

Intraspecific Interactions

“Populační ekologie živočichů“

Stano Pekár

Density-dependent growth

Discrete (difference) model

- ▶ logistic growth due to density dependent changes in fecundity and survival
- ▶ K .. carrying capacity, upper limit of population growth, where $\lambda = 1$
- ▶ change in λ depends on N

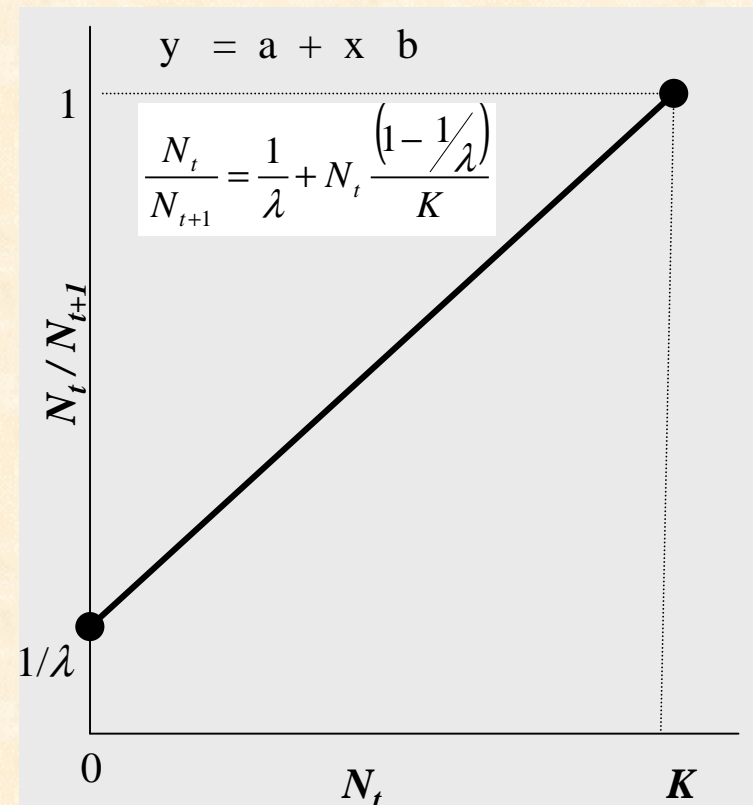
$$N_{t+1} = N_t \lambda$$

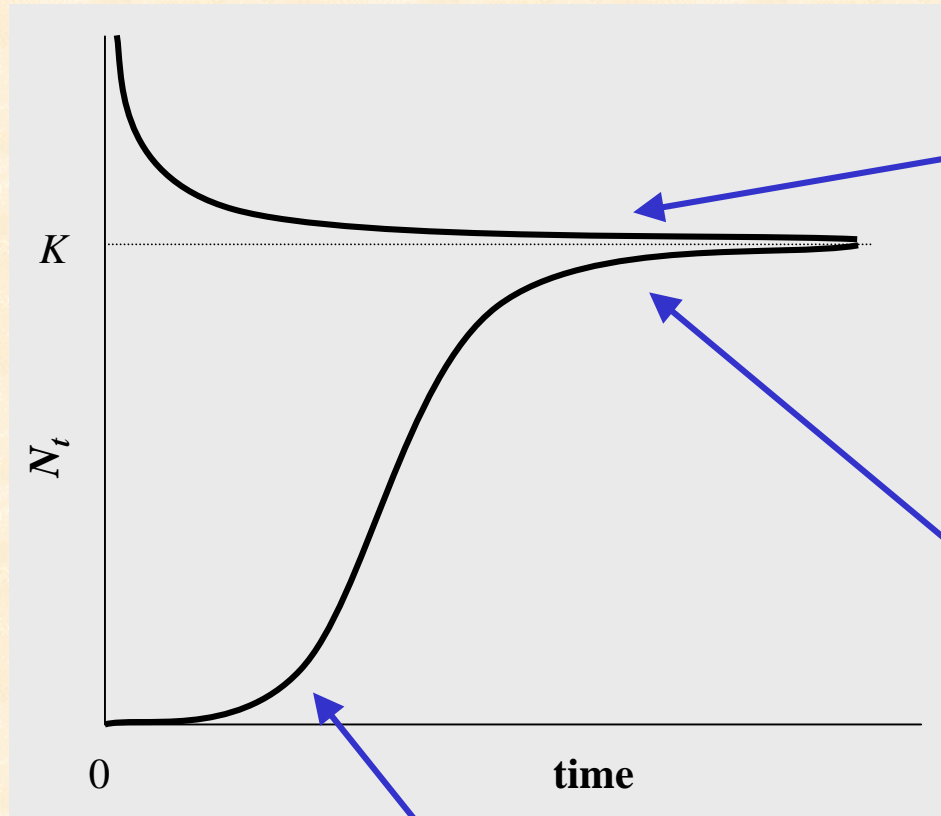
$$\frac{N_t}{N_{t+1}} = \frac{1}{\lambda}$$

