Intraspecific Interactions

"Populační ekologie živočichů"

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Density-dependent growth

Discrete (difference) model

- logistic growth due to density dependent changes in fecundity and survival
- *K* .. carrying capacity, upper limit of population growth, where λ = 1
- change in λ depends on *N*

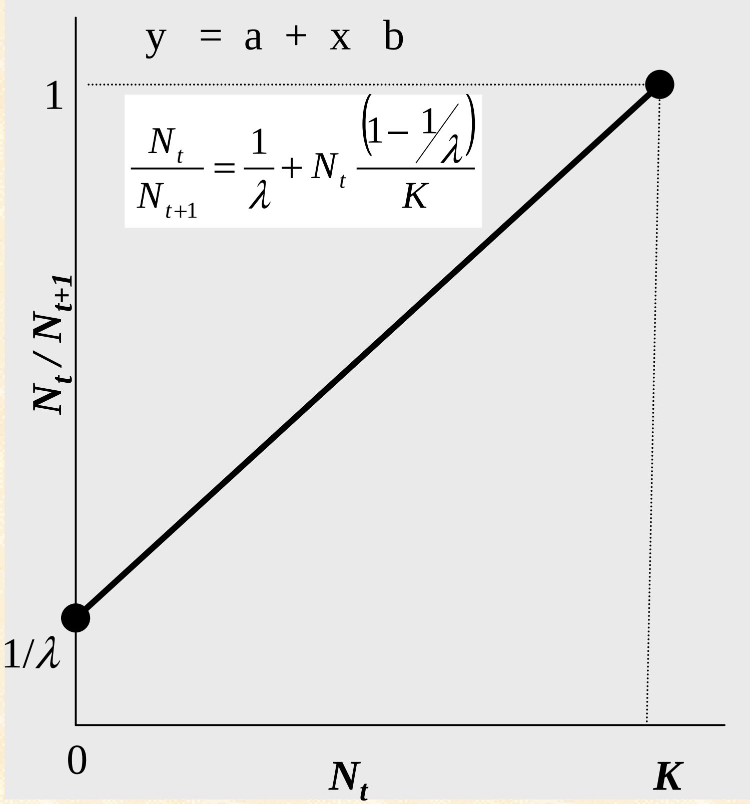
$$N_{t+1} = N_t \lambda$$

$$\frac{N_t}{N_{t+1}} = \frac{1}{\lambda}$$

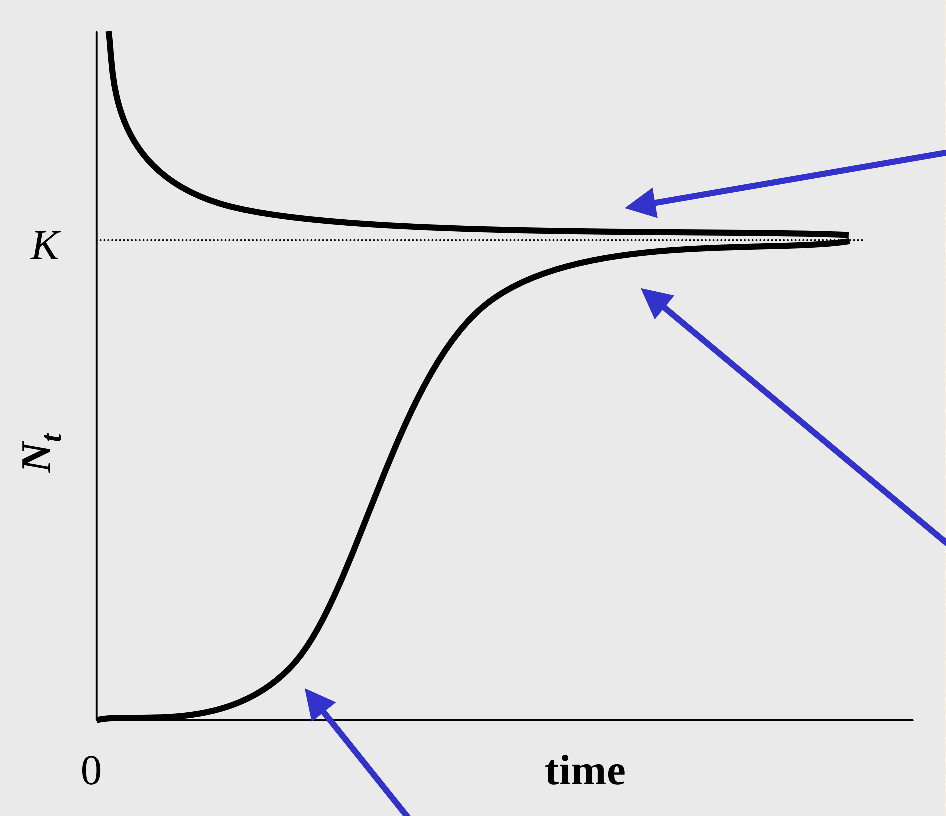
$$N_{t+1} = \frac{N_t \lambda}{1 + \frac{(\lambda - 1) N_t}{K}}$$

