



"Populační ekologie živočichů"

Stano Pekár

Types of interactions

Effect of species 2 on fitness of species 1

Effect of species 1 on fitness of species 2

ecies		Increase	Neutral	Decrease
fitness of spe	Increase	+ +		
	Neutral	0 +	00	
	Decrease	+ -	- 0	

- ++ .. mutualism (plants and pollinators)
- **0** + .. commensalism (saprophytism, parasitism, phoresis)
- + .. predation (herbivory, parasitism), Batesian mimicry
- 0 .. amensalism (allelopathy)
- - .. competition

Model of competition

based on the logistic model of Lotka (1925) and Volterra (1926)

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right)$$

▶ assumptions:

- all parameters are constant
- individuals of the same species are identical
- environment is homogenous, differentiation of niches is not possible
- only exact compensation is present

species 1: N_1 , K_1 , r_1 **species 2**: N_2 , K_2 , r_2

$$\frac{dN_1}{dt} = N_1 r_1 \left(1 - \frac{N_1 + N_2}{K_1} \right)$$
$$\frac{dN_2}{dt} = N_2 r_2 \left(1 - \frac{N_1 + N_2}{K_2} \right)$$

total competitive effect (intra + inter-specific)

 $(N_1 + \alpha N_2)$ where α .. coefficient of competition $\alpha = 0$.. no interspecific competition

 $\alpha < 1$.. species 2 has lower effect on species 1 than species 1 on itself $\alpha = 0.5$.. one individual of species 1 is equivalent to 0.5 individuals of species 2)

 $\alpha = 1$... both species has equal effect on the other one

 $\alpha > 1$... species 2 has greater effect on species 1 than species 1 on itself

species 1:
$$\frac{dN_1}{dt} = N_1 r_1 \left(1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right)$$

species 2:
$$\frac{dN_2}{dt} = N_2 r_2 \left(1 - \frac{\alpha_{21}N_1 + N_2}{K_2} \right)$$

• if competing species use the same resource then interspecific competition is equal to intraspecific

Analysis of the model

• examination of the model behaviour on a phase plane

• used to describe change in any two variables in coupled differential equations by projecting orthogonal vectors

• identification of isoclines: a set of abundances for which the growth rate is 0







▶ species 1 $r_1N_1 (1 - [N_1 + \alpha_{12}N_2] / K_1) = 0$ $r_1N_1 ([K_1 - N_1 - \alpha_{12}N_2] / K_1) = 0$ if $r_1, N_1, K_1 = 0$ and if $K_1 - N_1 - \alpha_{12}N_2 = 0$ then $N_1 = K_1 - \alpha_{12}N_2$

if
$$N_1 = 0$$
 then $N_2 = K_1 / \alpha_{12}$
if $N_2 = 0$ then $N_1 = K_1$

• species 2 $r_2 N_2 (1 - [N_2 + \alpha_{21} N_1] / K_2) = 0$ $N_2 = K_2 - \alpha_{21} N_1$

if
$$N_2 = 0$$
 then $N_1 = K_2 / \alpha_{21}$
if $N_1 = 0$ then $N_2 = K_2$

Isoclines



- above isocline i_1 and below i_2 competition is weak
- in-between i_1 and i_2 competition is strong

1. Species 2 drives species 1 to extinction

K and α determine the model behaviour
disregarding initial densities species 2 (stronger competitor) will outcompete species 1 (weaker competitor)



$$K_1 = K_2$$
 $r_1 = r_2$
 $\alpha_{12} > \alpha_{21}$ $N_{01} = N_{02}$



2. Species 1 drives species 2 to extinction

species 1 (stronger competitor) will outcompete species 2 (weaker competitor)

$$K_1 > \frac{K_2}{\alpha_{21}}$$
 $K_2 < \frac{K_1}{\alpha_{12}}$

$$r_1 = r_2$$
 $K_1 = K_2$
 $N_{01} = N_{02}$ $\alpha_{12} < \alpha_{21}$



3. Stable coexistence of species

• disregarding initial densities both species will coexist at stable equilibrium (where isoclines cross)

- ▶ at at equilibrium population density of both species is reduced
- both species are weak competitors





Test of the model

Tribolium versus Oryzaephilus

• when *Tribolium* and *Oryzaephilus* were reared separately both species increased to 420-450 individuals (= K)

• when reared together *Tribolium* reached $K_1 = 360$, while *Oryzaephilus* $K_2 = 150$ individuals

• combination resulted in more efficient conversion of grain ($K_{12} = 510$ individuals)

 three combinations of densities converged to the same stable equilibrium

prediction of
 Lotka-Volterra model is correct



Crombie (1947)