$$N_{t+1} = \frac{N_t \lambda}{1 + \frac{(\lambda - 1)N_t}{K}}$$

if $a = \frac{\lambda - 1}{K}$ then

$$N_{t+1} = \frac{N_t \lambda}{1 + aN_t}$$





when $N_t > K$ then

$$\frac{\lambda}{1 + aN_t} < 1$$

- population returns to K

when $N_t \rightarrow K$ then

$$\frac{\lambda}{1 + aN_t} \approx 1$$

- density-dependent control
- S-shaped (sigmoid) growth

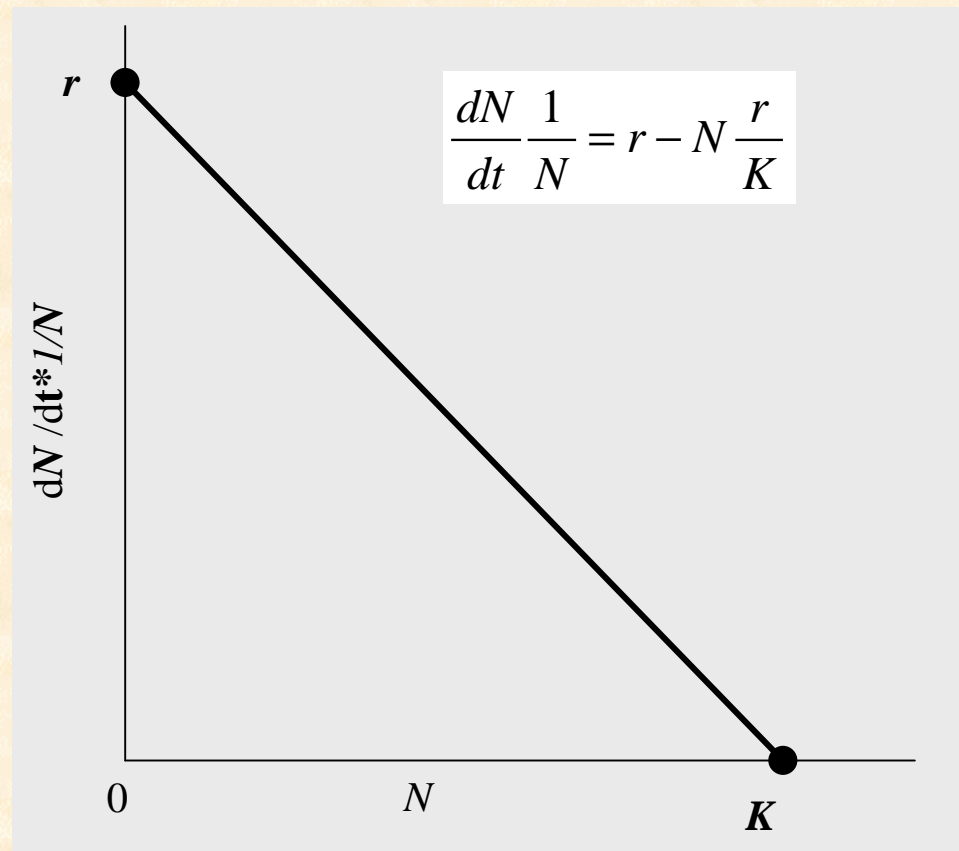
when $N_t \rightarrow 0$ then

$$\frac{\lambda}{1 + aN_t} \approx \lambda$$

- no competition
- exponential growth

Continuous (differential) model

- ▶ logistic growth
- ▶ first used by Verhulst (1838) to describe growth of human population



