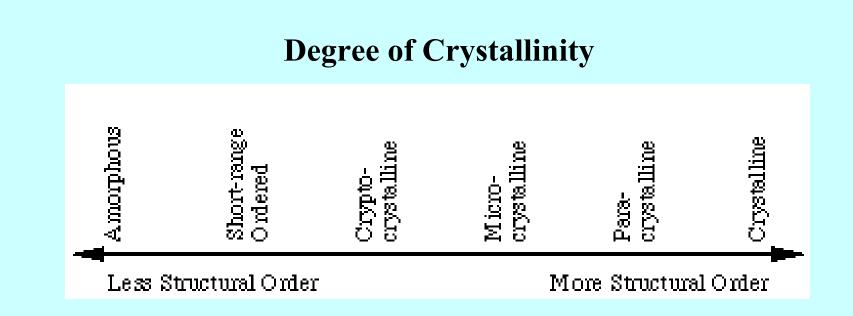
### **Basic Structural Chemistry**

**Crystalline state** 

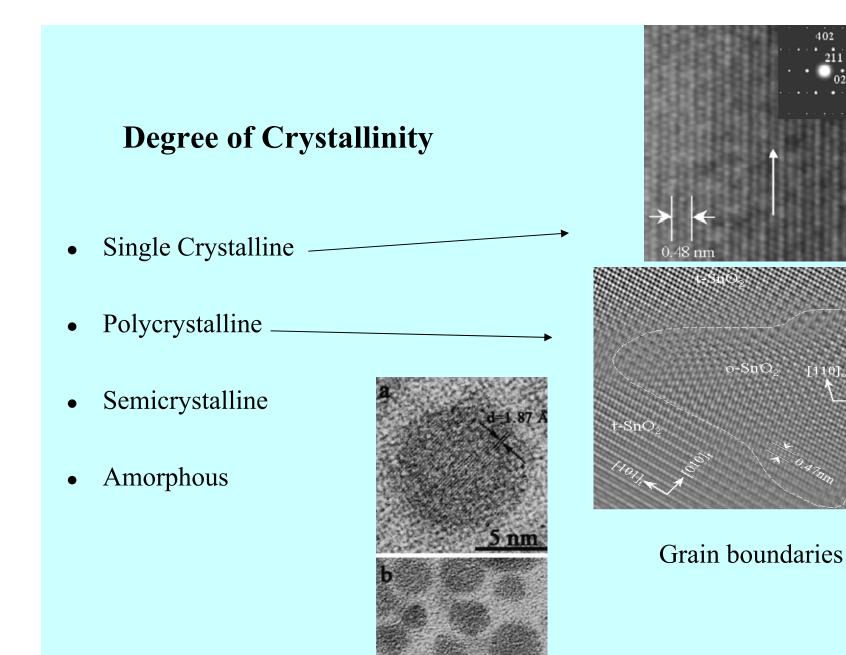
**Structure types** 



Crystalline – 3D long range order Single-crystalline Polycrystalline - many crystallites of different sizes and orientations (random, oriented)

Paracrystalline - short and medium range order, lacking long range order

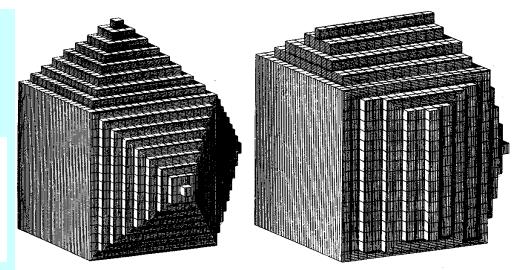
Amorphous – no order, random



t-SnO

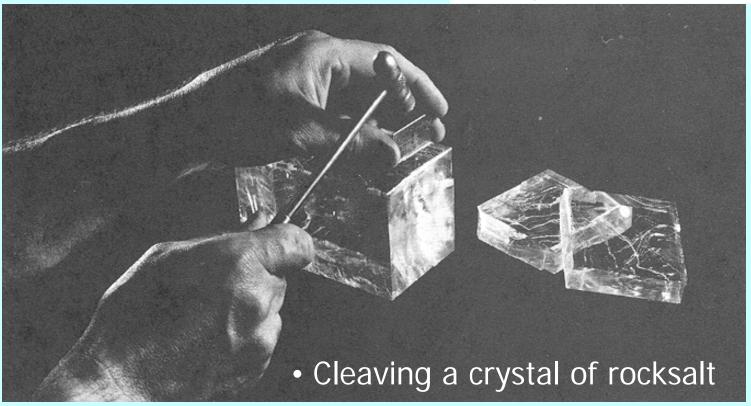
### **Crystal Structure**

•The building blocks of these two are identical, but different crystal faces are developed



(a)

(b)



## Crystals

- Crystal consist of a periodic arrangement of structural motifs = building blocks
- Building block is called *a basis*: an atom, a molecule, or a group of atoms or molecules
- Such a periodic arrangement must have translational symmetry such that if you move a building block by a distance:

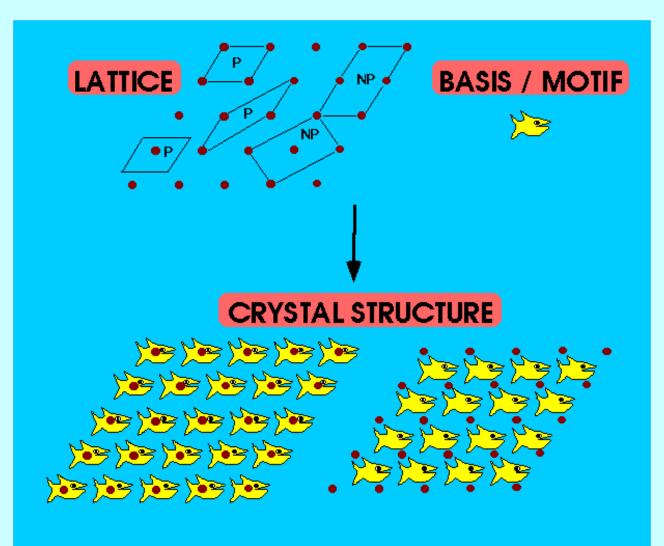
 $\overline{T} = n_1 \overline{a} + n_2 \overline{b} + n_3 \overline{c}$ 

