

# Nanotechnologie v bioanalýze

**Karel Klepárník**

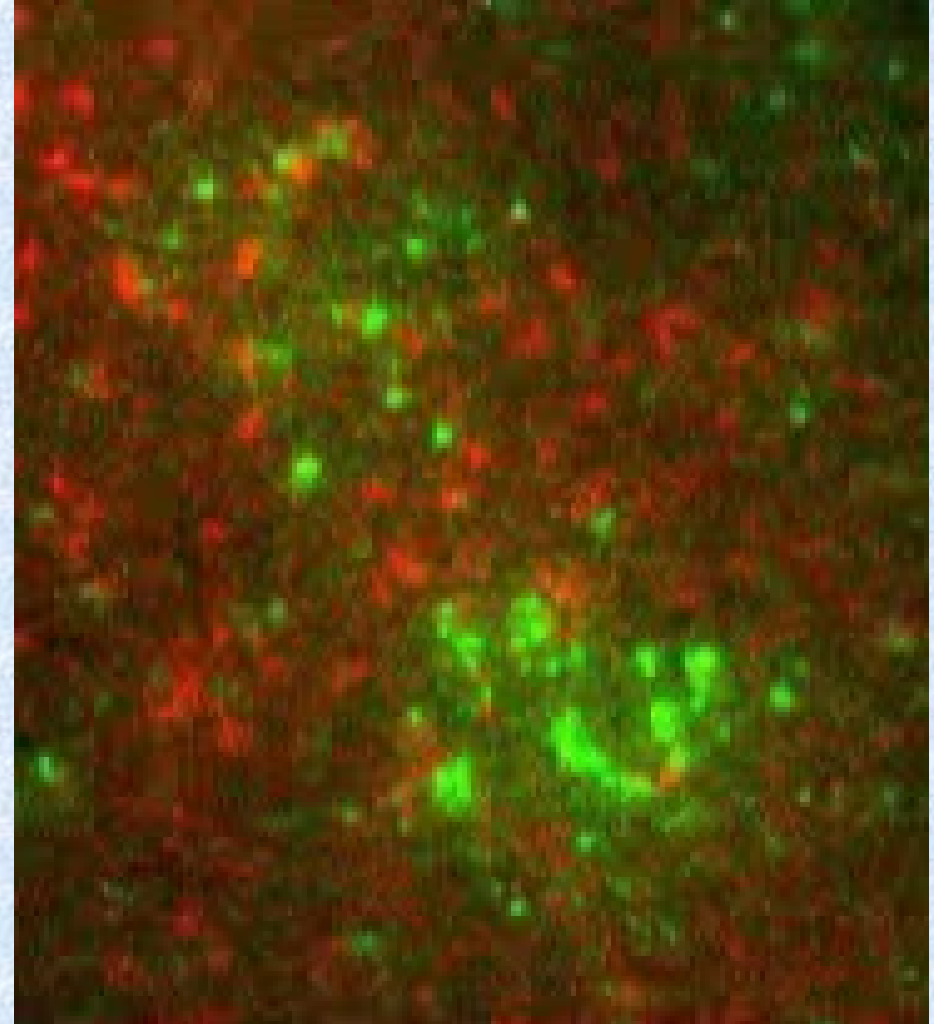
*Oddělení bioanalytické instrumentace  
Ústav analytické chemie  
Akademie věd České republiky  
Brno*



# Single molecule imaging

## Issues:

- ❖ space resolution – diffraction limit  
 $D = \lambda / (2 \times \text{N.A.}) \approx 180 \text{ nm}$  (for 500 nm)
- ❖ time resolution – brownian motion
- ❖ photo bleaching
- ❖ narrow excited layer (TIRF)
- ❖ two lasers: 514, 633 nm



Membrane proteins  
CD58-Cy3 (*green*)  
ICAM-1-Cy5 (*red*)  
in a glass-bound planar phospholipid bilayer  
under two PMA/ionomycin-treated Jurkat cells.

