1. $f(x,y) = \sqrt{|3x| + |y-2| + 1} = c \quad /^2$

$|3x| + |y-2| + 1 = c^2$

$\boxed{c^2 - 1 = |3x| + |y-2|} \Rightarrow$

$c^2 - 1 < 0 \Rightarrow \emptyset$

$c^2 - 1 \geq 0 \Rightarrow \checkmark$

m.b.: $x = 0, \ y = 2$

PRO $c=0$ VRST. NEEX.

PRO $c=1$ $\Rightarrow$ $0 = |3x| + |y-2| \Rightarrow x=0, y=2$

$\Rightarrow$ VRST. JE BOD $[0,2]$

PRO $c>1$ $\Rightarrow$ 4 MOŽNOSTI:

(ii) $\boxed{x \geq 0 \wedge y \geq 2} \Rightarrow c^2 - 1 = 3x + y - 2$

$$y = -3x + c^2 + 1$$

$\boxed{x \geq 0} \wedge y \geq 2 \Rightarrow -3x + c^2 + 1 \geq 2$

$$-3x \geq 1 - c^2$$

$$x \leq \frac{c^2-1}{3}$$

$$x \in \left[0, \frac{c^2-1}{3}\right]$$

(ii) $x \geq 0 \wedge y \leq 2 \Rightarrow c^2 - 1 = 3x - y + 2$

$$y = 3x - c^2 + 3$$

$x \geq 0 \wedge y \leq 2 \Rightarrow 3x - c^2 + 3 \leq 2$

$$3x \leq c^2 - 1$$

$$x \leq \frac{c^2-1}{3}$$

$$x \in \left[0, \frac{c^2-1}{3}\right]$$

(iii) $x \leq 0 \wedge y \geq 2 \Rightarrow c^2 - 1 = -3x + y - 2$

$y = 3x + c^2 + 1$

$x \leq 0 \wedge y \geq 2 \Rightarrow 3x + c^2 + 1 \geq 2$

$3x \geq 1 - c^2$

$x \geq \frac{1-c^2}{3}$

$x \in \left[\frac{1-c^2}{3}, 0\right]$

(iv) $x \leq 0 \wedge y \leq 2 \Rightarrow c^2 - 1 = -3x - y + 2$

