## 3.12 Exercises

**E 3–1**

A spectrometer operates with a Larmor frequency of 600 MHz for protons. For a particular set up the RF field strength, $\omega_1/(2\pi)$ has been determined to be 25 kHz. Suppose that the transmitter is placed at 5 ppm; what is the offset (in Hz) of a peak at 10 ppm? Compute the tilt angle, $\theta$, of a spin with this offset.

For the normal range of proton shifts (0 – 10 ppm), is this 25 kHz field strong enough to give what could be classed as hard pulses?

**E 3–2**

In an experiment to determine the pulse length an operator observed a positive signal for pulse widths of 5 and 10 $\mu$s; as the pulse was lengthened further the intensity decreased going through a null at 20.5 $\mu$s and then turning negative.

Explain what is happening in this experiment and use the data to determine the RF field strength in Hz and the length of a 90° pulse.

A further null in the signal was seen at 41.0 $\mu$s; to what do you attribute this?

**E 3–3**

Use vector diagrams to describe what happens during a spin echo sequence in which the 180° pulse is applied about the $y$ axis. Also, draw a phase evolution diagram appropriate for this pulse sequence.

In what way is the outcome different from the case where the refocusing pulse is applied about the $x$ axis?

What would the effect of applying the refocusing pulse about the $-x$ axis be?

**E 3–4**

The gyromagnetic ratio of phosphorus-31 is $1.08 \times 10^8$ rad s$^{-1}$ T$^{-1}$. This nucleus shows a wide range of shifts, covering some 700 ppm.

Estimate the minimum 90° pulse width you would need to excite peaks in this complete range to within 90% of the their theoretical maximum for a spectrometer with a $B_0$ field strength of 9.4 T.

**E 3–5**

A spectrometer operates at a Larmor frequency of 400 MHz for protons and hence 100 MHz for carbon-13. Suppose that a 90° pulse of length 10 $\mu$s is applied to the protons. Does this have a significant effect of the carbon-13 nuclei? Explain your answer carefully.

**E 3–6**

Referring to the plots of Fig. 3.26 we see that there are some offsets at which the transverse magnetization goes to zero. Recall that the magnetization is rotating about the *effective field*, $\omega_{\text{eff}}$; it follows that these nulls in the excitation come about when the magnetization executes complete 360° rotations about the effective field. In such a rotation the magnetization is returned to the $z$ axis. Make a sketch of a "grapefruit" showing this.

The effective field is given by

$$\omega_{\text{eff}} = \sqrt{\omega_1^2 + \Omega^2}.$$

Suppose that we express the offset as a multiple $\kappa$ of the RF field strength:

$$\Omega = \kappa\omega_1.$$

Show that with this values of $\Omega$ the effective field is given by:

$$\omega_{\text{eff}} = \omega_1\sqrt{1+\kappa^2}.$$

(The reason for doing this is to reduce the number of variables.)

Let us assume that on-resonance the pulse flip angle is $\pi/2$, so the duration of the pulse, $\tau_{\text{p}}$, is give from

$$\omega_1\tau_{\text{p}} = \pi/2 \quad \text{thus} \quad \tau_{\text{p}} = \frac{\pi}{2\omega_1}.$$

The angle of rotation about the effective field for a pulse of duration $\tau_{\text{p}}$ is $(\omega_{\text{eff}}\tau_{\text{p}})$. Show that for the effective field given above this angle, $\beta_{\text{eff}}$ is given by

$$\beta_{\text{eff}} = \frac{\pi}{2}\sqrt{1+\kappa^2}.$$

The null in the excitation will occur when $\beta_{\text{eff}}$ is $2\pi$ i.e. a complete rotation. Show that this occurs when $\kappa = \sqrt{15}$ i.e. when $(\Omega/\omega_1) = \sqrt{15}$. Does this agree with Fig. 3.26?

Predict other values of $\kappa$ at which there will be nulls in the excitation.

**E 3–7**

When calibrating a pulse by looking for the null produced by a 180° rotation, why is it important to choose a line which is close to the transmitter frequency (i.e. one with a small offset)?

**E 3–8**

Use vector diagrams to predict the outcome of the sequence:

$$90° - \text{delay } \tau - 90°$$

applied to equilibrium magnetization; both pulses are about the $x$ axis. In your answer, explain how the $x$, $y$ and $z$ magnetizations depend on the delay $\tau$ and the offset $\Omega$.

**E 3–9**

Consider the spin echo sequence to which a 90° pulse has been added at the end:

$$90°(x) - \text{delay } \tau - 180°(x) - \text{delay } \tau - 90°(\phi).$$

The axis about which the pulse is applied is given in brackets after the flip angle. Explain in what way the outcome is different depending on whether the phase $\phi$ of the pulse is chosen to be $x$, $y$, $-x$ or $-y$.

**E 3–10**

The so-called "1–$\bar{1}$" sequence is:

$$90°(x) - \text{delay } \tau - 90°(-x)$$

For a peak which is on resonance the sequence does not excite any observable magnetization. However, for a peak with an offset such that $\Omega\tau = \pi/2$ the sequence results in all of the equilibrium magnetization appearing along the $x$ axis. Further, if the delay is such that $\Omega\tau = \pi$ no transverse magnetization is excited.

Explain these observations and make a sketch graph of the amount of transverse magnetization generated as a function of the offset for a fixed delay $\tau$.

The sequence has been used for suppressing strong solvent signals which might otherwise overwhelm the spectrum. The solvent is placed on resonance, and so is not excited; $\tau$ is chosen so that the peaks of interest are excited. How does one go about choosing the value for $\tau$?

**E 3–11**

The so-called "1–1" sequence is:

$$90°(x) - \text{delay } \tau - 90°(y).$$

Describe the excitation that this sequence produces as a function of offset. How it could be used for observing spectra in the presence of strong solvent signals?