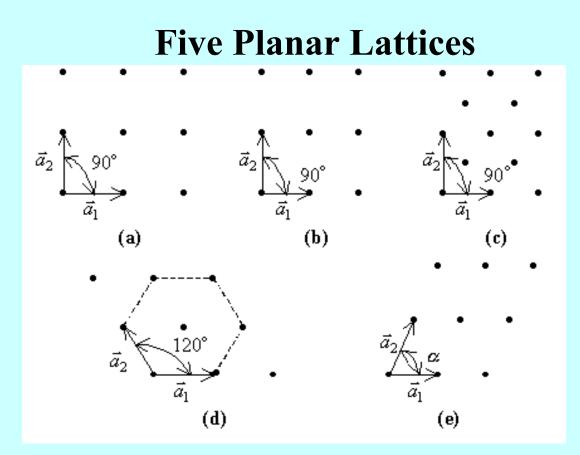
where  $n_1, n_2$ , and  $n_3$  are integers, and  $\overline{a}, \overline{b}, \overline{c}$  are vectors.

then it falls on another identical building block with the same orientation.

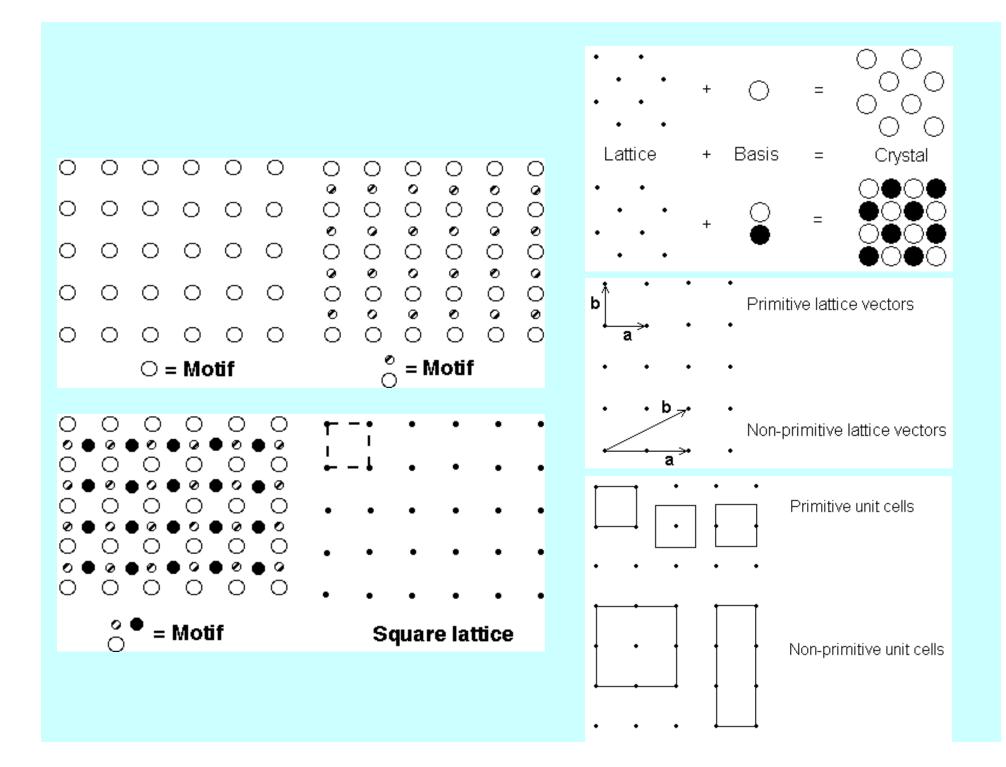
• If we remove the building blocks and replace them with points, then we have *a point lattice* or Bravais lattice.

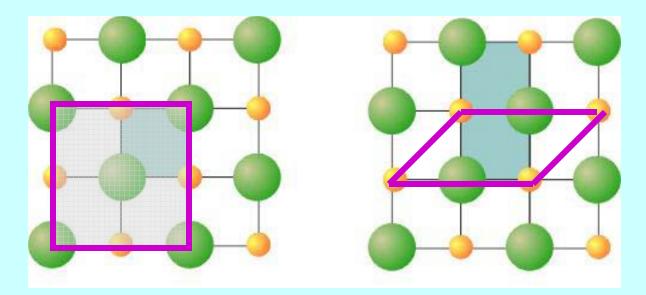
#### **Planar Lattice 2D**





Name	Number of Bravais lattices	Conditions
Square	1	$a_1 = a_2$ , $\alpha = 90^{\circ}$
Rectangular	2	$a_1 \neq a_2$ , $\alpha = 90^\circ$
Hexagonal	1	$a_1 = a_2$ , $\alpha = 120^{\circ}$
Oblique	1	a <sub>1</sub> ≠a <sub>2</sub> ,α≠120°,α≠90°



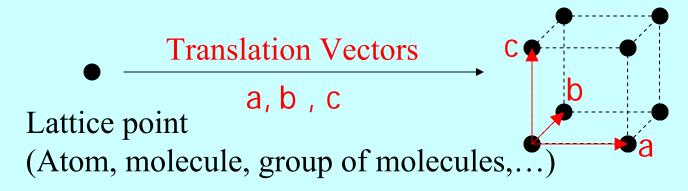


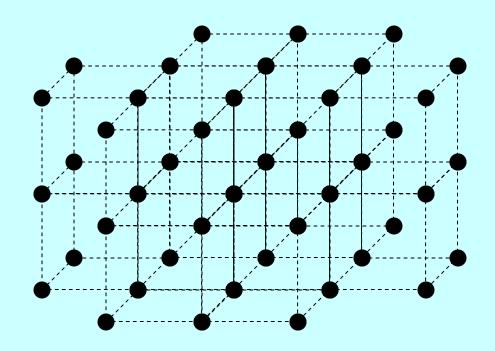
**Unit Cell**: An "imaginary" parallel sided region of a structure from which the entire crystal can be constructed by purely translational displacements Contents of unit cell represents chemical composition

