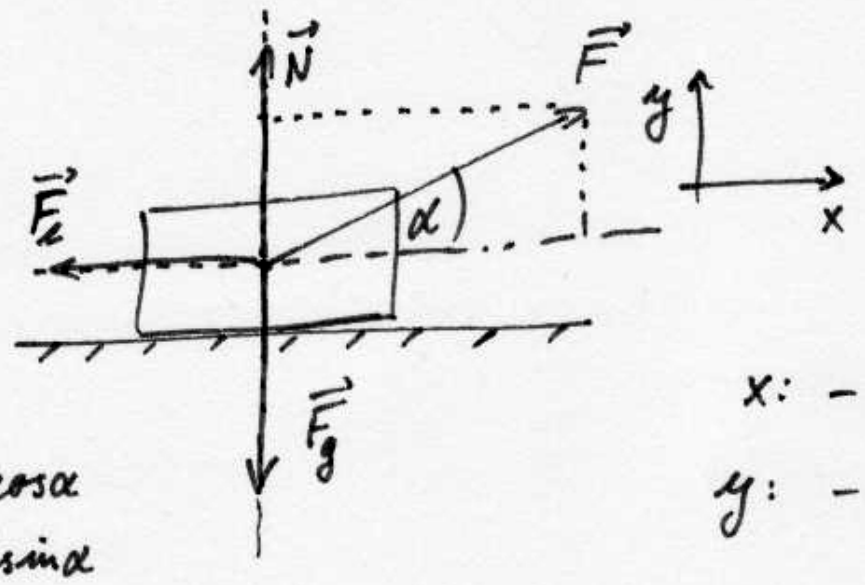
2. kápočtová písemka – A

1. $m = 1\,kg$
$F = 10\,N$
$\alpha = 60°$
$\mu = 0{,}58$
$a = ?$



$F_x = F\cos\alpha$
$F_y = F\sin\alpha$

$\vec{F} = (F\cos\alpha,\ F\sin\alpha)$
$\vec{F}_g = (0,\ -F_g)$
$\vec{N} = (0,\ N)$
$\vec{F}_t = (-F_t,\ 0)$

$F_t = \mu . N$,

$\vec{F} = m\vec{a}$:

$x: \ -F_t + F\cos\alpha = m a_x$

$y: \ -F_g + N - F\sin\alpha = m a_y$ ($a_y = 0$)

$\Rightarrow N = F_g - F\sin\alpha$

$\Rightarrow F_t = \mu(mg - F\sin\alpha)$

$\Rightarrow -\mu(mg - F\sin\alpha) + F\cos\alpha = m a_x$

$a_x = \dfrac{F(\cos\alpha - \mu\sin\alpha) - \mu mg}{m} = 4{,}22\,m/s^2$

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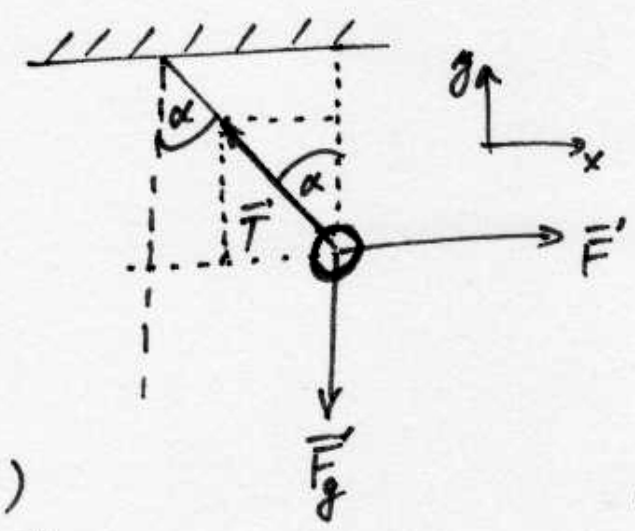
2. $m = 30\,g = 0{,}03\,kg$
$\alpha = 40°$
$F = ?$
$T = ?$