if $a = \frac{\lambda - 1}{K}$ then

$$N_{t+1} = \frac{N_t \lambda}{1 + a N_t}$$



$$\frac{N_t}{N_{t+1}} = \frac{1}{\lambda} + N_t \frac{\left(1 - \frac{1}{\lambda}\right)}{K}$$



when $N_t > K$ then

$$\frac{\lambda}{1+aN_t} < 1$$

- population returns to K

when $N_t \to K$ then

$$\frac{\lambda}{1+aN_t} \approx 1$$

- density-dependent control
- S-shaped (sigmoid) growth

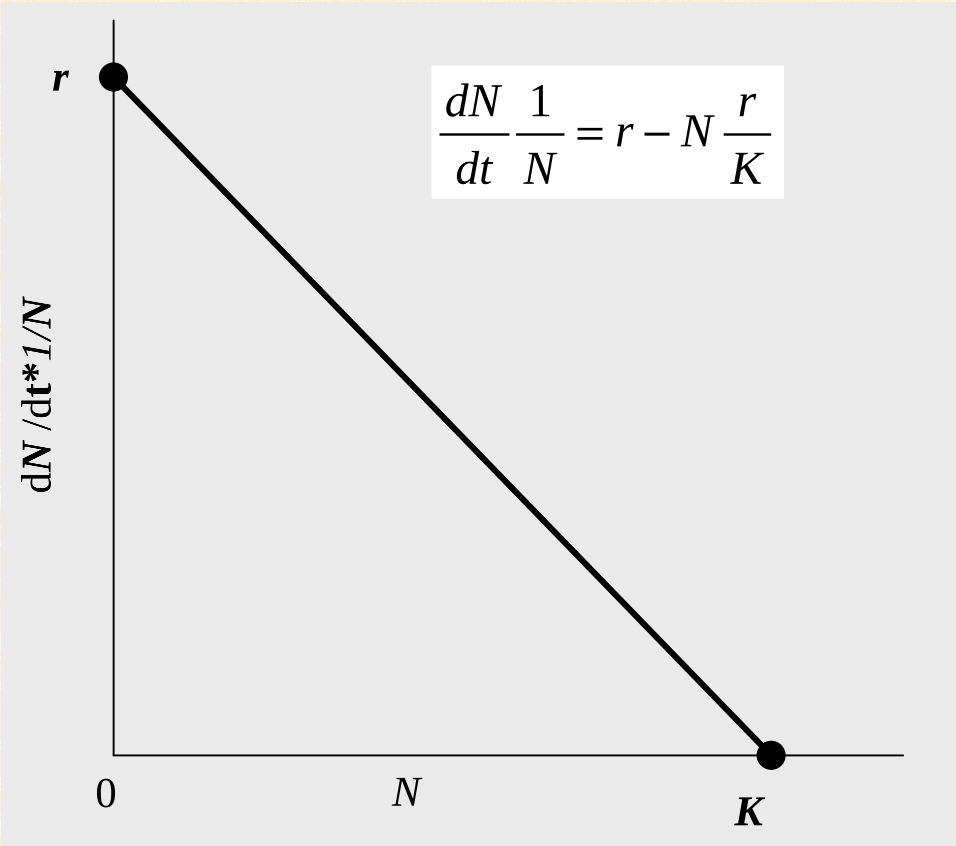
when $N_t \to 0$ then

$$\frac{\lambda}{1+aN_t} \approx \lambda$$

- no competition
- exponential growth

Continuous (differential) model

- logistic growth
- first used by Verhulst (1838) to describe growth of human population