$y = -3x - c^2 + 3$

$x \leq 0 \wedge y \leq 2 \Rightarrow -3x - c^2 + 3 \leq 2$

$-3x \leq c^2 - 1$

$x \geq \frac{1-c^2}{3}$

$x \in \left[\frac{1-c^2}{3}, 0\right]$

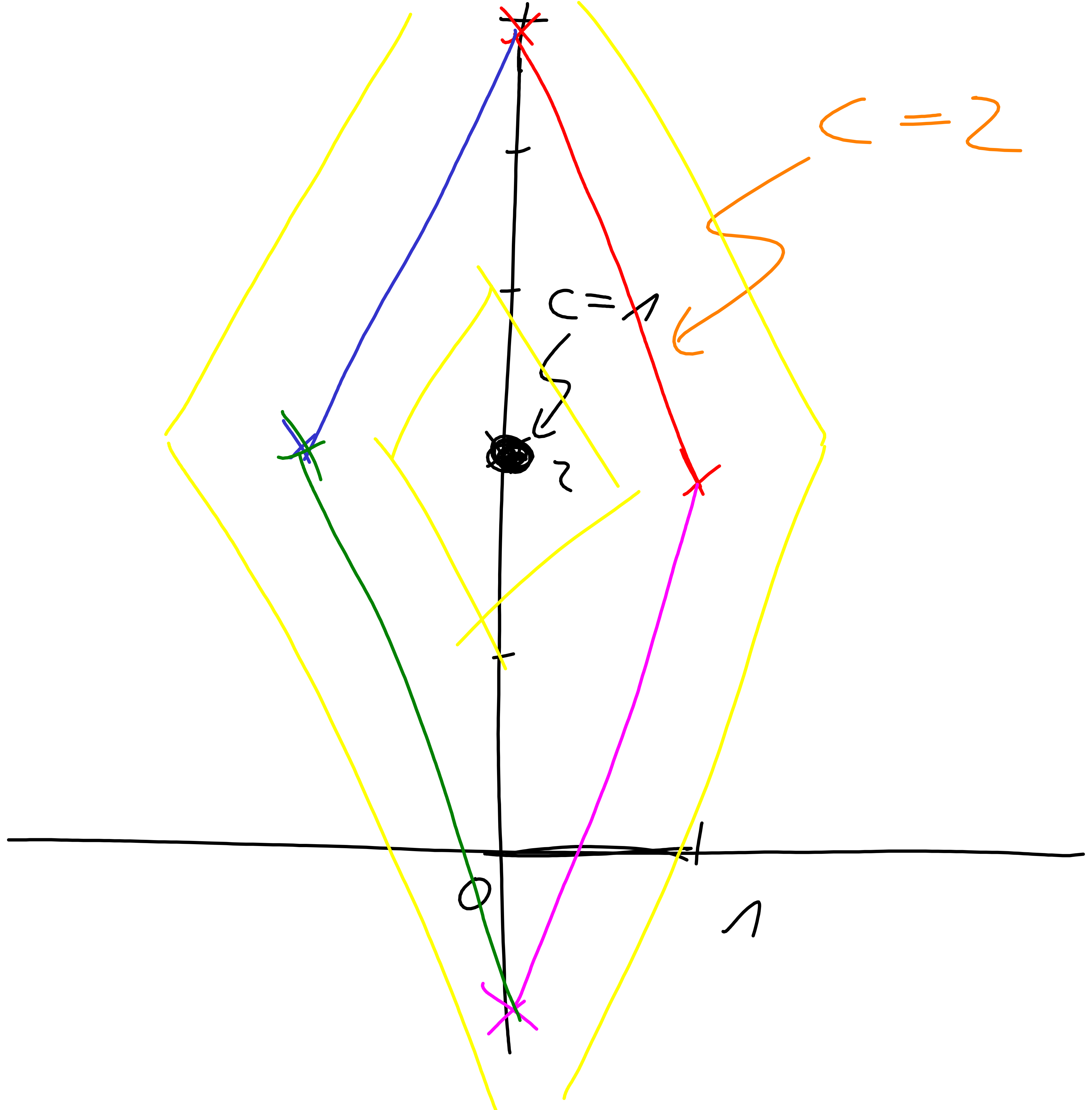
$c = 2 \Rightarrow$

(i) $y = -3x + 5, \quad x \in [0, 1]$

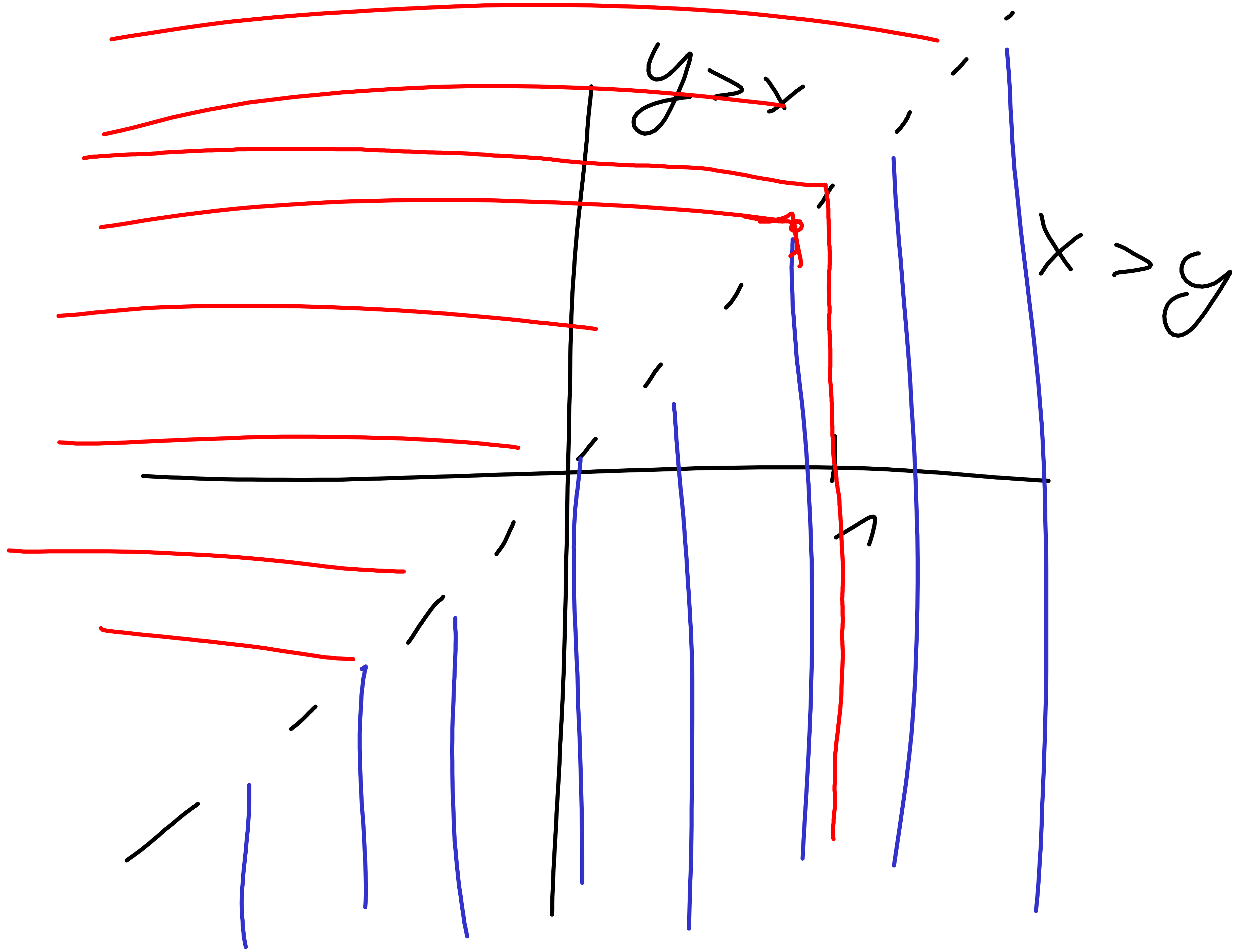
(ii) $y = 3x - 1, \quad x \in [0, 1)$

(iii) $y = 3x + 5, \quad x \in [-1, 0]$

(iv) $y = -3x - 1, \quad x \in [-1, 0)$



$$f(x,y) = \sup\{x, y\}$$



$$x = y$$

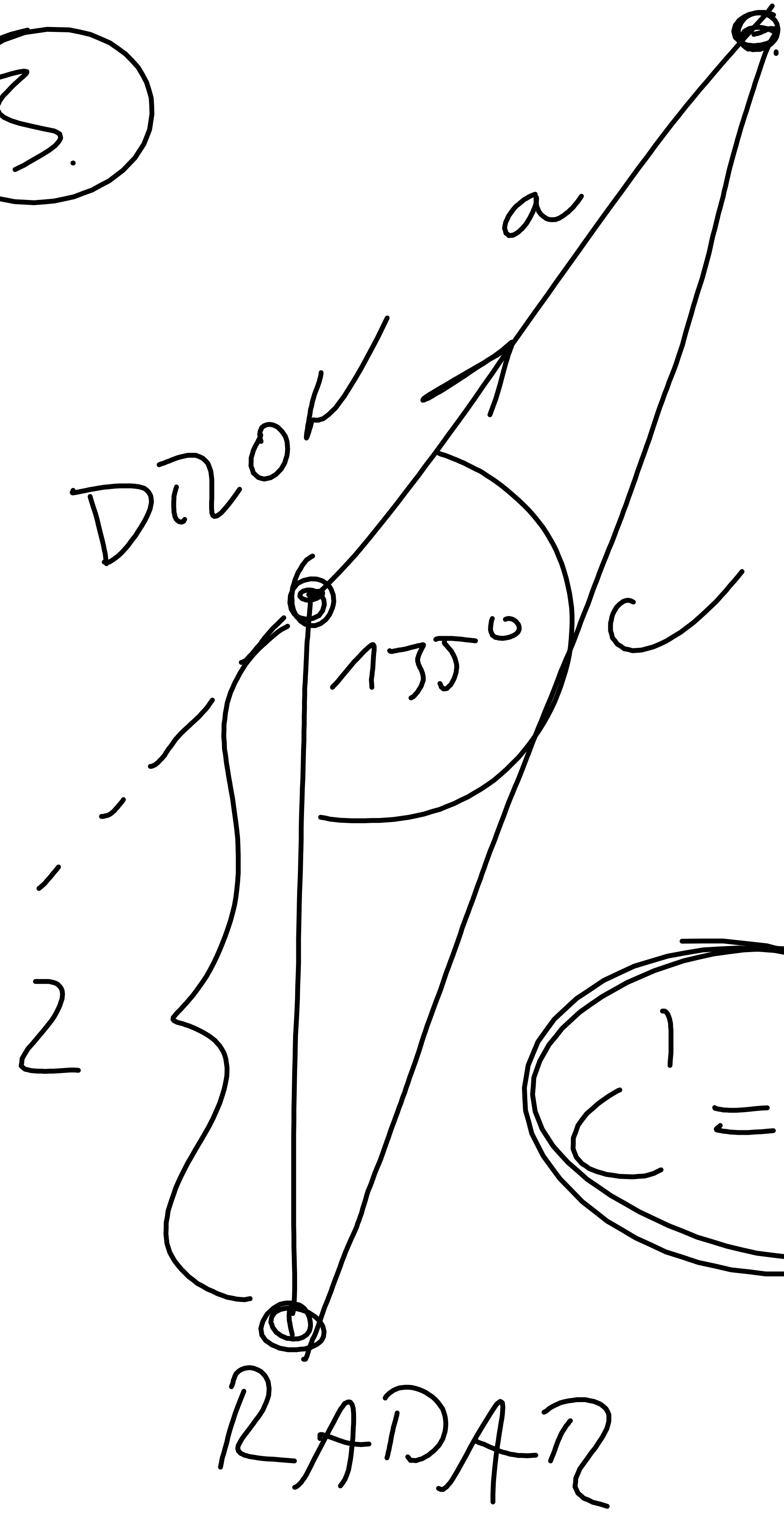
$y > x$

$f = y = c$

$x > y$

$\Leftrightarrow f(x,y) = x = c$

3.



