Some exercises to practise (c8601-05)

If not stated differently, references are to the text „Základy fyziky hvězdných atmosfér" (version 19. května 2024) stored in the IS. HM refers to the book Theory of Stellar Atmospheres (Hubeny and Mihalas, 2014). M78 refers to the book Stellar Atmospheres (Mihalas, 1978). LC refers to the book Introduction to Stellar Winds (Lamers and Cassinelli, 1999).

1. Assume an isothermal wind. Find a function $v(r)$ from an analytical solution (integration) of the momentum equation

$$\left(1 - \frac{a_\mathrm{s}^2}{v^2}\right) v \frac{\mathrm{d}v}{\mathrm{d}r} = \frac{2a_\mathrm{s}^2}{r} - \frac{GM_*}{r^2}. \tag{19.9}$$

   The value of the integration constant is to be determined from the solution at the critical point $r = r_\mathrm{c}$.

   What is the value v_0 of the velocity at the base of the wind? Using this value, write an expression for the ratio $v(r)/v_0$.

   (section 19.1.1; LC sections 3.1.3)

2. The velocity dependence on radius for a spherically symmetric isothermal wind can be expressed as

$$\frac{v}{v_0} \exp\left(-\frac{v^2}{2a_\mathrm{s}^2}\right) = \left(\frac{r_0}{r}\right)^2 \exp\left\{\frac{GM_*}{a_\mathrm{s}^2}\left(\frac{1}{r_0} - \frac{1}{r}\right)\right\}, \tag{19.18}$$

   where v_0 is the value of the velocity at r_0 (the base of the subsonic region). Let ρ_0 is the value of the density at r_0. Using this boundary condition and the continuity equation

$$\frac{1}{r^2}\frac{\mathrm{d}(r^2 \rho v)}{\mathrm{d}r} = 0 \tag{19.1}$$

   find an expression for $\rho(r)$.

   Derive an expression for the density dependence which follows from an equation for a hydrostatic atmosphere,

$$\frac{1}{\rho}\frac{\mathrm{d}p_\mathrm{g}}{\mathrm{d}r} + \frac{GM_*}{r^2} = 0 \tag{19.21}$$

   and compare both expressions for $\rho(r)$.

   Express the wind mass-loss rate $\dot{M}$ using quantities at the bottom of the subsonic region.

(section 19.1.1; LC sections 3.1.3, 3.1.4)

3. Assuming that the external force in an isothermal wind is proportional to $v\mathrm{d}v/\mathrm{d}r$, the equation of motion can be written in the form

$$v\frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{1}{\rho}\frac{\mathrm{d}p_\mathrm{g}}{\mathrm{d}r} - \frac{GM_*}{r^2} + Bv\frac{\mathrm{d}v}{\mathrm{d}r}. \tag{19.37}$$

Combining this with the continuity equation to derive the parameters of the critical point and the mass-loss rate.

(section 19.1.2; LC section 3.3)

4. Consider the equation of motion for the radiatively driven wind

$$\rho v\frac{\mathrm{d}v}{\mathrm{d}r} + \frac{\mathrm{d}p_\mathrm{g}}{\mathrm{d}r} = \rho\frac{GM_*}{r^2}\left(\Gamma - 1\right). \tag{20.7}$$

Integrating this equation over the outflowing wind matter $\mathrm{d}m = 4\pi r^2\rho\,\mathrm{d}r$ derive the equation for wind outflowing momentum

$$\dot{M}v_\infty = \frac{L_*}{c}\frac{\Gamma - 1}{\Gamma}\tau_W \tag{20.15}$$

using the expression for the optical depth of the wind

$$\tau_W = \int_{r_\mathrm{c}}^{\infty} \bar{\varkappa}_F\rho\,\mathrm{d}r, \tag{20.14}$$

where r_c is the critical point.

(section 20.2.1; LC section 7.2.2)