$$\frac{dN}{dt} = Nr \quad \rightarrow \quad \frac{dN}{dt}\frac{1}{N} = r$$

- when $N \rightarrow K$ then $r \rightarrow 0$

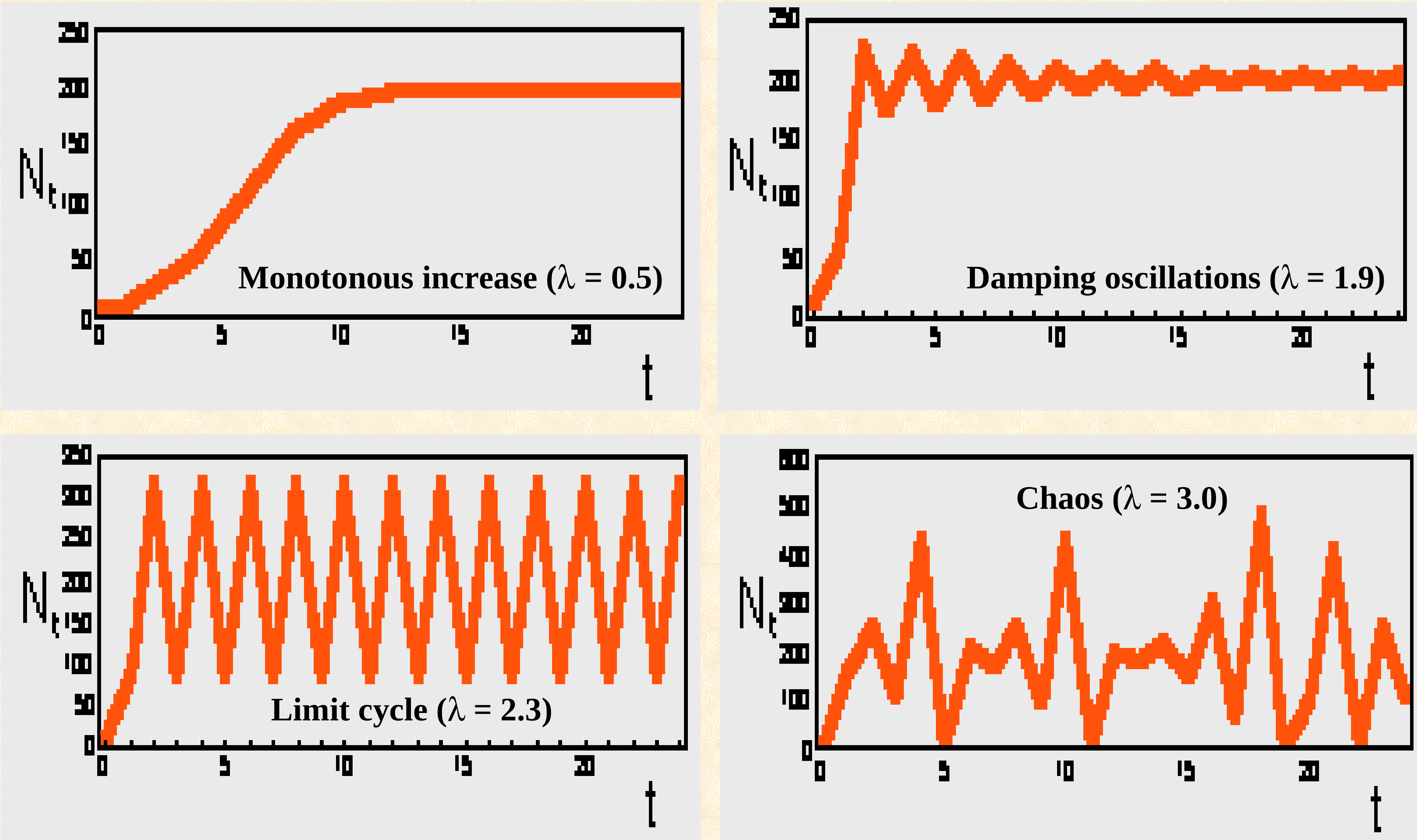
$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right)$$

<u>Solution of the differential equation</u>

$$N_t = \frac{K}{1 + e^{a-rt}} \qquad a = \ln\left(\frac{K - N_0}{N_0}\right)$$

Examination of the logistic model

$$N_{t+1} = \frac{N_t \lambda}{1 + aN_t}$$



Model equilibria

1. $N = 0$ .. unstable equilibrium
2. $N = K$ .. stable equilibrium .. if $0 < \lambda < 2$

