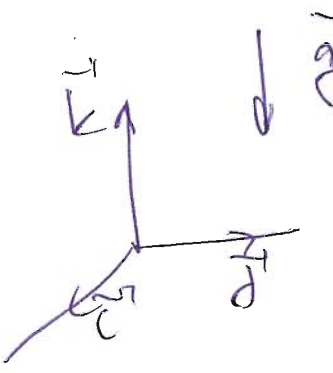
DÚ:

$\vec{a} = (0, 0, -g)$ $m s^{-2}$

$\vec{g} \cdot \vec{i}$ | $\vec{g} \cdot \vec{j}$ | $\vec{g}\vec{k} = g \cdot 1 \cdot \cos 180°$



$\vec{i} \times \vec{j} = \vec{k}$ pravá

řešení

$0 = \ddot{x}(t) \Rightarrow ? \quad \dot{x}(t) = C_1 \Rightarrow ? \quad x(t) = C_1 t + B_1$ ($m s^{-1}$, $x(0) = B_1$, m)

$0 = \ddot{y}(t) \Rightarrow ? \quad \dot{y}(t) = C_2 \Rightarrow y(t) = C_2 t + B_2$

$-g = \ddot{z}(t) \Rightarrow ? \quad \dot{z}(t) = -gt + C_3$

$\dot{z}(0) = -g \cdot 0 + C_3$

z

$$z(t) = -\frac{1}{2} g t^2 + C_3 t + B_3$$

$\vec{r}_0 = \vec{r}(t=0) = \vec{r}(0) = (B_1, B_2, B_3)$ m

$\vec{v}_0 = \vec{v}(t=0) = \vec{v}(0) = (C_1, C_2, C_3)$ m/s

$\vec{r}(t) = (x(t), y(t), z(t)) = (C_1 t + B_1, C_2 t + B_2, -\frac{1}{2} g t^2 + C_3 t + B_3)$

$\vec{v}(t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t)) = (C_1, C_2, -gt + C_3)$

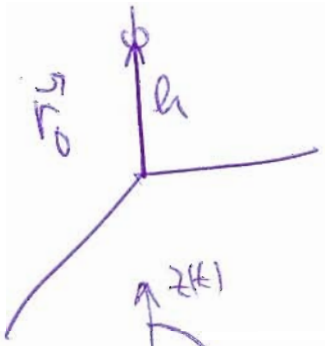
VRHY

$\{c_1, c_2, c_3\}$

$\vec{v}_0 = (0, 0, 0)$

$\vec{r}_0 = (0, 0, h)$

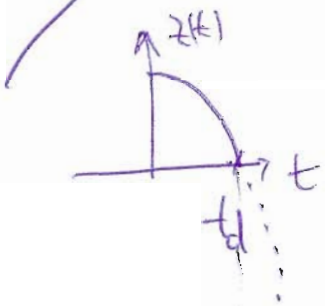
(B_1, B_2, B_3)