**Space Lattice:** A pattern that is formed by the lattice points that have identical environment.

**Coordination Number (CN):** Number of direct neighbours of a given atom (first coordination sphere)

### **Crystal = Periodic Arrays of Atoms**





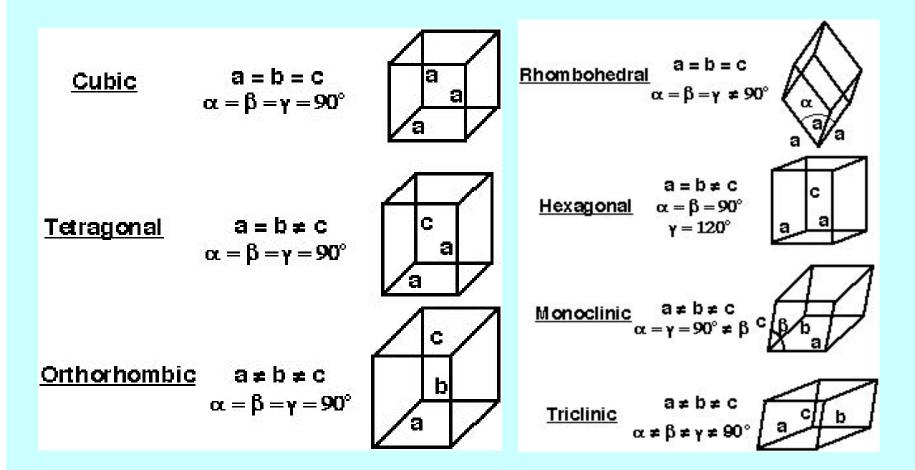
#### Primitive Cell:

- Smallest building block for the crystal lattice.
- Repetition of the primitive cell gives a crystal lattice

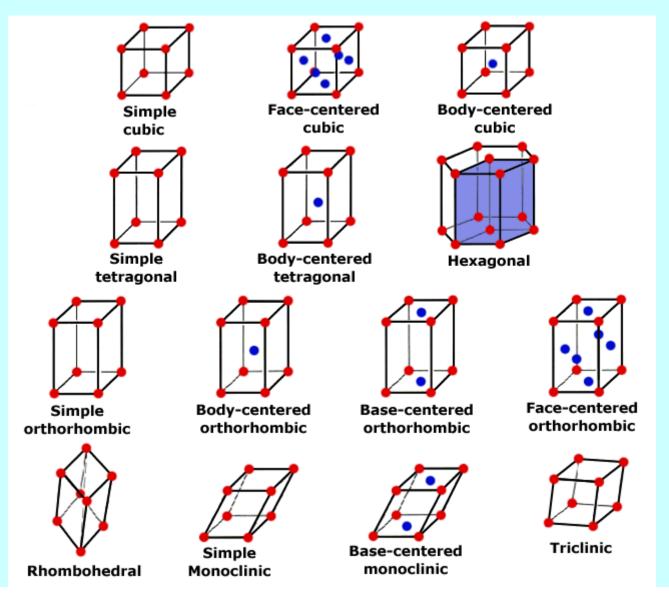
## **Lattices and Space Groups**

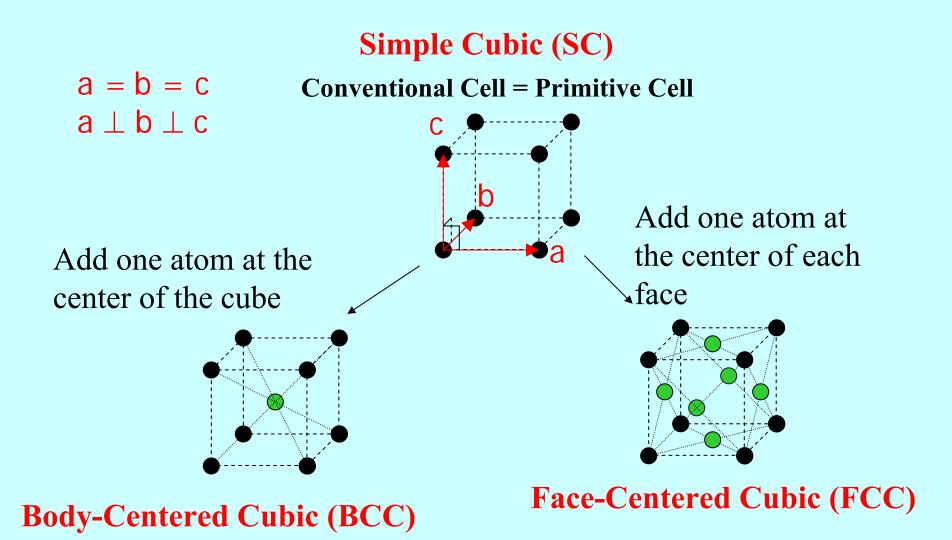
	Bravais Lattice (Lattice point = Basis of Spherical Symmetry)	Crystal Structure (Structural motif = Basis of Arbitrary Symmetry)
Number of point groups:	7 (7 crystal systems)	32 (32 crystallographic point groups)
Number of space groups:	14 (14 Bravais lattices)	230 (230 space groups)

### Seven Crystal Systems



### **Fourteen Bravais Lattices**





**Conventional Unit Cell ≠ Primitive Cell** 

## **Primitive Cell**

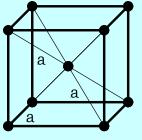
A *primitive cell* of the lattice = volume of space translated through all the vectors in a lattice that just fills all of space without overlapping or leaving voids.

A primitive cell contains just one Bravais lattice point.

The primitive cell is the smallest cell that can be translated throughout space to completely recreate the entire lattice.

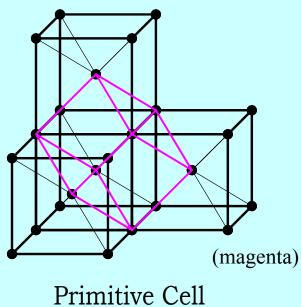
There is not one unique shape of a primitive cell, many possible shapes.

The primitive cell for the simple cubic lattice is equal to the simple cubic unit cell (they are identical).



