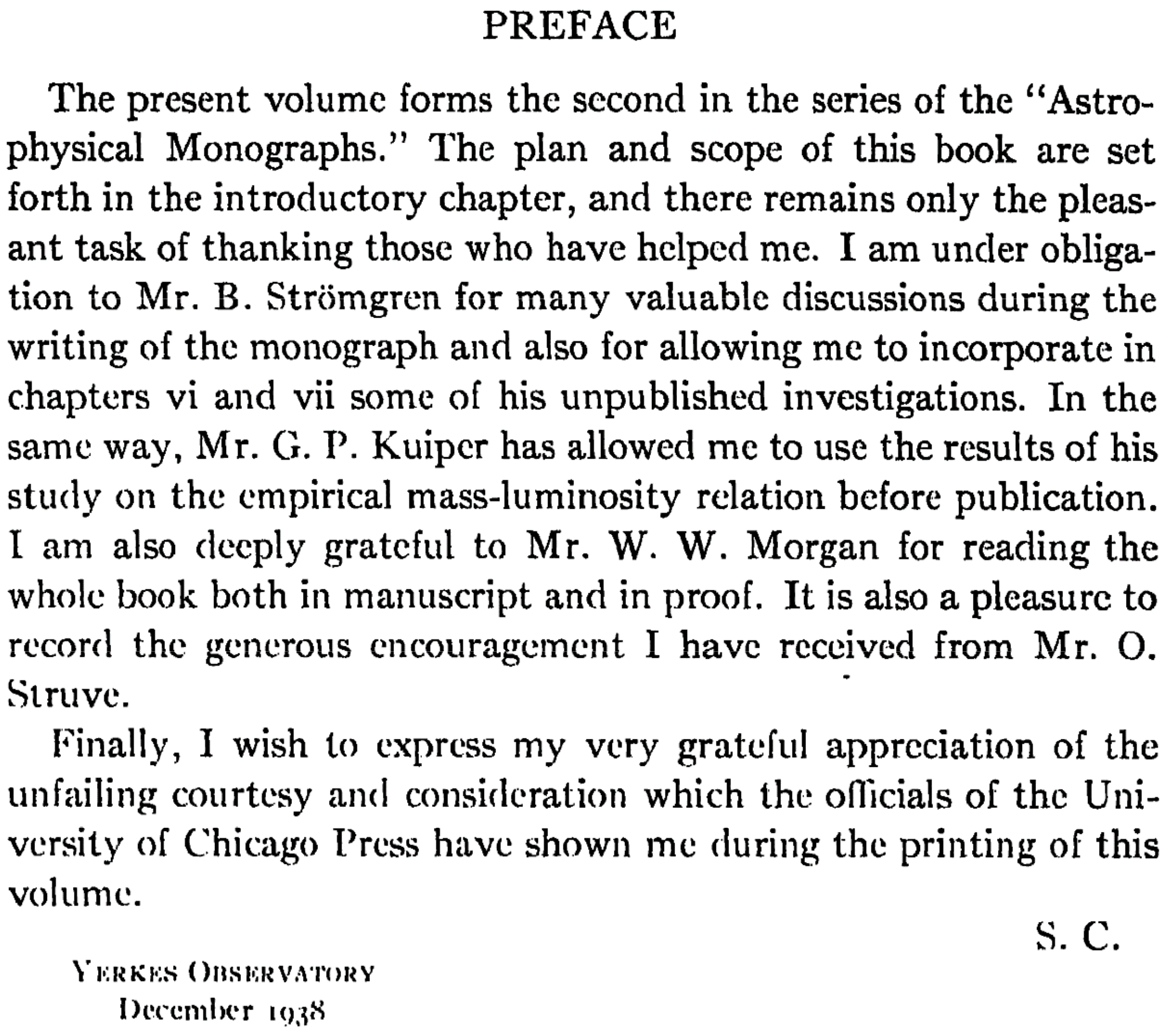


PREFACE

The present volume forms the second in the series of the "Astrophysical Monographs." The plan and scope of this book are set forth in the introductory chapter, and there remains only the pleasant task of thanking those who have helped me. I am under obligation to Mr. B. Strömgren for many valuable discussions during the writing of the monograph and also for allowing me to incorporate in chapters vi and vii some of his unpublished investigations. In the same way, Mr. G. P. Kuiper has allowed me to use the results of his study on the empirical mass-luminosity relation before publication. I am also deeply grateful to Mr. W. W. Morgan for reading the whole book both in manuscript and in proof. It is also a pleasure to record the generous encouragement I have received from Mr. O. Struve.

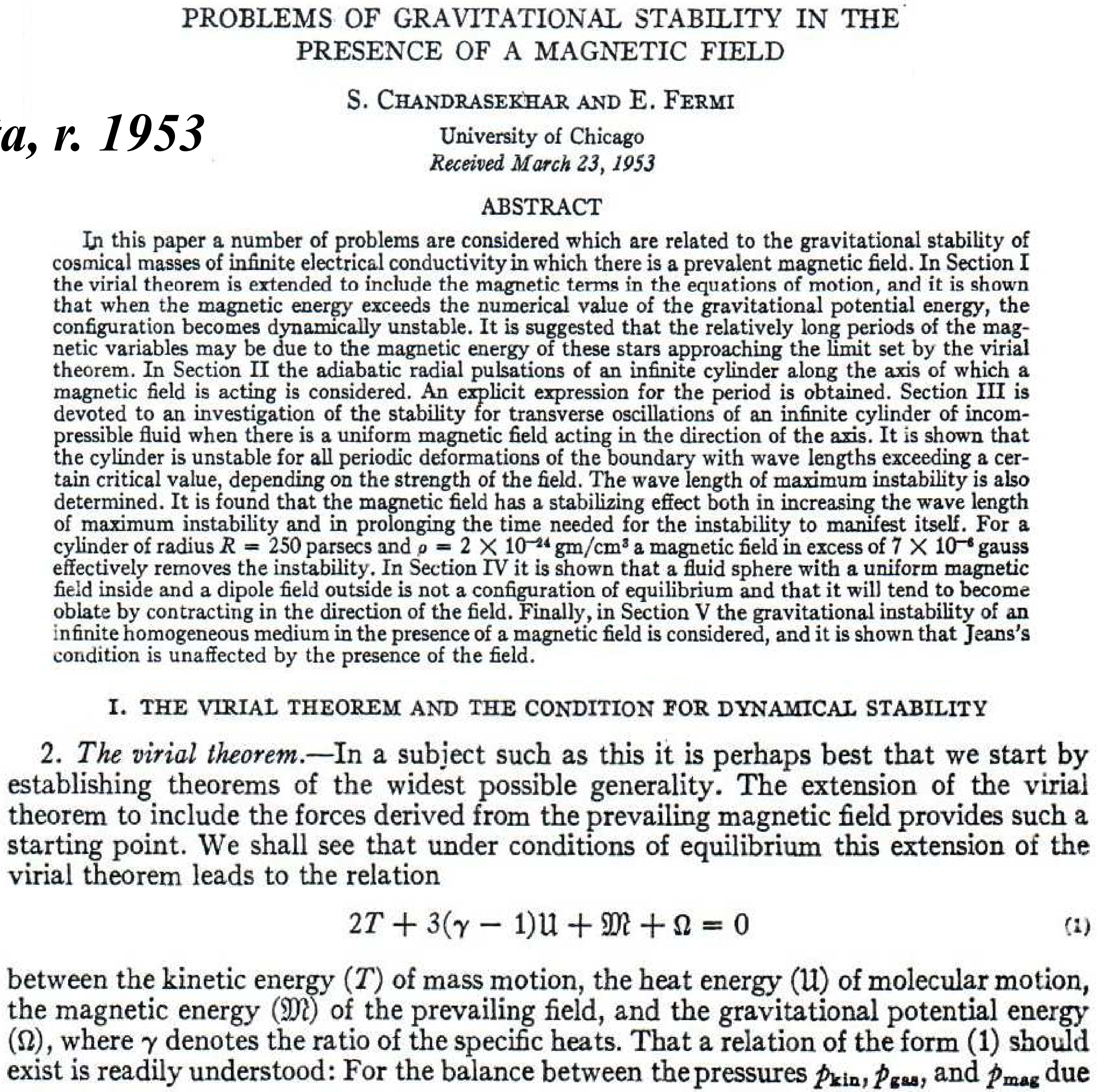
Finally, I wish to express my very grateful appreciation of the unfailing courtesy and consideration which the officials of the University of Chicago Press have shown me during the printing of this volume.

