$\gamma(t)$ nějaká par. (zadaná)

$\gamma(s)$ par. oblouku

$t = t(s)$

$$\frac{d\gamma}{ds} = \frac{d\gamma}{dt} \cdot \frac{dt}{ds} \quad / \; \|\dots\|$$

$$1 = \underbrace{\left\|\frac{d\gamma}{dt}\right\|}_{\sqrt{1+c_2^2}} \cdot \underbrace{\frac{dt}{ds}}_{>0}$$

předpoklad

$$\frac{ds}{dt} = \sqrt{1+c_2^2}$$

$$\Leftarrow \quad s = \int_0^t \sqrt{1+c_2^2}\, du$$

$$\gamma(t) = f(t, c_2)$$

$$j(s) = f\left(\frac{s}{\sqrt{1+c_2^2}}, c_2\right)$$

$$j'(s) = \frac{dj(s)}{ds} = \frac{1}{\sqrt{1+c_2^2}} f_1 + 0 \cdot f_2 = A(s)$$

$$\boxed{\begin{array}{ll} u_1(s) = \dfrac{s}{\sqrt{1+c_2^2}}, & u_2(s) = c_2 \\[2ex] U_1(s) = \dfrac{1}{\sqrt{1+c_2^2}}, & U_2(s) = 0 \end{array}}$$

$$\frac{\nabla j'(s)}{ds} = \left(\frac{\nabla U_1(s)}{ds}, \frac{\nabla U_2(s)}{ds}\right)$$

$$= \frac{\nabla U_1(s)}{ds} \cdot f_1(u_1(s), u_2(s))$$

$$+ \frac{\nabla U_2(s)}{ds} \cdot f_2(u_1(s), u_2(s))$$

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$$\Gamma^2_{11}(u_1, u_2) = -u_2, \quad \Gamma^1_{12}(u_1, u_2) = \frac{u_2}{u_2^2+1}$$

$$\frac{\nabla v_1(s)}{ds} = \underbrace{\frac{d v_1(s)}{ds}}_{=0} + \Gamma^1_{12}\Big(\underbrace{v_1}_{\frac{1}{\sqrt{1+c_2^2}}}\underbrace{\frac{du_2}{ds}}_{=0} + \underbrace{v_2}_{=0}\frac{du_1}{ds}\Big)$$

$$= \Gamma^1_{12}\left(\frac{s}{\sqrt{1+c_2^2}}, c_2\right)\cdot \frac{1}{\sqrt{1+c_2^2}}\cdot 0 = 0$$

$$\frac{\nabla v_2(s)}{ds} = \frac{d v_2(s)}{ds} + \Gamma^2_{11}\left(\frac{s}{\sqrt{1+c_2^2}}, c_2\right) v_1 \frac{du_1}{ds}$$

$$= -c_2 \frac{1}{\sqrt{1+c_2^2}}\cdot \frac{1}{\sqrt{1+c_2^2}} = -\frac{c_2}{1+c_2^2}$$

$$\boxed{\text{Závěr: } \frac{\nabla \dot\gamma(s)}{ds} = -\frac{c_2}{1+c_2^2}\cdot f_2\left(\frac{s}{\sqrt{1+c_2^2}}, c_2\right)}$$