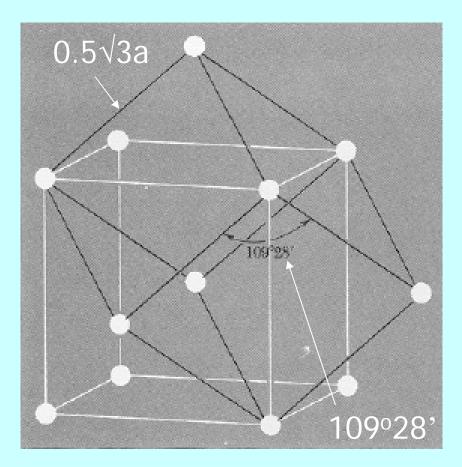
Body-Centered Cubic (I)

Unit Cell



### **Primitive Cell of BCC**

#### •Rhombohedron primitive cell

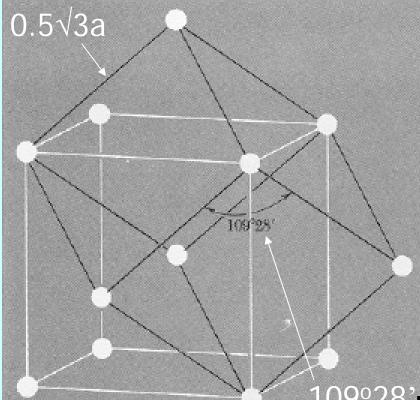


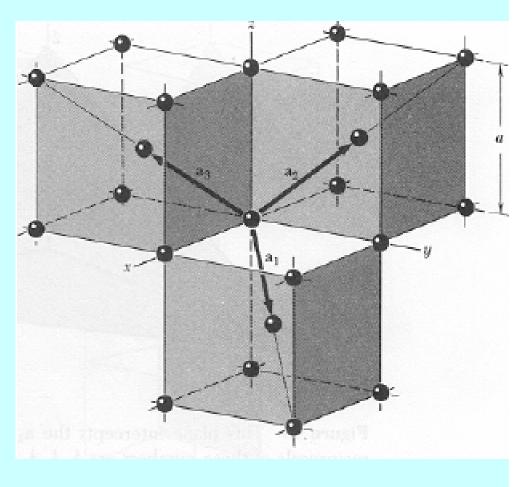
The primitive cell is smaller or equal in size to the unit cell.

The unit cells possesses the highest symmetry present in the lattice (for example Cubic).

#### **Primitive Cell of BCC**

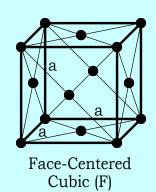
#### •Rhombohedron primitive cell



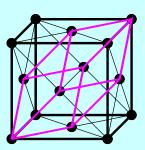


109°28. Primitive Translation Vectors:  $a_1 = \frac{1}{2}a(\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}}) ; \qquad a_2 = \frac{1}{2}a(-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}) ; \\
a_3 = \frac{1}{2}a(\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}) .$ 

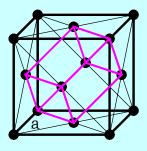
### Nonprimitive Unit Cell vs. Primitive Cell



Unit Cell



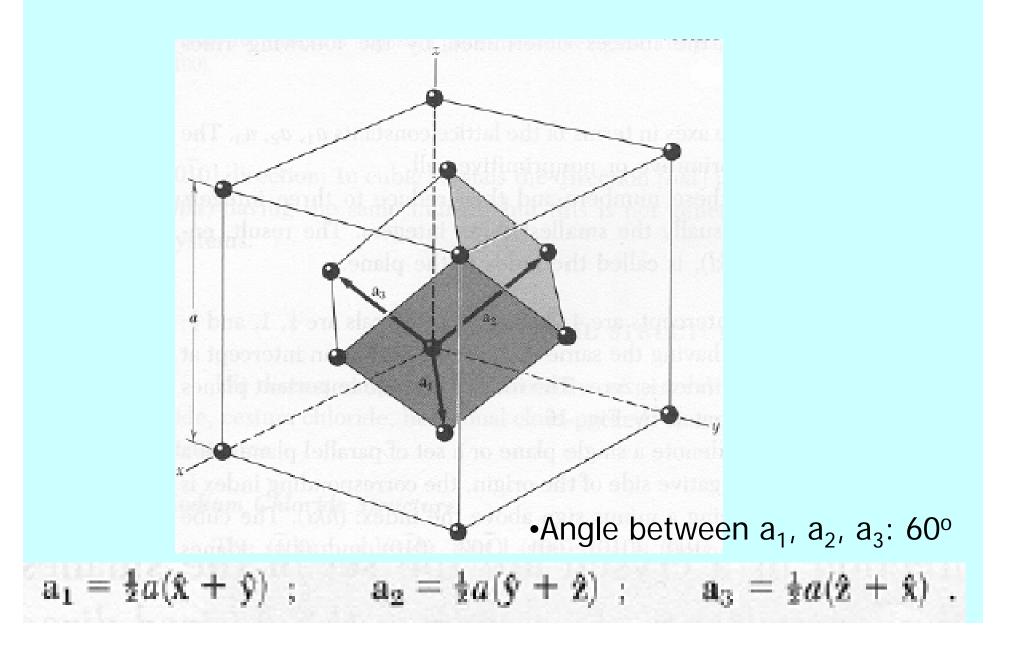
#### Primitive Cell



Rotated 90°

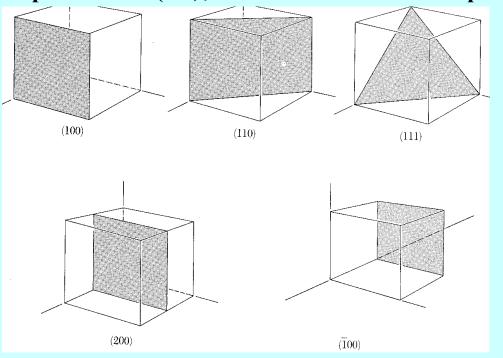
The primitive cell is smaller or equal in size to the unit cell. The unit cells possesses the highest symmetry present in the lattice (for example Cubic).