① Volný pád

$$\vec{r}(t) = \left(0, 0, -\frac{1}{2}gt^2 + h\right)$$

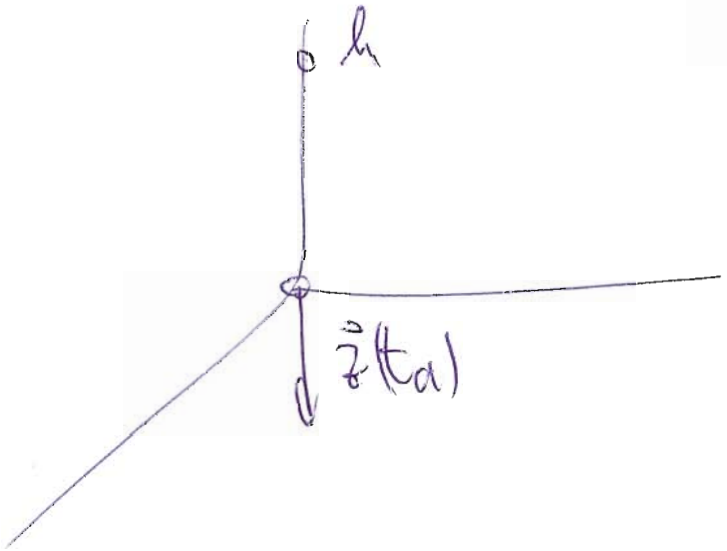
$$\vec{v}(t) = (0, 0, -gt)$$

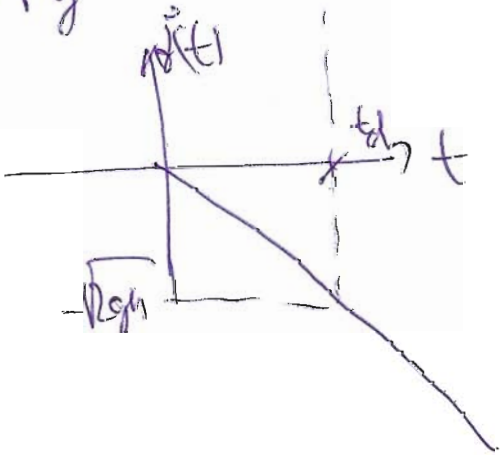


② ~~Vodorovný vrh~~

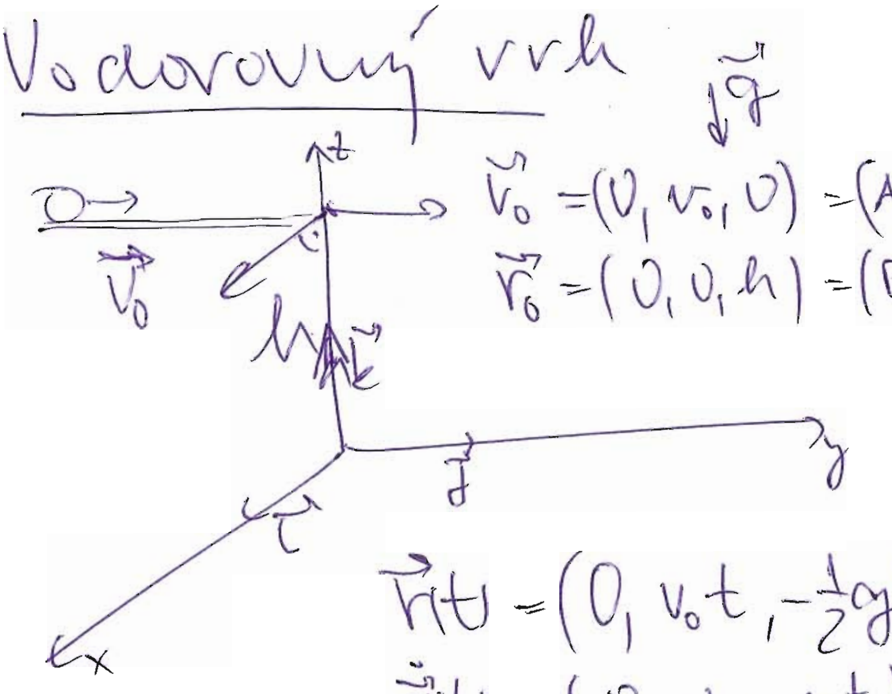
$$z(t_d) = 0 = -\frac{1}{2}gt_d^2 + h \Rightarrow t_d = \sqrt{\frac{2h}{g}} \quad \sqrt{\frac{m}{m s^{-2}}} = s$$

$$\dot{z}(t_d) = -gt_d = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh} \quad \sqrt{m s^{-2} m} = \sqrt{m^2 s^{-2}} = \frac{m}{s}$$