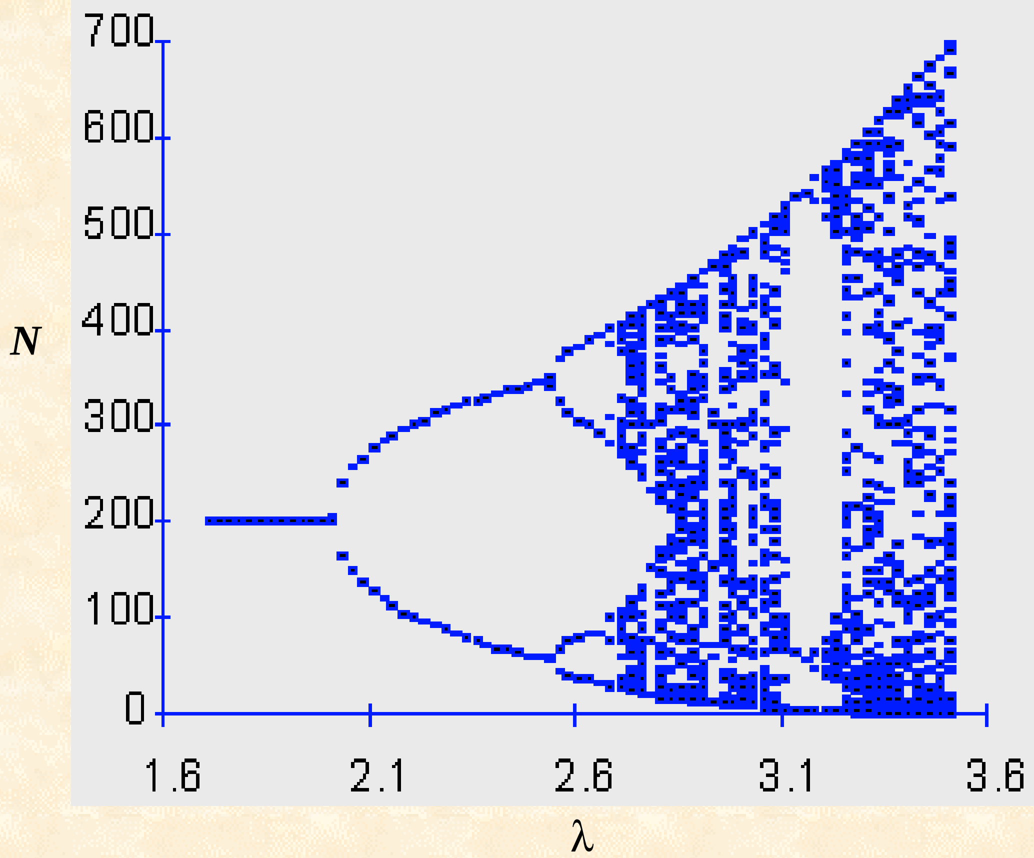
▸ "Monotonous increase" and "Damping oscillations" has a stable equilibrium

▸ "Limit cycle" and "Chaos" has no equilibrium

$\lambda < 2$ .. stable equilibrium
$\lambda = 2$ .. 2-point limit cycle
$\lambda = 2.5$ .. 4-point limit cycle
$\lambda = 2.692$ .. chaos

▸ chaos can be produced by deterministic process

▸ density-dependence is stabilising only when λ is rather low



Observed population dynamics

a) yeast (logistic curve)