$$\frac{dN}{dt} = Nr \rightarrow \frac{dN}{dt} \frac{1}{N} = r$$

- when $N \rightarrow K$ then $r \rightarrow 0$

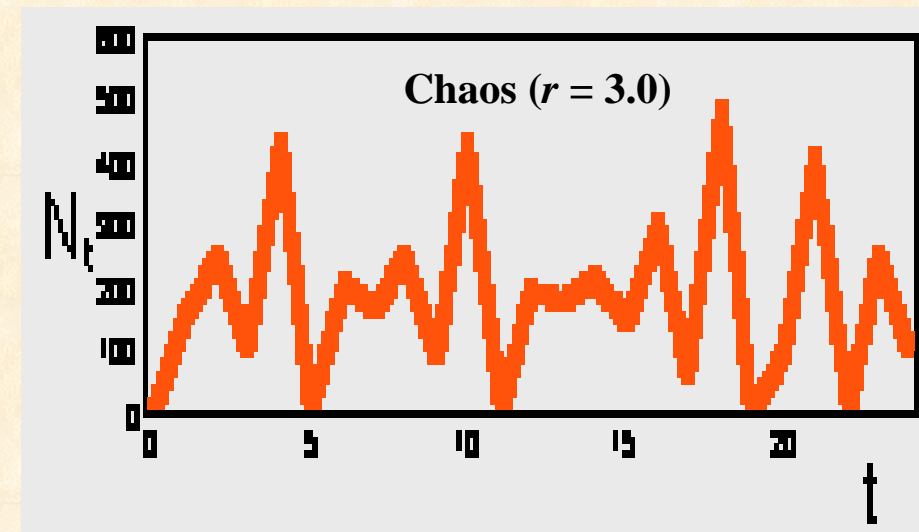
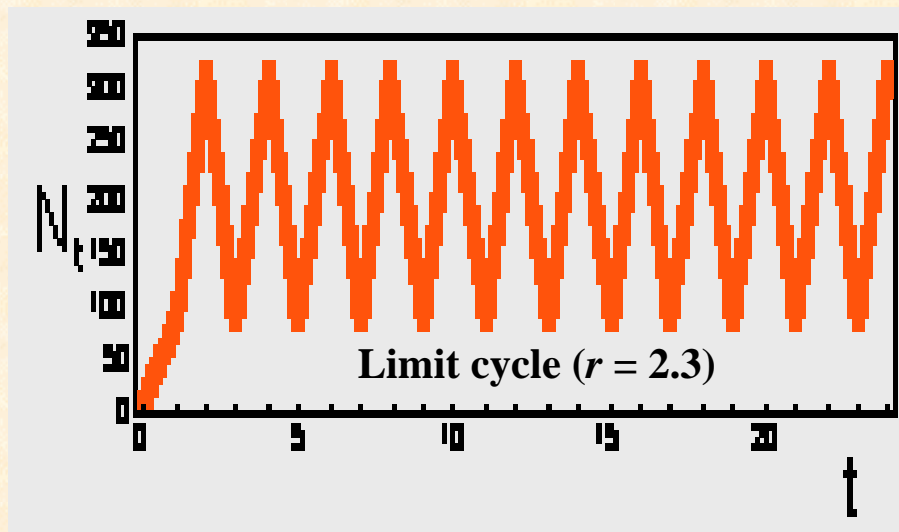
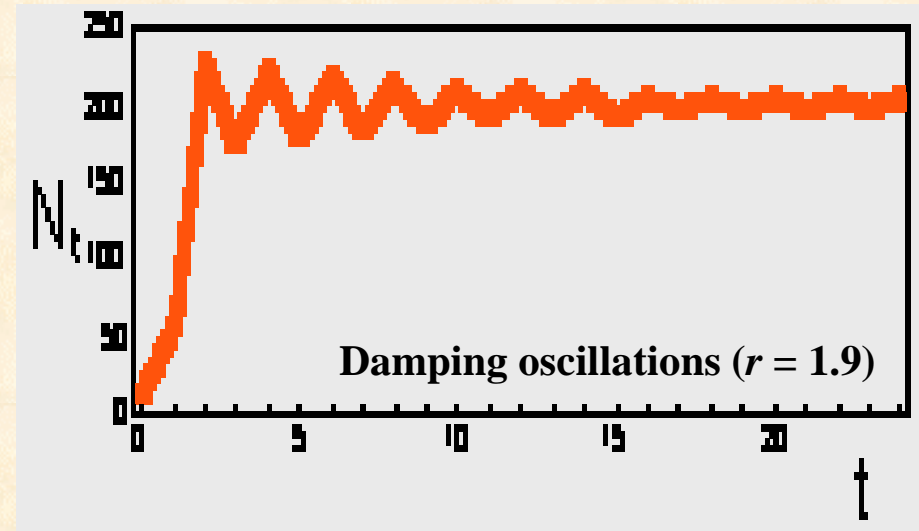
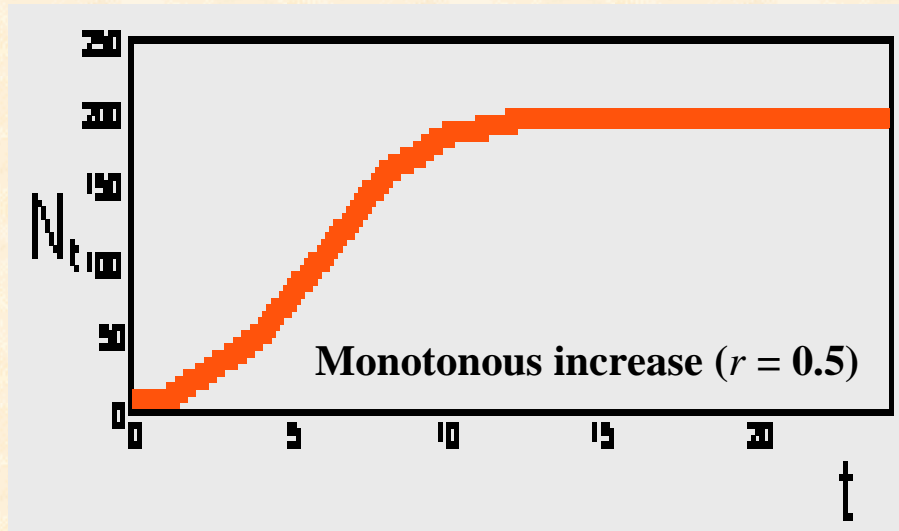
$$\frac{dN}{dt} = Nr \left(1 - \frac{N}{K} \right)$$