$c' = ?$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos 135^\circ$$

$$c^2 = a^2 + 4 - 4a \cdot \left(-\frac{\sqrt{2}}{2}\right)$$

$$c^2 = a^2 + 4 + 2\sqrt{2}\,a \quad \Big/ \frac{d}{dt}$$

$c = c(t)$

$a = a(t)$

$$\cancel{2}c \cdot c' = \cancel{2} \cdot a \cdot a' + 0 + \cancel{2}\sqrt{2} \cdot a'$$

$$c' = \frac{a \cdot a' + \sqrt{2} \cdot a'}{c}$$

$a = ?$ $\quad a' = 180\sqrt{2}$, $2\,min = \frac{2}{60} = \frac{1}{30}\,h.$

$\Rightarrow a = \frac{1}{30} \cdot 180 \cdot \sqrt{2} = 6\sqrt{2}\,km$

$c = ?$ $\quad c^2 = (6\sqrt{2})^2 + 4 + 2\sqrt{2} \cdot 6\sqrt{2} = 72 + 4 + 24 =$

$= 100$

$\Rightarrow \boxed{c = 10}$

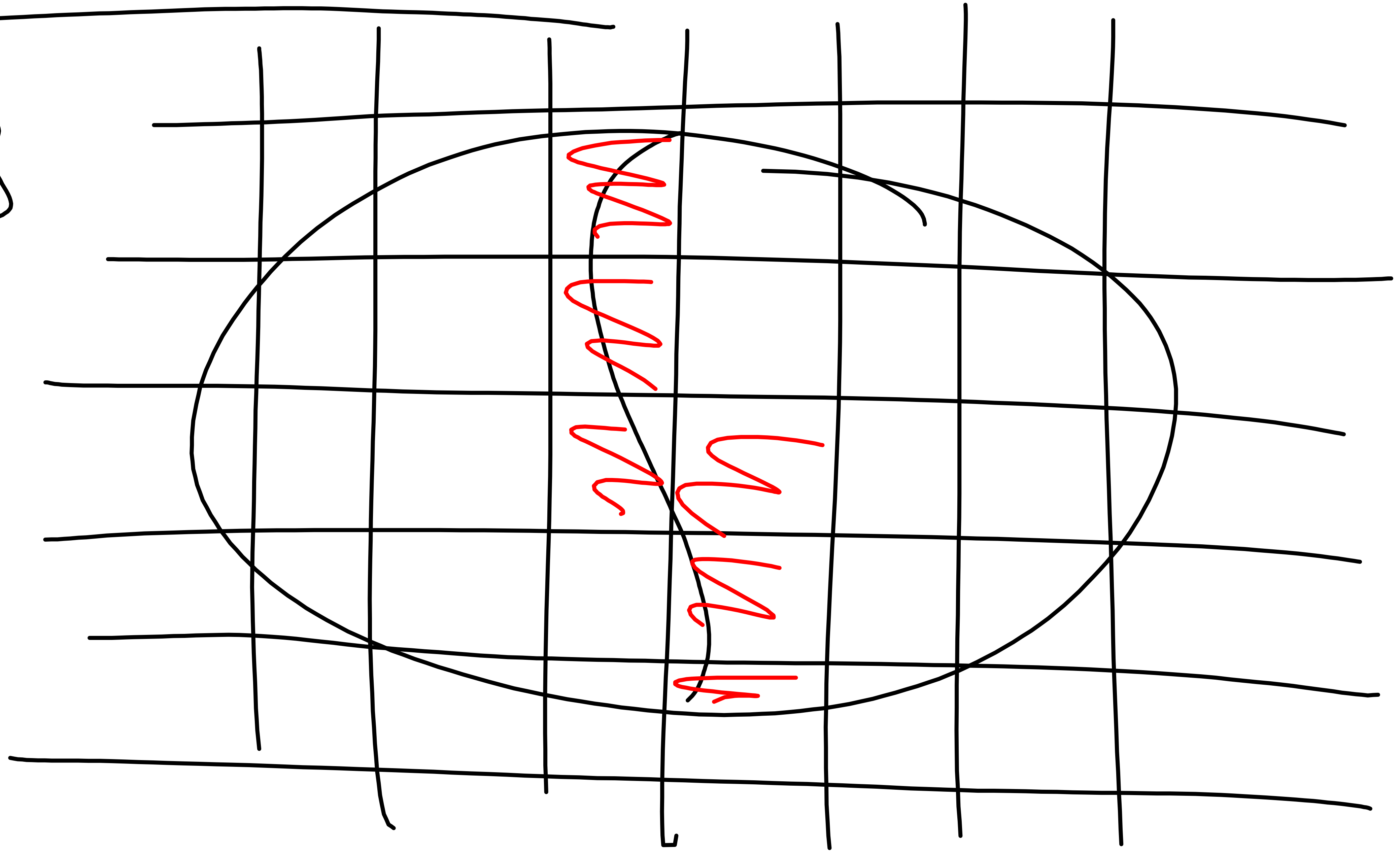
$$c' = \frac{6\sqrt{2} \cdot 18\cancel{0}\sqrt{2} + \sqrt{2} \cdot 18\cancel{0}\sqrt{2}}{1\cancel{0}} =$$

