Exercises 1

(1) We showed that the free commutative R-algebra on $\{x_1, \ldots, x_n\}$ is $R[x_1, \ldots, x_n]$.
What about on a general set X?
What about the non-commutative case?

(2) Show that the ring of continuous functions is not Noetherian.
(Hint: consider eventually constant functions)

(3) An ideal I is irreducible if it cannot be written as $I = A \cap B$ for two ideals $I \subset A, B \subset R$

Show that in an Noetherian ring, each ideal has a decomposition

$I = I_1 \cap \ldots \cap I_n$ where the I_j are irreducible.

(4) Relate the above to decomposition of integers into prime powers when $R = \mathbb{Z}$.

(5) Show that commutative R-algebras A are in bijection with homomorph. of commutative rings $R \longrightarrow A$.