Marked assignment 3 - Deadline May 20'th

Let S_3 be the symmetric group of permutations on 3 elements $\{\bar{1}, \bar{2}, \bar{3}\}$.
Consider $\mathbb{C}[\bar{1}, \bar{2}, \bar{3}]$, the permutation S_3-module, with elements $a\bar{1} + b\bar{2} + c\bar{3}$ for $a, b, c \in \mathbb{C}$ & action $g.(a\bar{1} + b\bar{2} + c\bar{3}) = a.\overline{g1} + b.\overline{g2} + c.\overline{g3}$ where $g \in S_3$.

a) Show that the vector subspace $\langle \bar{1} + \bar{2} + \bar{3} \rangle \leq \mathbb{C}[\bar{1}, \bar{2}, \bar{3}]$ is a S_3-submodule & isomorphic to the trivial S_3-module $\mathbb{C}$ (ie. with trivial action $g.a = a$ all $a \in \mathbb{C}$). (1)

b) Show that the vector subspace $\langle \bar{2} - \bar{1}, \bar{3} - \bar{2} \rangle \leq \mathbb{C}[\bar{1}, \bar{2}, \bar{3}]$ is also an S_3-submodule and that (2)

$$\langle \bar{2} - \bar{1}, \bar{3} - \bar{2} \rangle \oplus \langle \bar{1} + \bar{2} + \bar{3} \rangle = \mathbb{C}[\bar{1}, \bar{2}, \bar{3}]$$

c) Prove that $\langle \bar{2} - \bar{1}, \bar{3} - \bar{2} \rangle$ is an irreducible S_3-module. (4)

d) Using results in the lectures (not W14) calculate all irreducible S_3-modules up to iso. (3)