S. C.

YERKES OBSERVATORY
December 1938

Subrahmanyan Chandrasekhar, Enrico Fermi

Viriálová věta, r. 1953



PROBLEMS OF GRAVITATIONAL STABILITY IN THE PRESENCE OF A MAGNETIC FIELD

S. CHANDRASEKHAR AND E. FERMI
University of Chicago

Received March 23, 1953

ABSTRACT

In this paper a number of problems are considered which are related to the gravitational stability of cosmical masses of infinite electrical conductivity in which there is a prevalent magnetic field. In Section I the virial theorem is extended to include the magnetic terms in the equations of motion, and it is shown that when the magnetic energy exceeds the numerical value of the gravitational potential energy, the configuration becomes dynamically unstable. It is suggested that the relatively long periods of the magnetic variables may be due to the magnetic energy of these stars approaching the limit set by the virial theorem. In Section II the adiabatic radial pulsations of an infinite cylinder along the axis of which a magnetic field is acting is considered. An explicit expression for the period is obtained. Section III is devoted to an investigation of the stability for transverse oscillations of an infinite cylinder of incompressible fluid when there is a uniform magnetic field acting in the direction of the axis. It is shown that the cylinder is unstable for all periodic deformations of the boundary with wave lengths exceeding a certain critical value, depending on the strength of the field. The wave length of maximum instability is also determined. It is found that the magnetic field has a stabilizing effect both in increasing the wave length of maximum instability and in prolonging the time needed for the instability to manifest itself. For a cylinder of radius $R = 250$ parsecs and $\rho = 2 \times 10^{-24}$ gm/cm^3 a magnetic field in excess of 7×10^{-6} gauss effectively removes the instability. In Section IV it is shown that a fluid sphere with a uniform magnetic field inside and a dipole field outside is not a configuration of equilibrium and that it will tend to become oblate by contracting in the direction of the field. Finally, in Section V the gravitational instability of an infinite homogeneous medium in the presence of a magnetic field is considered, and it is shown that Jeans's condition is unaffected by the presence of the field.

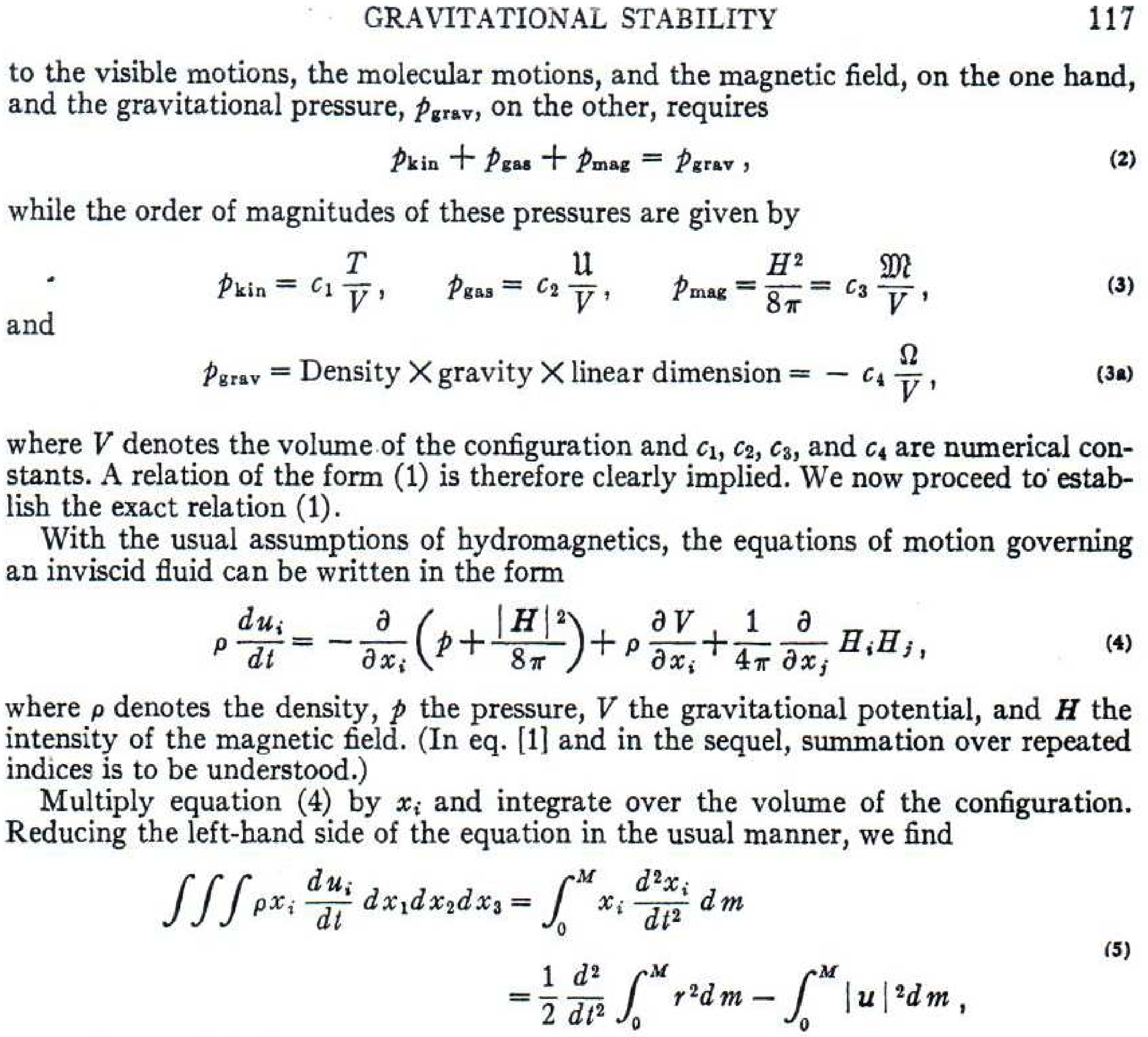
I. THE VIRIAL THEOREM AND THE CONDITION FOR DYNAMICAL STABILITY

2. *The virial theorem.*—In a subject such as this it is perhaps best that we start by establishing theorems of the widest possible generality. The extension of the virial theorem to include the forces derived from the prevailing magnetic field provides such a starting point. We shall see that under conditions of equilibrium this extension of the virial theorem leads to the relation

$$2T + 3(\gamma - 1)\mathfrak{U} + \mathfrak{M} + \Omega = 0 \qquad (1)$$

between the kinetic energy (T) of mass motion, the heat energy ($\mathfrak{U}$) of molecular motion, the magnetic energy ($\mathfrak{M}$) of the prevailing field, and the gravitational potential energy (Ω), where γ denotes the ratio of the specific heats. That a relation of the form (1) should exist is readily understood: For the balance between the pressures $p_{\rm kin}$, $p_{\rm gas}$, and $p_{\rm mag}$ due

Hvězdy - Subrahmanyan Chandrasekhar, Enrico Fermi



to the visible motions, the molecular motions, and the magnetic field, on the one hand, and the gravitational pressure, $p_{\rm grav}$, on the other, requires

$$p_{\rm kin} + p_{\rm gas} + p_{\rm mag} = p_{\rm grav}\,, \qquad (2)$$

while the order of magnitudes of these pressures are given by

$$p_{\rm kin} = c_1 \frac{T}{V}\,, \qquad p_{\rm gas} = c_2 \frac{\mathfrak{U}}{V}\,, \qquad p_{\rm mag} = \frac{H^2}{8\pi} = c_3 \frac{\mathfrak{M}}{V}\,, \qquad (3)$$

and

$$p_{\rm grav} = \text{Density} \times \text{gravity} \times \text{linear dimension} = -\,c_4 \frac{\Omega}{V}\,, \qquad (3a)$$

where V denotes the volume of the configuration and c_1, c_2, c_3, and c_4 are numerical constants. A relation of the form (1) is therefore clearly implied. We now proceed to establish the exact relation (1).