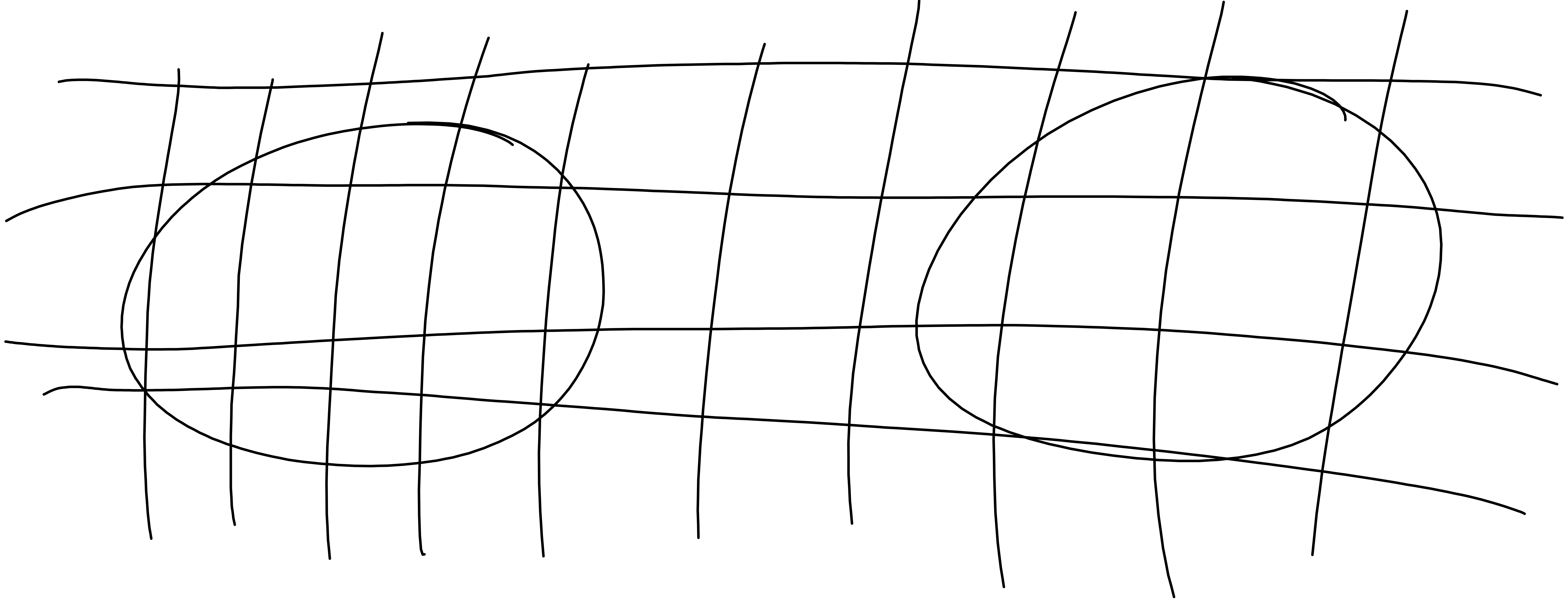
$$= 12 \cdot 18 + 2 \cdot 18 = 14 \cdot 18 = 252 \ \mathrm{km/h}$$

(5.) f ... integr.

A, B ... MĚŘ., OMEZ., DISJ.

