

Algebra 4 - Ex 5

① Consider the simplicial complex

$$K = \left\{ \begin{array}{ccc} & a & b \\ & / & \backslash \\ a & & c \\ & \backslash & / \\ & c & b \end{array} \right\} \quad \left(\begin{array}{l} \text{more formally,} \\ \{a\}, \{b\}, \{c\}, \{ab\}, \{bc\}, \\ \{a, c\} \end{array} \right)$$

- Calculate the chain complex $C(K)$ and its homology.

② Do the same for

$$\left\{ \begin{array}{ccc} & a & b \\ & / & \backslash \\ a & & c \\ & \backslash & / \\ & c & b \end{array} \right\}$$

③

$$\left\{ \begin{array}{cccc} & & & d \\ & & \frac{bd}{\quad} & / \\ & a & b & \backslash \\ & / & \backslash & / \\ a & & c & \\ & \backslash & / & \\ & c & b & \end{array} \right\}$$

④

$$\left\{ \begin{array}{cc} a & b \\ \bullet & \bullet \end{array} \right\}$$

⑤ Prove that if K is a simplicial complex then $C(K)$ is in fact a chain complex - i.e.

why does $d_n \circ d_{n+1} = 0$ hold?
 To begin with, think about why

$$C(K)_2 \xrightarrow{d_0 - d_1 + d_2} C(K)_1, \quad d_0 - d_1 \xrightarrow{\quad} C(K)_0$$

equals 0.

⑥ Prove that every ses is isomorphic to one of the form $0 \rightarrow A \hookrightarrow B \rightarrow B/A \rightarrow 0$ for A a submodule of B .