With the usual assumptions of hydromagnetics, the equations of motion governing an inviscid fluid can be written in the form

$$\rho \frac{du_i}{dt} = -\frac{\partial}{\partial x_i}\left(p + \frac{|H|^2}{8\pi}\right) + \rho \frac{\partial V}{\partial x_i} + \frac{1}{4\pi} \frac{\partial}{\partial x_j} H_i H_j\,, \qquad (4)$$

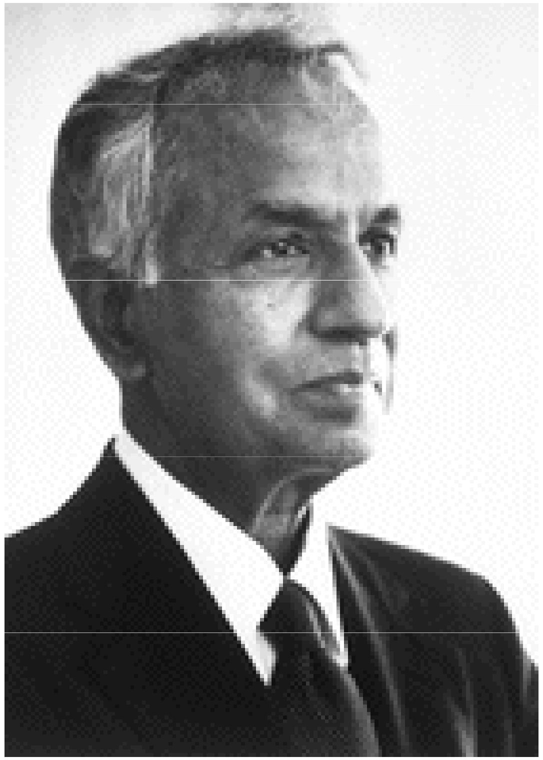
where ρ denotes the density, p the pressure, V the gravitational potential, and $\boldsymbol{H}$ the intensity of the magnetic field. (In eq. [1] and in the sequel, summation over repeated indices is to be understood.)

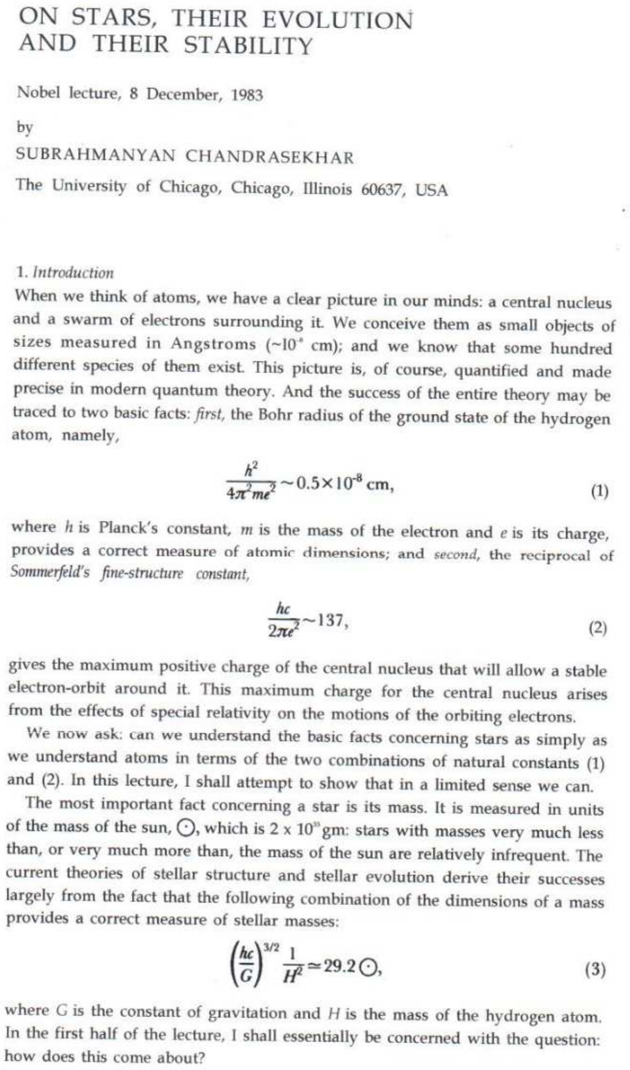
Multiply equation (4) by x_i and integrate over the volume of the configuration. Reducing the left-hand side of the equation in the usual manner, we find

$$\begin{aligned} \iiint \rho x_i \frac{du_i}{dt}\, dx_1 dx_2 dx_3 &= \int_0^M x_i \frac{d^2 x_i}{dt^2}\, dm \\ &= \frac{1}{2} \frac{d^2}{dt^2} \int_0^M r^2 dm - \int_0^M |u|^2 dm\,, \end{aligned} \qquad (5)$$

Hvězdy
Subrahmanyan Chandrasekhar

americký astrofyzik indického původu, fyzikální podmínky v degenerovaných hvězdách, **N.c. 1983**





ON STARS, THEIR EVOLUTION AND THEIR STABILITY

Nobel lecture, 8 December, 1983

by

SUBRAHMANYAN CHANDRASEKHAR

The University of Chicago, Chicago, Illinois 60637, USA

1. *Introduction*

When we think of atoms, we have a clear picture in our minds: a central nucleus and a swarm of electrons surrounding it. We conceive them as small objects of sizes measured in Angstroms ($\sim 10^{-8}$ cm); and we know that some hundred different species of them exist. This picture is, of course, quantified and made precise in modern quantum theory. And the success of the entire theory may be traced to two basic facts: *first*, the Bohr radius of the ground state of the hydrogen atom, namely,

$$\frac{h^2}{4\pi^2 m e^2} \sim 0.5 \times 10^{-8}\ \text{cm}, \qquad (1)$$

where h is Planck's constant, m is the mass of the electron and e is its charge, provides a correct measure of atomic dimensions; and *second*, the reciprocal of *Sommerfeld's fine-structure constant*,

$$\frac{hc}{2\pi e^2} \sim 137, \qquad (2)$$

gives the maximum positive charge of the central nucleus that will allow a stable electron-orbit around it. This maximum charge for the central nucleus arises from the effects of special relativity on the motions of the orbiting electrons.

We now ask: can we understand the basic facts concerning stars as simply as we understand atoms in terms of the two combinations of natural constants (1) and (2). In this lecture, I shall attempt to show that in a limited sense we can.

The most important fact concerning a star is its mass. It is measured in units of the mass of the sun, $\odot$, which is 2×10^{33} gm: stars with masses very much less than, or very much more than, the mass of the sun are relatively infrequent. The current theories of stellar structure and stellar evolution derive their successes largely from the fact that the following combination of the dimensions of a mass provides a correct measure of stellar masses:

$$\left(\frac{hc}{G}\right)^{3/2} \frac{1}{H^2} \simeq 29.2\,\odot, \qquad (3)$$

where G is the constant of gravitation and H is the mass of the hydrogen atom. In the first half of the lecture, I shall essentially be concerned with the question: how does this come about?

