Alg 4 – Exercises 4

(1) Check that the operations $+, \times$ for the ring structure on $U^{-1}R$

$$r/u + s/v = \frac{rv+su}{uv} \quad \& \quad r/u \cdot s/v = rs/uv$$

are well defined.

(2) Prove that R is a local ring $\iff$ its set of non-units form an ideal, which is then the unique maximal ideal.

(3) The following gives an alternative construction of $U^{-1}R$. Show it is the quotient of the polynomial ring $R[\{x_u\}_{u \in U}]$ by the ideal generated by $\langle ux_u - 1 : u \in U \rangle$. (In other words, we freely add an inverse x_u for each $u \in U$)