Solution of the differential equation

$$N_t = \frac{K}{1 + e^{a-rt}}$$

$$a = \ln \left(\frac{K - N_0}{N_0} \right)$$

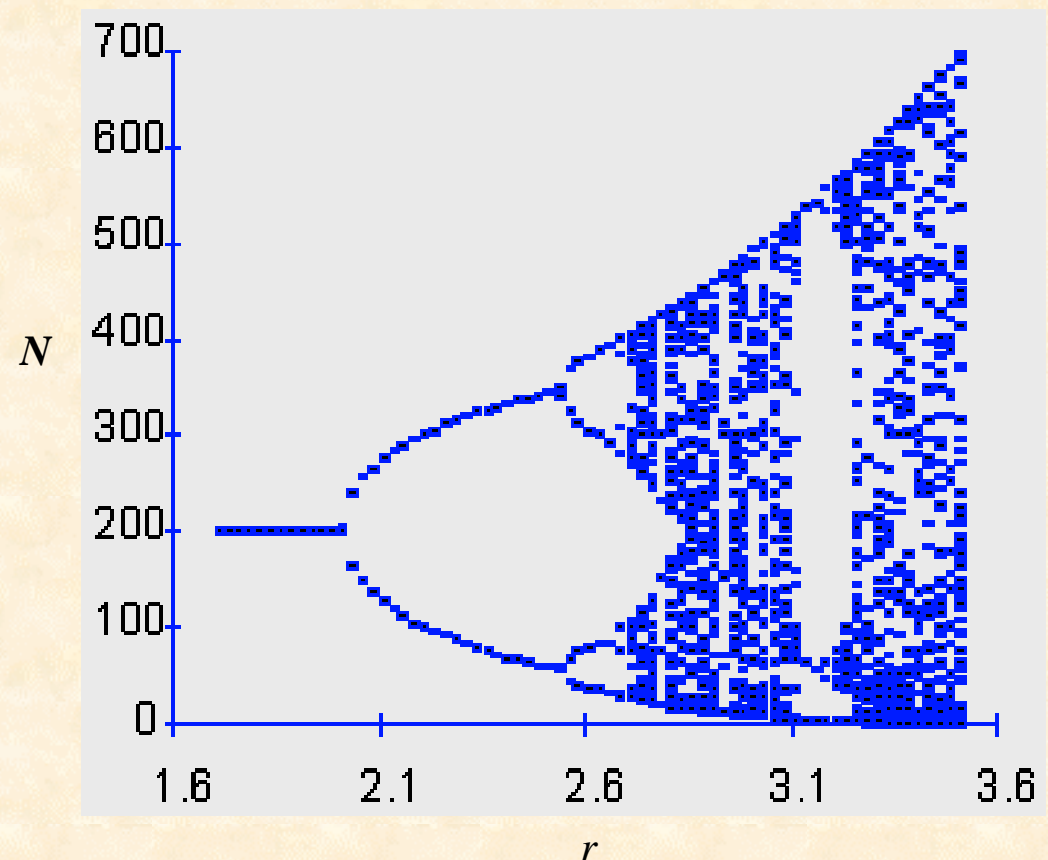
Examination of the logistic model



Model equilibria

1. $N = 0$.. unstable equilibrium
2. $N = K$.. stable equilibrium .. if $0 < r < 2$
 - ▶ “Monotonous increase” and “Damping oscillations” has a stable equilibrium
 - ▶ “Limit cycle” and “Chaos” has no equilibrium

- $r < 2$.. stable equilibrium
- $r = 2$.. 2-point limit cycle
- $r = 2.5$.. 4-point limit cycle
- $r = 2.692$.. chaos
 - ▶ chaos can be produced by deterministic process
 - ▶ density-dependence is stabilising only when r is rather low



Observed population dynamics

a) yeast (logistic curve)

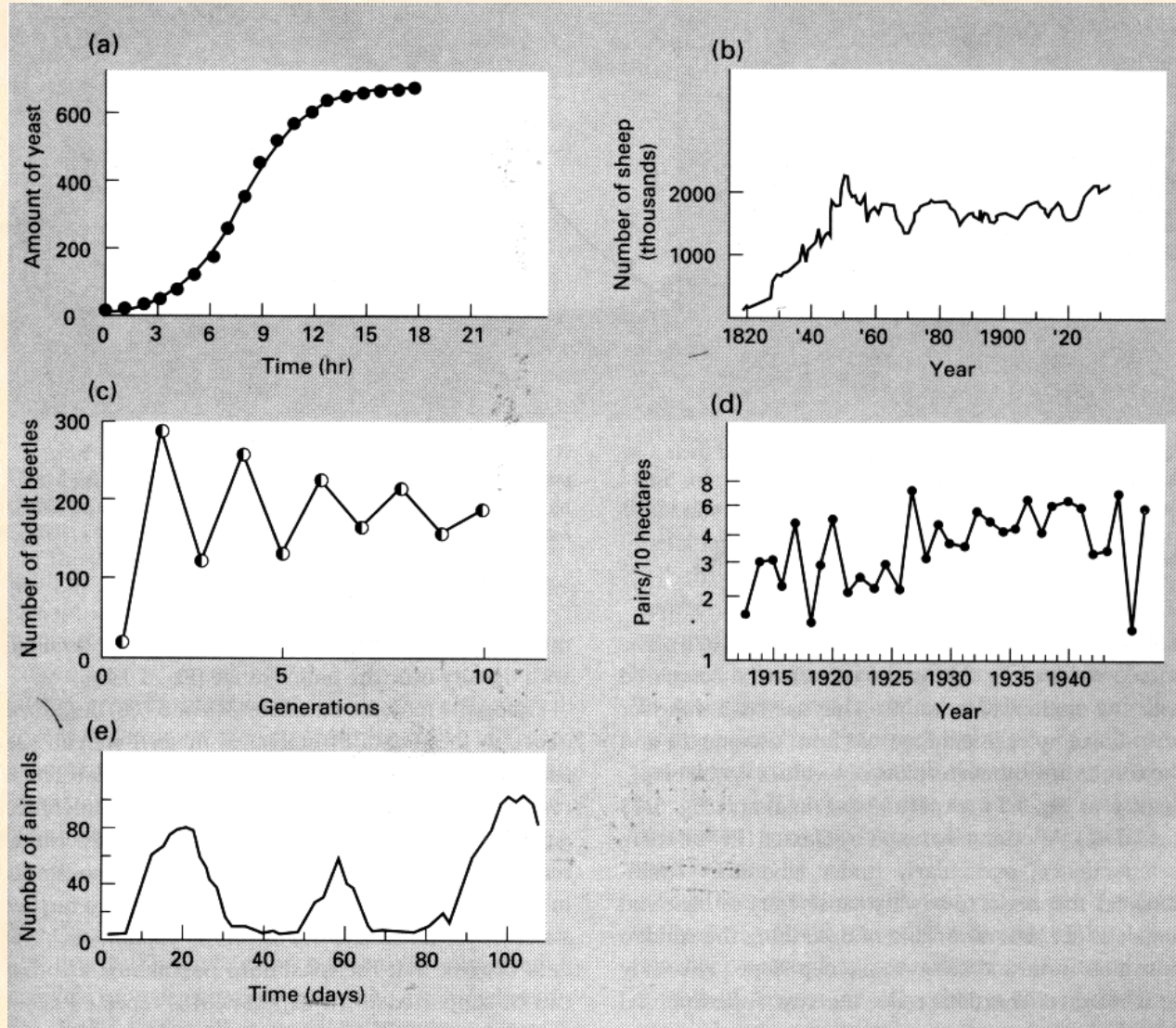
b) sheep (logistic curve with oscillations)