#### **Primitive Cell of FCC**

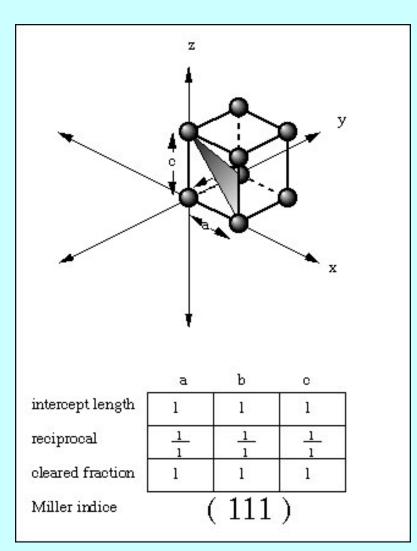


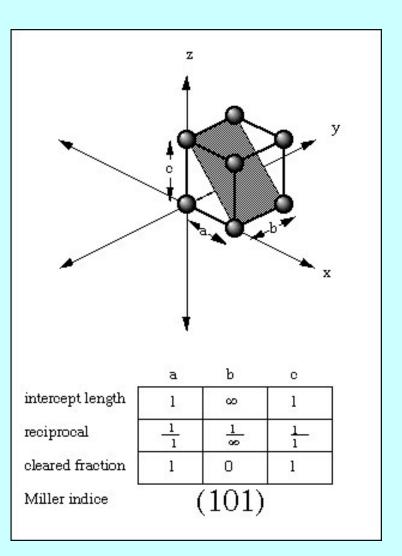
### **Index System for Crystal Planes (Miller Indices)**

- 1) Find the intercepts on the axes in terms of the lattice constants a, b, c. The axes may be those of a primitive or nonprimitive unit cell.
- 2) Take the reciprocals of these numbers and then reduce to three integers having the same ratio, usually the smallest three integers. The result enclosed in parenthesis (*hkl*), is called the index of the plane.

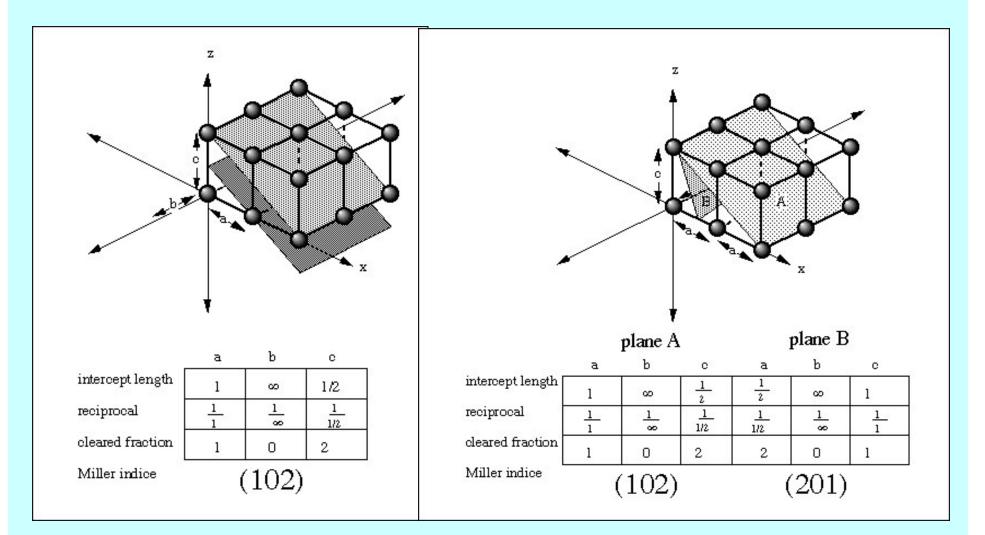


### **Miller Indices**





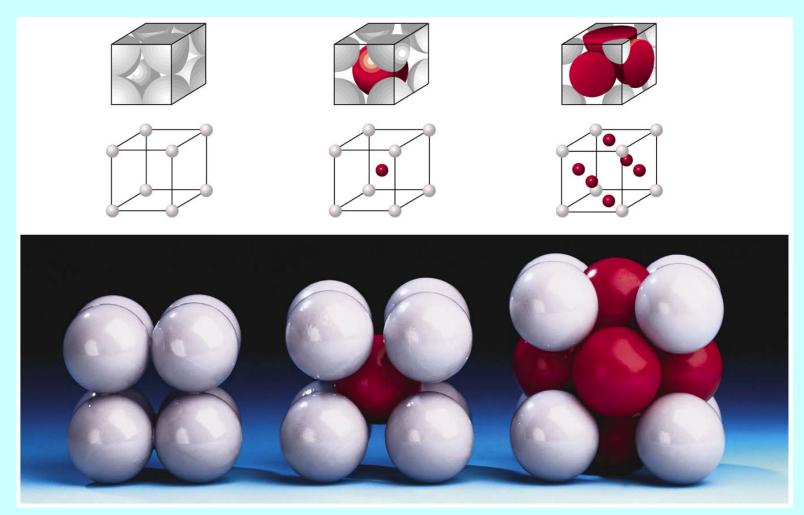
#### **Miller Indices**



### **Crystals and Crystal Bonding**

- metallic (Cu, Fe, Au, Ba, alloys ) metallic bonding
- ionic (NaCl, CsCl, CaF<sub>2</sub>, ... )
   Ionic bonds, cations and anions, electrostatic interactions
- covalent (diamond, graphite, SiO<sub>2</sub>, AlN,...) atoms, covalent bonding
- molecular (Ar, C<sub>60</sub>, HF, H<sub>2</sub>O, organics, proteins ) molecules, van der Waals and hydrogen bonding

### **Three Cubic Cells**



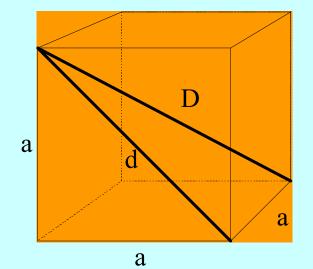
SC or Primitive (P) BCC (I)

FCC (F)

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	Simple		
	······	an an the sure of the second definitions	and a strength of the state of
Volume, conventional cell	$a^3$	$a^3$	$a^3$
Lattice points per cell	1	2	4
Volume, primitive cell	$a^3$	$\frac{1}{2}a^3$	$\frac{1}{4}a^{3}$
Lattice points per unit volur	ne $1/a^3$	$2/a^3$	$4/a^{3}$
Number of nearest neighbor	s <sup>a</sup> 6	8.	12
Nearest-neighbor distance	a	$3^{1/2}a/2 = 0.866a$	$a/2^{1/2} = 0.707a$
Number of second neighbor	s 12	6	6
Second neighbor distance	$2^{1/2}a$	a _	<u>a</u>
Packing fraction <sup>b</sup>	$rac{1}{6}\pi$	$\frac{1}{8}\pi\sqrt{3}$ .	$\frac{1}{6}\pi\sqrt{2}$
	=0.524	=0.680	= 0.740