# Quantum dots

Properties of semiconductor quantum dots:

- ❖ High photo-stability
- ❖ Broad excitation curve
- ❖ Narrow emission spectra
- ❖ Easy tunability
- ❖ High quantum efficiency



# Quantum effects

Helmholtz  
Planck  
Einstein  
deBroglie

## Time dependent one-dimensional Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x) \Psi(x, t)$$

$\Psi(x, t)$  wave function

$i$  imaginary unit

$\hbar$  reduced Planck constant ( $\hbar = h/2\pi$ ;  $E = h\nu$ )

$x$  space

$t$  time

$m$  mass

$V(x)$  time independent potential energy at  $x$



**Erwin Schrödinger**  
1887 – 1961 Vienna

# Separation of Variables – Eigenfunction-Eigenvalue Problem

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x) \Psi(x, t) \quad 1/[\psi(x) T(t)]$$

$$\Psi(x, t) = \psi(x) T(t)$$

$$\frac{1}{T} i\hbar \frac{dT(t)}{dt} = \frac{1}{\psi} \left[ -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x) \right] = E = \hbar \omega$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x) \quad \text{time-independent equation}$$

# Solution to the time independent equation – quantum well

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi(x)$$

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2} E\psi(x)$$

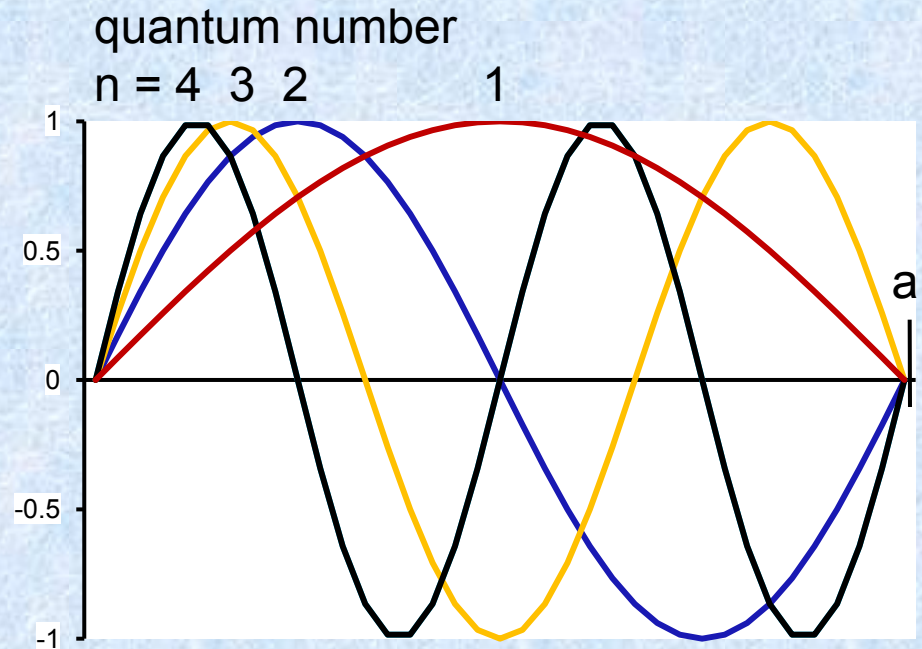
$$\psi(x) = \sin(kx)$$

$$k = n\pi/a \text{ (wavenumber)}$$

