



"Populační ekologie živočichů"

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## **Types of interactions**

- T		Effect of species 1 on fitness of species 2				
Effect of species 2 fitness of species		Increase	Neutral	Decrease		
	Increase	+ +				
	Neutral	0 +	0 0			
	Decrease	+ -	- 0			

- + + .. mutualism (plants and pollinators)
- 0 + .. commensalism (saprophytism, parasitism, phoresis)
- + .. predation (herbivory, parasitism), mimicry
- 0 .. amensalism (allelopathy)
- - .. competition

## Model of competition

based on the logistic model of Lotka (1925) and Volterra (1926)

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right)$$

▶ assumptions:

- all parameters are constant
- individuals of the same species are identical
- environment is homogenous, differentiation of niches is not possible
- only exact compensation is present

**species 1**:  $N_1$ ,  $K_1$ ,  $r_1$ **species 2**:  $N_2$ ,  $K_2$ ,  $r_2$ 

$$\frac{dN_1}{dt} = N_1 r_1 \left( 1 - \frac{N_1 + N_2}{K_1} \right)$$
$$\frac{dN_2}{dt} = N_2 r_2 \left( 1 - \frac{N_1 + N_2}{K_2} \right)$$

total competitive effect (intra + inter-specific)

 $(N_1 + \alpha N_2)$  where  $\alpha$ .. coefficient of competition  $\alpha = 0$ .. no interspecific competition

 $\alpha < 1$ .. species 2 has lower effect on species 1 than species 1 on itself  $\alpha = 0.5$ .. one individual of species 1 is equivalent to 0.5 individuals of species 2)

 $\alpha = 1$ ... both species has equal effect on the other one

 $\alpha > 1$ .. species 2 has greater effect on species 1 than species 1 on itself

species 1: 
$$\frac{dN_1}{dt} = N_1 r_1 \left( 1 - \frac{N_1 + \alpha_{12} N_2}{K_1} \right)$$
  
species 2: 
$$\frac{dN_2}{dt} = N_2 r_2 \left( 1 - \frac{\alpha_{21} N_1 + N_2}{K_2} \right)$$

• if competing species use the same resource then interspecific competition is equal to intraspecific

### Analysis of the model

• examination of the model behaviour on a phase plane

• used to describe change in any two variables in coupled differential equations by projecting orthogonal vectors

• identification of isoclines: a set of abundances for which the growth rate is 0







# ▶ species 1 $r_1N_1 (1 - [N_1 + \alpha_{12}N_2] / K_1) = 0$ $r_1N_1 ([K_1 - N_1 - \alpha_{12}N_2] / K_1) = 0$ if $r_1, N_1, K_1 = 0$ and if $K_1 - N_1 - \alpha_{12}N_2 = 0$ then $N_1 = K_1 - \alpha_{12}N_2$

if 
$$N_1 = 0$$
 then  $N_2 = K_1 / \alpha_{12}$   
if  $N_2 = 0$  then  $N_1 = K_1$ 

• species 2  $r_2 N_2 (1 - [N_2 + \alpha_{21} N_1] / K_2) = 0$  $N_2 = K_2 - \alpha_{21} N_1$ 

if 
$$N_2 = 0$$
 then  $N_1 = K_2 / \alpha_{21}$   
if  $N_1 = 0$  then  $N_2 = K_2$ 

## Isoclines



- above isocline  $i_1$  and below  $i_2$  competition is weak
- in-between  $i_1$  and  $i_2$  competition is strong

#### 1. Species 2 drives species 1 to extinction

K and α determine the model behaviour
disregarding initial densities species 2 (stronger competitor) will outcompete species 1 (weaker competitor)



$$K_1 = K_2$$
  $r_1 = r_2$   
 $\alpha_{12} > \alpha_{21}$   $N_{01} = N_{02}$ 



#### 2. Species 1 drives species 2 to extinction

species 1 (stronger competitor) will outcompete species 2 (weaker competitor)

$$K_1 > \frac{K_2}{\alpha_{21}}$$
  $K_2 < \frac{K_1}{\alpha_{12}}$ 

$$r_1 = r_2$$
  $K_1 = K_2$   
 $N_{01} = N_{02}$   $\alpha_{12} < \alpha_{21}$ 



#### **3. Stable coexistence of species**

• disregarding initial densities both species will coexist at stable equilibrium (where isoclines cross)

- ▶ at at equilibrium population density of both species is reduced
- both species are weak competitors





## Test of the model

• when *Rhizopertha* and *Oryzaephilus* were reared separately both species increased to 420-450 individuals (= K)

• when reared together *Rhizopertha* reached  $K_1 = 360$ , while *Oryzaephilus*  $K_2 = 150$  individuals

• combination resulted in more efficient conversion of grain ( $K_{12} = 510$  individuals)

 three combinations of densities converged to the same stable equilibrium

prediction of
 Lotka-Volterra model is correct



Crombie (1947)