$T_x = T\sin\alpha$
$T_y = T\cos\alpha$

$\vec{F}' = (F,\ 0)$
$\vec{F}_g = (0,\ -F_g)$
$\vec{T} = (-T\sin\alpha,\ T\cos\alpha)$

$\vec{F} = m\vec{a}$:

$x: \ F - T\sin\alpha = \underbrace{m a_x}_{0}$

$y: \ -F_g + T\cos\alpha = \underbrace{m a_y}_{0}$

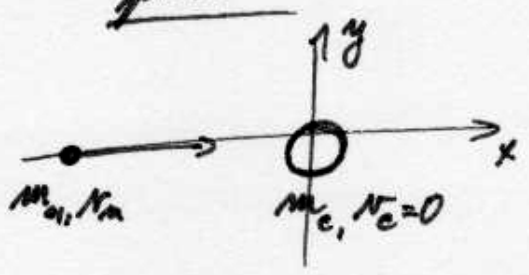
$\Rightarrow T\cos\alpha = mg \Rightarrow T = \dfrac{mg}{\cos\alpha} = 0{,}39\,N$

$\Rightarrow F = T\sin\alpha = \dfrac{mg}{\cos\alpha}\cdot\sin\alpha = mg\,tg\alpha = 0{,}25\,N$

---

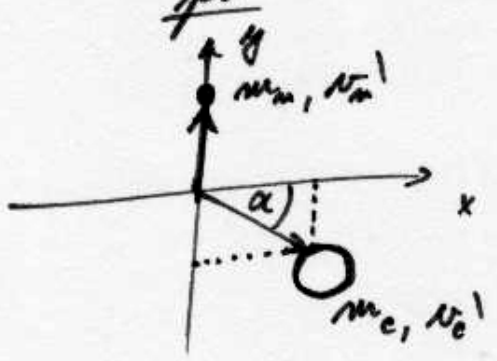
3. $m_m = 1{,}67 . 10^{-27}\,kg$
$v_m = 18000\,km/h = 5000\,m/s$
$m_c = 12 . m_m = 2 . 10^{-26}\,kg$
$v_c = 0\,m/s$
$v_m' = ?$
$v_c' = ?$

před:



⇒

po:



$\vec{p}_m = m_m\vec{v}_m = (m_m v_m,\ 0)$
$\vec{p}_{mc} = m_c v_c = (0,\ 0)$ } před

$\vec{p}_m' = m_m\vec{v}_m' = (0,\ m_m v_m')$
$\vec{p}_c' = m_c . v_c' = (m_c v_c'\cos\alpha,\ -m_c v_c'\sin\alpha)$ } po

(1)

ZZH: $\vec{p}_{před} = \vec{p}_{po}$:

$x: \quad m_m v_m = m_c v_c' \cos\alpha$

$y: \quad 0 = m_m v_m' - m_c v_c' \sin\alpha$

$\Rightarrow v_c' = \frac{m_m v_m}{m_c \cos\alpha} = 443\ m/s$

$v_m' = \frac{m_c v_c' \sin\alpha}{m_m} = \frac{m_c v_m \sin\alpha}{m_c m_m \cos\alpha} = v_m \operatorname{tg}\alpha = 1819\ m/s$

---

4. $m = 80\,g = 0{,}08\,kg$

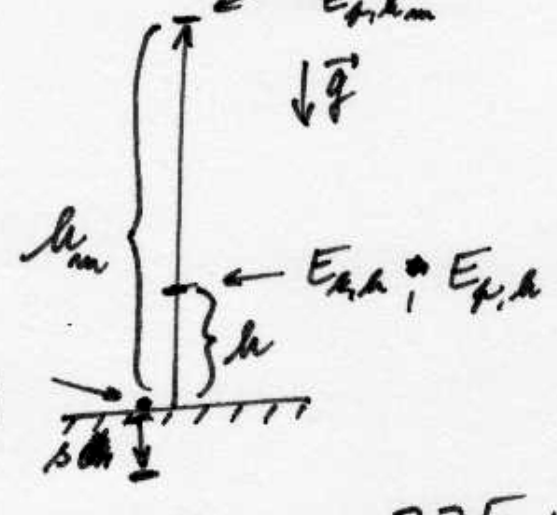
$h = 45\ m$

$E_{k,h} = 110\,J$

$F_{odp} = 600\,N$

$h_m = ?$

$s = ?$



$E_{k,h} = 110\,J$

$E_{p,h} = mgh$

$E_{k,h_m} = 0\,J$

$E_{p,h_m} = mgh_m$

ZZE: $E_{celk,1} = E_{celk,2}$:

