Exercise: The Gravity Model

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1. General Information

The location of economic activities in generates a transport demand that is represented geographically by a set of origin-destination pairs. Gravity-type spatial interactions models offer a methodology to appraise the transport demand between a set of locations. For a transport company, knowing this information is very useful to the processes of scheduling vehicles along routes. This exercise involve the **application of a gravity spatial interaction model** for scheduling the activities of a fictive airline company.

A large airline company (Air Modal inc.) hired a consultant in order to help them reorganize their international links and to develop new service strategies. Indeed, the company wants to know the potential demand of air transport between and inside continents. Knowing this demand, it will be able to assign its flights more efficiently and see where growth perspectives are the most attractive. The following information is available to apply the spatial interaction model

Emission and Attraction Data			
	Urban population (2000)	**Lambda**	**Alpha**
Europe	545,000,000	1.00	1.00
North America	239,000,000	1.08	1.08
Oceania	21,000,000	1.07	1.07
Latin America	391,000,000	1.01	1.01
Africa	295,000,000	0.91	0.91
Asia	1,352,000,000	0.99	0.99

Both the lambda and alpha indexes have the same value, since it is assumed that on a yearly basis all the passengers are going back to their place of origin. The reality is more complex as some passengers may do multiple destination trips (e.g. North America to Europe to Asia and then back to North America), while others are doing a one-way flight only (immigration). The values of lambda and alpha are higher for Latin America than for Europe, which reflects better land based transport alternatives for Europe (mainly train) as well as a more compact location of the population.

Average Distance Between Destinations						
Distance (in km)	Europe	North America	Oceania	Latin America	Africa	Asia
Europe	1,000	8,000	15,000	9,000	4,000	12,000
North America	8,000	2,000	14,000	8,000	11,000	12,000
Oceania	15,000	14,000	2,000	14,000	14,000	13,000
Latin America	9,000	8,000	14,000	3,000	7,000	17,000
Africa	4,000	11,000	14,000	7,000	3,000	10,000
Asia	12,000	12,000	13,000	17,000	10,000	3,000

The value of the k coefficient is **0.00001** (yearly value) and the friction of space (beta) coefficient is **1.34** between all origins and destinations.

2. Results

Results must be presented as a report to the board of directors of Air Modal inc. This report must include the following:

- **Calculation of spatial interactions**. By using the available data (urban population, lambda, alpha and k indexes), appraise the origin-destination matrix of air transport passengers within and between continents.
- **Mapping**. Put on a map the 10 most important origin-destination pairs of this matrix as proportional symbols. A basemap is available here.
- **Allocation of flights**. Knowing that a typical Air Modal flight contains 280 people, how many flights would be necessary (considering that Air Modal services 6% of the market) to fill the demand for one day between all the O-D pairs (matrix of required flights)?
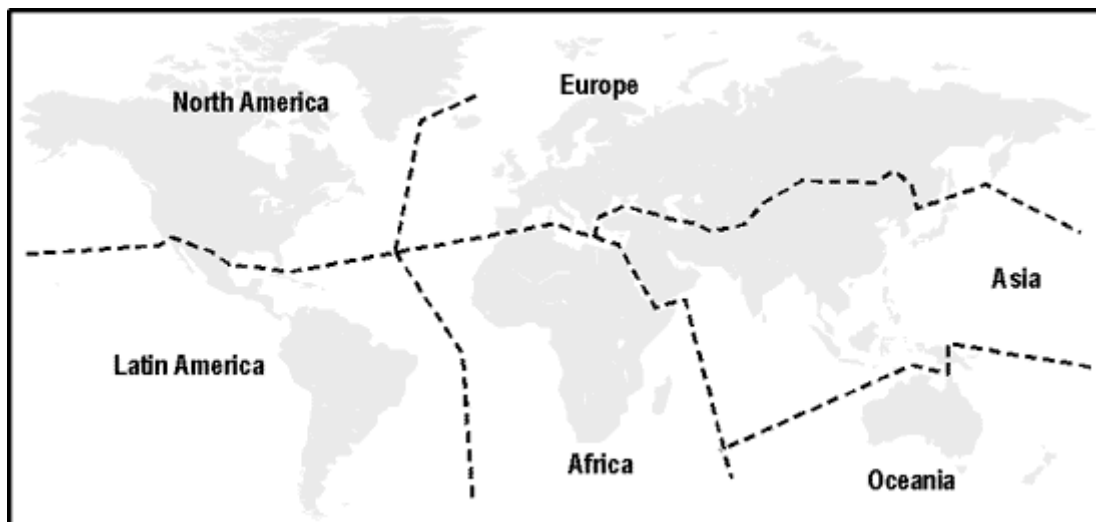
Gravity Model Exercise Basemap