## Model for discrete generations

solution of the differential model:

$$N_{1,t+1} = N_{1,t} e^{r_1 \left(\frac{K_1 - N_{1,t} - \alpha_{12}N_{2,t}}{K_1}\right)} N_{2,t+1} = N_{2,t} e^{r_2 \left(\frac{K_2 - N_{2,t} - \alpha_{21}N_{1,t}}{K_2}\right)}$$

 multiple regression analysis is used to estimate parameters from abundances

$$\ln\left(\frac{N_{1,t+1}}{N_{1,t}}\right) = r_1 - N_{1,t} \frac{r_1}{K_1} - N_{2,t} \frac{r_1 \alpha_{12}}{K_1} \left| \ln\left(\frac{N_{2,t+1}}{N_{2,t}}\right) = r_2 - N_{1,t} \frac{r_2}{K_2} - N_{1,t} \frac{r_2 \alpha_{21}}{K_2} \right| \\ \ln\left(\frac{N_{1,t+1}}{N_{1,t}}\right) = a + bN_{1,t} + cN_{2,t} \left| \ln\left(\frac{N_{2,t+1}}{N_{2,t}}\right) = a + bN_{2,t} + cN_{1,t} \right|$$

$$r = a$$
  $\alpha = -\frac{Kc}{r}$   $K = -\frac{r}{b}$ 

## Excercise 16

Two species of *Tribolium* beetles were kept together in a jar with flour. The species breed in discrete periods. Their densities were recorded once a week. The following abundances were observed:

A: 10, 6, 5, 4, 3, 4, 6, 8, 10, 12, 15, 16 B: 20, 18, 16, 11, 6, 6, 5, 3, 2, 2, 1, 1

1. Estimate  $r_1, r_2, K_1, K_2, \alpha_{12}, \alpha_{21}$ .

2. Simulate the dynamics using difference model system for a period of 20 years. Use estimated values of parameters and initial densities of 20 individuals.

```
a<-c(10,6,5,4,3,4,6,8,10,12,15,16)
b<-c(20,18,16,11,6,6,5,3,2,2,1,1)
a1<-a[-1]/a[-12]
b1<-b[-1]/b[-12]</pre>
```

coef(lm(log(a1)~a[-12]+b[-12]))
0.60443/0.02992
20.20154\*0.04106/0.60443

```
coef(lm(log(b1)~b[-12]+a[-12]))
0.399980/0.005052
79.1726*0.011438/0.399980
```

```
N12<-data.frame(N1<-numeric(1:20),N2<-numeric(1:20))
N12[,1]<-20
N12[,2]<-20
for(t in 1:20) N12[t+1,]<-{
N1<-N12[t,1]*exp(0.6*(20.2-N12[t,1]-1.4*N12[t,2])/20.2)
N2<-N12[t,2]*exp(-0.4*(79.2-N12[t,2]-2.3*N12[t,1])/79.2)
c(N1,N2)}
matplot(N12, type="l",lty=1:2)
legend(1,80,c("N1","N2"),lty=1:2)</pre>
```



Two species of spiders, *Pardosa* and *Pachygnatha*, occur together and were found to feed in the field on the following prey:

Druh	Collembola	He mipte ra	Ensife ra	Dipte ra	Isopoda
Pardosa	0.61	0.15	0.12	0.07	0.05
Pachygnatha	0.93	0.05	0.01	0	0.01

- 1. Estimate and plot niche breadth (D) for each species.
- 2. Estimate niche overlap  $(a_{12}, a_{21})$  for each species.

$$D = \frac{1}{\sum_{k=1}^{n} p_k^2} \qquad a_{12} = \frac{\sum p_{1k} p_{2k}}{\sum p_{1k}^2} \qquad a_{21} = \frac{\sum p_{1k} p_{2k}}{\sum p_{2k}^2}$$

```
Par<-c(0.61,0.15,0.12,0.07,0.05)
Pach<-c(0.93,0.05,0.01,0,0.01)
both<-rbind(Par,Pach)
barplot(both,beside=T,legend.text=c("Par","Pach"))
1/sum(Par^2)
1/sum(Pach^2)</pre>
```

a12<-sum(Par\*Pach)/sum(Par^2); a12
a21<-sum(Par\*Pach)/sum(Pach^2); a21</pre>

## Excercise 18

An invasive ant species is spreading and may replace a native ant species as both have similar niches. The following parameters are know for the native (1) and invasive (2) species.

$r_1 = 0.2$	$r_2 = 0.9$
$K_1 = 200$	$K_2 = 300$
$\alpha_{12} = 1.1$	$\alpha_{21} = 0.7$

Simulate the population dynamic using differential model system for the period of 30 years. The initial densities are  $N_{01}$ =200 and  $N_{02}$ =10.

#### How to achieve stable coexistence?



```
comp<-function(t,y,param){
N1<-y[1]
N2<-y[2]
with(as.list(param),{
dN1.dt<-r1*N1*(1-(N1+a12*N2)/K1)
dN2.dt<-r2*N2*(1-(N2+a21*N1)/K2)
return(list(c(dN1.dt,dN2.dt)))})</pre>
```

```
N1<-200;N2<-10
param<-c(r1=0.2,r2=0.9,a12=1.1,a21=0.7,K1=200,K2=300)
time<-seq(0,30,0.1)
library(deSolve)
out<-data.frame(ode(c(N1,N2),time,comp,param))
matplot(time,out[,-1],type="l",lty=1:2,col=1)
legend("right",c("N1","N2"),lty=1:2)</pre>
```

```
N1<-200;N2<-10
param<-c(r1=0.2,r2=0.9,a12=0.5,a21=0.7,K1=200,K2=300)
time<-seq(0,30,0.1)
library(deSolve)
out<-data.frame(ode(c(N1,N2),time,comp,param))
matplot(time,out[,-1],type="1",lty=1:2,col=1)
legend("right",c("N1","N2"),lty=1:2)</pre>
```