Some exercises to practise (c8601-07)

If not stated differently, references are to the text „Základy fyziky hvězdných atmosfér" (version 19. května 2024) stored in the IS. HM refers to the book Theory of Stellar Atmospheres (Hubeny and Mihalas, 2014). M78 refers to the book Stellar Atmospheres (Mihalas, 1978). LC refers to the book Introduction to Stellar Winds (Lamers and Cassinelli, 1999).

1. Using the equation of motion

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p_{\mathrm{g}}}{\mathrm{d}r} - \frac{GM_*\rho}{r^2} + f^{\mathrm{rad}}, \tag{19.2}$$

the continuity equation

$$\frac{1}{r^2}\frac{\mathrm{d}(r^2\rho v)}{\mathrm{d}r} = 0 \tag{19.1}$$

and the expression for the radiatiove acceleration in the CAK approximation

$$g_{\mathrm{R}}^{\mathrm{L}} = g_{\mathrm{e}} M(t), \tag{22.17}$$

derive after substituting for

$$g_{\mathrm{e}} = \frac{\varkappa_{\mathrm{e}} L_*}{4\pi r^2 c}, \tag{22.18}$$

$$M(t) = k t^{-\alpha} \left(\frac{10^{-11} n_{\mathrm{e}}}{W} \right)^{\delta} \tag{22.19}$$

and

$$t = \varkappa_{\mathrm{e}} v_{\mathrm{th}} \rho \left(\frac{\mathrm{d}v}{\mathrm{d}r} \right)^{-1} \tag{22.20}$$

the momentum equation,

$$\left(1 - \frac{a_{\mathrm{s}}^2}{v^2}\right) v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{GM_*(1-\Gamma_{\mathrm{e}})}{r^2} + \frac{2a_{\mathrm{s}}^2}{r} - \frac{\mathrm{d}a_{\mathrm{s}}^2}{\mathrm{d}r} + \frac{C}{r^2}\left(r^2 v \frac{\mathrm{d}v}{\mathrm{d}r}\right)^{\alpha} \tag{22.28}$$

and determine the constant C.

(section 22.4.1; LC section 8.7)