Vmitvo semestralka

(1) $g(t) = \left(\cos t, \sin t, \frac{2}{3} t^{\frac{3}{2}}\right)$

(a) param. obloka

$t = t(s)$

$\frac{dg}{ds} = \frac{dg}{dt} \cdot \frac{dt}{ds} \quad / \|..\|$

$1 = \|g'(t)\| \cdot \frac{dt}{ds}$

$g'(t) = (-\sin t, \cos t, \sqrt{t})$

$ds = \|g'\| dt$

$\|g'\| = \sqrt{t+1}$

$s = \int \|g'\| dt$

$s = \int \sqrt{t+1}\, dt = \frac{2}{3}(t+1)^{\frac{3}{2}}$

$\frac{3}{2} s = (t+1)^{\frac{3}{2}}$

$\left(\frac{3}{2} s\right)^{\frac{2}{3}} = t+1$

$t = \left(\frac{3}{2} s\right)^{\frac{2}{3}} - 1$

$g(s) = \left(\cos\left(\left(\frac{3}{2} s\right)^{\frac{2}{3}} - 1\right), \sin(\dots), \dots\right)$

(b) $\int_3^8 \|g'(t)\| \, dt = \int_3^8 \sqrt{t+1} \, dt$

$= \left[\frac{2}{3}(t+1)^{\frac{3}{2}}\right]_3^8 = \frac{2}{3}\left[27 - 8\right]$

(c) Bod na sféře, který náleží C_1:

$\|g(t)\| = \sqrt{13}$

$\sqrt{1 + \frac{4}{9}t^3} = \sqrt{13}$

$\frac{4}{9}t^3 = 12$

$t^3 = 27 \Rightarrow \underline{\underline{t = 3}}$

Bod na sféře: $\left[\cos 3, \sin 3, \frac{2}{3}\sqrt{27}\right]$

$\Rightarrow$ normál. vektor $(\cos 3, \sin 3, \frac{2}{3}\sqrt{27})$

tečný vektor C_1 $(-\sin 3, \cos 3, \sqrt{3})$

$$\cos\alpha = \frac{\frac{2}{3}\cdot 9}{\sqrt{1+\frac{4}{9}\cdot 27}\cdot\sqrt{1+3}} = \frac{6}{\sqrt{13}\cdot 2}$$

$$= \frac{3}{\sqrt{13}}$$

Výsledek: $\frac{\pi}{2} - \arccos\frac{3}{\sqrt{13}}$

2) $f(t) = (\cos t, \sin t, e^t)$

(a) $f'(t) = (-\sin t, \cos t, e^t)$

$f''(t) = (-\cos t, -\sin t, e^t)$

$f'''(t) = (\sin t, -\cos t, e^t)$

$$\kappa = \frac{\|f' \times f''\|}{\|f'\|^3} = \dots = \frac{\sqrt{2e^{2t}+1}}{\left(\sqrt{e^{2t}+1}\right)^3}$$

$$\tau = \frac{2e^t}{2e^{2t}+1}$$

$$\varkappa(0) = \frac{\sqrt{3}}{(\sqrt{2})^3} \qquad \tau(0) = -\frac{2}{3}$$

(b) $\omega = \langle f', f'' \rangle$

má normálový vektor

$$n = f' \times f'' =$$

$$= (e^t(\sin t + \cos t), e^t(\sin t - \cos t), 1)$$

$n \perp$ osa y

$\hookrightarrow (0, 1, 0)$

$$\sin t = \cos t$$

$$t = \frac{\pi}{4}$$