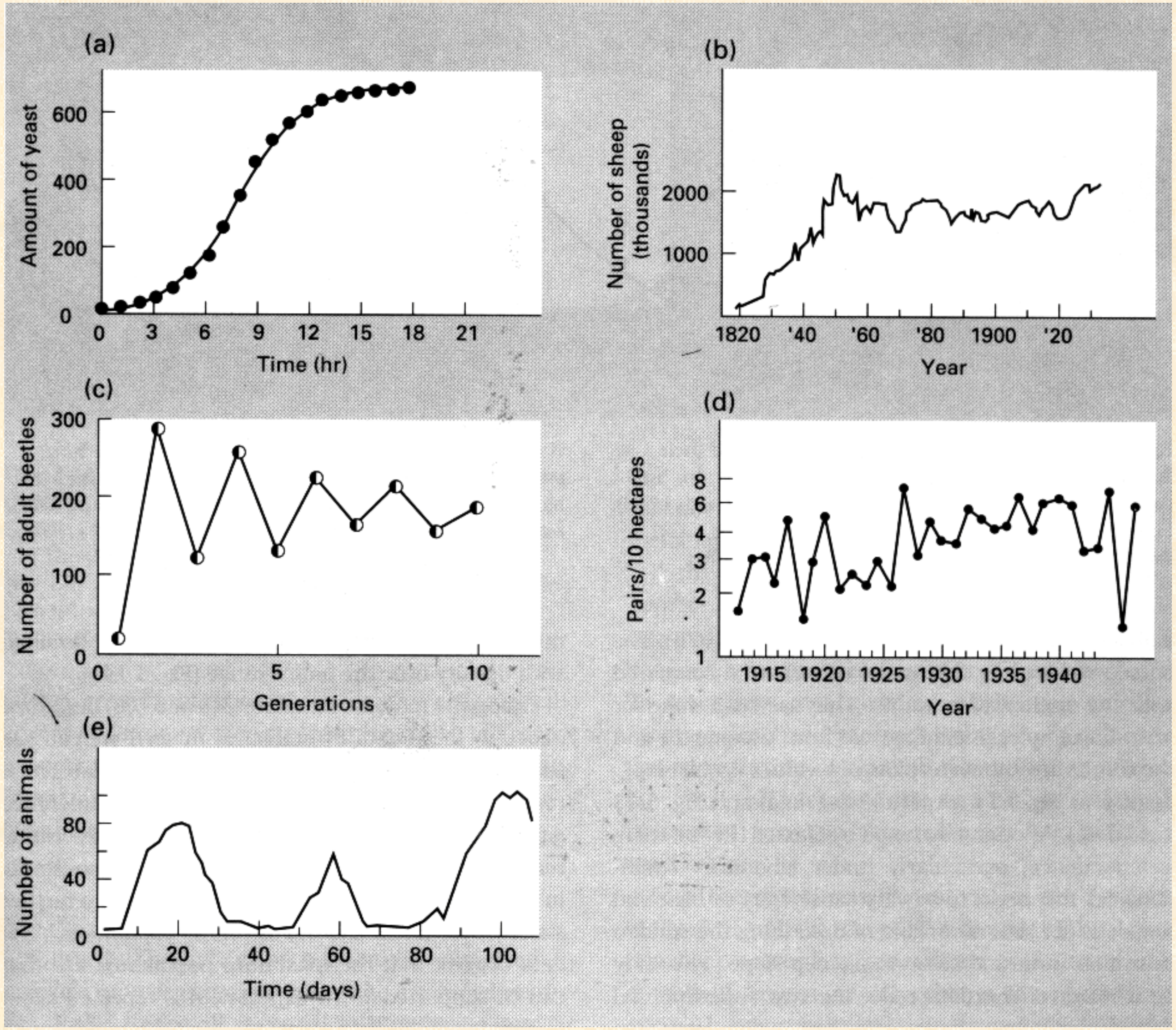
b) sheep (logistic curve with oscillations)

c) *Callosobruchus* (damping oscillations)

d) *Parus* (chaos)

e) *Daphnia*

- of 28 insect species in one species chaos was identified, one other showed limit cycles, all other were in stable equilibrium



General model

- Hassell (1975) proposed general model for DD

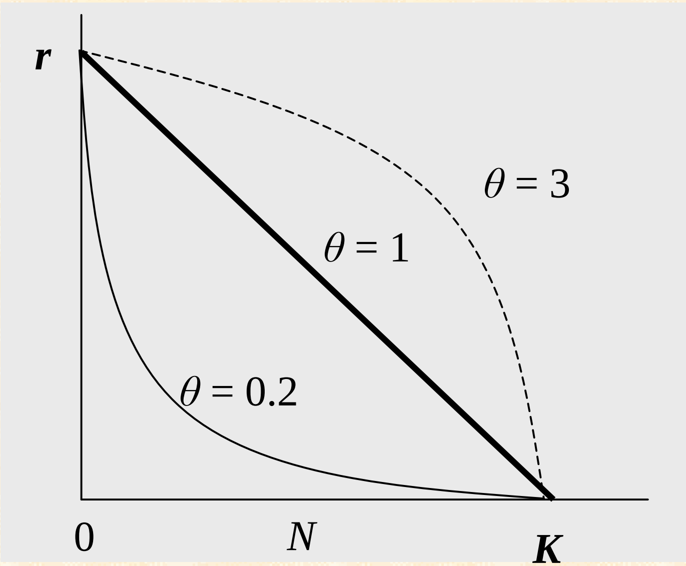
$$N_{t+1} = \frac{N_t \lambda}{(1 + aN_t)^\theta}$$