$E_{k,h} + E_{p,h} = E_{k,h_m} + E_{p,h_m}$

$E_{k,h} + mgh = mgh_m$

$\Rightarrow h_m = \frac{E_{k,h} + mgh}{mg} = 182{,}5\ m$

- při dopadu je celková energie rovna kinetické, ta se celá přeměnní na práci odpor. sil:

$E_{celk,0} = E_{k,0} + \underbrace{E_{p,0}}_{0} = \underbrace{E_{k,h_m}}_{0} + E_{p,h_m}$

$\rightarrow E_{k,0} = mgh_m$

$\Delta E_k = W \quad ; \quad W = F.s$

$E_{k,0} - 0 = F_{odp} s \Rightarrow s = \frac{E_{k,0}}{F_{odp}} = \frac{mgh_m}{F_{odp}} = \frac{E_{k,h} + mgh}{F_{odp}} = 0{,}24\ m$

BONUS:

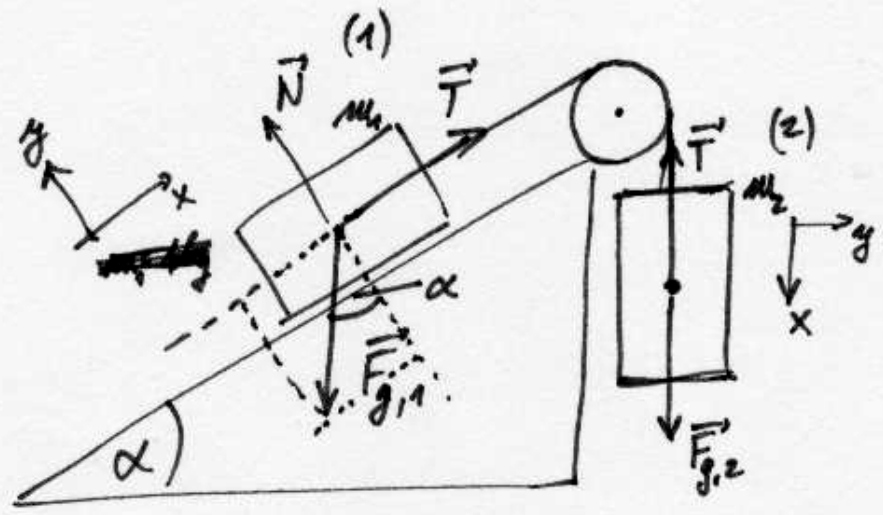
$m_1 = 1\,kg$

$m_2 = 2\,kg$

$\alpha = 30°$

$E_{k,celk} = ?$

$s = 25\,cm = 0{,}25\,m$



- osy volíme tak, aby pohyb jednoho tělesa v určitém směru přímo odpovídal pohybu druhého tělesa v témže směru (zde podél osy x)

$F_{g,1,x} = F_{g,1} \sin\alpha$

$F_{g,1,y} = F_{g,1} \cos\alpha$

Těleso (1):

$\vec{F}_{g,1} = (F_{g,1} \sin\alpha, -F_{g,1} \cos\alpha)$

$\vec{N} = (0, N)$

$\vec{T} = (T, 0)$

Těleso (2):

$\vec{F}_{g,2} = (F_{g,2}, 0)$

$\vec{T} = (-T, 0)$

(2)

(1): $\vec{F} = m_1 \vec{a}_1$ (2): $\vec{F} = m_2 \vec{a}_2$

těleso (1):