$$\psi(x) = \sin(kx)$$

$$-k^2 = -\frac{2m}{\hbar^2} E$$

$$E = \frac{k^2 \hbar^2}{2m} = \frac{n^2 h^2}{2ma^2}$$



# Infinately deep quantum well

$$E = \frac{\hbar^2 \pi^2 n^2}{2m a^2}$$

$$E_{n+1} - E_n = (2n+1)h^2 / (2ma^2)$$

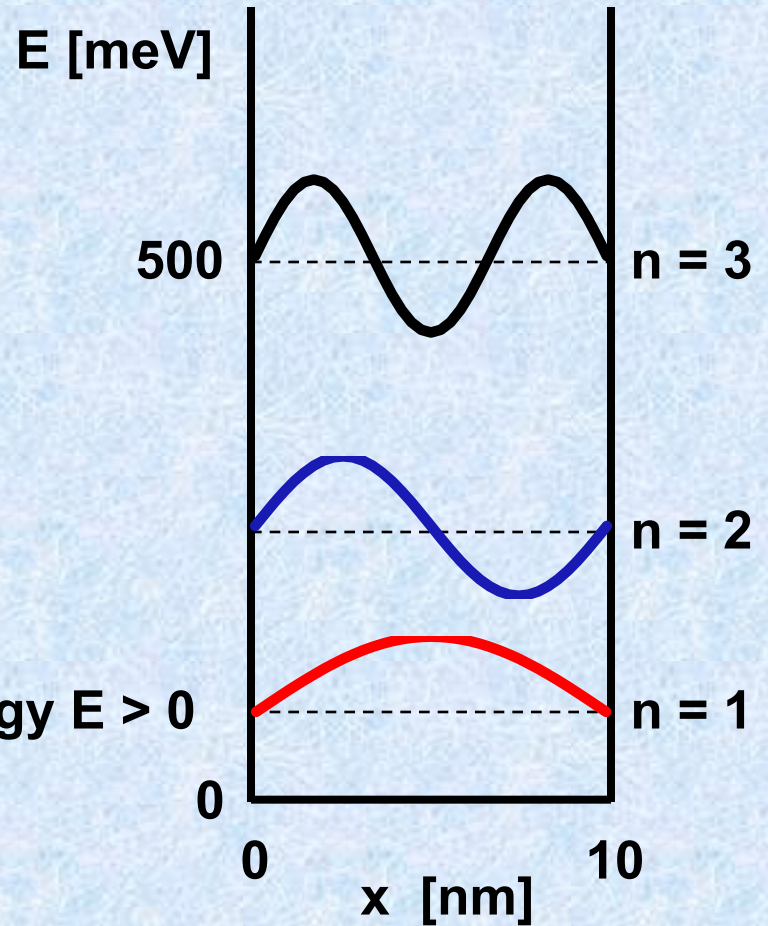
3

233, 700 nm

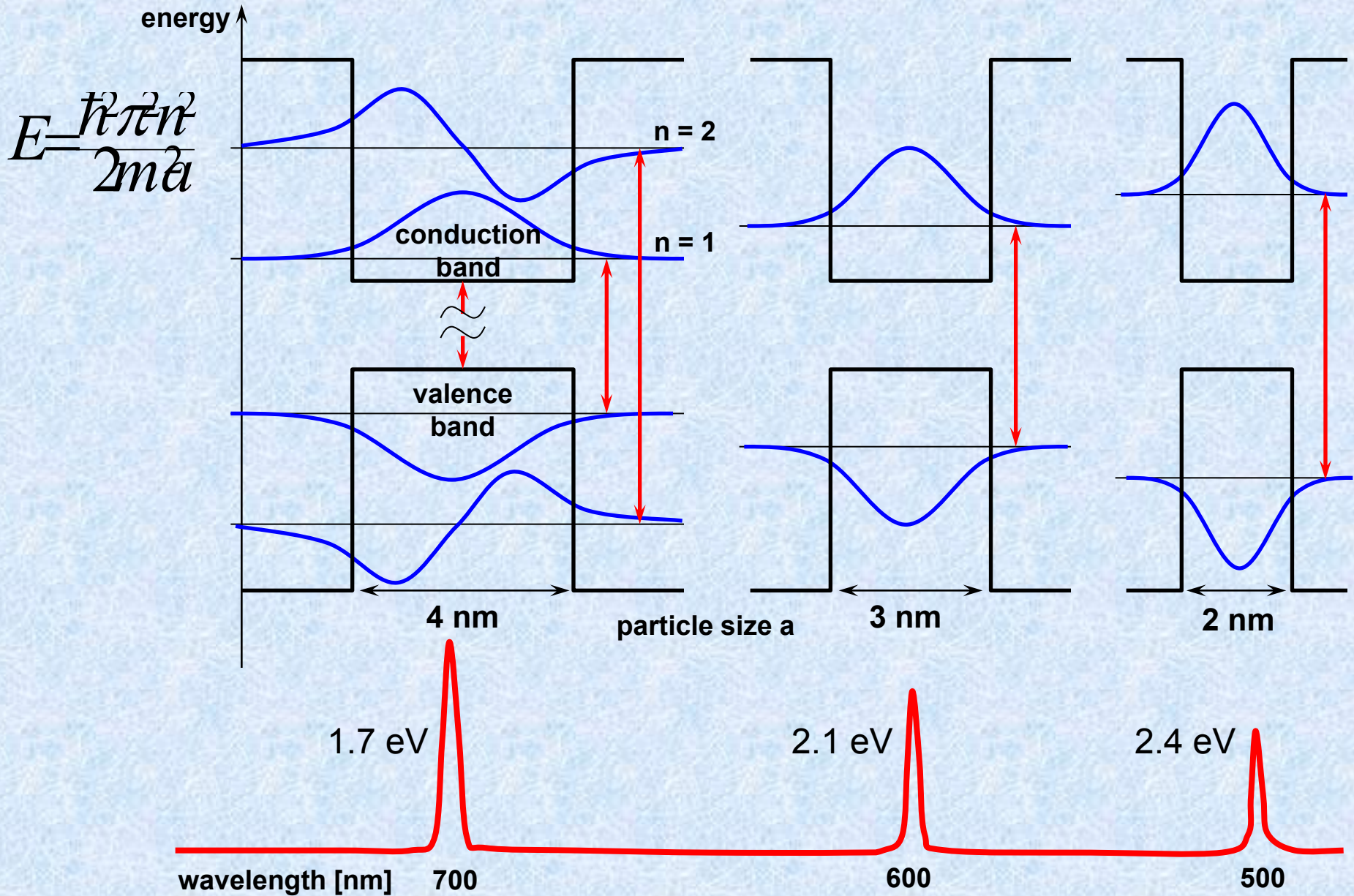
5

7

lowest energy  $E > 0$



# Quantum dots - size effect of optical properties





# Optical properties of quantum dots

$$E = \underbrace{\frac{\hbar^2}{8R^2} \left( \frac{1}{m_e} + \frac{1}{m_h} \right)}_{\text{exciton quantization}} - \underbrace{\frac{1.8e^2}{4\pi\epsilon_0 R}}_{\text{electrostatic attraction}}$$