- where θ.. the strength of competition

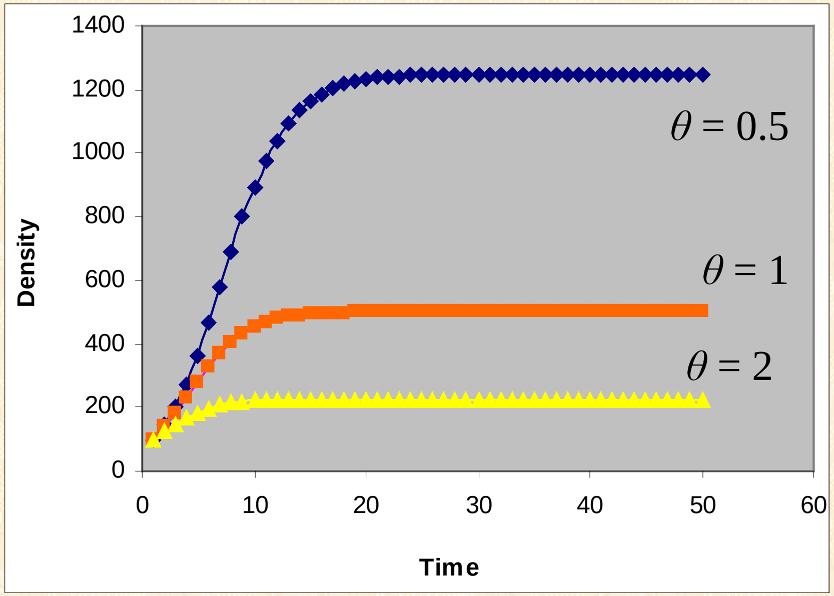
$\theta >> 1$ .. scramble competition (over-compensation)

$\theta = 1$ .. contest competition (exact compensation)

$\theta < 1$ .. under-compensation



Effect of θ on population density



Models with time-lags

- species response to resource change is not immediate but delayed due to maternal effect, seasonal effect
- appropriate for species with long generation time where reproductive rate is dependent on density of a previous generation
- time lag (d, τ) .. negative feedback of the 2nd order

discrete model

$$N_{t+1} = \frac{N_t \lambda}{1 + aN_{t-d}}$$

continuous model

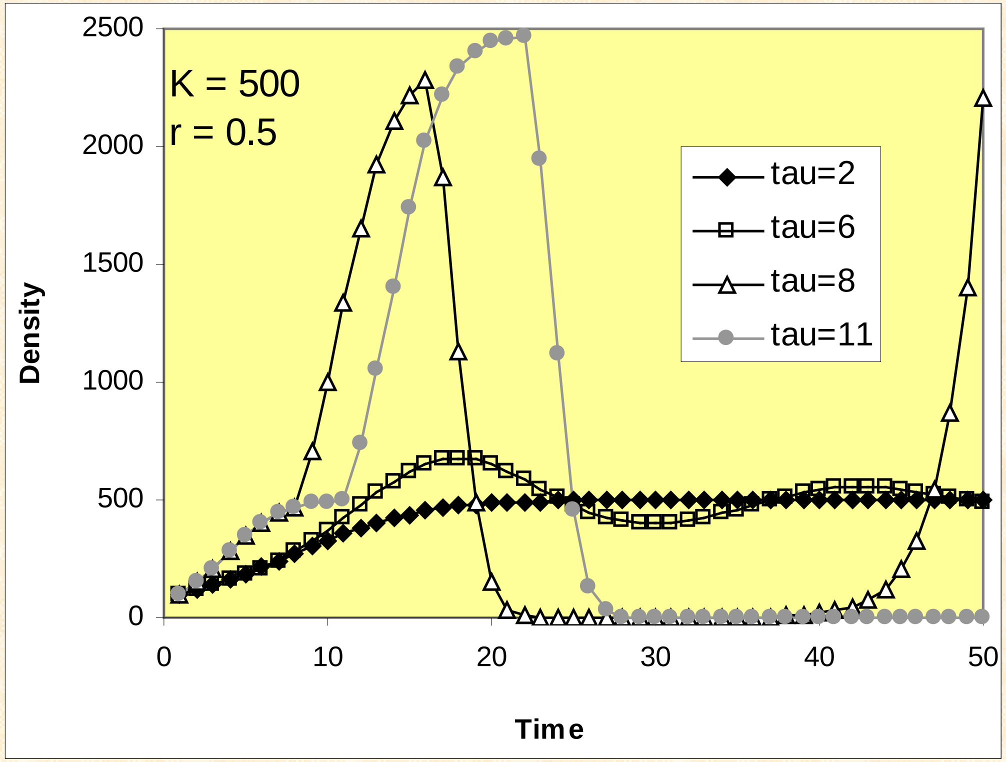
$$\frac{dN}{dt} = N_t r \frac{K - N_{t-\tau}}{K}$$

- many populations of mammals cycle with 3-4 year periods
- time-lag provokes fluctuations of certain amplitude at certain periods
- period of the cycle in continuous model is always 4τ