$x: -F_{g,1} \sin\alpha + T = m_1 a_{x,1}$

$y: -F_{g,1} \cos\alpha + N = \underbrace{m_1 a_{g,1}}_{0}$

$\Rightarrow N = m_1 g \cos\alpha$
(to nás ale nezajímá)

těleso (2):

$x: F_{g,2} - T = m_2 a_{x,2}$

$y: \quad 0 = \underbrace{m_2 a_{g,2}}_{0}$

- protože tělesa jsou spojená lanem, pohybují se stejně a platí $a_{x,1} = a_{x,2}$

$T = m_1 g \sin\alpha + m_1 a_{x,1} \longrightarrow m_2 g - m_1 g \sin\alpha - m_1 a_{x,1} = m_2 a_{x,2}$

$\Rightarrow g(m_2 - m_1 \sin\alpha) = (m_1 + m_2) \cdot a_{x,2}$

$$a_{x,2} = \frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \cdot g = 0{,}5\ m/s^2$$

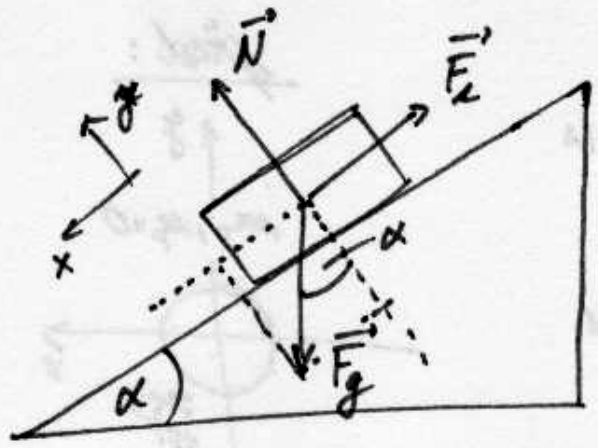
$s = \underbrace{v_0 t}_{0} + \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$

$v = \underbrace{v_0}_{0} + at \Rightarrow v = at = \sqrt{2s \cdot a} = 0{,}5\ m/s$

$E_{k,celk} = E_{k,1} + E_{k,2} = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 = \frac{1}{2}(m_1 + m_2) v^2 = 0{,}375\ J$

2. kápočtová písemka – B

1. $m = 400\,t = 400000\,kg$
$\alpha = 65°$
$\mu = 0{,}17$
$a = ?$



$F_t = \mu . N$
$F_{g,x} = F_g \sin\alpha$
$F_{g,y} = F_g \cos\alpha$

$\vec{F_g} = (F_g \sin\alpha , -F_g \cos\alpha)$
$\vec{F_t} = (-F_t , 0)$
$\vec{N} = (0 , N)$

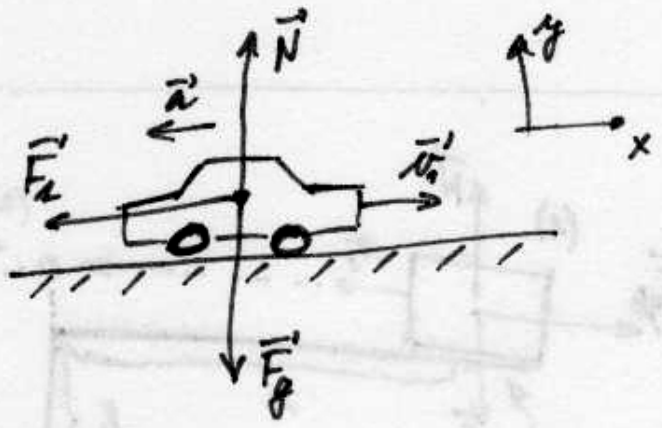
$\vec{F} = m\vec{a}$:

$x: F_g \sin\alpha - F_t = m a_x$
$y: -F_g \cos\alpha + N = \underbrace{m a_y}_{0}$