Obecná teorie relativity

průběhu svého pobytu v Praze Einstein začal pracovat nad problematikou OTR, proč by měla existovat privilegovaná soustava souřadnic spojená s rovnoměrně přímočaře se pohybujícími soustavami?

Fyzikální zákony musí být stejné ve všech v libovolně se pohybující soustavě souřadnic, tedy i v neinerciálních

r. 1915 dospěl Einstein ke správnému tvaru rovnic obecné teorie relativity, gravitace popisována soustavou 10 nelineárních parciálních diferenciálních rovnic pro 10 potenciálů

experimentální potvrzení teorie

I. stáčení perihélia dráhy Merkuru

II. zakřivení dráhy světelných paprsků v gravitačním poli Slunce - *1,75“*

III. gravitační rudý posuv - bílý trpaslík Sírius 1928

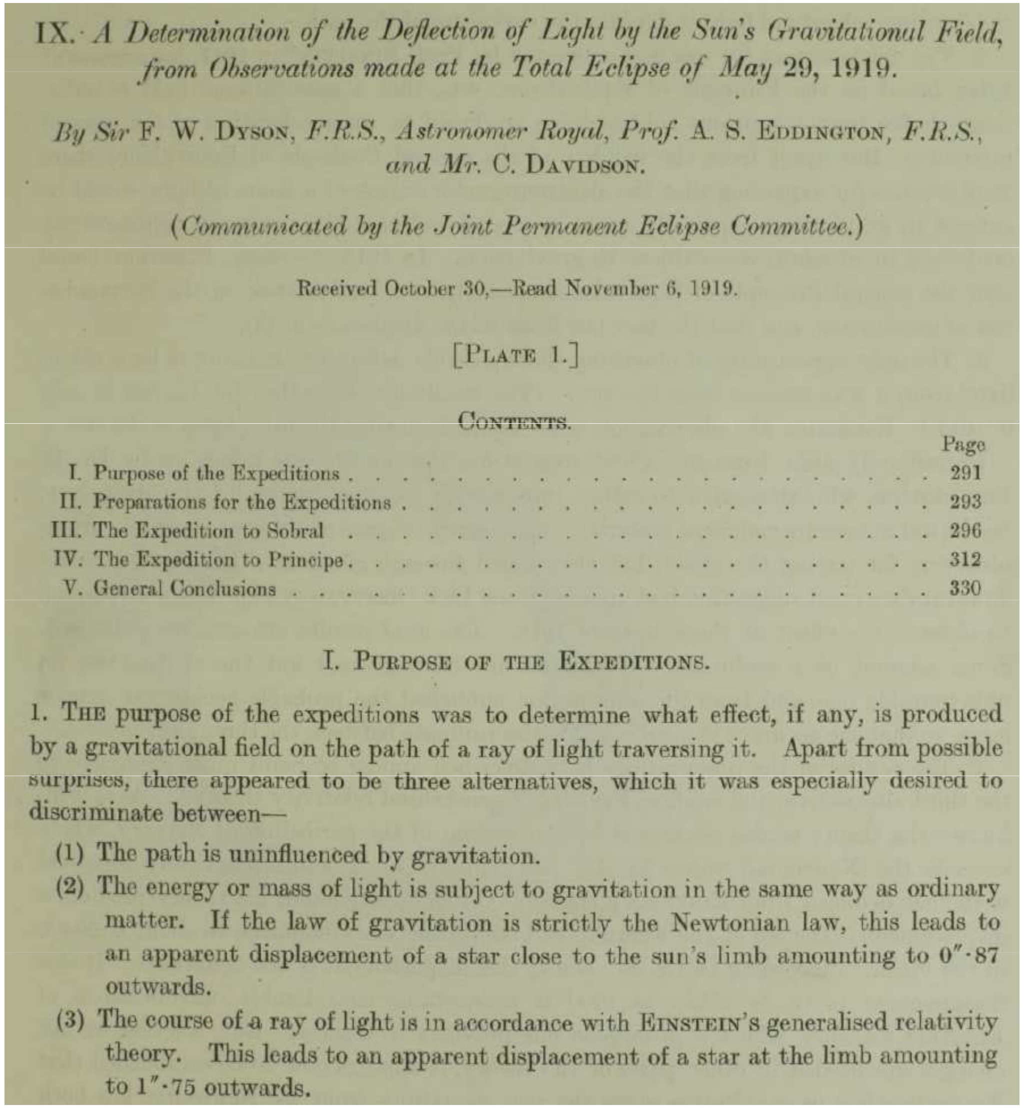
II. ***potvrzeno při úplném zatmění Slunce 1919***

Arthur Stanley Eddington 1882-1944 anglický astrofyzik, zakladatel cykloturistiky

OTR

Určení ohybu světelných paprsků v gravitačním poli Slunce z pozorování úplného zatmění r. 1919

F. W. Dyson, A. S. Eddington, C. Davison



IX. *A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations made at the Total Eclipse of May 29, 1919.*

By Sir F. W. Dyson, *F.R.S., Astronomer Royal, Prof.* A. S. Eddington, *F.R.S., and Mr.* C. Davidson.

(*Communicated by the Joint Permanent Eclipse Committee.*)

Received October 30,—Read November 6, 1919.

[Plate 1.]

Contents.

I. Purpose of the Expeditions.

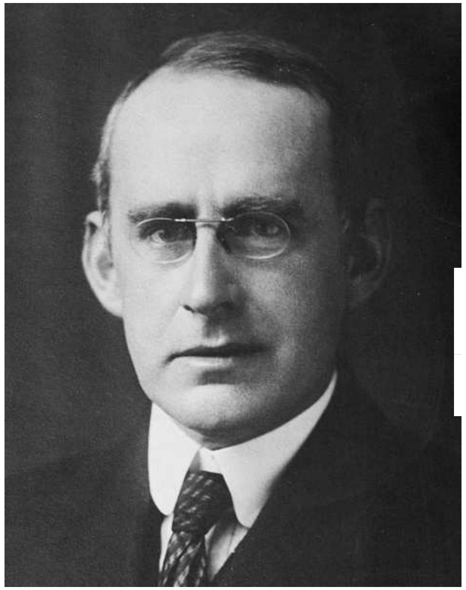
1. The purpose of the expeditions was to determine what effect, if any, is produced by a gravitational field on the path of a ray of light traversing it. Apart from possible surprises, there appeared to be three alternatives, which it was especially desired to discriminate between—

(1) The path is uninfluenced by gravitation.

(2) The energy or mass of light is subject to gravitation in the same way as ordinary matter. If the law of gravitation is strictly the Newtonian law, this leads to an apparent displacement of a star close to the sun's limb amounting to $0''\cdot 87$ outwards.