$E$  energy of exciton

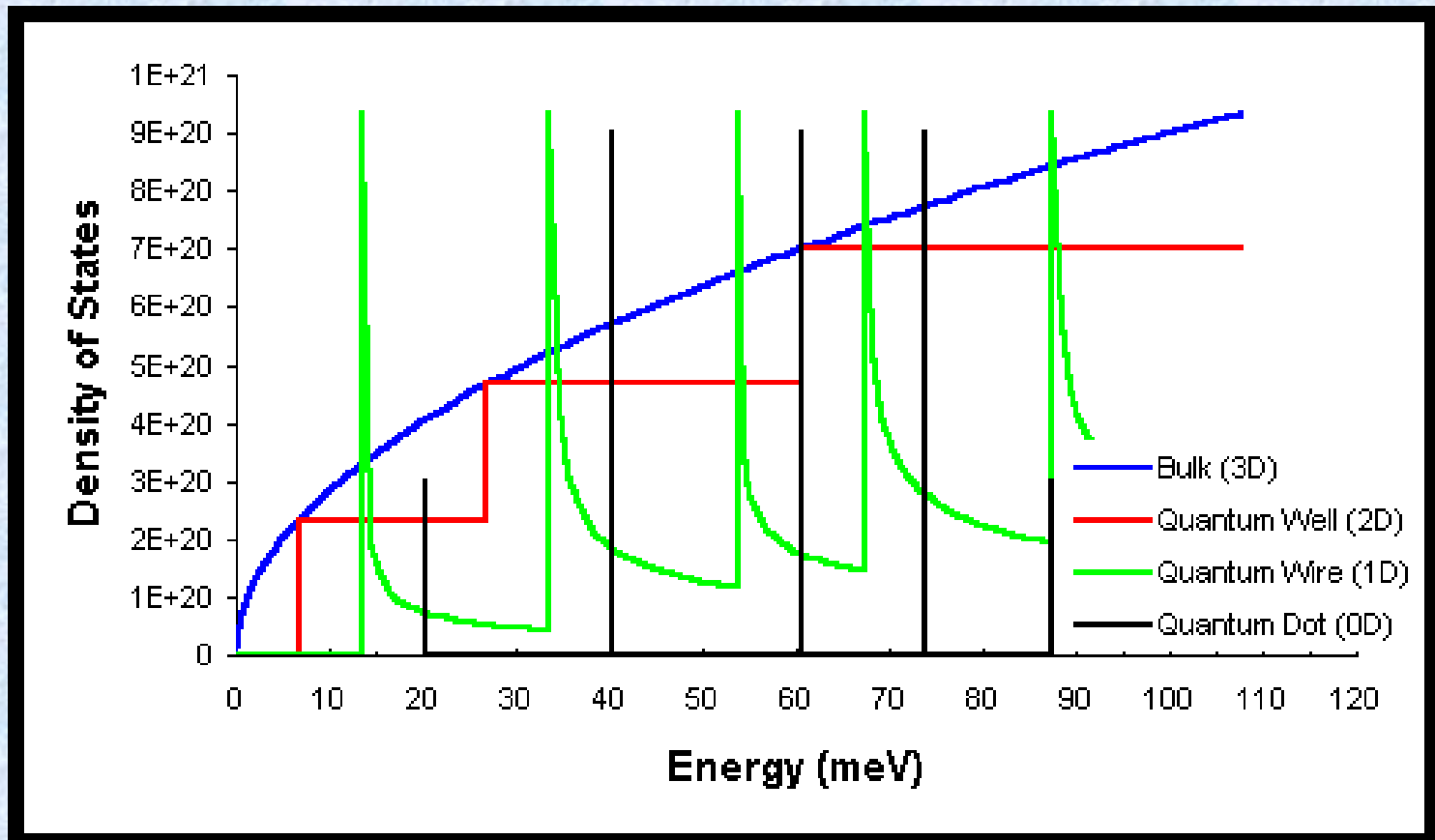
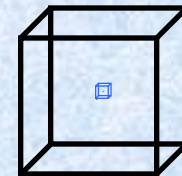
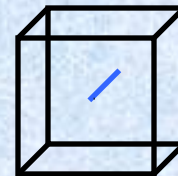
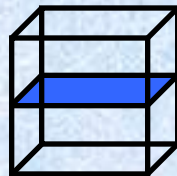
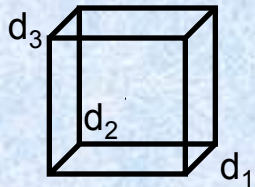
$R$  particle radius

$m_e$  mass of electron

$m_h$  mass of hole

# Density of states: bulk, quantum well, wire and dot

$$E = E_c + \frac{\hbar^2(n_1\pi/a_1)^2}{2m} + \frac{\hbar^2(n_2\pi/a_2)^2}{2m} + \frac{\hbar^2(n_3\pi/a_3)^2}{2m}$$



# Energy absorption

wide excitation spectra

vs.

narrow emission spectra

