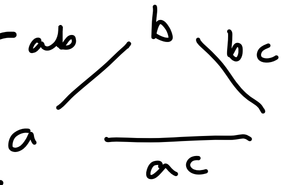
Algebra 4 - Ex 5

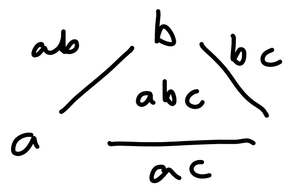
① Consider the simplicial complex

$K = \{$ 

$\}$ (more formally, $\{a\}, \{b\}, \{c\}, \{ab\}, \{bc\}, \{a, c\}$

- Calculate the chain complex $C(K)$ and its homology.

② Do the same for

$\{$ 

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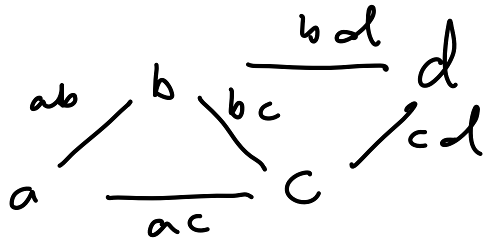
③ $\{$ 

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④ $\{$ 

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(5) Prove that if K is a simplicial complex then $C(K)$ is in fact a chain complex - ie.

why does $d_n \circ d_{n+1} = 0$ hold? To begin with, think about why

$$C(K)_2 \xrightarrow{d_0 - d_1 + d_2} C(K)_1 \xrightarrow{d_0 - d_1} C(K)_0$$

equals 0.

(6) Prove that every ses is isomorphic to one of the form $0 \to A \hookrightarrow B \to B/A \to 0$ for A a submodule of B.