c) *Callosobruchus* (damping oscillations)

d) *Parus* (chaos)

e) *Daphnia*

▶ of 28 insect species in one species chaos was identified, one other showed limit cycles, all other were in stable equilibrium



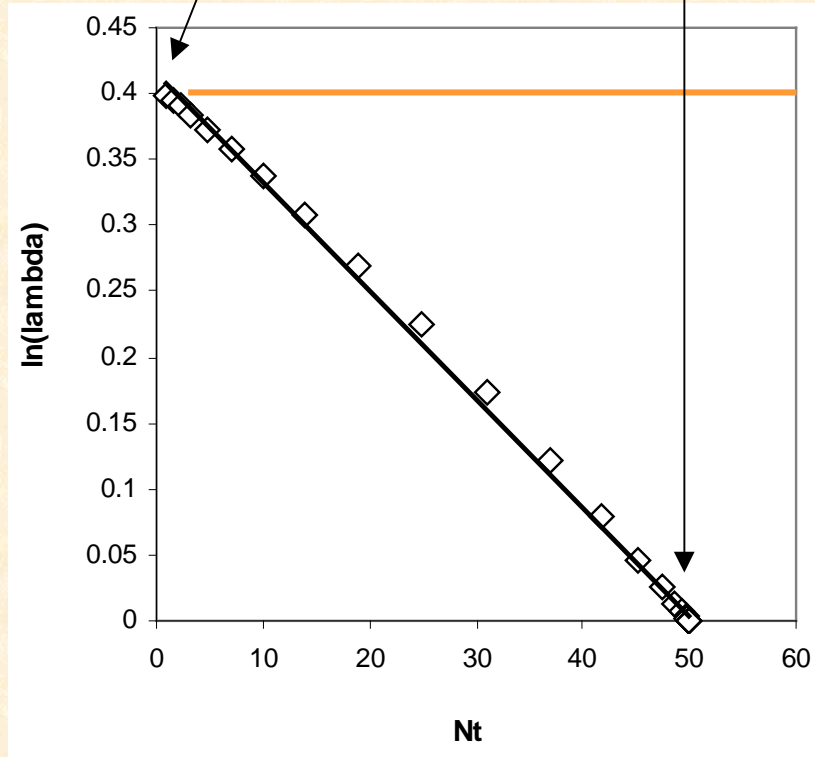
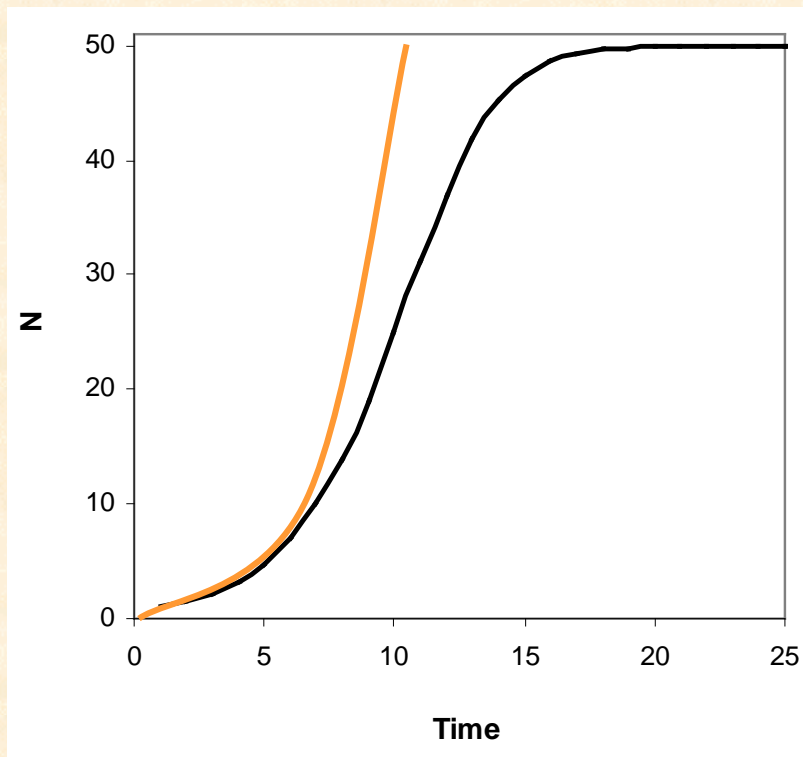
Estimation of lambda & K