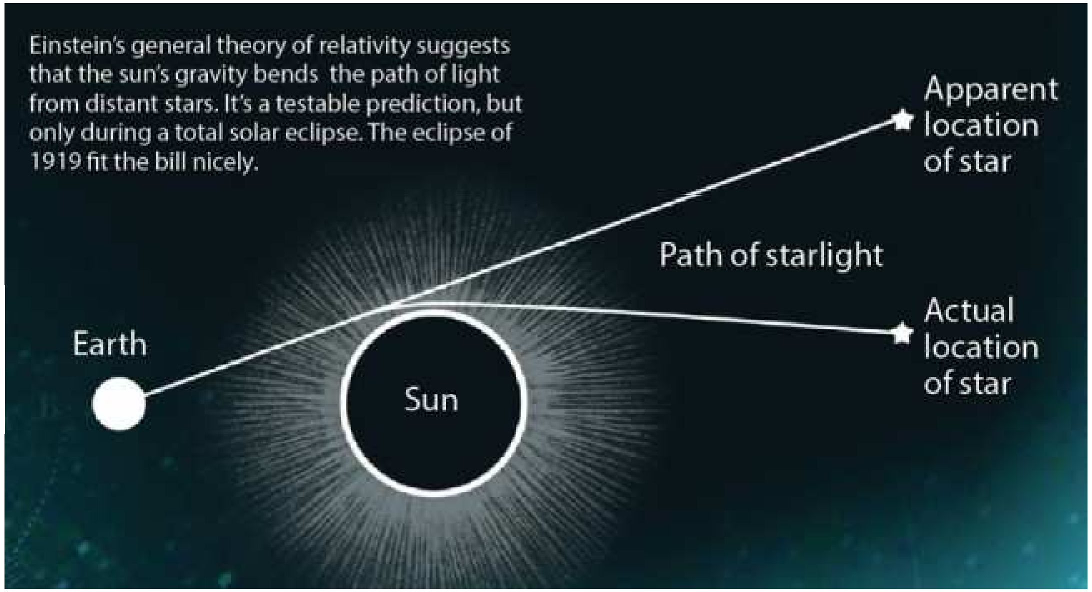
(3) The course of a ray of light is in accordance with Einstein's generalised relativity theory. This leads to an apparent displacement of a star at the limb amounting to $1''\cdot 75$ outwards.

OTR - Ohyb světelných paprsků - Slunce

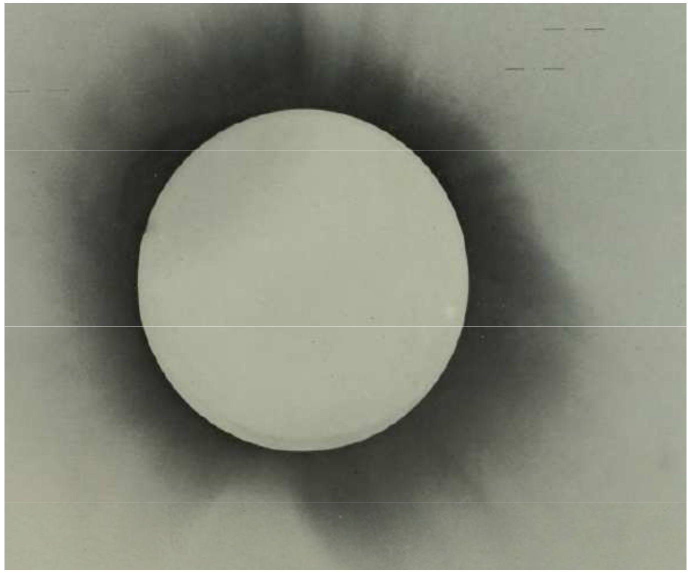


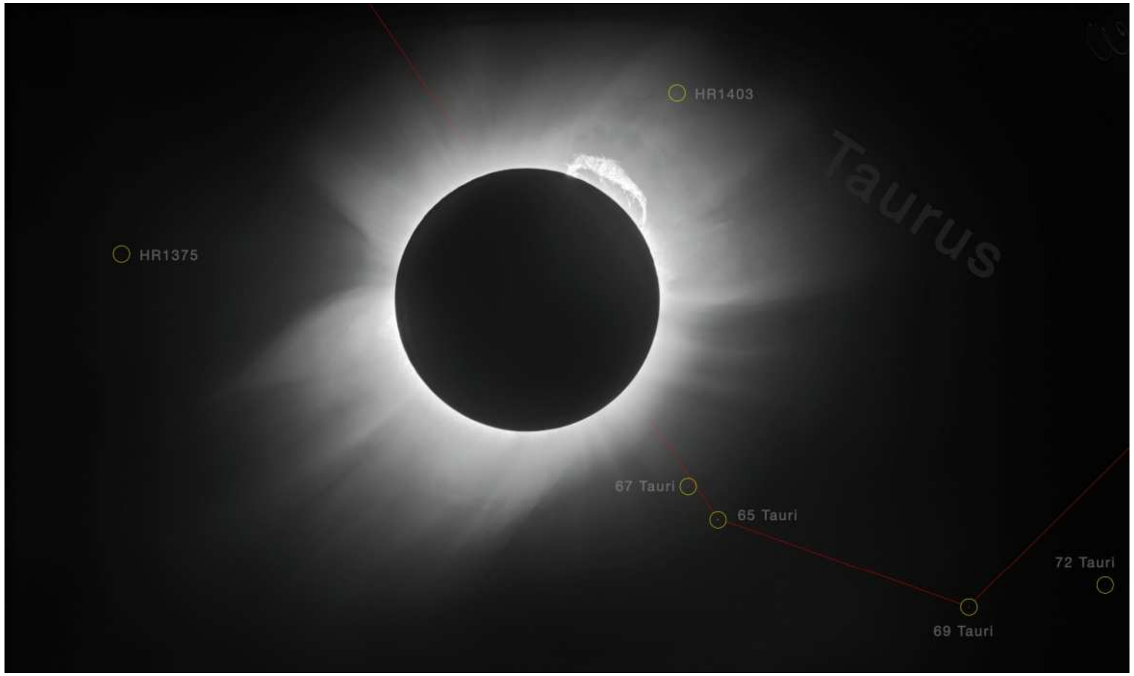
$$\alpha \simeq 4\,\frac{GM}{c^2 r}.$$

$\alpha \approx 1{,}75$“



Einstein's general theory of relativity suggests that the sun's gravity bends the path of light from distant stars. It's a testable prediction, but only during a total solar eclipse. The eclipse of 1919 fit the bill nicely.





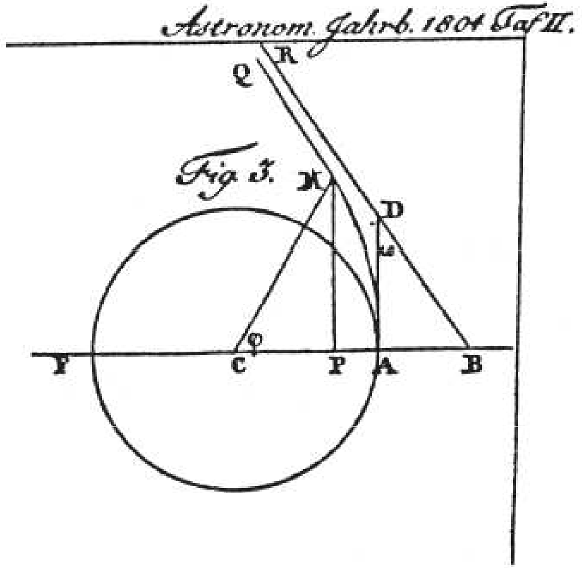
Klasická fyzika - ohyb světla

Nerelativistický vztah pro ohyb světla odvodil mnichovský astronom Johann von Soldner v práci *O odchylce světelného paprsku od jeho přímočarého pohybu v důsledku přitažlivosti tělesa, kolem něhož blízko prochází*, otištěné v berlínské ročence na rok 1804 [4]. Podle klasické mechaniky má těleso přilétající s rychlostí v a srážkovým parametrem r hyperbolickou dráhu a jeho směr letu se přítomností tělesa hmotnosti M odchýlí o úhel α daný vztahem

$$\operatorname{tg}\frac{\alpha}{2} = \frac{GM}{v^2 r},$$

kde G je gravitační konstanta, což při malé hodnotě argumentu — v celé této problematice jde o vteřiny nebo ještě řádově menší úhly — přejde ve vztah

