Interspecilic Intractions

## Types of interactions

|  | Effect of species 1 on fitness of species 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Increase | Neutral | Decrease |
|  | Increase | + + |  |  |
| ¢ | Neutral | $0+$ | 00 |  |
| 㐌 | Decrease | + - | - 0 | -- |

++ .. mutualism (plants and pollinators)
$0+$.. commensalism (saprophytism, parasitism, phoresis)

-     + .. predation (herbivory, parasitism), Batesian mimicry
- 0 .. amensalism (allelopathy)
-     - .. competition


## Model of competition

- based on the logistic model of Lotka (1925) and Volterra (1926)

$$
\frac{d N}{d t}=N r\left(1-\frac{N}{K}\right)
$$

- assumptions:
- all parameters are constant
- individuals of the same species are identical
- environment is homogenous, differentiation of niches is not possible
- only exact compensation is present
species 1: $N_{1}, K_{1}, r_{1}$

$$
\begin{aligned}
& \frac{d N_{1}}{d t}=N_{1} r_{1}\left(1-\frac{N_{1}+N_{2}}{K_{1}}\right) \\
& \frac{d N_{2}}{d t}=N_{2} r_{2}\left(1-\frac{N_{1}+N_{2}}{K_{2}}\right)
\end{aligned}
$$

- total competitive effect (intra + inter-specific)

$$
\left(N_{1}+\alpha N_{2}\right) \quad \text { where } \alpha \text {.. coefficient of competition }
$$

$\alpha=0$.. no interspecific competition
$\alpha<1$.. species 2 has lower effect on species 1 than species 1 on itself $\alpha=0.5$.. one individual of species 1 is equivalent to 0.5 individuals of species 2)
$\alpha=1$.. both species has equal effect on the other one
$\alpha>1$.. species 2 has greater effect on species 1 than species 1 on itself

$$
\begin{array}{ll}
\text { species 1: } & \frac{d N_{1}}{d t}=N_{1} r_{1}\left(1-\frac{N_{1}+\alpha_{12} N_{2}}{K_{1}}\right) \\
\text { species 2: } & \frac{d N_{2}}{d t}=N_{2} r_{2}\left(1-\frac{\alpha_{21} N_{1}+N_{2}}{K_{2}}\right)
\end{array}
$$

- if competing species use the same resource then interspecific competition is equal to intraspecific


## Analysis of the model

- examination of the model behaviour on a phase plane
- used to describe change in any two variables in coupled differential equations by projecting orthogonal vectors
- identification of isoclines: a set of abundances for which the growth rate is 0

$$
\frac{d N}{d t}=0
$$




- species 1

$$
\begin{array}{ll} 
& r_{1} N_{1}\left(1-\left[N_{1}+\alpha_{12} N_{2}\right] / K_{1}\right)=0 \\
& r_{1} N_{1}\left(\left[K_{1}-N_{1}-\alpha_{12} N_{2}\right] / K_{1}\right)=0 \\
\text { if } & r_{1}, N_{1}, K_{1}=0 \\
\text { and if } & K_{1}-N_{1}-\alpha_{12} N_{2}=0 \\
\text { then } & N_{1}=K_{1}-\alpha_{12} N_{2}
\end{array}
$$

if $N_{1}=0$ then $N_{2}=K_{1} / \alpha_{12}$
if $N_{2}=0$ then $N_{1}=K_{1}$

- species 2

$$
\begin{aligned}
& r_{2} N_{2}\left(1-\left[N_{2}+\alpha_{21} N_{1}\right] / K_{2}\right)=0 \\
& N_{2}=K_{2}-\alpha_{21} N_{1}
\end{aligned}
$$

$$
\text { if } N_{2}=0 \text { then } N_{1}=K_{2} / \alpha_{21}
$$

$$
\text { if } N_{1}=0 \text { then } N_{2}=K_{2}
$$

- above isocline $i_{1}$ and below $i_{2}$ competition is weak
- in-between $i_{1}$ and $i_{2}$ competition is strong


## 1. Species 2 drives species 1 to extinction

- $K$ and $\alpha$ determine the model behaviour
- disregarding initial densities species 2 (stronger competitor) will outcompete species 1 (weaker competitor)

$$
K_{2}>\frac{K_{1}}{\alpha_{12}} \quad K_{1}<\frac{K_{2}}{\alpha_{21}}
$$

$$
\begin{aligned}
K_{1} & =K_{2} & r_{1} & =r_{2} \\
\alpha_{12} & >\alpha_{21} & N_{01} & =N_{02}
\end{aligned}
$$




## 2. Species $\mathbf{1}$ drives species $\mathbf{2}$ to extinction

- species 1 (stronger competitor) will outcompete species 2 (weaker competitor)

$$
K_{1}>\frac{K_{2}}{\alpha_{21}} \quad K_{2}<\frac{K_{1}}{\alpha_{12}}
$$

$$
\begin{aligned}
r_{1} & =r_{2} & K_{1}=K_{2} \\
N_{01} & =N_{02} & \alpha_{12}<\alpha_{21}
\end{aligned}
$$




## 3. Stable coexistence of species

- disregarding initial densities both species will coexist at stable equilibrium (where isoclines cross)
- at at equilibrium population density of both species is reduced
- both species are weak competitors

$$
K_{1}<\frac{K_{2}}{\alpha_{21}} \quad K_{2}<\frac{K_{1}}{\alpha_{12}}
$$

$$
\begin{array}{cc}
r_{1}<r_{2} & K_{1}=K_{2} \\
N_{01}=N_{02} & \alpha_{12}, \alpha_{21}<1
\end{array}
$$




