Exercises 10

(1) The cyclic group $C_3 = \langle g \mid g^3 = e \rangle$ has regular representation the vector space

$K[C_3] = \{ \lambda_1 e + \lambda_2 g + \lambda_3 g^2 : \lambda_i \in K \}$,

with standard basis $\{e, g, g^2\}$.

Describe the matrices for the regular representation (ie. the corresp. matrix representation).

(2) Show that if $\sigma : G \longrightarrow Mat_n(K)$ is a matrix representation of G & T is an invertible $n \times n$-matrix, then

$g \longmapsto T^{-1}\sigma(g)T$ is also a matrix representation. (Two matrix reps related in this way are said to be equivalent.)

(3) There is a forgetful funtor

$U : K\text{-Alg} \longrightarrow Mon$ sending a K-algebra $(A, +, \cdot, 0, 1)$ to its underlying monoid $(A, \cdot, 1)$.

Describe its left adjoint.

(4) A group G gives rise to a category ΣG with one object $\bullet$ & morphisms the elements of G. Show that

$G\text{-Mod} \cong [\Sigma G, Vect]$, the

category of functors from ΣG to k-vector spaces.