$$\alpha \simeq 2\,\frac{GM}{v^2 r}.$$

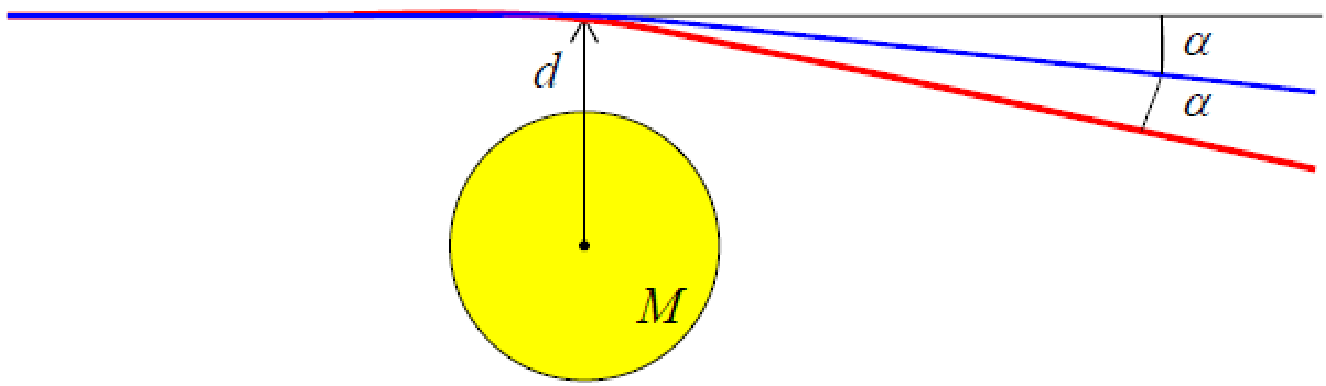


Obr. 1. Změna směru světelného paprsku v gravitačním poli podle J. Soldnera [4].

(Na obrázku 1 je Soldnerův náčrtek, polovina odchylky je zde označena jako ω a $r = CA$, resp. CB, protože odchylka α je velmi malý úhel.) V případě světla dostaneme „newtonovskou“ hodnotu ohybu dosazením $v = c$. Pro paprsky procházející těsně kolem povrchu Slunce tak vyjde hodnota $0{,}85''$. Soldner proto v závěru práce připomíná, že odchylka je tak malá, že *„při současném stavu praktické astronomie nevzniká potřeba brát v úvahu perturbace světelných paprsků v důsledku přitažlivosti nebeských těles“*, a doufá, že mu nikdo nebude mít za zlé, že zacházel se světlem jako s hmotným objektem.

Klasická fyzika + OTR
Ohyb světelných paprsků v gravitačním poli

Průchod světla gravitačním polem



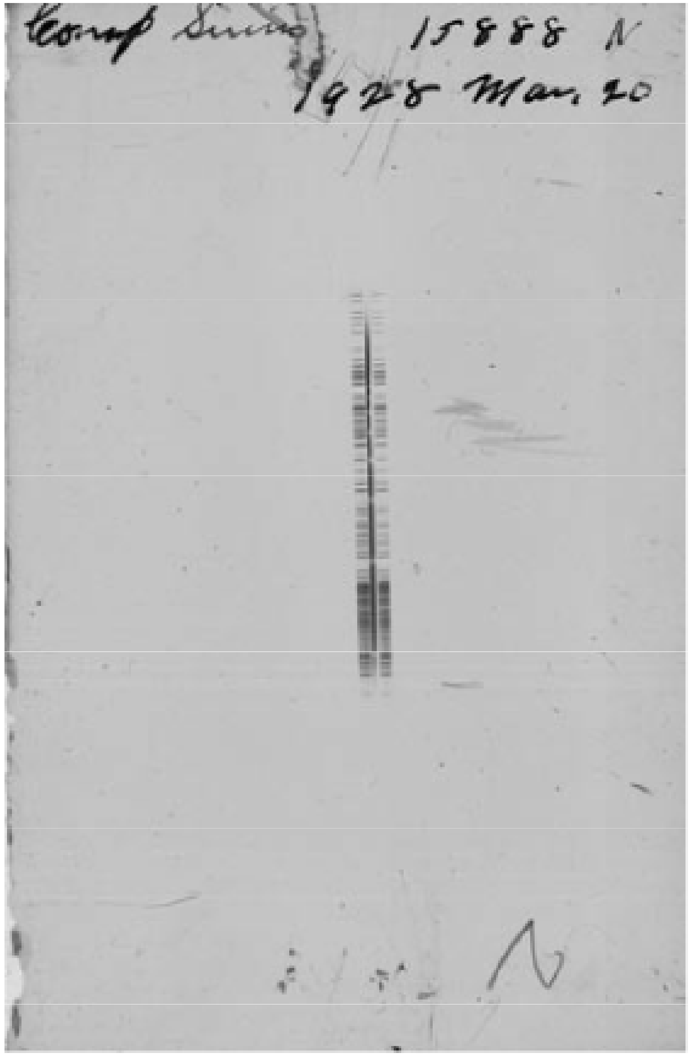
Newtonovská gravitace + nehmotné fotony: $\alpha = 0$

Newtonovská gravitace + hmotné fotony: $\alpha = \dfrac{2GM}{c^2 d}$

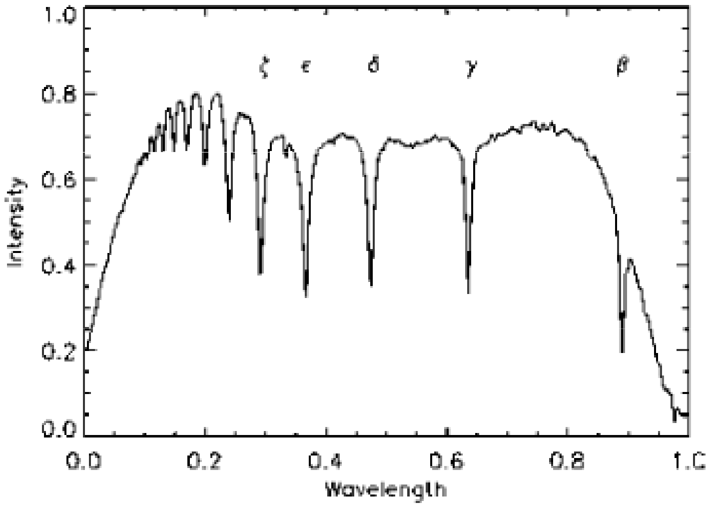
Obecná teorie relativity: $\alpha = \dfrac{4GM}{c^2 d}$

OTR gravitační rudý posuv - bílí trpaslíci

Sirius B
40 Eri B
van Maanen 2

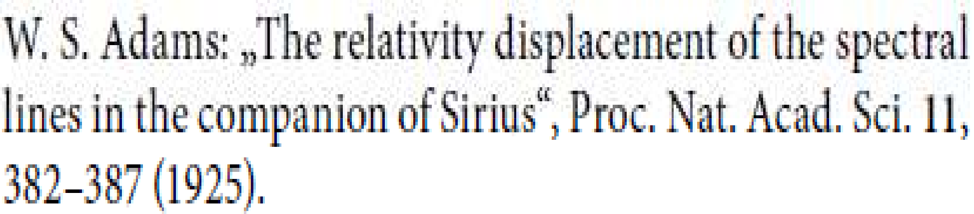


An image of the 20 March 1928 spectroscopic plate of Sirius B obtained by Joseph Moore. The dispersed 'stellar' image is the dark vertical central line. The sequence of short horizontal lines on each side are emission lines from the comparison lamp. Some of the 'stellar' lines measured by Moore can be seen as faint notches in the stellar spectrum. The physical size of the plate is approximately 22 mm by 34 mm. Courtesy of Lick Observatory.