MAJME MNOŽINY A, B VELMI BLÍZKO, VIZ OBR.
V SÍTI JE JEDINÝ PROBLÉM – ČTV., KT. SE ZAPOČÍTÁVAJÍ 2x





6

$y = x$

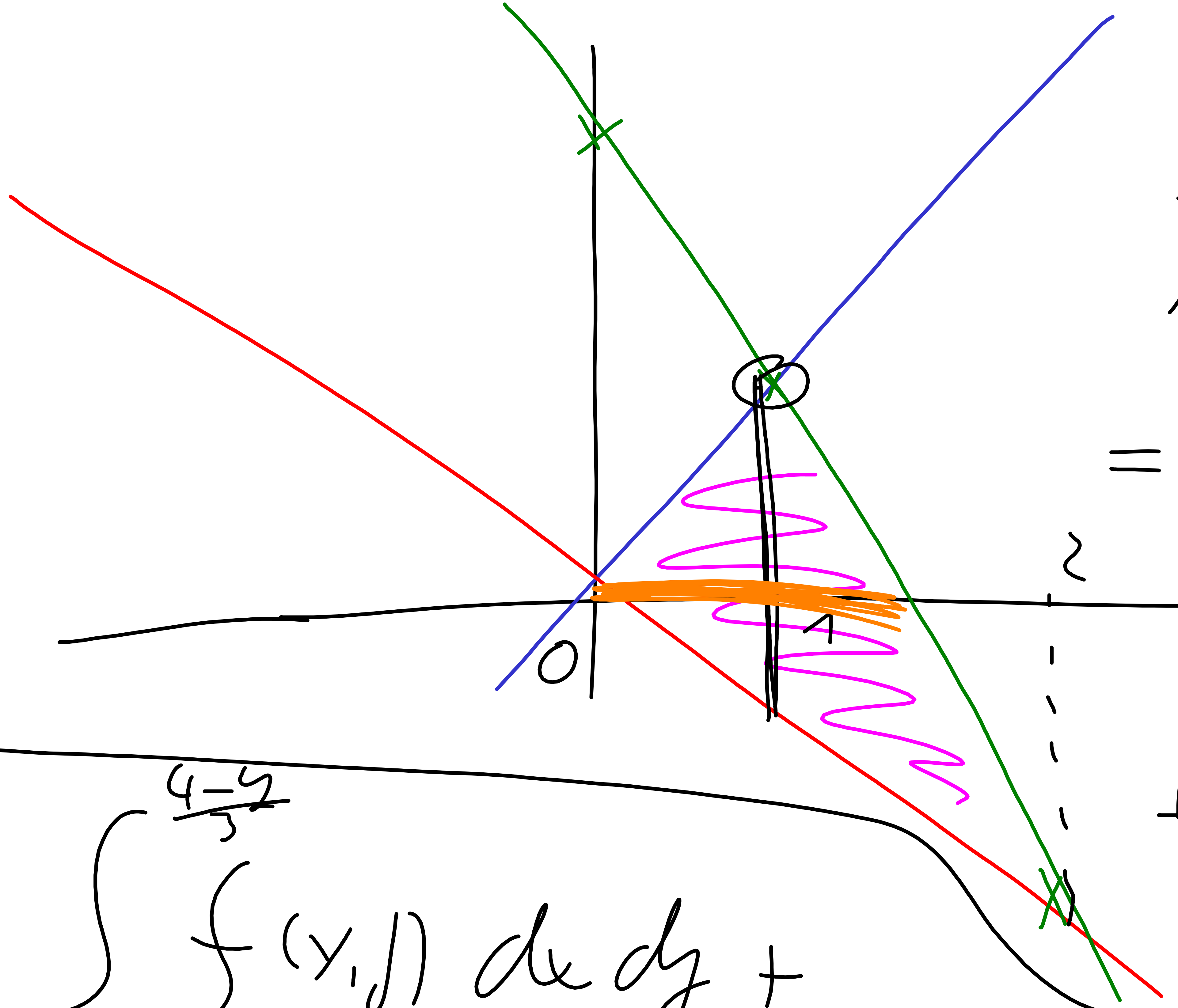
$y = -x$

$y = -3x + 4$

$x = 0 \Rightarrow y = 4$

$x = 1 \Rightarrow y = 1$

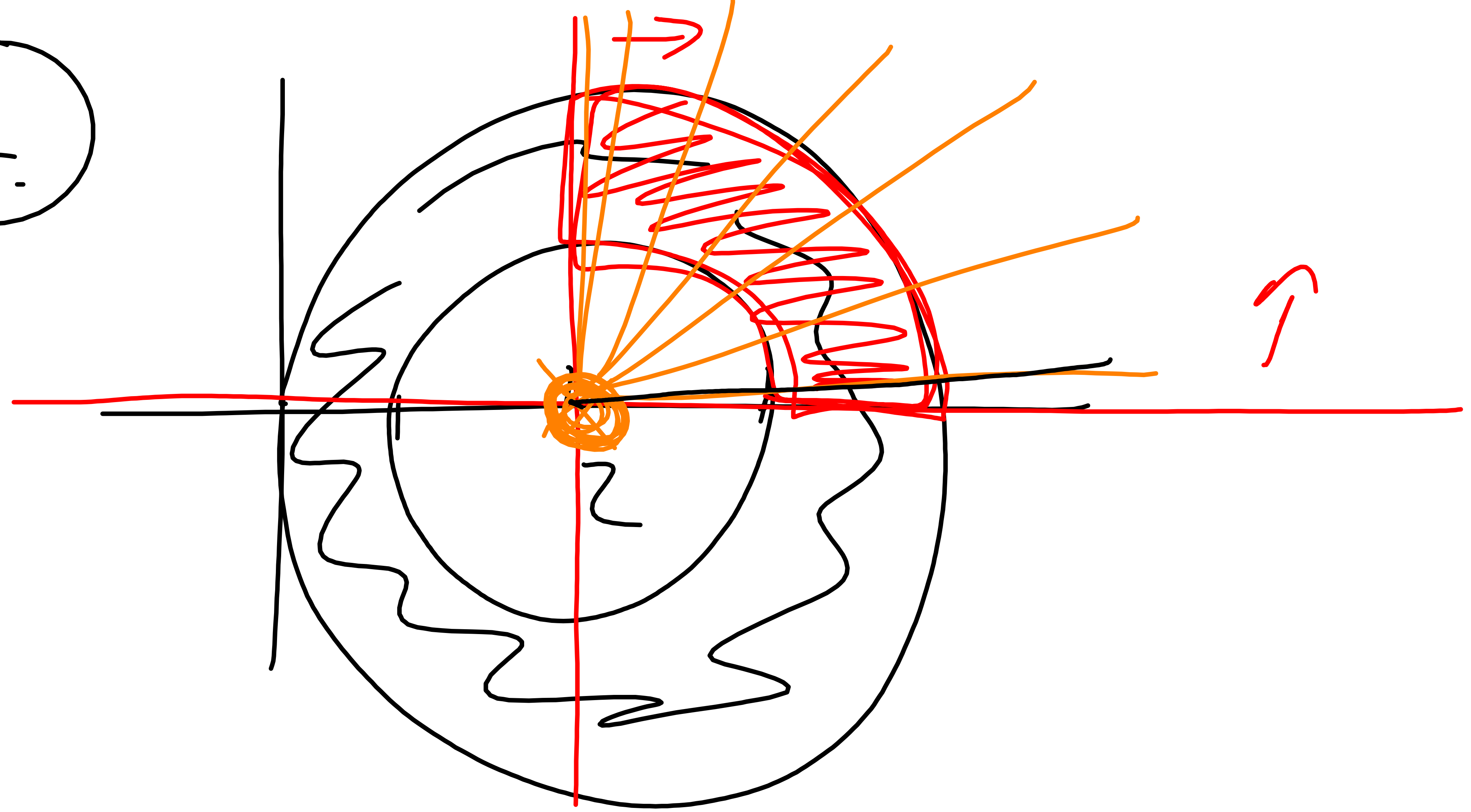
$x = 2 \Rightarrow y = -2$



$$\iint_A f(x,y)\,dx\,dy = \int_0^1 \int_{-x}^{x} f(x,y)\,dy\,dx + \int_1^2 \int_{-x}^{-3x+4} f(x,y)\,dy\,dx$$