$\Rightarrow N = mg\cos\alpha$
$\Rightarrow F_t = \mu mg\cos\alpha$
$\Rightarrow mg\sin\alpha - \mu mg\cos\alpha = m a_x$
$a_x = g(\sin\alpha - \mu\cos\alpha) = 6{,}1\,m/s^2$

---

2. $m = 1{,}5\,t = 1500\,kg$
$v_1 = 60\,km/h = 16{,}6\,m/s$
$t_1 = 2{,}5\,s$
$\mu_1 = ?$
$\mu_2 = 0{,}85$
$v_2 = 80\,km/h = 22{,}\bar{2}\,m/s$
$t_2 = ?$



$F_t = \mu . N$
$\vec{F_g} = (0 , -F_g)$
$\vec{F_t} = (-F_t , 0)$
$\vec{N} = (0 , N)$

$\underbrace{v(t)}_{0} = v_1 - at \Rightarrow a = \frac{v_1}{t} = -a_x$

$\vec{F} = m\vec{a}$: $x: -F_t = m a_x$
$y: -F_g + N = \underbrace{m a_y}_{0} \Rightarrow N = mg$
$\Rightarrow F_t = \mu_1 mg$

$\Rightarrow -\mu_1 mg = -m\frac{v_1}{t}$
$\mu_1 = \frac{v_1}{gt} = 0{,}66$

$\underbrace{v(t)}_{0} = v_2 - at_2$, $-\mu_2 mg = -ma$
$t_2 = \frac{v_2}{a} = \frac{v_2}{\mu_2 g} = 2{,}61\,s$

3. $m_{Si} = 30 . m_m = 5 . 10^{-26}\,kg$

$v_{Si} = 500\,km/h = 138{,}8\,m/s$

$m_{e^+} = 1{,}67 . 10^{-27}\,kg$

$v_{e^+} = 7200\,km/h = 2000\,m/s$

$m_2 = ?$

$v_2 = 90\,000\,km/h = 25000\,m/s$

$\alpha = ?$

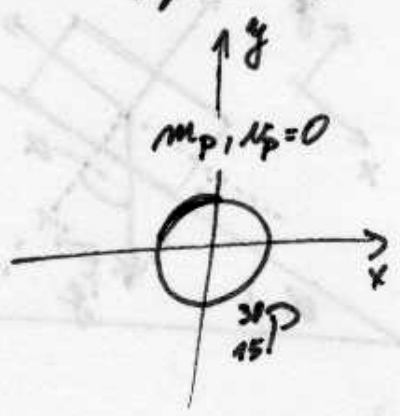
$\vec{p}_{Si} = m_{Si}\,\vec{v}_{Si} = (m_{Si} v_{Si},\ 0)$

$\vec{p}_{e^+} = m_{e^+} v_{e^+} = (0,\ m_{e^+} v_{e^+})$

$\vec{p}_2 = m_2\,\vec{v}_2 = (-m_2 v_2 \cos\alpha,\ -m_2 v_2 \sin\alpha)$

$\Rightarrow\ m_2 = \dfrac{m_{Si} v_{Si}}{v_2 \cos\alpha} = 2{,}78 . 10^{-28}\,kg$

před:



$\Rightarrow$

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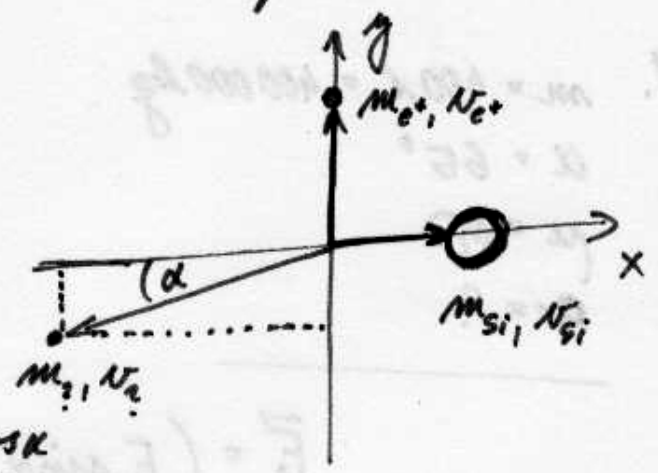


