Some exercises to practise (c8601-01)

If not stated differently, references are to the text „Základy fyziky hvězdných atmosfér" (version 19. května 2024) stored in the IS. HM refers to the book Theory of Stellar Atmospheres (Hubeny and Mihalas, 2014). M78 refers to the book Stellar Atmospheres (Mihalas, 1978). LC refers to the book Introduction to Stellar Winds (Lamers and Cassinelli, 1999).

1. Using the expression for the total stellar luminosity

$$L_* = \int \mathfrak{F}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{S} \tag{13.1}$$

($\mathfrak{F}$ is the total radiative flux) express stellar effective temperature T_{eff} and surface gravity g using stellar luminosity L_*, stellar mass M_*, and stellar radius R_*.

(section 13; HM section 18.1, also 3.3)

2. Combining the continuity equation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0 \tag{13.14a}$$

and the equation of motion

$$\rho \frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = \boldsymbol{f}^{\text{ext}} - \boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot \sigma \tag{13.15}$$

derive the momentum equation

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \boldsymbol{\nabla} \cdot [\rho \boldsymbol{v}\boldsymbol{v} + p 1 - \sigma] = \boldsymbol{f}^{\text{ext}} \tag{13.19}$$

Consider that the external force consists of radiation and gravity,

$$\boldsymbol{f}^{\text{ext}} = \rho \boldsymbol{g} + \boldsymbol{f}^{\text{rad}}, \tag{13.20}$$

and using the first moment radiative transfer equation

$$\frac{1}{c^2} \frac{\partial \mathfrak{F}}{\partial t} + \boldsymbol{\nabla} \cdot \mathcal{P}_{\text{R}} = \frac{1}{c} \int_0^\infty \mathrm{d}\nu \oint \boldsymbol{n} \left[\eta_\nu(\boldsymbol{n}) - \chi_\nu(\boldsymbol{n}) I_\nu(\boldsymbol{n})\right] \mathrm{d}\varpi = -\boldsymbol{f}^{\text{rad}}. \tag{3.36b}$$

rewrite the momentum equation.

(section 13.2.2, HM section 16.1)

3. Start with the 3-D continuity equation (HM 16.13)

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0, \tag{13.14a}$$

and the radiating fluid momentum equation (HM 16.23)

$$\frac{\partial}{\partial t}\left(\rho \boldsymbol{v} + \frac{\boldsymbol{\mathfrak{F}}}{c^2}\right) + \boldsymbol{\nabla} \cdot [\rho \boldsymbol{v}\boldsymbol{v} + p\mathsf{1} - \mathsf{\sigma} + \mathcal{P}_{\mathrm{R}}] = \rho \boldsymbol{g} \tag{13.22}$$

and from the energy equation for matter with radiation (HM 16.27)

$$\begin{aligned} &\frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}v^2\right) + \mathcal{E}_{\mathrm{R}}\right] \\ &\quad + \boldsymbol{\nabla} \cdot \left[\rho\left(e + \frac{1}{2}v^2\right)\boldsymbol{v} + (p\mathsf{1} - \mathsf{\sigma}) \cdot \boldsymbol{v} + \boldsymbol{q} + \boldsymbol{\mathfrak{F}}\right] = \boldsymbol{v} \cdot \boldsymbol{f}^{\mathrm{ext}} + \epsilon_N \end{aligned} \tag{13.26}$$

and write their simplified forms for

(a) one-dimensional time-variable flow,

(b) one-dimensional stationary flow, and

(c) one-dimensional static atmosphere

for the plane-parallel and spherically symmetric approximations. Assume that the bulk viscosity is zero (ideal fluid).

(section 13.3; HM end of sections 16.1, 16.2, 16.3, and 16.4)