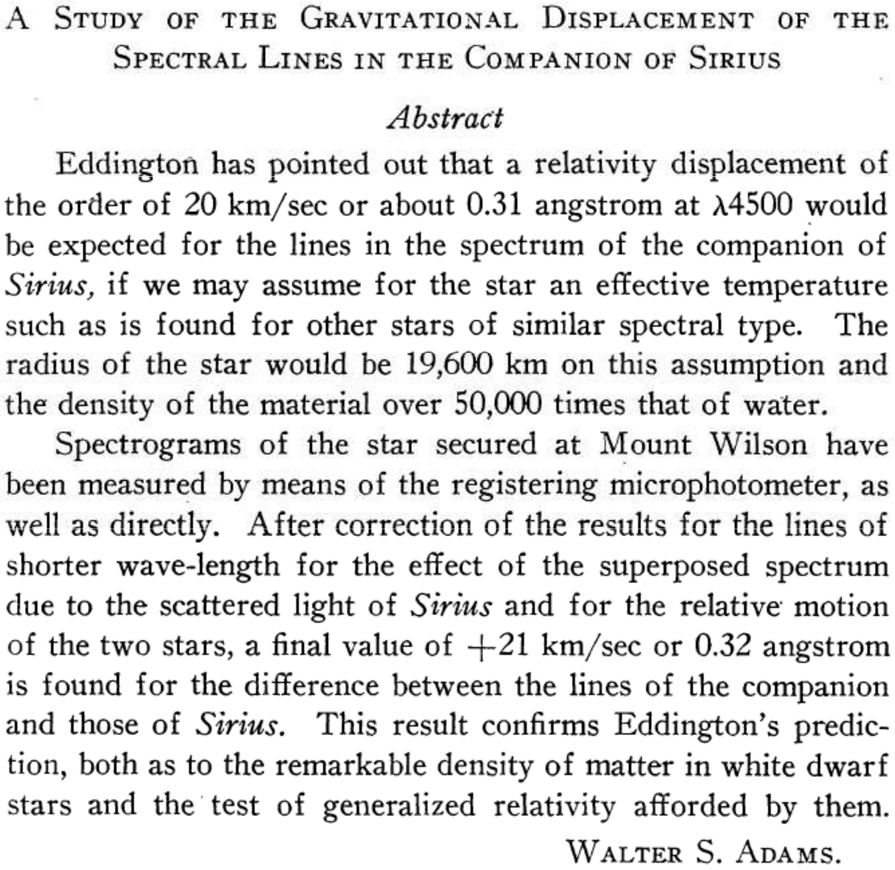


OTR gravitační rudý posuv Sirius B

Sirius B
W. C. Adams 1876 - 1956



W. S. Adams: „The relativity displacement of the spectral lines in the companion of Sirius“, Proc. Nat. Acad. Sci. 11, 382–387 (1925).



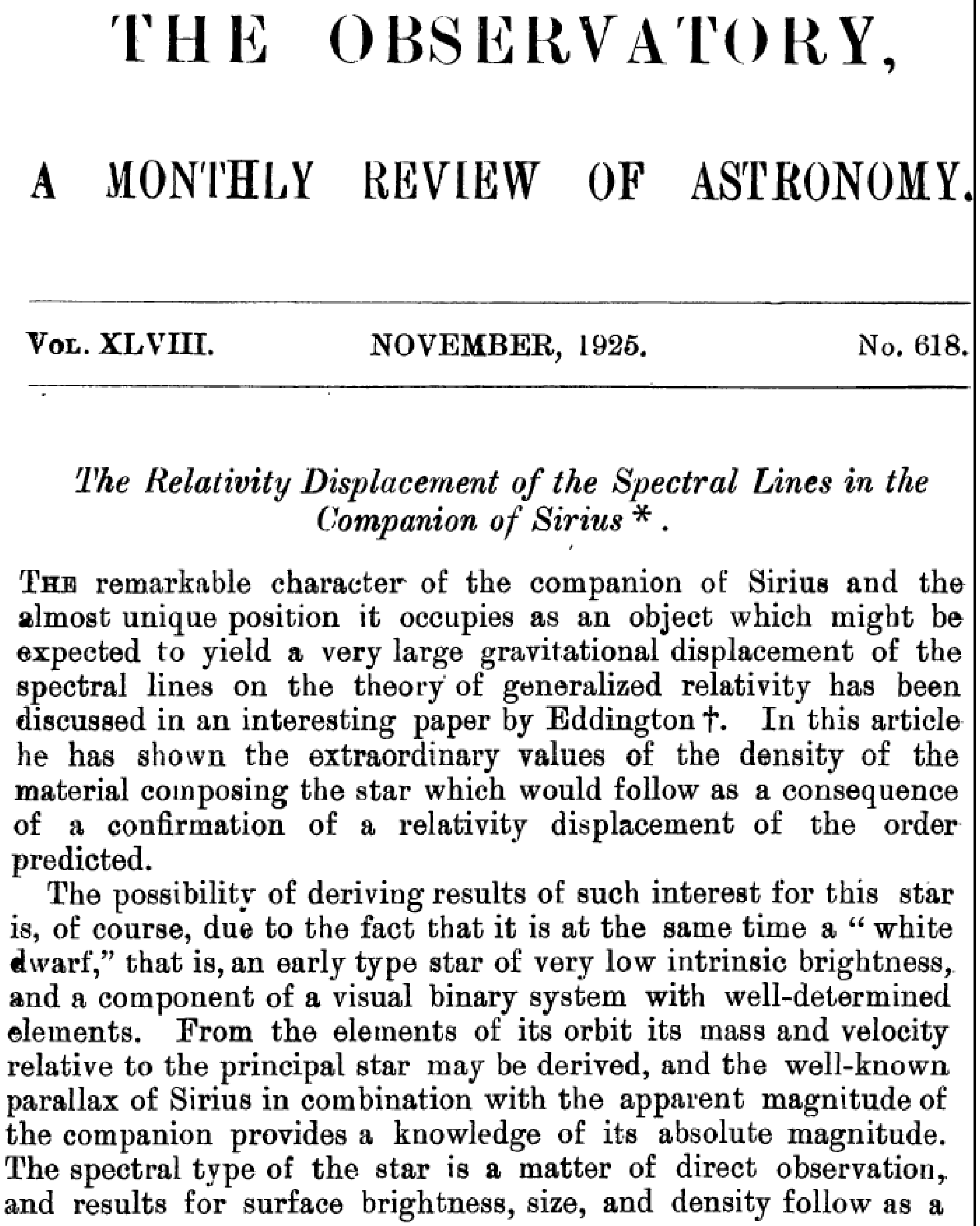
A STUDY OF THE GRAVITATIONAL DISPLACEMENT OF THE SPECTRAL LINES IN THE COMPANION OF SIRIUS

Abstract

Eddington has pointed out that a relativity displacement of the order of 20 km/sec or about 0.31 angstrom at λ4500 would be expected for the lines in the spectrum of the companion of *Sirius,* if we may assume for the star an effective temperature such as is found for other stars of similar spectral type. The radius of the star would be 19,600 km on this assumption and the density of the material over 50,000 times that of water.

Spectrograms of the star secured at Mount Wilson have been measured by means of the registering microphotometer, as well as directly. After correction of the results for the lines of shorter wave-length for the effect of the superposed spectrum due to the scattered light of *Sirius* and for the relative motion of the two stars, a final value of +21 km/sec or 0.32 angstrom is found for the difference between the lines of the companion and those of *Sirius.* This result confirms Eddington's prediction, both as to the remarkable density of matter in white dwarf stars and the test of generalized relativity afforded by them.

WALTER S. ADAMS.



THE OBSERVATORY,

A MONTHLY REVIEW OF ASTRONOMY.