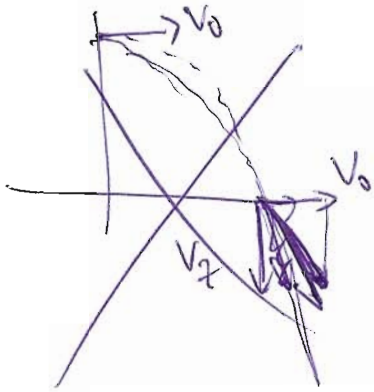
Vodorovný vrh

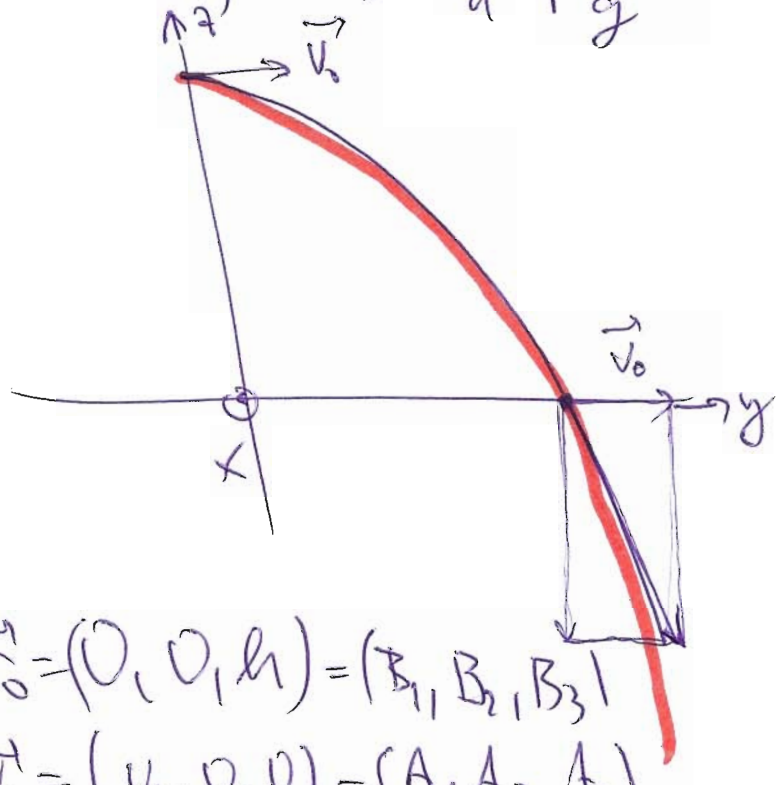


$$\vec{v}_0 = (0, v_0, 0) = (A_1, A_2, A_3)$$
$$\vec{r}_0 = (0, 0, h) = (B_1, B_2, B_3)$$

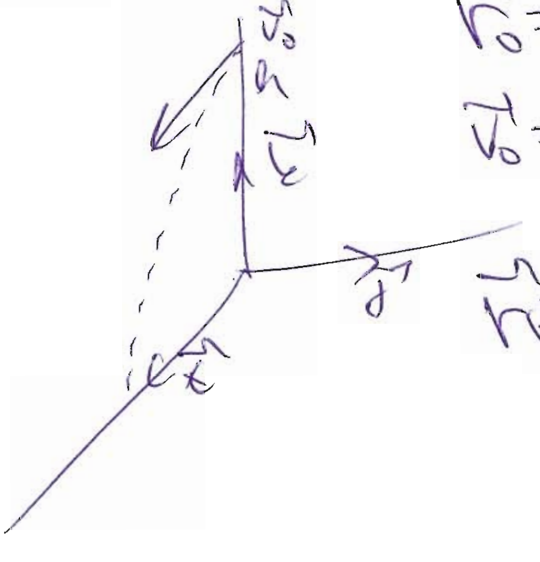
$$\vec{r}(t) = \left(0, v_0 t, -\tfrac{1}{2} g t^2 + h\right)$$
$$\vec{v}(t) = (0, v_0, -gt)$$