#### Table 2 Characteristics of cubic lattices<sup>a</sup>

### Cube



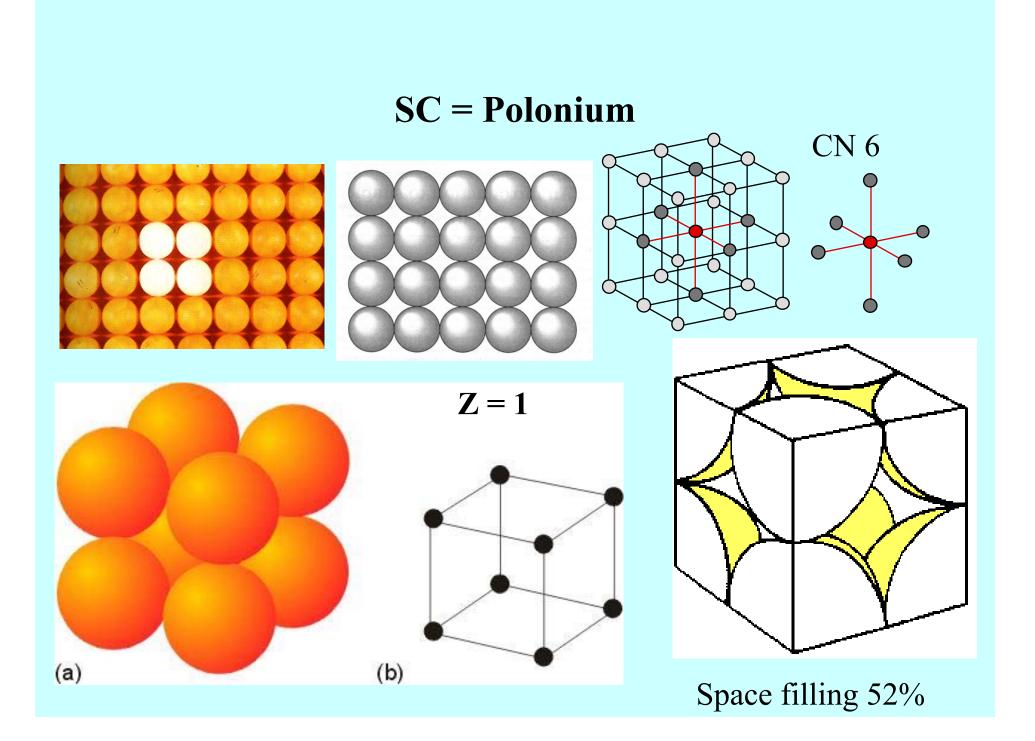
$$a = edge$$

d = face diagonl $(d^2 = a^2 + a^2 = 2a^2)$ 

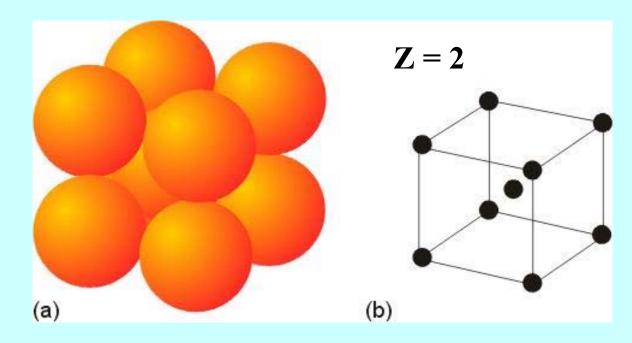
$$D = body diagonal$$
$$(D2 = d2 + a2 = 2a2 + a2 = 3a2)$$

