Homework problems #1

1. Helmholtz free energy of liquids can by approximated by

$$F(T,V,N) = -kT \ln \left\{ \left(\frac{h^2}{2\pi mkT} \right)^{-3N/2} \left[\frac{4}{3}\pi \left(\left(\sqrt{2}\frac{V}{N} \right)^{1/3} - \sigma \right)^3 \right]^N \right\},$$

   where h, m, k and σ are constants. Determine the equation of state of this gas $p = p(T,V)$, the difference of specific heats $c_p - c_V$, and show that both formulae correspond to ideal gas for $V/N \gg \sigma$.

2. *Computer problem*: The number of microstates of the system of N classical noninteracting particles with energy lower than E in a volume V is given by

$$\Omega(E) = \left(\frac{2\pi mE}{h^2} \right)^{\frac{3}{2}N} \frac{V^N}{N!\,\Gamma(\frac{3}{2}N+1)}.$$

   Let us study two such systems in a thermal interaction with total energy $2E$. We shall study fluctuations in these systems, within which one of the system has energy $E+\Delta E$ and the second one $E-\Delta E$. Plot the number of microstates of combined system as a function of ΔE for $N = 10,\ 10^2,\ 10^4$. (It is advisable to express the energy in units of $\varepsilon = h^2/(2\pi m)$ an number of states in units of $\gamma = V^N/(N!\,\Gamma(\frac{3}{2}N+1))$.) Let us assume that ΔE changes discountinuously by the value of $\varepsilon/100$ and determine an error introduced by assuming that the entropy is calculated only from the states corresponding to the equilibrium. (calculate for $N = 10,\ 10^2,\ 10^4,\ 10^5$).

The solution should be submitted not later than on April 2nd.