doba dopadu t_d : $z(t_d) = 0 \Rightarrow t_d = \sqrt{\dfrac{2h}{g}}$





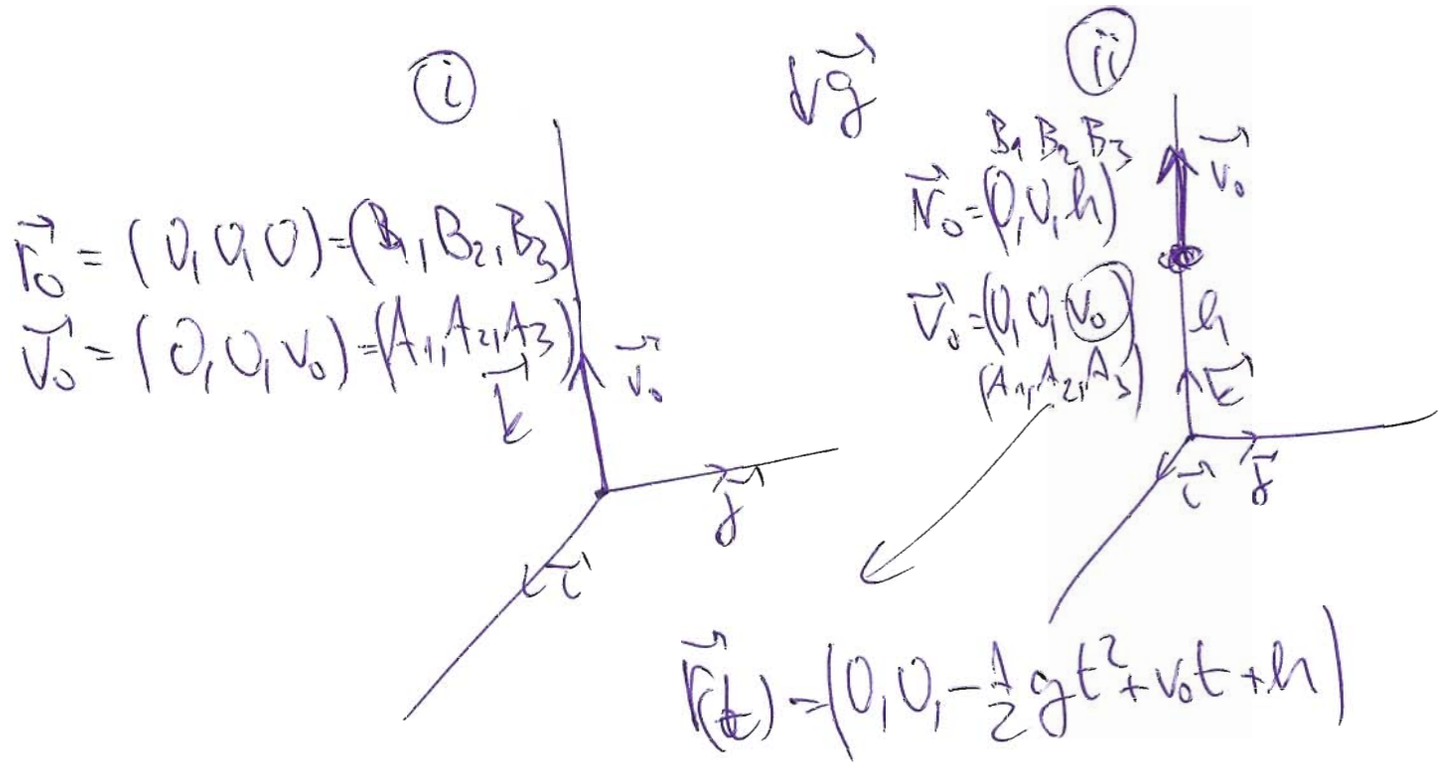
Modifikace



$$\vec{r}_0 = (0, 0, h) = (B_1, B_2, B_3)$$
$$\vec{v}_0 = (v_0, 0, 0) = (A_1, A_2, A_3)$$

$$\vec{r}(t) = \left(v_0 t, 0, -\tfrac{1}{2} g t^2 + h\right)$$

Svislý vrh



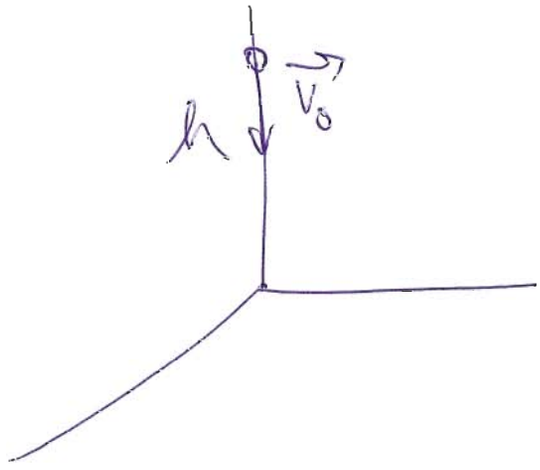
$$\vec{r}(t) = \left(0, 0, -\frac{1}{2}gt^2 + v_0 t + h\right)$$

Tj.

$$\vec{r}(t) = \left(A_1 t + B_1,\ A_2 t + B_2,\ -\frac{1}{2}gt^2 + A_3 t + B_3\right)$$