Solution of the continuous model:

$$N_{t+1} = N_t e^{r\left(1 - \frac{N_{t-\tau}}{K}\right)}$$

$r\,\tau < 0.4 \rightarrow$ monotonous increase
$r\,\tau < 1.6 \rightarrow$ damping fluctuations
$r\,\tau < 2 \rightarrow$ limit cycle fluctuations
$r\,\tau > 2 \rightarrow$ extinction



Harvesting

- to attain maximum sustainable yield

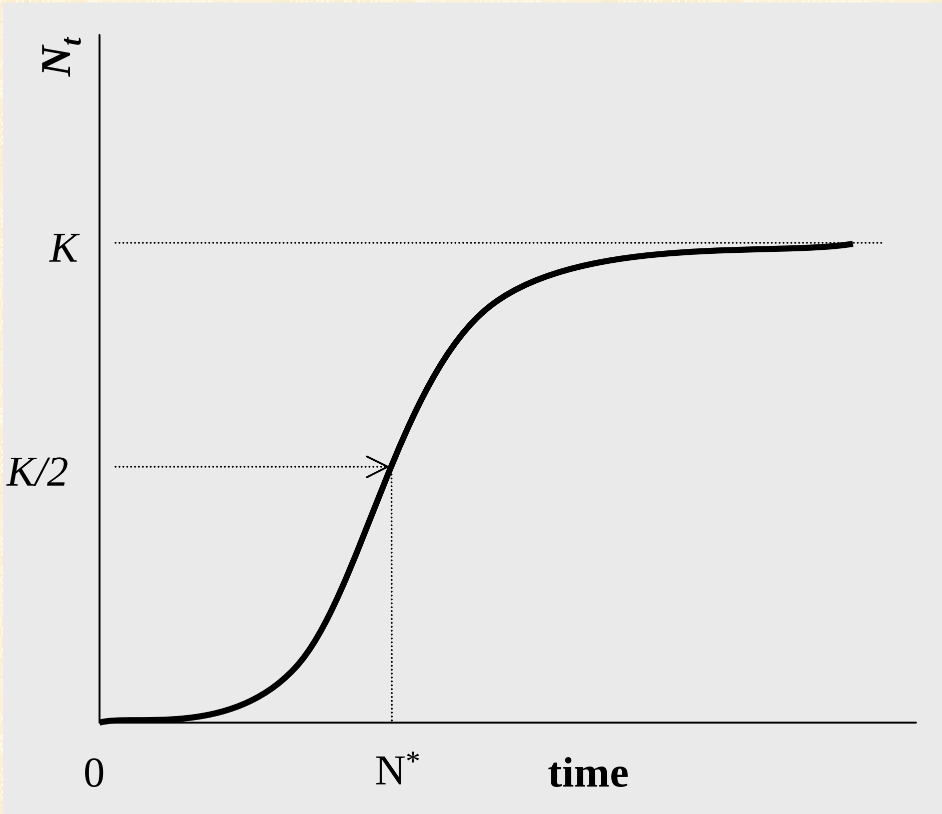
$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right) = 0$$