- plot $\ln(\lambda)$ against N_t
- estimate λ and K using

$$\ln(\lambda) = a + bN_t$$

$$\lambda_{\max} = e^a$$

$$K = -\frac{a}{b}$$



General logistic model

- ▶ Hassell (1975) proposed general model for DD

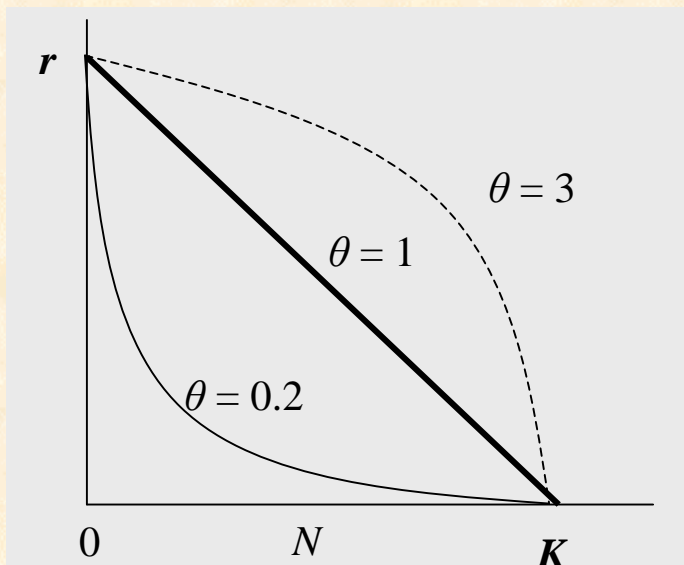
$$N_{t+1} = \frac{N_t \lambda}{(1 + aN_t)^\theta}$$

- where θ .. the strength of competition

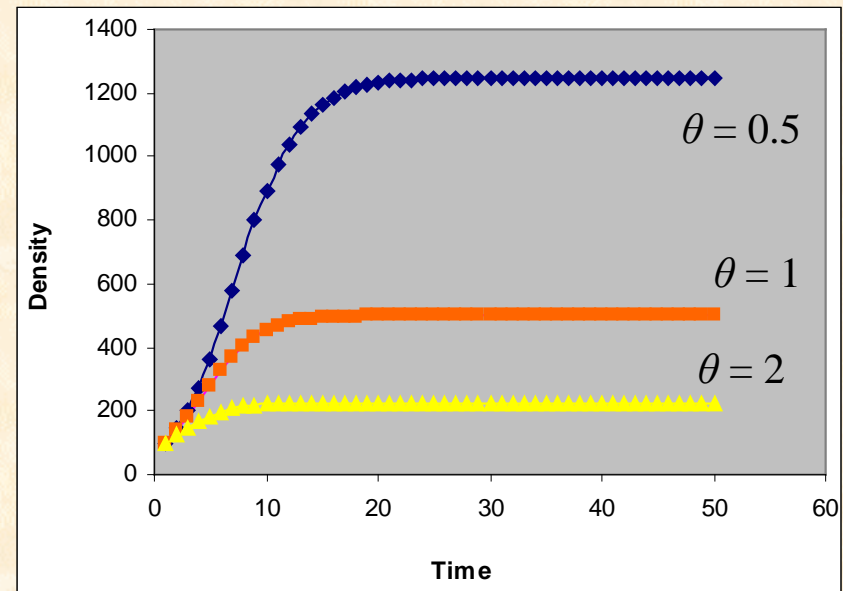
$\theta \gg 1$.. scramble competition (over-compensation)

$\theta = 1$.. contest competition (exact compensation)