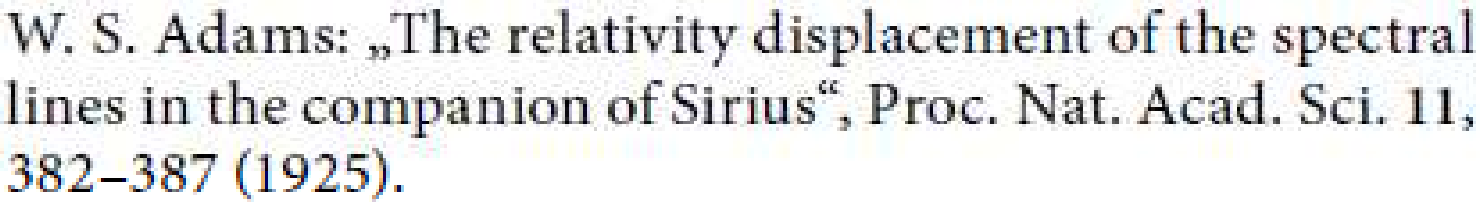
VOL. XLVIII. NOVEMBER, 1925. No. 618.

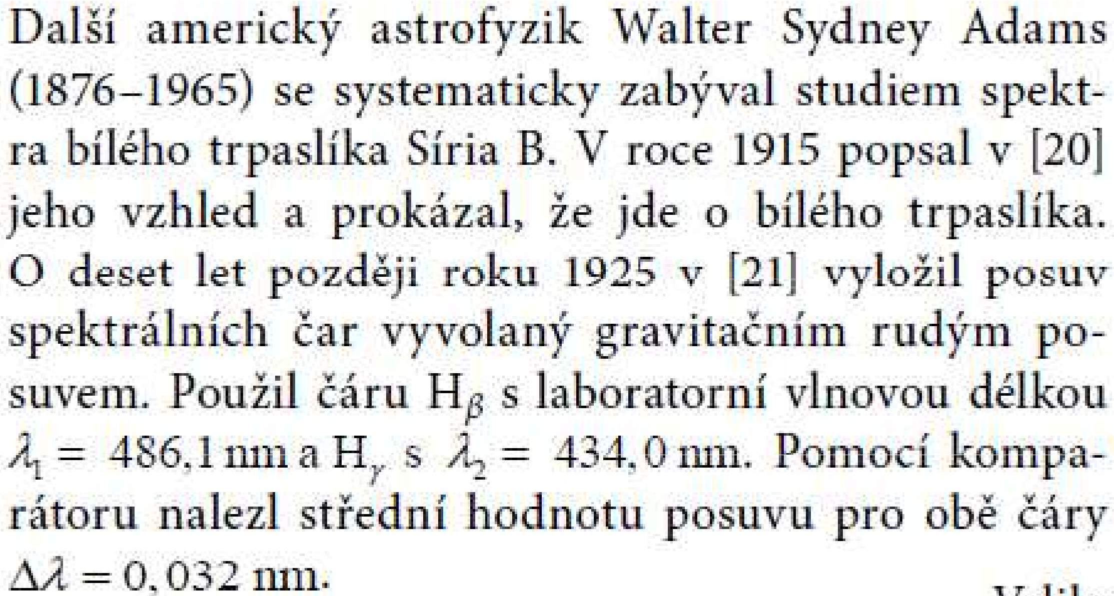
The Relativity Displacement of the Spectral Lines in the Companion of Sirius *.

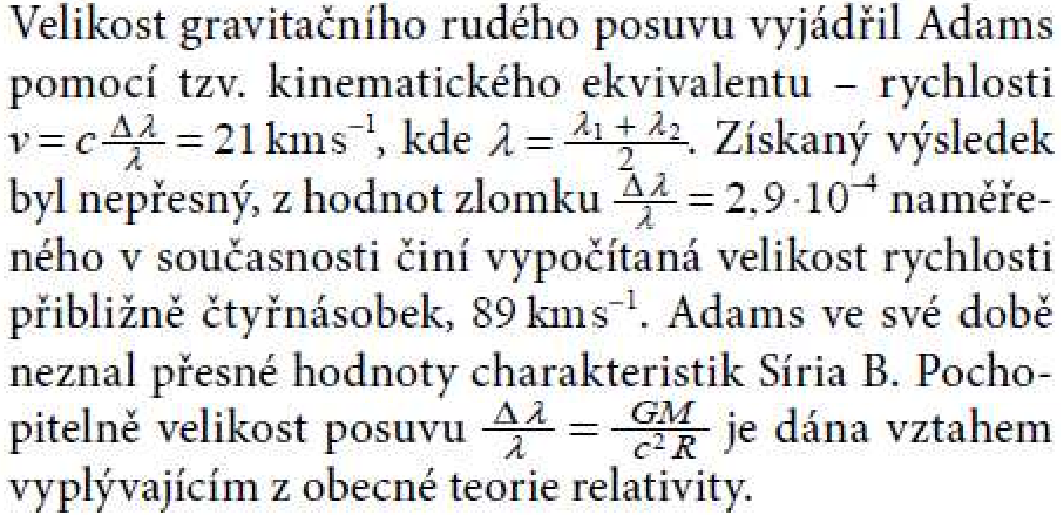
THE remarkable character of the companion of Sirius and the almost unique position it occupies as an object which might be expected to yield a very large gravitational displacement of the spectral lines on the theory of generalized relativity has been discussed in an interesting paper by Eddington †. In this article he has shown the extraordinary values of the density of the material composing the star which would follow as a consequence of a confirmation of a relativity displacement of the order predicted.

The possibility of deriving results of such interest for this star is, of course, due to the fact that it is at the same time a "white dwarf," that is, an early type star of very low intrinsic brightness, and a component of a visual binary system with well-determined elements. From the elements of its orbit its mass and velocity relative to the principal star may be derived, and the well-known parallax of Sirius in combination with the apparent magnitude of the companion provides a knowledge of its absolute magnitude. The spectral type of the star is a matter of direct observation, and results for surface brightness, size, and density follow as a

OTR gravitační rudý posuv Sirius B


W. S. Adams: „The relativity displacement of the spectral lines in the companion of Sirius“, Proc. Nat. Acad. Sci. 11, 382–387 (1925).


Další americký astrofyzik Walter Sydney Adams (1876–1965) se systematicky zabýval studiem spektra bílého trpaslíka Síria B. V roce 1915 popsal v [20] jeho vzhled a prokázal, že jde o bílého trpaslíka. O deset let později roku 1925 v [21] vyložil posuv spektrálních čar vyvolaný gravitačním rudým posuvem. Použil čáru H_β s laboratorní vlnovou délkou $\lambda_1 = 486{,}1\,\mathrm{nm}$ a H_γ s $\lambda_2 = 434{,}0\,\mathrm{nm}$. Pomocí komparátoru nalezl střední hodnotu posuvu pro obě čáry $\Delta\lambda = 0{,}032\,\mathrm{nm}$.


Velikost gravitačního rudého posuvu vyjádřil Adams pomocí tzv. kinematického ekvivalentu – rychlosti $v = c\frac{\Delta\lambda}{\lambda} = 21\,\mathrm{km\,s^{-1}}$, kde $\lambda = \frac{\lambda_1 + \lambda_2}{2}$. Získaný výsledek byl nepřesný, z hodnot zlomku $\frac{\Delta\lambda}{\lambda} = 2{,}9 \cdot 10^{-4}$ naměřeného v současnosti činí vypočítaná velikost rychlosti přibližně čtyřnásobek, $89\,\mathrm{km\,s^{-1}}$. Adams ve své době neznal přesné hodnoty charakteristik Síria B. Pochopitelně velikost posuvu $\frac{\Delta\lambda}{\lambda} = \frac{GM}{c^2 R}$ je dána vztahem vyplývajícím z obecné teorie relativity.