$$d = \sqrt{2} \cdot a$$

$$D = \sqrt{3} \cdot a$$

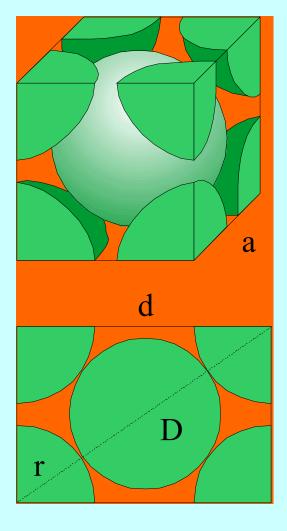


### BCC = W, Tungsten

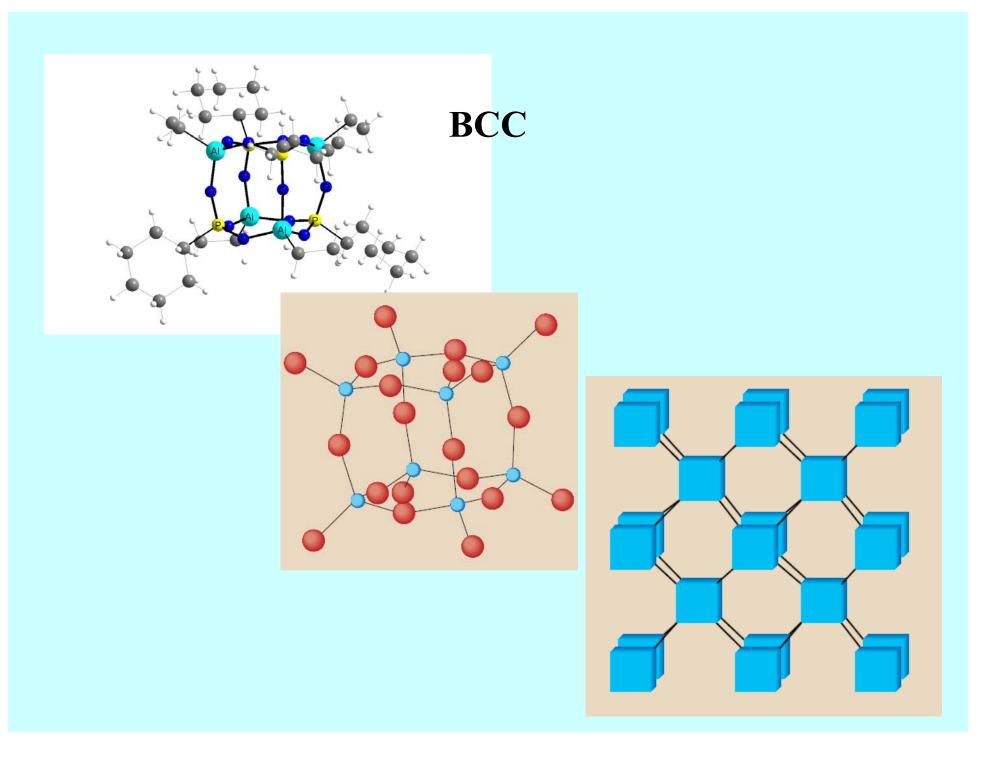


Space filling 68% CN 8

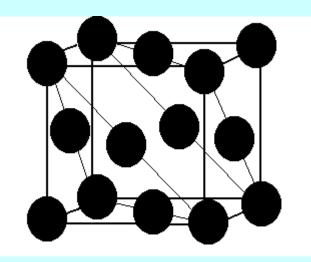
Fe, Cr, V, Li-Cs, Ba

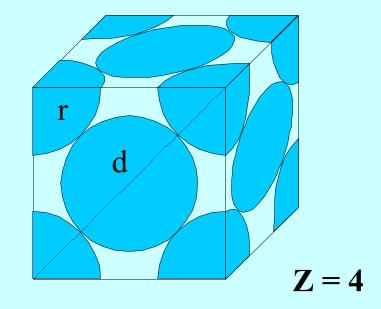


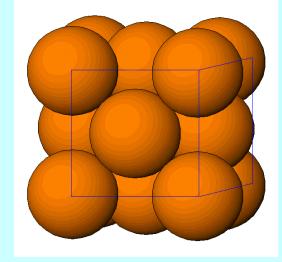




## FCC = Copper, Cu = CCP

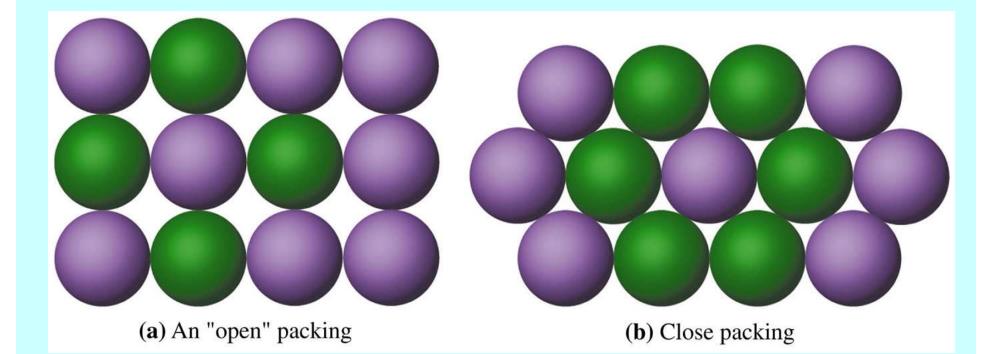


