- Let $0 \to A \overset{i}{\hookrightarrow} H \xrightarrow{p} G \to 0$ (subgroup)

  be ses of groups with A abelian.

- We want to give an action of G on A (ie. G-module structure) which is equally to give a homomorphism $G \to \mathrm{Aut}(A)$.

- Now H acts on itself by conjugation

$$h * a = h.a.h^{-1}$$ & if $a \in A$

then $h * a \in A$ since $A = \ker(p)$ &

$$p(h*a) = ph.pa.(ph)^{-1} = ph.1.(ph)^{-1} = 1.$$

- Therefore H acts on A

- Then

$$A \xrightarrow{i} H \xrightarrow{\varphi} \mathrm{Aut}(A)$$
$$h \longmapsto h * -$$

sends everything to the identity $1 : A \to A$ since if $b \in A$, $b * a = b.a.b^{-1} = b.b^{-1}.a = a$.

Now G is cokernel of i

(ie as p surj., by First iso them, $G \cong H/A$)

so

$$\begin{tikzcd} A \arrow[r, "i"] & H \arrow[r, "\varphi"] \arrow[d, "p"] & \mathrm{Aut}(A) \\ & G \arrow[ur, dashed, "\exists!"'] & \end{tikzcd}$$

giving the required action. $\square$