Global analysis. Exercises 7

1) Check if the differential 1-form w is exact in the domain D. If it is exact, find all functions φ such that $d\varphi = w$:

- $w = xydx + \frac{x^2}{2}dy,\ D = \mathbb{R}^2;$
- $w = xdx + xzdy + xydz,\ D = \mathbb{R}^3;$
- $w = (\frac{1}{x^2} + \frac{1}{y^2})(ydx - xdy),\ D = \{(x, y) \in \mathbb{R}^2 | xy \neq 0\}.$

2) Prove that the differential form $w = r^{-3}(xdy \wedge dz + ydz \wedge dx + zdx \wedge dy)$, where $r = \sqrt{x^2 + y^2 + z^2}$, is closed in the domain $\mathbb{R}^3 \backslash \{(0, 0, 0)\}$, but it is not exact there.

3) Find dw, g^*w and $g^*(dw)$, where $g : \mathbb{R}^2 \to \mathbb{R}^3$, $g(u, v) = (uv, u \cos v, e^v)$:

- $w = xdy$
- $w = ydz \wedge dx$
- $w = dx \wedge dy \wedge dz$