0 0 0 0 v_0 h

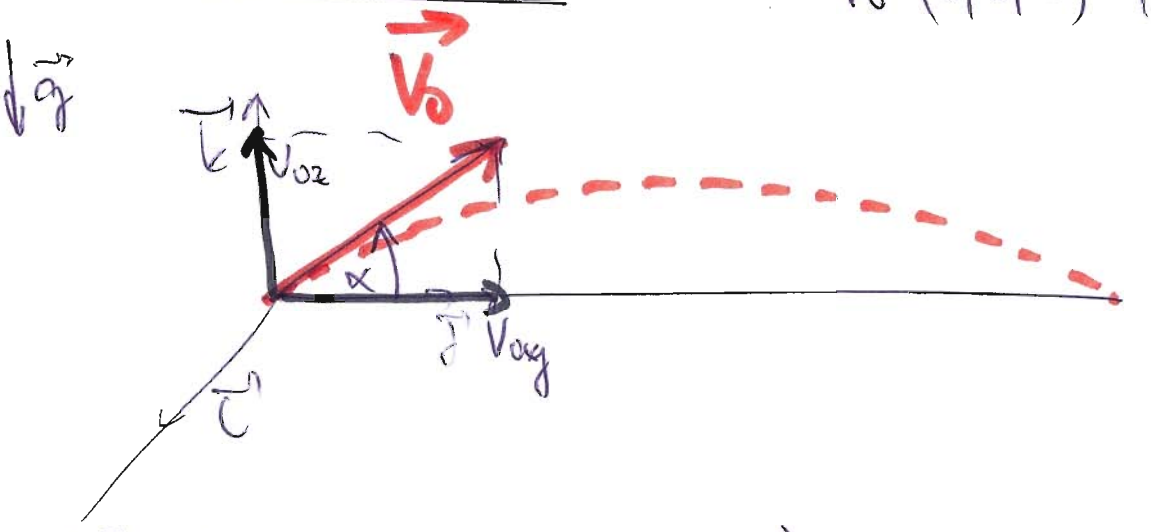
iii)



SIKMÝ VRH

⑤/9

$\vec{r}_0 = (0,0,0) = (B_1, B_2, B_3)$



$\vec{v}_0 = (0, v_0 \cos\alpha, v_0 \sin\alpha)$

(A_1, A_2, A_3)

$\vec{r}(t) = \left(0, v_0 t \cos\alpha, -\frac{1}{2} g t^2 + v_0 t \sin\alpha\right)$

$\vec{v}(t) = (0, v_0 \cos\alpha, -gt + v_0 \sin\alpha)$

$t_d = ?$ — dĺžka výstupu $t_v = ?$

$z_{max} \Rightarrow z(t_v) = 0$ — extrém fce: $z(t)$

$-g t_v + v_0 \sin\alpha = 0$

$t_v = \frac{v_0 \sin\alpha}{g}$

$2t_v$

$z(t_d) = 0 \qquad -\frac{1}{2} g t_d^2 + v_0 t_d \sin\alpha = 0$

Dĺžka $R = y(t_d)$

$g t_d^2 = 2 v_0 t_d \sin\alpha \quad / : t_d$

$t_d = \frac{2 v_0 \sin\alpha}{g} = 2 t_v$

dolet R

$$y(t_d) = v_0 t_d \cos\alpha = v_0 \frac{2 v_0 \sin\alpha}{g} \cdot \cos\alpha =$$

$$= \frac{v_0^2}{g} 2 \sin\alpha \cos\alpha = \frac{v_0^2}{g} \sin 2\alpha$$

$R(\alpha)$ Max. dolet?

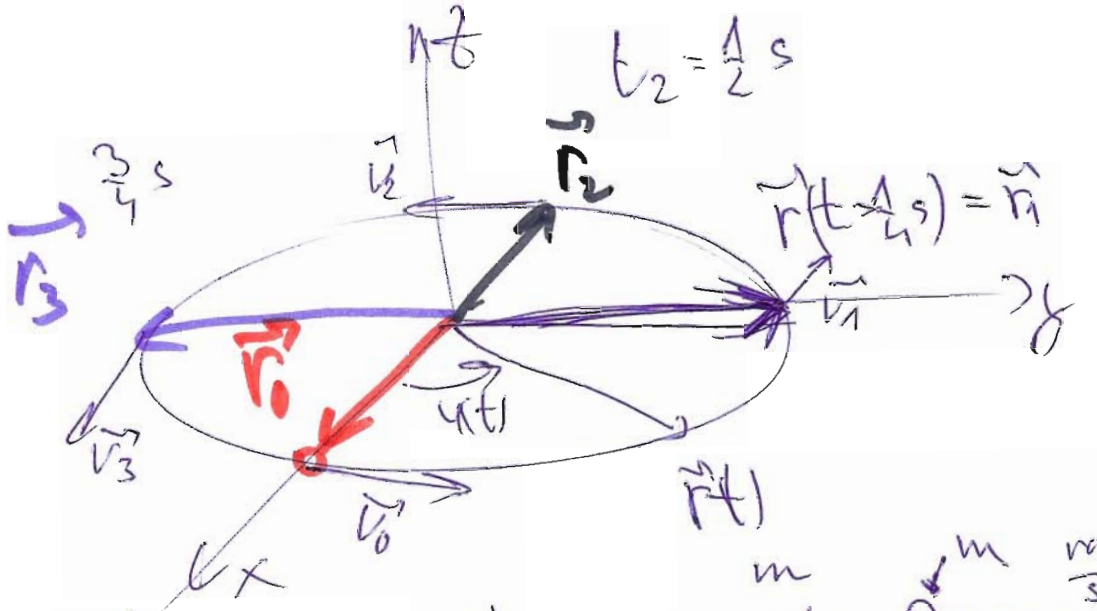
$$\left. \frac{dR(\alpha)}{d\alpha} \right|_{\alpha_0} = 0$$