Competition model for discrete generations

solution of the differential model:

$$N_{1,t+1} = N_{1,t}e^{r_1\left(1 - \frac{N_{1,t} - \alpha_{12}N_{2,t}}{K_1}\right)} \qquad N_{2,t+1} = N_{2,t}e^{r_2\left(1 - \frac{N_{2,t} - \alpha_{21}N_{1,t}}{K_2}\right)}$$

 dynamic regression analysis is used to estimate parameters from abundances

$$\ln \left(\frac{N_{1,t+1}}{N_{1,t}} \right) = r_1 - N_{t,1} \frac{r_1}{K_1} - N_{2,t} \frac{r_1 \alpha_{12}}{K_1} \left| \ln \left(\frac{N_{2,t+1}}{N_{2,t}} \right) = r_2 - N_{t,1} \frac{r_2}{K_2} - N_{1,t} \frac{r_2 \alpha_{21}}{K_2} \right|$$

$$\ln \left(\frac{N_{1,t+1}}{N_{1,t}} \right) = a + b N_{1,t} + c N_{2,t}$$

$$\ln \left(\frac{N_{2,t+1}}{N_{2,t}} \right) = a + b N_{2,t} + c N_{1,t}$$

$$r = a$$
 $\alpha = -\frac{Kc}{r}$ $K = -\frac{r}{b}$

Excercise 13

Two species of *Tribolium* beetles were kept together in a jar with flour. Their densities were recorded once a week. The following abundances were observed:

A: 10, 6, 5, 4, 3, 4, 6, 8, 10, 12, 15, 16 B: 20, 18, 16, 11, 6, 6, 5, 3, 2, 2, 1, 1

1. Estimate $r_1, r_2, K_1, K_2, \alpha_{12}, \alpha_{21}$.

2. Simulate the dynamics using estimated parameters and initial densities of 20 individuals for each species within POPULUS.

3. Find such combinations of r_1 , r_2 and α_{12} , α_{21} so that the two species would coexist.

a<-c(10, 6, 5, 4, 3, 4, 6, 8, 10, 12, 15, 16)
b<-c(20, 18, 16, 11, 6, 6, 5, 3, 2, 2, 1, 1)
a1<-a[-1]/a[-12]
b1<-b[-1]/b[-12]</pre>

summary(lm(log(a1)~a[-12]+b[-12]))
0.60443/0.02992
20.20154*0.04106/0.60443

summary(lm(log(b1)~b[-12]+a[-12]))
0.399980/0.005052
79.1726*0.011438/0.399980

Excercise 14

Two species of spiders, *Pardosa* and *Ero*, occur together and were found to feed on the following prey:

	Araneae	Collembola	Isopoda	Hemiptera	Ensifera
Pardosa	0.61	0.15	0.12	0.07	0.05
Ero	0.93	0.05	0.01	0	0.01

1. Estimate and plot niche breadth for each species.

2. Estimate niche overlap (a_{12}, a_{21}) for each species.

3. Simulate the population dynamic of the two species using estimated α_{12} , α_{21} and given $r_1=0.7$, $r_2=0.8$, $K_1=300$, $K_2=200$, $N_{01}=20$, $N_{02}=20$.

breadth =
$$\frac{1}{\sum_{k=1}^{n} p_k^2}$$
 $a_{12} = \frac{\sum p_{1k} p_{2k}}{\sum p_{1k}^2}$ $a_{21} = \frac{\sum p_{1k} p_{2k}}{\sum p_{2k}^2}$

```
Par<-c(0.61,0.15,0.12,0.07,0.05)
Ero<-c(0.93,0.05,0.01,0,0.01)
both<-rbind(Par,Ero)
barplot(both,beside=T,legend.text=c("Par","Ero"))</pre>
```

```
1/sum(Par^2)
1/sum(Ero^2)
al2<-sum(Par*Ero)/sum(Par^2); al2
a21<-sum(Par*Ero)/sum(Ero^2); a21</pre>
```

```
comp<-function(t,y,param){
N1<-y[1]
N2<-y[2]
with(as.list(param),{
dN1.dt<-r1*N1*(1-(N1+a12*N2)/K1)
dN2.dt<-r2*N2*(1-(N2+a21*N1)/K2)
return(list(c(dN1.dt,dN2.dt)))})</pre>
```

```
N1<-20;N2<-20
param<-c(r1=0.7,r2=0.8,a12=1.4,a21=0.7,K1=300,K2=200)
time<-seq(0,50,0.1)
library(deSolve)
out<-data.frame(ode(c(N1,N2),time,comp,param))
matplot(time,out[,-1],type="l",lty=1:2,col=1)
legend("right",c("N1","N2"),lty=1:2)</pre>
```