$\theta < 1$.. under-compensation



Effect of θ on population density



Models with time-lags

- ▶ species response to resource change is not immediate but delayed due to maternal effect, seasonal effect
- ▶ appropriate for species with long generation time where reproductive rate is dependent on density of a previous generation
- ▶ time lag (d, τ) .. negative feedback of the 2nd order

discrete model

$$N_{t+1} = \frac{N_t \lambda}{1 + aN_{t-d}}$$

continuous model

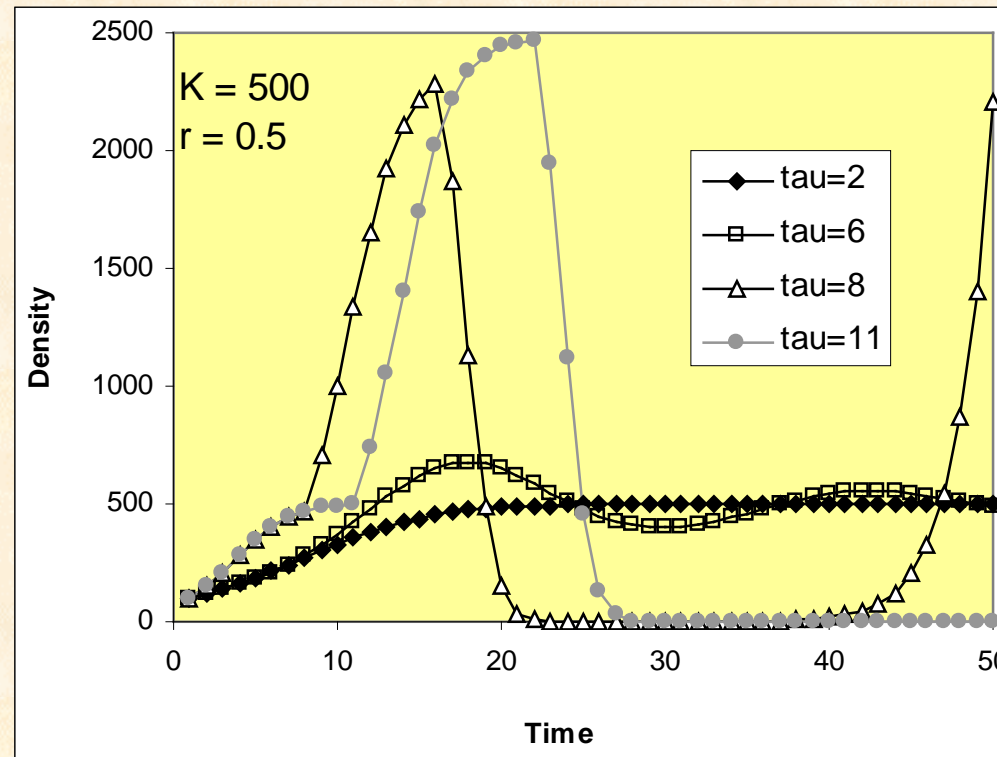
$$\frac{dN}{dt} = N_t r \frac{K - N_{t-\tau}}{K}$$

- ▶ many populations of mammals cycle with 3-4 year periods
- ▶ time-lag provokes fluctuations of certain amplitude at certain periods
- ▶ period of the cycle in continuous model is always 4τ

Solution of the continuous model:

$$N_{t+1} = N_t e^{r \left(1 - \frac{N_{t-\tau}}{K} \right)}$$

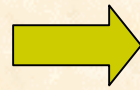
- $r \tau < 1 \rightarrow$ monotonous increase
- $r \tau < 3 \rightarrow$ damping fluctuations
- $r \tau < 4 \rightarrow$ limit cycle fluctuations
- $r \tau > 5 \rightarrow$ extinction



Harvesting

- ▶ to attain maximum sustainable yield (*MSY*)
- ▶ local maximum of the model for N

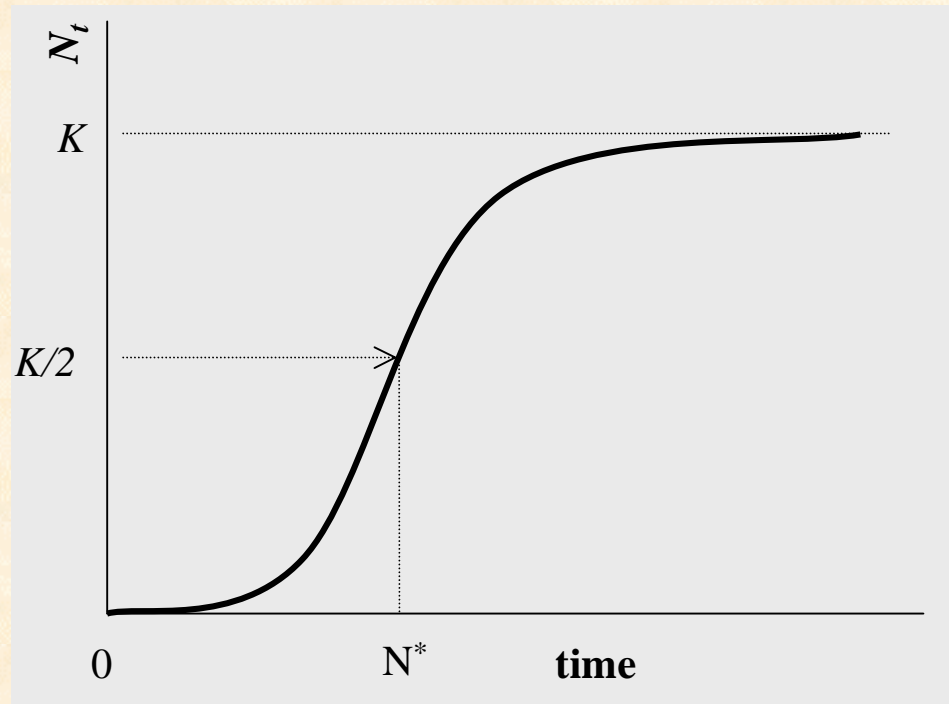
$$\frac{dN}{dt} = Nr \left(1 - \frac{N}{K} \right) = 0$$