$$\frac{v_0^2}{g} 2 \cdot \cos 2\alpha_0 = 0$$

$$2\alpha_0 = \pi/2 \Rightarrow \alpha_0 = 45^\circ$$

polomer zakrivenosti trajektorie

Pohyb po kružnici (jednoduchý)

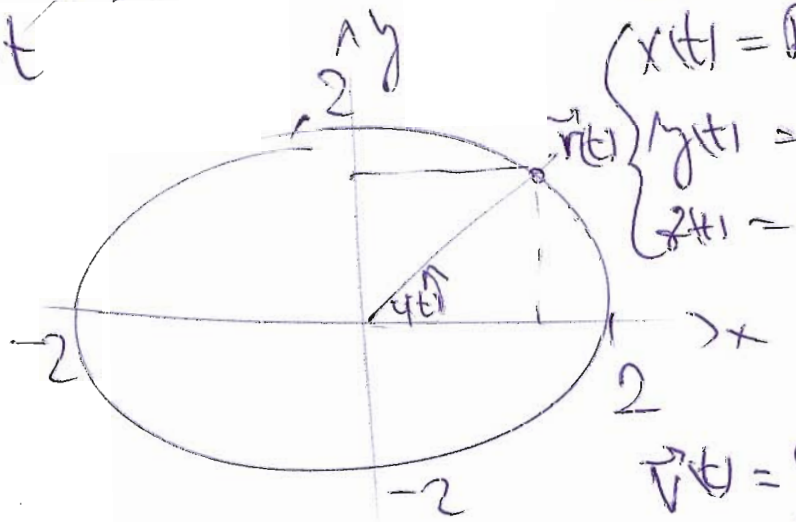


$\varphi(t) = \omega t$

$\omega = 2\pi$ rad/s

$\vec{r}(t)$ [m]

$$\begin{cases} x(t) = R\cos\omega t \\ y(t) = R\sin\omega t \\ z(t) = 0 \end{cases}$$



$$\vec{v}(t) = \frac{d\vec{r}}{dt} = (\dot{x}(t), \dot{y}(t), \dot{z}(t))$$

$$x^2 + y^2 = R^2$$

$$\vec{v}(t) \begin{cases} \dot{x}(t) = R(-\sin\omega t)\,\omega = -R\omega\sin\omega t \\ \dot{y}(t) = R\omega\cos\omega t \\ \dot{z}(t) = 0 \end{cases}$$

$\omega = \frac{2\pi}{T}$

$$\vec{r}_0 = \vec{r}(0) =$$

$$(R\cos\omega 0, R\sin\omega 0, 0) = (R\cos 0, R\sin 0, 0) = (R, 0, 0)$$

$$\vec{r}(t_1 = \tfrac{1}{4}) = (R\cos 2\pi\tfrac{1}{4}; R\sin 2\pi\tfrac{1}{4}; 0) = (0, R, 0)$$