$$= \int_{-2}^{0} \int_{-y}^{\frac{4-y}{3}} f(x,y)\,dx\,dy + \int_{0}^{1} \int_{y}^{\frac{4-y}{3}} f(x,y)\,dx\,dy$$

7.



$x = 2 + \rho \cos\varphi$

$y = \rho \cdot \sin\varphi, \quad |J| = \rho$

$\rho \in [1, 2], \quad \varphi \in [0, \frac{\pi}{2}]$

$$m = \int_1^2 \int_0^{\frac{\pi}{2}} 4\rho + 4\rho^2 \cdot \cos\varphi + \rho^3 \, d\varphi \, d\rho = \frac{28}{3} + \frac{39}{8}\pi$$

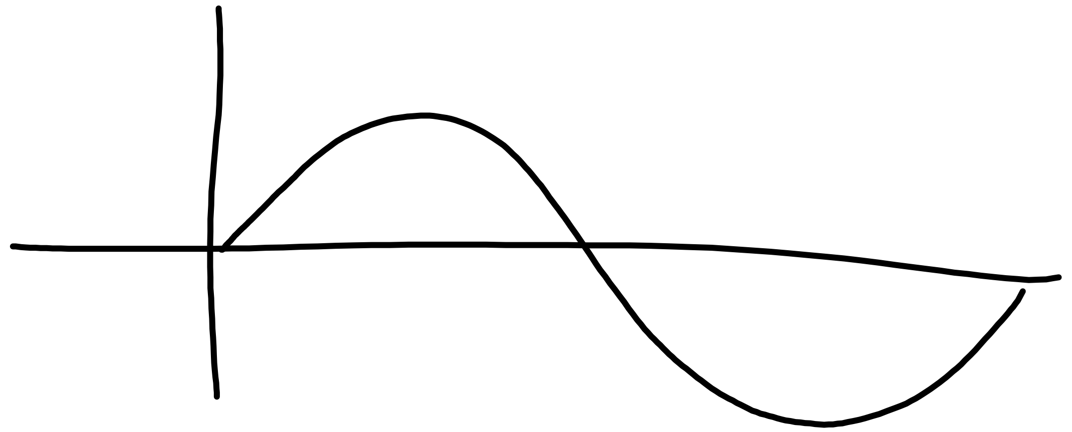
$$x^2 + y^2 = (2 + \rho \cos\varphi)^2 + (\rho \sin\varphi)^2 =$$

