Global analysis. Exercises 6

1) Let T be a tensor field of type (r, s) on $\mathbb{R}^n$. Let $x^1, ..., x^n$ and $x^{1'}, ..., x^{n'}$ be two coordinate systems. Find the relation between the components of the tensor field T in these coordinates.

2) Let x^1, x^2 and $x^{1'}, x^{2'}$ be two coordinate systems related by

$$x^{1'} = \frac{rx^1}{(x^1)^2 + (x^2)^2}, \quad x^{2'} = \frac{rx^2}{(x^1)^2 + (x^2)^2},$$

where $r > 0$ is a fixed number. Knowing the components of the following tensors in the first coordinate system, find the components of these tensors in the second coordinate system.

- $a_{11} = (x^1)^2 + (x^2)^2, \quad a_{12} = x^1, \quad a_{21} = x^2, \quad a_{22} = \frac{1}{(x^1)^2+(x^2)^2};$
- $a^1_1 = x^1, \quad a^2_1 = x^1 + x^2, \quad a^1_2 = x^1 - x^2, \quad a^2_2 = x^2.$

3) For $w = (x^2 + y^2)dx + xzdz$ and $\theta = zdy \wedge dx + xdz \wedge dx$ find dw, $d\theta$, $w \wedge w$, $\theta \wedge \theta$, $w \wedge \theta$, $d(w \wedge w)$, $d(\theta \wedge \theta)$, $d(w \wedge \theta)$.

4) Find dw for

- $w = x^2ydy - xy^2dx;$
- $w = xdy + ydx;$
- $w = f(x)dx + g(y)dy;$
- $w = xdy \wedge dz + ydz \wedge dx + zdx \wedge dy.$