$$\vec{v}_0 = \vec{v}(t=0) = (0, R\omega, 0) \qquad v_0 = R\omega$$

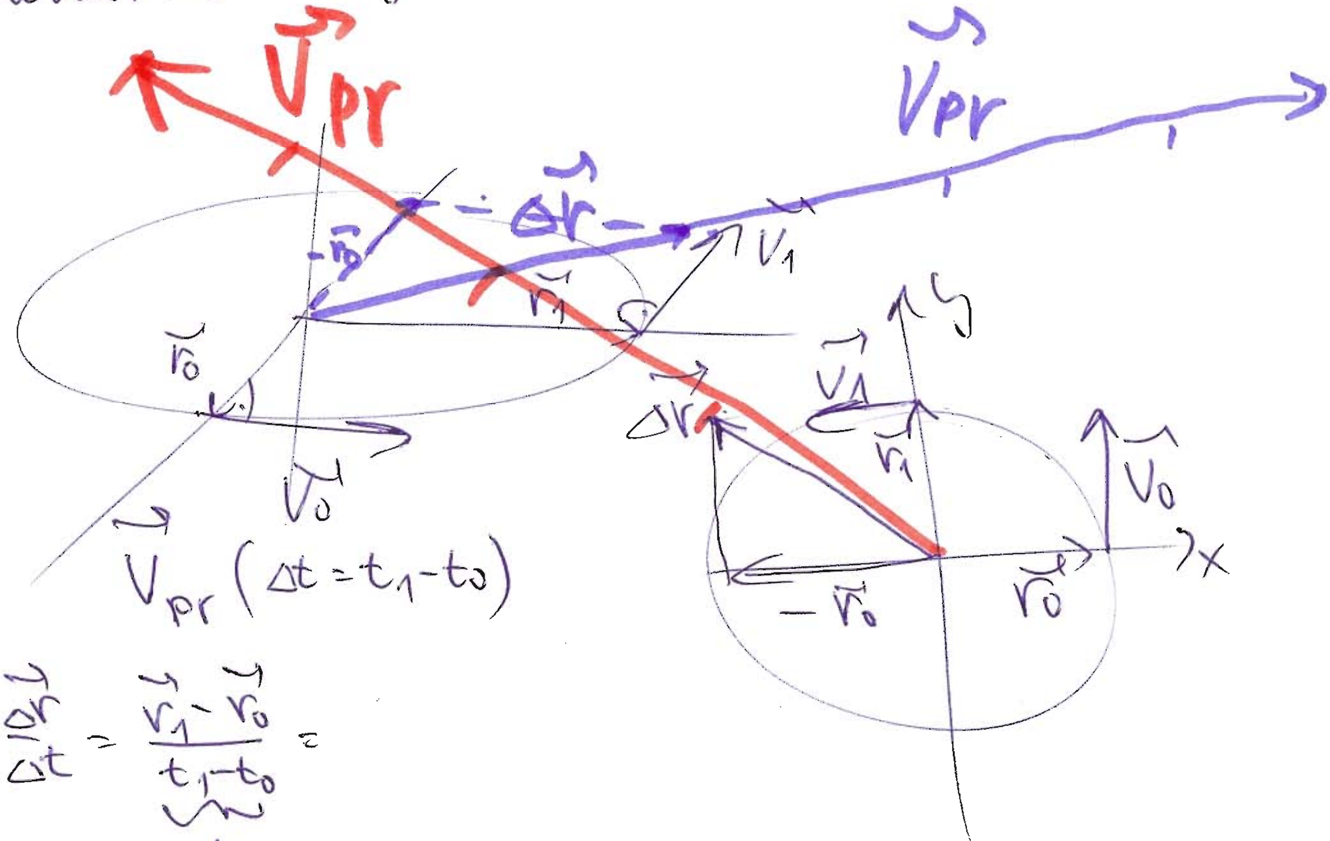
$\vec{v}_1 = \vec{v}(t_1 = 1/4\,s) =$

$\vec{v}(t) = (-R\omega \sin 2\pi t;\ R\omega \cos 2\pi t,\ 0)$

$\uparrow 1/4 \qquad \uparrow 1/4$

$= R\omega\left(-\sin\frac{2\pi}{4},\ \cos\frac{2\pi}{4},\ 0\right) = R\omega(-1, 0, 0)$