$$= 4 + 4 \cdot \rho \cdot \cos\varphi + \underbrace{\rho^2 \cdot \cos^2\varphi + \rho^2 \cdot \sin^2\varphi}$$

$$= 4 + 4 \cdot \rho \cdot \cos\varphi + \rho^2$$

8.

$$\int_C \frac{1}{\sqrt{2-y^2}}\, dl = \left| \begin{array}{l} x = t \\ y = \sin t \\ t \in [0, 2\pi] \end{array} \right| =$$



$$dx = 1\,dt$$

$$dy = \cos t\, dt$$

$$dl = \sqrt{1+\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \cancel{\frac{1}{\sqrt{2-\sin^2 t}}}\, \cancel{\sqrt{1+\cos^2 t}}\, dt = \underline{\underline{2\pi}}$$

$$1 + \boxed{1-\sin^2 t} = 1+\cos^2 t$$

$$④\ L(x, y, \lambda) = \cos^2 x + \cos^2 y + \lambda \cdot (4x - 4y - \pi)$$

$$\left.\begin{aligned}
L_x &= -2 \cdot \sin x \cdot \cos x + 4\lambda = 0 \\
L_y &= -2 \cdot \sin y \cdot \cos y - 4\lambda = 0 \\
L_\lambda &= 4x - 4y - \pi = 0
\end{aligned}\right\}
\boxed{\begin{aligned}
4\lambda &= \sin 2x \ \ldots (1) \\
-4\lambda &= \sin 2y \ \ldots (2) \\
4x - 4y &= \pi \ \ldots (3)
\end{aligned}}$$

$$(1)+(2) \Rightarrow \quad 0 = \sin 2x + \sin 2y$$

$$\sin 2x = -\sin 2y$$

$$\sin 2x = \sin(-2y) \Rightarrow x = -y + k\pi$$

DoS. Do (3) $\Rightarrow \quad 4\cdot(-y+k\pi) - 4y = \pi$

$$-8y = \pi - 4k\pi \Rightarrow y = \frac{-\pi}{8} + k\frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{8} - k \cdot \frac{\pi}{2} + k\pi = \frac{\pi}{8} + k\frac{\pi}{2}$$

$$\Rightarrow \text{STAC. B.} \quad \left[\frac{\pi}{8} + k\frac{\pi}{2}, -\frac{\pi}{8} + k\frac{\pi}{2}\right], \quad k \in \mathbb{Z}$$

$$\lambda = ? \quad (1) \Rightarrow \lambda = \frac{1}{4} \cdot \sin\left(\frac{\pi}{4} + k\pi\right) = \begin{cases} = \frac{1}{4} \cdot \frac{\sqrt{2}}{2} & \text{k-sudo} \\ = \frac{1}{4} \cdot \left(-\frac{\sqrt{2}}{2}\right) & \text{k-liho} \end{cases}$$

$L_{xx} = -\cos 2x \cdot 2 = -2 \cdot \cos 2x$

$L_{xy} = 0 \quad // \quad L_{yy} = -\cos 2y \cdot 2 = -2 \cdot \cos 2y$

SUDÉ $k \Rightarrow k = 2n \ (n \in \mathbb{Z})$

$\Rightarrow$

$$\Rightarrow L_{xx} = -2 \cdot \cos\left(\frac{\pi}{4} + 2n\pi\right) = -2 \cdot \frac{\sqrt{2}}{2} = -\sqrt{2}$$

$$L_{yy} = -2 \cdot \cos\left(-\frac{\pi}{4} + 2n\pi\right) = -2 \cdot \frac{\sqrt{2}}{2} = -\sqrt{2}$$

$$D^2 L = \begin{pmatrix} -\sqrt{2} & 0 \\ 0 & -\sqrt{2} \end{pmatrix}$$

JE NEG. DEF.

$\Rightarrow$ MAXIMA

$$f(\ldots) = \cos^2\left(\frac{\pi}{8} + n\pi\right) + \cos^2\left(-\frac{\pi}{8} + n\pi\right)$$

$$= 2 \cdot \left(\frac{1}{2} + \frac{\sqrt{2}}{4}\right) = 1 + \frac{\sqrt{2}}{2}$$