$$N^* = \frac{K}{2}$$

$$MSY = a \left(\frac{\lambda K - K}{2} \right)$$

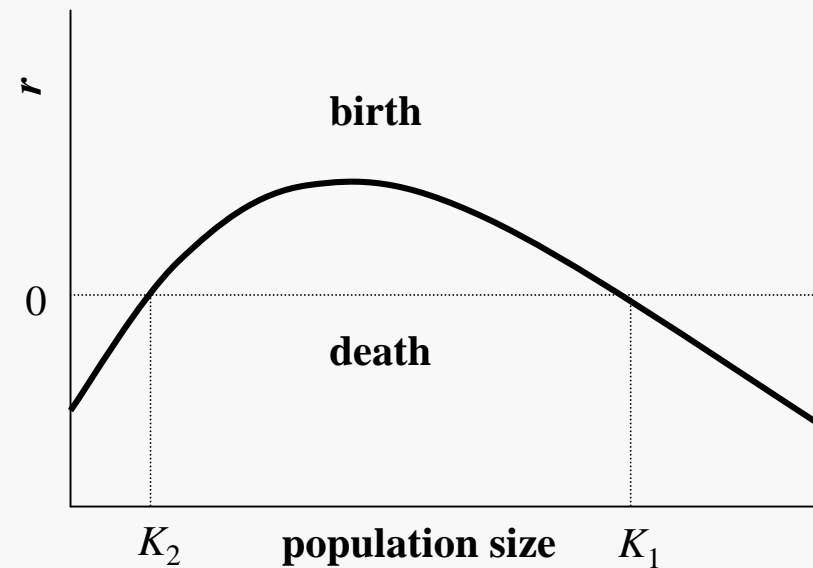
where $a = 0.6$ for $L < 5$
 $a = 0.4$ for $L = (5, 10)$
 $a = 0.2$ for $L > 10$



Alee effect

- ▶ K_2 .. extinction threshold, unstable equilibrium
- ▶ population increase is slow at low density but fast at high density

$$\frac{dN}{dt} = Nr \left(1 - \frac{N}{K_1} \right) \left(\frac{N}{K_2} - 1 \right)$$



Excercise 10

Simulate population dynamics using density-dependent model for discrete population growth for a period of 40 generations with $N_0=10$.

1. With deterministic $\lambda (=1.2)$ and $K (=500)$.
2. With stochastic $\lambda (=1.2 \pm 0.2)$ but deterministic $K (=500)$.
3. With stochastic $K (=500 \pm 50)$ but deterministic $\lambda (=1.2)$.
4. With stochastic $\lambda (=1.2 \pm 0.2)$ and $K (=500 \pm 50)$.


```
N<-41
for(t in 1:40) N[t+1]<-{
N[t]*1.2/(1+N[t]*(1.2-1)/500)}
plot(0:40,N,type="b")
```

```
for(t in 1:40) N[t+1]<-{
N[t]* runif(1,1,1.4)/(1+N[t]*(runif(1,1,1.4)-1)/500)}
plot(0:40,N,type="b")
```

```
for(t in 1:40) N[t+1]<-{
N[t]*1.2/(1+N[t]*(1.2-1)/runif(1,450,550))}
plot(0:40,N,type="b")
```

```
for(t in 1:40) N[t+1]<-{
N[t]* runif(1,1,1.4)/(1+N[t]*(runif(1,1,1.4)-
1)/runif(1,450,550))}
plot(0:40,N,type="b")
```

Excercise 11

You have observed the following population dynamic of yearly censuses of aphids:

180, 531, 277, 296, 828, 329, 397, 772, 625, 318, 567, 881, 386

1. Plot the population dynamic. Is there evidence for density-dependence?
3. Estimate λ_{\max} and K .

```
aphid<-c(180, 531, 277, 296, 828, 329, 397, 772, 625, 318, 567,  
881, 386)
```

```
plot(aphid,type="b")
```

```
lambda1<-aphid[-1]/aphid[-13]
```

```
plot(aphid[-13], log(lambda1))
```

```
m2<-lm(log(lambda1)~aphid[-13])
```

```
coef(m2)
```

```
abline(m2)
```

```
exp(1.3057703)
```

```
-1.30577030/-0.00248398
```


Excercise 12

On an African market wild game animals are sold. You know carrying capacities (K), finite growth rates (λ), and longevities (L) for each species:

	K	lambda	Longevity	Harvest
monkey	49000	1.17	31	781
pangolin	22000	2.01	13	192
porcupine	110000	1.82	23	1580
duiker	45000	1.63	7	732

1. Compute MSY for each species:
2. Is the observed harvest sustainable in each species?

$$0.2*(1.17*49000-49000)/2$$

$$0.2*(2.01*22000-22000)/2$$

$$0.2*(1.82*110000-110000)/2$$

$$0.4*(1.63*45000-45000)/2$$