průměrná rychlost

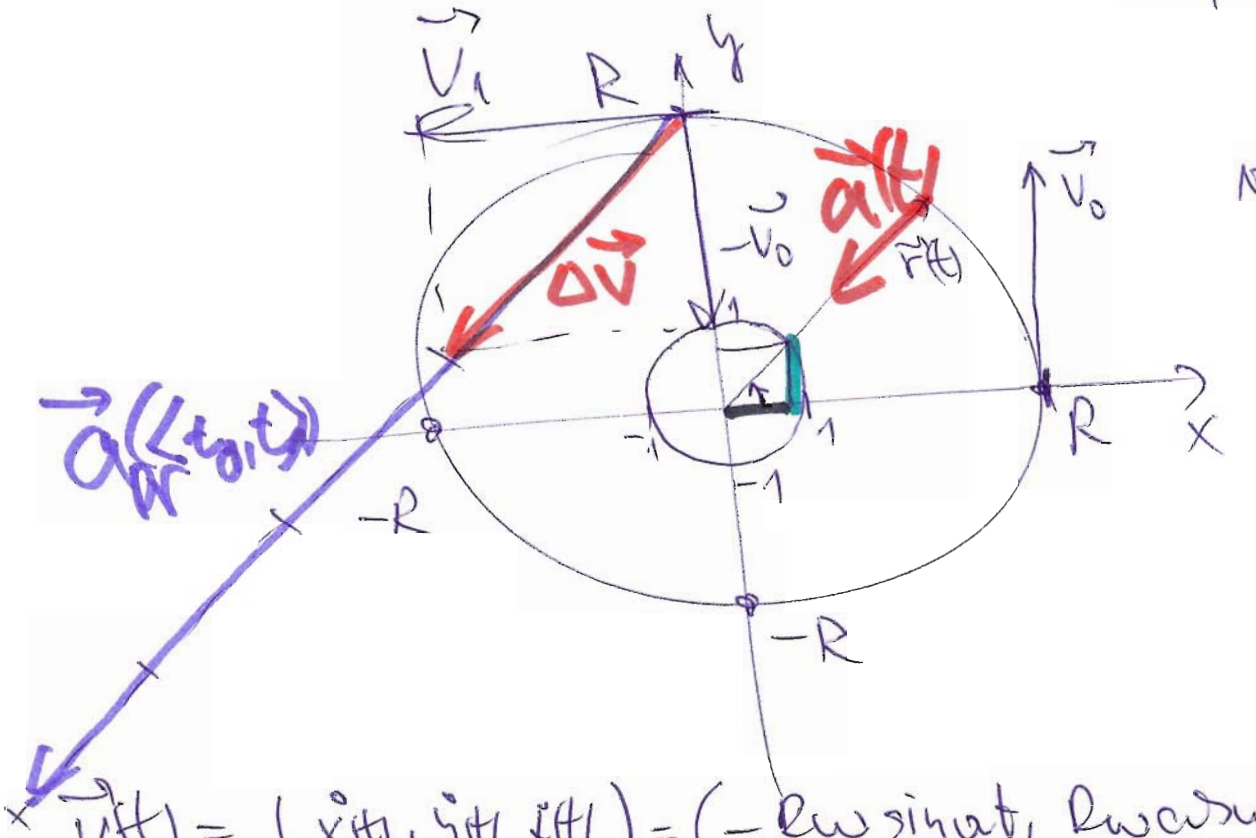


$\vec{V}_{pr}(\Delta t = t_1 - t_0)$

$$\vec{V}_{pr} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_1 - \vec{r}_0}{\underbrace{t_1 - t_0}_{1/4}} =$$

Průměrné zrychlení

$$\vec{a}_{pr}(\langle t_0, t_1 \rangle) = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_1 - \vec{v}_0}{t_1 - t_0} = \frac{\Delta \vec{v}}{\frac{1}{4}}$$



$v_0 = R\omega$

$$\vec{v}(t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t)) = (-R\omega \sin\omega t, R\omega\cos\omega t, 0)$$

$$|\vec{v}(t)| = \sqrt{v^2(t)} = v(t) = \sqrt{\dot{x}^2(t) + \dot{y}^2(t) + \dot{z}^2(t)} = R\omega$$

Okamžité zrychlení

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2} = \ddot{\vec{r}}(t) = (\ddot{x}(t), \ddot{y}(t), \ddot{z}(t))$$

$$= (-R\omega^2\cos\omega t, -R\omega^2 \sin\omega t, 0) = -R\omega^2(\cos\omega t, \sin\omega t, 0)$$

$$|\vec{a}(t)| = R\omega^2$$

$$\vec{a}(t) = -R\omega^2(\cos\omega t, \sin\omega t, 0) = -\omega^2 \vec{r}(t)$$

$$\vec{r}(t) = R(\cos\omega t, \sin\omega t, 0)$$