Global analysis. Exercises 8

1) Find $\int_M \omega$ for

- $\omega = (x-y)dx + (x+y)dy$ and M is the segment AB with $A = (2,3)$ and $B = (3,5)$;
- $\omega = ydx + xdy$ and M is a quarter of the circle $g(t) = (R\cos t, R\sin t)$, $t \in (0, \frac{\pi}{2})$;
- $\omega = xdx + ydy + (x+y-1)dz$ and M is the segment AB with $A = (1,1,1)$ and $B = (2,3,4)$.

2) Find

$$\int_M dx_3 \wedge dx_4 + x_1 x_3 dx_2 \wedge dx_4,$$

where $M = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 | x_1^2 + x_2^2 = 1,\ x_3^2 + x_4^2 = 1\}$ and the orientation is given by the parameterization $g(u,v) = (\cos u, \sin u, \cos v, \sin v)$, $(u,v) \in (0, 2\pi) \times (0, 2\pi)$.

3) Using the Stokes Theorem, find $\int_M \omega$, where

- $\omega = (x^2+y^2)dx + (x^2-y^2)dy$ and M is the circuit of the triangle with the vertexes $A = (0,0)$, $B = (1,0)$, $C = (0,1)$;
- $\omega = (x^2+y^2)dx + (x^2-y^2)dy$ and M is the graph of the function $y = 1 - |1-x|$, $x \in (0,2)$, starting at $(0,0)$;
- $\omega = xydy \wedge dz + yzdz \wedge dx + xzdx \wedge dy$ and M is the surface of the pyramid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.