## 4. Competitive exclusion

- one species will drive other to extinction depending on the initial conditions
- coexistence for a short time
- both species are strong competitors

$$
K_{1}>\frac{K_{2}}{\alpha_{21}} \quad K_{2}>\frac{K_{1}}{\alpha_{12}}
$$





## Test of the model

## Tribolium versus Oryzaephilus

- when Tribolium and Oryzaephilus were reared separately both species increased to 420-450 individuals $(=K)$
- when reared together Tribolium reached $K_{1}=360$, while Oryzaephilus $K_{2}=150$ individuals
- combination resulted in more efficient conversion of grain ( $K_{12}=510$ individuals)
- three combinations of densities converged to the same stable equilibrium
- prediction of

Lotka-Volterra model is correct


## Competition model for discrete generations

- solution of the differential model:

$$
N_{1, t+1}=N_{1, t} e^{r_{1}\left(1-\frac{N_{1, t}-\alpha_{12} N_{2, t}}{K_{1}}\right)} \quad N_{2, t+1}=N_{2, t} e^{r_{2}\left(1-\frac{N_{2, t}-\alpha_{21} N_{1, t}}{K_{2}}\right)}
$$

- dynamic regression analysis is used to estimate parameters from abundances

$$
\begin{gathered}
\ln \left(\frac{N_{1, t+1}}{N_{1, t}}\right)=r_{1}-N_{t, 1} \frac{r_{1}}{K_{1}}-N_{2, t} \frac{r_{1} \alpha_{12}}{K_{1}} \ln \left(\frac{N_{2, t+1}}{N_{2, t}}\right)=r_{2}-N_{t, 1} \frac{r_{2}}{K_{2}}-N_{1, t} \frac{r_{2} \alpha_{21}}{K_{2}} \\
\ln \left(\frac{N_{1, t+1}}{N_{1, t}}\right)=a+b N_{1, t}+c N_{2, t} \quad \ln \left(\frac{N_{2, t+1}}{N_{2, t}}\right)=a+b N_{2, t}+c N_{1, t} \\
r=a \quad \alpha=-\frac{K c}{r} \quad K=-\frac{r}{b}
\end{gathered}
$$

## Excercise 13

Two species of Tribolium beetles were kept together in a jar with flour. Their densities were recorded once a week. The following abundances were observed:

A: $10,6,5,4,3,4,6,8,10,12,15,16$
B: $20,18,16,11,6,6,5,3,2,2,1,1$

1. Estimate $r_{1}, r_{2}, K_{1}, K_{2}, \alpha_{12}, \alpha_{21}$.
2. Simulate the dynamics using estimated parameters and initial densities of 20 individuals for each species within POPULUS.
3. Find such combinations of $r_{1}, r_{2}$ and $\alpha_{12}, \alpha_{21}$ so that the two species would coexist.
```
a<-c(10, 6, 5, 4, 3, 4, 6, 8, 10, 12, 15, 16)
b<-c(20, 18, 16, 11, 6, 6, 5, 3, 2, 2, 1, 1)
a1<-a[-1]/a [-12]
b1<-b[-1]/b[-12]
summary(lm(log(a1) ~a[-12]+b[-12]))
0.60443/0.02992
20.20154*0.04106/0.60443
summary(lm(log(b1) ~b[-12]+a [-12]))
0.399980/0.005052
79.1726*0.011438/0.399980
```


## Excercise 14

Two species of spiders, Pardosa and Ero, occur together and were found to feed on the following prey:

|  | Araneae | Collembola | Isopoda | Hemiptera | Ensifera |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Pardosa | 0.61 | 0.15 | 0.12 | 0.07 | 0.05 |
| Ero | 0.93 | 0.05 | 0.01 | 0 | 0.01 |

1. Estimate and plot niche breadth for each species.
2. Estimate niche overlap $\left(a_{12}, a_{21}\right)$ for each species.
3. Simulate the population dynamic of the two species using estimated $\alpha_{12}, \alpha_{21}$ and given $r_{1}=0.7, r_{2}=0.8, K_{1}=300, K_{2}=200, N_{01}=20$, $N_{02}=20$.

$$
\text { breadth }=\frac{1}{\sum_{k=1}^{n} p_{k}^{2}}
$$

$$
a_{12}=\frac{\sum p_{1 k} p_{2 k}}{\sum p_{1 k}^{2}}
$$

$$
a_{21}=\frac{\sum p_{1 k} p_{2 k}}{\sum p_{2 k}^{2}}
$$

```
Par<-c(0.61,0.15,0.12,0.07,0.05)
Ero<-c(0.93,0.05,0.01,0,0.01)
both<-rbind (Par, Ero)
barplot(both,beside=T, legend.text=c("Par", "Ero"))
1/sum(Par^2)
1/sum(Ero^2)
a12<-sum(Par*Ero) /sum(Par^2); a12
a21<-sum(Par*Ero)/sum(Ero^2); a21
comp<-function(t,y,param) {
N1<-y[1]
N2<-y[2]
with(as.list (param), {
dN1.dt<-r1*N1*(1-(N1+a12*N2)/K1)
dN2.dt<-r2*N2*(1-(N2+a21*N1)/K2)
return(list(c(dN1.dt,dN2.dt)))})}
N1<-20;N2<-20
param<-c(r1=0.7,r2=0.8,a12=1.4,a21=0.7, K1=300,K2=200)
time<-seq(0, 50,0.1)
library(deSolve)
out<-data. frame (ode (c (N1,N2), time, comp, param))
matplot(time, out[,-1],type="l",lty=1:2,col=1)
legend("right", c("N1", "N2"), lty=1:2)
```