$$Nr - \frac{N^2 r}{K} = 0$$

$$r - \frac{2rN}{K} = 0$$

$$N^* = \frac{rK}{2}$$

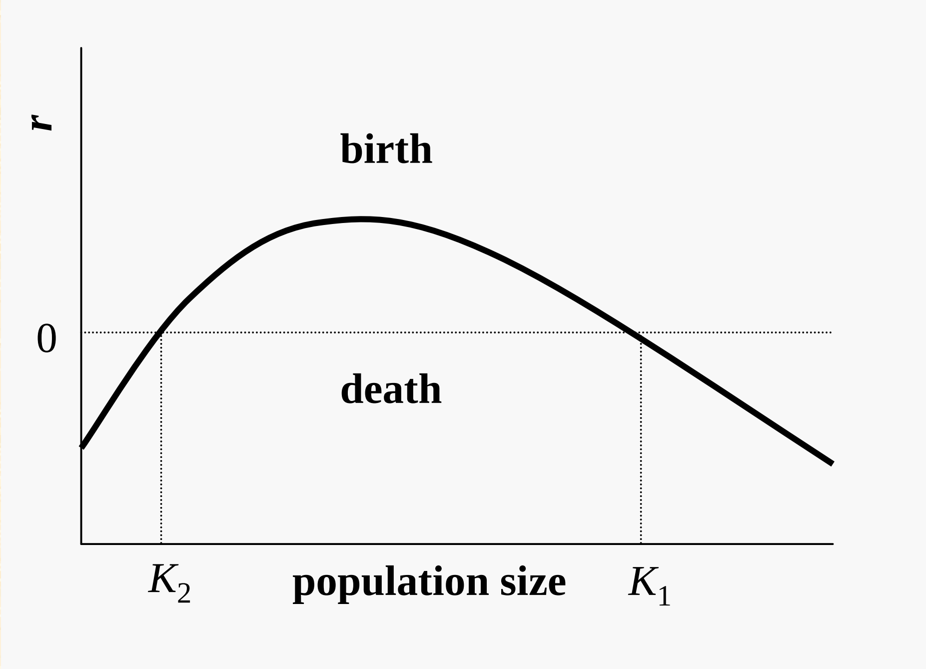
$$N^* = \frac{K}{2}$$



Alee effect

- K_2 .. extinction threshold, unstable equilibrium
- population increase is slow at low density but fast at high density

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K_1}\right)\left(\frac{N}{K_2} - 1\right)$$



Excercise 10

Simulate population dynamics using density-dependent model for discrete population growth for a period of 40 generations with N_0=10.

1. With deterministic λ and K (λ = 1.2, K =500).
2. With stochastic λ but deterministic K (λ = 1.2 ±0.12, K = 500).
3. With stochastic K but deterministic λ (λ = 1.2, K = 500 ±50).
4. With stochastic λ and K (λ = 1.2 ±0.12, K = 500 ±50).
5. Make a function for a density-dependent model for discrete population growth using N_0=100, K=500, λ =1.2, t=50.
6. Plot logarithm of lambda against N_t and estimate λ_{max} and K using the following formulas:

$$\ln(\lambda) = a - bN_t$$

$$\lambda_{max} = e^a$$

$$K = -\frac{a}{b}$$

```
N<-10
for(t in 1:40) N[t+1]<-{
N[t]*1.2/(1+N[t]*(1.2-1)/500)}
plot(0:40,N,type="b")

for(t in 1:40) N[t+1]<-{
N[t]*1.2/(1+N[t]*(1.2-1)/runif(1,450,550))}
plot(0:40,N,type="b")

for(t in 1:40) N[t+1]<-{
N[t]* runif(1,1,1.5)/(1+N[t]*(runif(1,1,1.5)-1)/500)}
plot(0:40,N,type="b")

for(t in 1:40) N[t+1]<-{
N[t]* runif(1,1,1.5)/(1+N[t]*(runif(1,1,1.5)-1)/runif(1,450,550))}
plot(0:40,N,type="b")
```

```
logistic<-function(N0=100,K=500,R=1.2,t=50){
N<-c(N0,numeric(t))
for(t in 1:t) N[t+1]<-{
N[t]*R/(1+N[t]*(R-1)/K)}
return(N)}

Nt<-logistic()
plot(Nt)

lambda<-Nt[-1]/Nt[-51]
plot(Nt[-51],log(lambda),ylim=c(0,0.5))
m1<-lm(log(lambda)~Nt[-51])
coef(m1)
exp(0.1763726056)
-0.1763726056/-0.0003536496
```

Excercise 11

You have observed the following population dynamic of yearly censuses of aphids:

180, 531, 277, 296, 828, 329, 397, 772, 625, 318, 567, 881, 386

1. Plot the population dynamic.
2. Is there evidence for density-dependence?
3. Estimate λ_{max} and K.

```
aphid<-c(180, 531, 277, 296, 828, 329, 397, 772, 625, 318, 567,
881, 386)
plot(aphid,type="b")

lambda1<-aphid[-1]/aphid[-13]
plot(aphid[-13], log(lambda1))
m2<-lm(log(lambda1)~aphid[-13])
coef(m2)
abline(m2)
exp(1.3057703)
-1.30577030/-0.00248398
```

Excercise 12

On an African market wild game animals are sold. You know carrying capacities (*K*), finite growth rates (*R*), and longevities (*L*) for each species:

	K	R	Longevity	Harvest
monkey	49000	1.17	31	781
pangolin	22000	2.01	13	192
porcupine	110000	1.82	23	1580
duiker	45000	1.63	7	732

1. Compute MSY for each species using the following formula:

$$MSY = a\left(\frac{RK - K}{2}\right)$$

where

$a = 0.6$ for $L < 5$

$a = 0.4$ for $L = (5,10)$

$a = 0.2$ for $L > 10$

2. Is the observed harvest sustainable in each species?

```
monkey<-0.2*(1.17*49000-49000)/2; monkey

pangolin<-0.2*(2.01*22000-22000)/2; pangolin

porcupine<-0.2*(1.82*110000-110000)/2; porcupine

duiker<-0.4*(1.63*45000-45000)/2; duiker
```