THE GRAVITY MODEL

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1. Formulation

The gravity model offers a good application of the spatial interaction method. It is named as such because it uses a similar formulation than Newton's gravity model. Accordingly, the attraction between two objects is **proportional to their mass and inversely proportional to their respective distance**. Consequently, the general formulation of spatial interactions can be adapted to reflect this basic assumption to form the **elementary formulation** of the gravity model:

$$T_{ij} = k \frac{P_i P_j}{d_{ij}}$$

- P_i and P_j : Importance of the location of origin and the location of destination.
- d_{ij} : Distance between the location of origin and then location of destination.
- k is a proportionality constant. Related to the rate of the event. For instance, if the same system of spatial interactions is considered, the value of k will be higher if interactions were considered for a year comparatively to the value of k for one week.

Thus, spatial interactions between locations i and j are proportional to their respective importance divided by their distance.

2. Extension

The gravity model can be extended to include several parameters:

$$T_{ij} = k \frac{P_i^{\lambda} P_j^{\alpha}}{d_{ij}^{\beta}}$$

- P, d and k refers to the same variable previously discussed.
- β (beta) : A parameter of transport friction related to the efficiency of the transport system between two locations. This friction is rarely linear as the further the movement the greater the friction of distance. For instance, a highway between two locations will have a weaker beta index than a road.
- λ (lambda) : Potential to generate movements (emissiveness). For movements of people, lambda is often related to an overall level of welfare. For instance, it is logical to infer that for retailing flows, a location having higher income levels will generate more movements.
- α (alpha) : Potential to attract movements (attractiveness). Related to the nature of economic activities at the destination. For instance, a center having important commercial activities will attract more movements.

3. Calibration

A significant challenge related to the usage of spatial interaction models, notably the gravity model, is related to their calibration. Calibration consists in finding the value of each

parameters of the model (constant and exponents) to insure that the estimated results are similar to the observed flows. If it is not the case, the model is almost useless as it predicts or explains little. It is impossible to know if the process of calibration is accurate without **comparing estimated results with empirical evidence**.

In the two formulations of the gravity model that has been introduced, the simple formulation offers a good flexibility for calibration since four parameters can be modified. Altering the value of beta, alpha and lambda will influence the estimated spatial interactions. Furthermore, the value of the parameters can change in time due to factors such as technological innovations and economic development. For instance, improvements in transport efficiency generally have the consequence of reducing the value of the beta exponent (friction of distance). Economic development is likely to influence the values of alpha and lambda, reflecting a growth in the mobility.

Often, a value of 1 is given to the parameters, and then they are progressively altered until the estimated results are similar to observed results. Calibration can also be considered for different O/D matrices according to age, income, gender, type of merchandise and modal choice. A great part of the scientific research in transport and regional planning aims at finding accurate parameters for spatial interaction models. This is generally a costly and time consuming process, but a very useful one. Once a spatial interaction model has been validated for a city or a region, it can then be used for simulation and prediction purposes, such as how many additional flows would be generated if the population increased or if better transport infrastructures (lower friction of distance) were provided.