Some exercises to practise (c8601-04)

If not stated differently, references are to the text „Základy fyziky hvězdných atmosfér" (version 19. května 2024) stored in the IS. HM refers to the book Theory of Stellar Atmospheres (Hubeny and Mihalas, 2014). M78 refers to the book Stellar Atmospheres (Mihalas, 1978). LC refers to the book Introduction to Stellar Winds (Lamers and Cassinelli, 1999).

1. Combining the continuity equation

$$\frac{1}{r^2}\frac{\mathrm{d}(r^2\rho v)}{\mathrm{d}r} = 0 \tag{19.1}$$

and equation of motion

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p_\mathrm{g}}{\mathrm{d}r} - \frac{GM_*\rho}{r^2} + f^\mathrm{rad} \tag{19.2}$$

for a spherically symmetric stationary stellar wind, and using the sound-speed

$$a_\mathrm{s} = \sqrt{\frac{p_\mathrm{g}}{\rho}} \tag{19.5}$$

derive the momentum equation

$$\left(1 - \frac{a_\mathrm{s}^2}{v^2}\right) v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a_\mathrm{s}^2}{r} - \frac{\mathrm{d}a_\mathrm{s}^2}{\mathrm{d}r} - \frac{GM_*}{r^2} + \frac{f^\mathrm{rad}}{\rho}. \tag{19.7}$$

(section 19; HM section 20.2)

2. The momentum equation for an isothermal wind is

$$\left(1 - \frac{a_\mathrm{s}^2}{v^2}\right) v\frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a_\mathrm{s}^2}{r} - \frac{GM_*}{r^2} \tag{19.9}$$

Write critical conditions for an existence of a transsonic solution. Evaluate $\mathrm{d}v/\mathrm{d}r$ at the critical point for the transsonic solution.

(section 19.1.1; LC section 3.1.1)