Remark. If (2.1.5) is replaced by

$$[I(f_0) < \infty, \sum_{i=1}^{N} (d_i - \bar{d})^2 \to 0],$$

then L_ν converges in P_ν-probability to 1, i.e. the distribution of $\log L_\nu$ converges to the degenerate normal distribution (0, 0). This could be proved as follows: First, we note that T_d given by (2.1.13) satisfies $\mathsf{E}T_d^2 \to 0$, and that considerations employed in the proofs of Lemmas 2.1.a and 2.1.b yield the relation $\mathsf{E}(W_d - T_d)^2 \to 0$. Thus $\mathsf{E}W_d^2 \to 0$, and, consequently, the distribution of W_d converges to the degenerate normal distribution (0, 0). Thus it remains to show that LeCam's second lemma extends to this case, too. However, the degenerate convergence of the distribution of W_d entails that (1.3.10), (1.3.11) and (1.3.12) hold with $\sigma^2 = 0$ (Loève (1955), p. 317). The rest of the proof needs no change, and we obtain that $(\log L_\nu - W_\nu) \to 0$ in probability, and hence $\log L_\nu \to 0$ in probability. Consequently the Theorem remains valid even for $b = 0$ and $I(f_0) < \infty$, and every statistic has the same limiting distribution, if any, under q_d as under p_d. Furthermore, the theorem remains valid even if we replace (2.1.5) by $I(f_0) < \infty$ and

(15) $$\sum_{i=1}^{N} (d_i - \bar{d})^2 \leqq b^2 < \infty .$$

For, if it were not valid, there would exist a sequence $\{d_\nu\}$ satisfying (15) and such that the theorem would not be valid for any of its subsequences. However, since we can draw a subsequence $\{d_j\} \subset \{d_\nu\}$ such that

$$\sum_{i=1}^{N} (d_{ji} - \bar{d}_j)^2 \to b_1^2 \leqq b^2 ,$$

this would contradict the theorem if $b_1^2 > 0$, and the above extension of the theorem, if $b_1^2 = 0$.

The results of the present section and related results were adapted and generalized from Hájek (1962).

2.5. Rank statistics for H_1. Consider

(1) $$q_{N\Delta} = \prod_{i=1}^{N} f_0(x_i - \Delta) ,$$

where f_0 is a known density *symmetric about zero*, and Δ satisfies

(2) $$N\Delta^2 \to b^2 , \quad 0 < b^2 < \infty .$$