Some exercises to practise (c8601-02)

If not stated differently, references are to the text „Základy fyziky hvězdných atmosfér" (version 19. května 2024) stored in the IS. HM refers to the book Theory of Stellar Atmospheres (Hubeny and Mihalas, 2014). M78 refers to the book Stellar Atmospheres (Mihalas, 1978). LC refers to the book Introduction to Stellar Winds (Lamers and Cassinelli, 1999).

1. For the grey atmosphere in radiative equilibrium derive the expression for the dependence of temperature on the optical depth $T(\tau)$ assuming Eddington approximation.

   Hint: Eddington flux in the plane-parallel atmosphere can be expressed as

   $$H_\nu(\tau_\nu) = \frac{1}{2}\int_{\tau_\nu}^{\infty} S_\nu(t) E_2(t-\tau_\nu)\,\mathrm{d}t - \frac{1}{2}\int_0^{\tau_\nu} S_\nu(t) E_2(\tau_\nu - t)\,\mathrm{d}t$$

   The numerical constant can be determined from the flux at the surface $H(0)$, and using relations valid for exponential integral functions (A.2, A.3, A.5)

   $$\begin{aligned} nE_{n+1}(x) &= e^{-x} - xE_n(x) \\ E'_{n+1}(x) &= -E_n(x) \\ E_n(0) = \int_0^1 \frac{\mathrm{d}t}{t^n} &= \frac{1}{n-1} \end{aligned}$$

   (section 14.1; M78 sections 3.1 and 3.3, also HM section 17.1)

2. Assume that the source function is a linear function of the optical depth. Solving the radiative transfer equation

   $$\mu\frac{\mathrm{d}I(\tau,\mu)}{\mathrm{d}\tau} = I(\tau,\mu) - S(\tau) \tag{3.15}$$

   derive an expression for the specific intensity $I(0,\mu)$ emerging from the semi-infinite atmosphere. In other words: derive the Eddington-Barbier relation.

   (beginning of the section 6; HM section 11.4)

3. Assuming linear dependence of the source function $S(\tau) = a + b\tau$, the emergent radiation can be expressed using the Eddington-Barbier relation as

   $$I(0,\mu) = a + b\mu = S(\tau = \mu).$$

Show that for zero incident radiation from space, the mean intensity J at the surface of the atmosphere $(\tau = 0)$ can be expressed as

$$J(\tau = 0) = \frac{1}{2} S \left(\tau = \frac{1}{2} \right).$$

(section 14.2; HM section 17.1)