$p_{2,x} = p_2 \cdot \cos\alpha$

$p_{2,y} = p_2 \sin\alpha$

ZZH: $\vec{p}_{před} = \vec{p}_{po}$:

$x:\ 0 = m_{Si} v_{Si} - m_2 v_2 \cos\alpha$

$y:\ 0 = m_{e^+} v_{e^+} - m_2 v_2 \sin\alpha$

$m_{Si} v_{Si} = m_2 v_2 \cos\alpha$

$m_{e^+} v_{e^+} = m_2 v_2 \sin\alpha$) :

$\dfrac{m_{Si} v_{Si}}{m_{e^+} v_{e^+}} = \operatorname{cotg}\alpha \Rightarrow \alpha = (2{,}61 . 10^{-4})^\circ$

---

4. $m = 0{,}4\,kg$

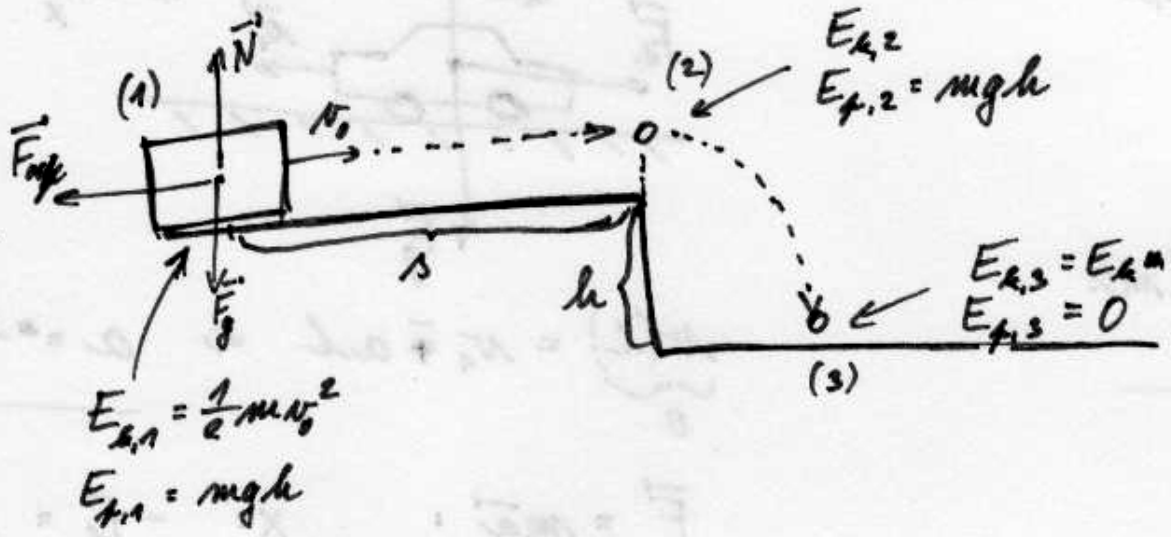
$v_0 = 6\,m/s$

$h = 5\,cm = 0{,}05\,m$

$E_k = 3\,J$

$F_{odp} = 2\,N$

$s = ?$



ZZE (2) $\rightarrow$ (3) : $E_{celk,2} = E_{celk,3}$:

$E_{k,2} + E_{p,2} = E_{k,3} + E_{p,3}$

$E_{k,2} + mgh = E_k$

$\Rightarrow E_{k,2} = E_k - mgh$

Práce odp. síly na úkor změny kinetické energie (1) $\rightarrow$ (2) : $W = F . s$, $\Delta E_k = E_{k,1} - E_{k,2}$

$W = \Delta E_k$:

$F . s = E_{k,1} - E_{k,2}$

$\Rightarrow s = \dfrac{E_{k,1} - E_{k,2}}{F} = \dfrac{\frac{1}{2} m v_0^2 - E_k + mgh}{F} = 2{,}2\,m$