Some exercises to practise (c8601-08)

If not stated differently, references are to the text „Základy fyziky hvězdných atmosfér" (version 19. května 2024) stored in the IS. HM refers to the book Theory of Stellar Atmospheres (Hubeny and Mihalas, 2014). M78 refers to the book Stellar Atmospheres (Mihalas, 1978). LC refers to the book Introduction to Stellar Winds (Lamers and Cassinelli, 1999).

1. Consider the equation of motion for the line driven wind

$$\rho v \frac{\mathrm{d}v}{\mathrm{d}r} = -\frac{\mathrm{d}p_{\mathrm{g}}}{\mathrm{d}r} - \rho \frac{GM_*\,(1-\Gamma_{\mathrm{e}})}{r^2} + f_L^{\mathrm{rad}} \tag{22.26}$$

Integrating this equation over the outflowing wind matter $\mathrm{d}m = 4\pi r^2 \rho\,\mathrm{d}r$ derive the equation for wind outflowing momentum

$$\dot{M} v_\infty = \frac{L_*}{c}\frac{\Gamma_{\mathrm{e}}-1}{\Gamma_{\mathrm{e}}}\tau_{\mathrm{e}} + B\frac{L_*}{c} \tag{22.44}$$

using the expression for the electron scattering optical depth of the wind

$$\tau_{\mathrm{e}} = \int_{r_{\mathrm{c}}}^{\infty} \varkappa_{\mathrm{e}} \rho\,\mathrm{d}r \tag{22.45}$$

where r_{c} is the critical point. Show that the line blocking factor B can be expressed as

$$B = \frac{1}{v_{\mathrm{th}}}\int_0^{\infty} M(t) t\,\mathrm{d}v. \tag{22.46}$$

(section 22.4.3; HM section 20.3)