

MODULARIZACE VÝUKY EVOLUČNÍ A EKOLOGICKÉ BIOLOGIE CZ.1.07/2.2.00/15.0204







INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Density-dependent growth

▶ includes all mechanisms of population growth that change with density

- population structure is ignored
- extrinsic effects are negligible
- response of r to N is immediate

▶ r decreases with population density either because natality decreases or mortality increases or both

- *K* .. carrying capacity
 upper limit of population growth where λ = 1 or r = 0
- is constant

Discrete (difference) model

- there is linear dependence of λ on N

$$N_{t+1} = N_t \lambda$$
 $\frac{N_t}{N_{t+1}} = \frac{1}{\lambda}$



$$N_{t+1} = \frac{N_t \lambda}{1 + \frac{(\lambda - 1)N_t}{K}}$$

if
$$a = \frac{\lambda - 1}{K}$$
 then

$$N_{t+1} = \frac{N_t \lambda}{1 + a N_t}$$



Continuous (differential) model

- logistic growth
- first used by Verhulst (1838) to describe growth of human population
- there is linear dependence of r on N



$$\frac{dN}{dt} = Nr \quad \rightarrow \quad \frac{dN}{dt} \frac{1}{N} = r$$

- when
$$N \to K$$
 then $r \to 0$

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right)$$

Solution of the differential equation

$$N_{t} = \frac{KN_{0}}{(K - N_{0})e^{-rt} + N_{0}}$$

Examination of the logistic model



Model equilibria

1. N = 0 .. unstable equilibrium

2. N = K .. stable equilibrium .. if 0 < r < 2

* "Monotonous increase" and "Damping oscillations" has a stable equilibrium

"Limit cycle" and "Chaos"
 has no equilibrium

r < 2 .. stable equilibrium r = 2 .. 2-point limit cycle r = 2.5 .. 4-point limit cycle r = 2.692 .. chaos

 chaos can be produced by deterministic process

 density-dependence is stabilising only when
 r is rather low



Observed population dynamics

a) yeast (logistic curve)
b) sheep (logistic curve with oscillations)
c) *Callosobruchus* (damping oscillations)
d) *Parus* (chaos)
e) *Daphnia*

of 28 insect species
 in one species chaos
 was identified, one
 other showed limit
 cycles, all other were in
 stable equilibrium



Estimation of lambda & K

plot ln(λ) against N_t
estimate λ and K using



General logistic model

Hassell (1975) proposed general model for DD *r* is not linearly dependent on N

where θ.. the strength of competition
θ >> 1 .. scramble competition (over-compensation)
strong DD, leads to fluctuations, oscillations around K

 $\theta = 1$.. contest competition (exact compensation) - stable density

 $\theta < 1$.. under-compensation - weak DD, population will return to K



 $N_{t+1} = \frac{N_t \lambda}{(1+aN_t)^{\theta}}$

Models with time-lags

▶ species response to resource change is not immediate but delayed due to maternal effect, seasonal effect, predator pressure

• appropriate for species with long generation time where reproductive rate is dependent on density of a previous generation

• time lag (d or τ) .. negative feedback of the 2nd order

discrete model

continuous model

$$N_{t+1} = \frac{N_t \lambda}{1 + a N_{t-d}}$$

$$\frac{dN}{dt} = N_t r \left(1 - \frac{N_{t-\tau}}{K} \right)$$

- many populations of mammals cycle with 3-4 year periods
- time-lag provokes fluctuations of certain amplitude at certain periods
- period of the cycle in continuous model is always 4τ

Solution of the continuous model:

$$N_{t+1} = N_t e^{r\left(1 - \frac{N_{t-\tau}}{K}\right)}$$

 $r \tau < 1 \rightarrow$ monotonous increase $r \tau < 3 \rightarrow$ damping fluctuations $r \tau < 4 \rightarrow$ limit cycle fluctuations $r \tau > 5 \rightarrow$ extinction



Harvesting

▶ Maximum Sustainable Harvest (*MSH*)

- to harvest as much as possible with the least negative effect on N
- ignore population structure
- ignore stochasticity

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K}\right) = 0$$



$$N = \frac{K}{2}$$

$$MSH = \frac{\pi K}{4}$$

Robinson & Redford (1991) Maximum Sustainable Yield (*MSY*)

$$MSY = a\left(\frac{\lambda K - K}{2}\right)$$

where a = 0.6 for longevity < 5 a = 0.4 for longevity = (5,10) a = 0.2 for longevity > 10

- Surplus production (catch-effort) models
- when r/λ and K are not known
- effort and catch over several years is known
- Schaefer quadratic model
- local maximum of the function identifies optimal effort



Alee effect

individuals in a population may cooperate in hunting, breeding – positive effect on population increase

- ▶ Allee (1931) discovered inverse DD
- genetic inbreeding decrease in fertility
- demographic stochasticity biased sex ratio
- small groups cooperation in foraging, defence, mating, thermoregulation
- K₂.. extinction threshold,
 unstable equilibrium
 population increase is slow
 at low density but fast at higher density

$$\frac{dN}{dt} = Nr\left(1 - \frac{N}{K_1}\right)\left(\frac{N}{K_2} - 1\right)$$