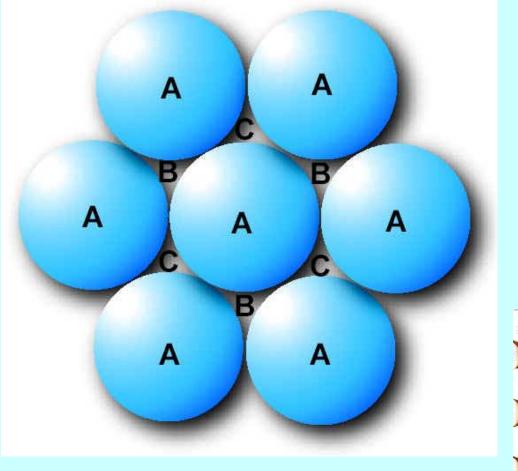


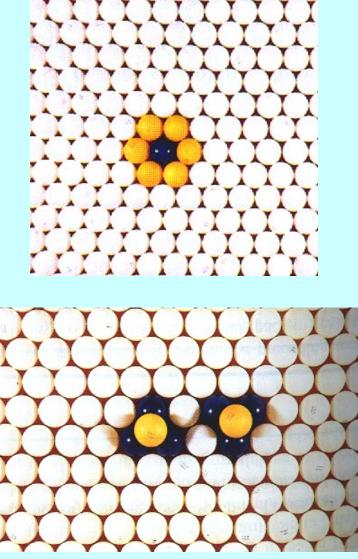


# Space filling 74% CN 12

### **Close Packing in Plane 2D**

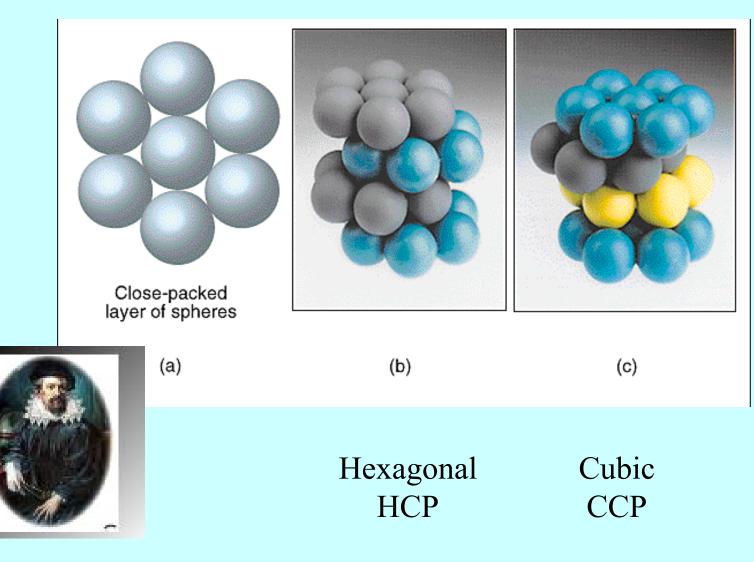


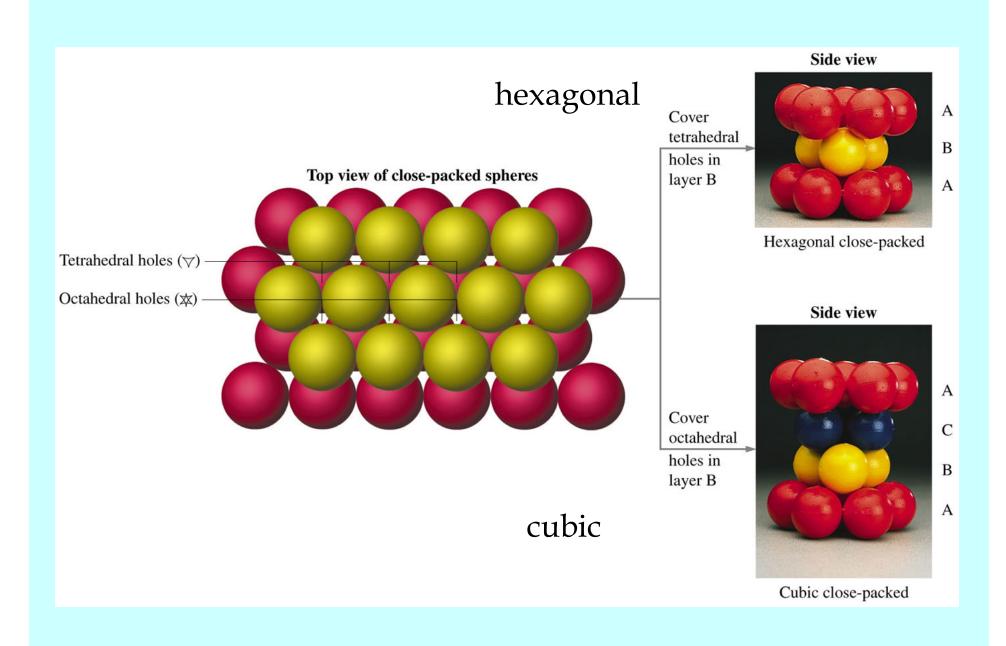


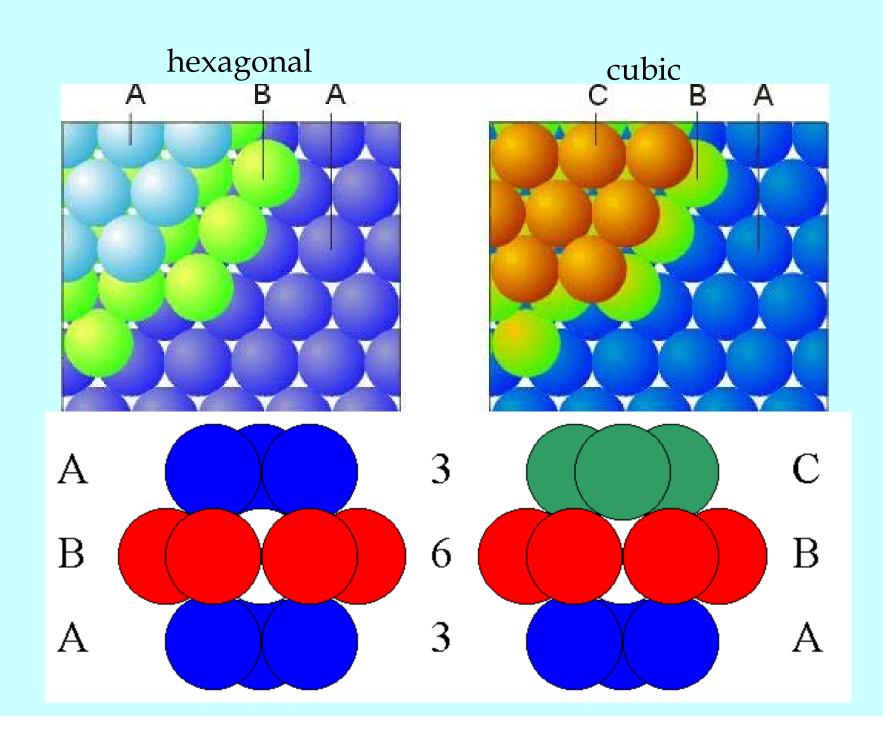


B and C holes cannot be occupied at the same time

### **Close Packing in Space 3D**

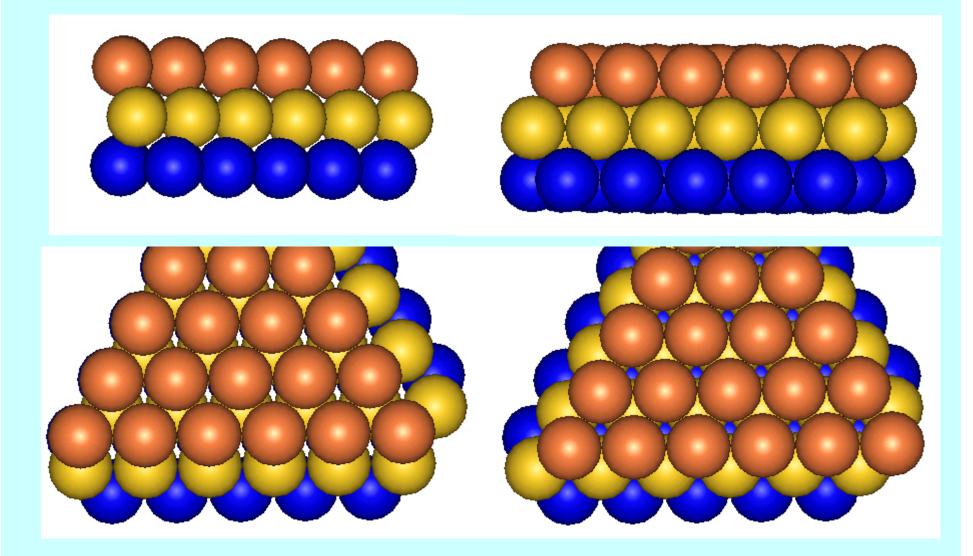


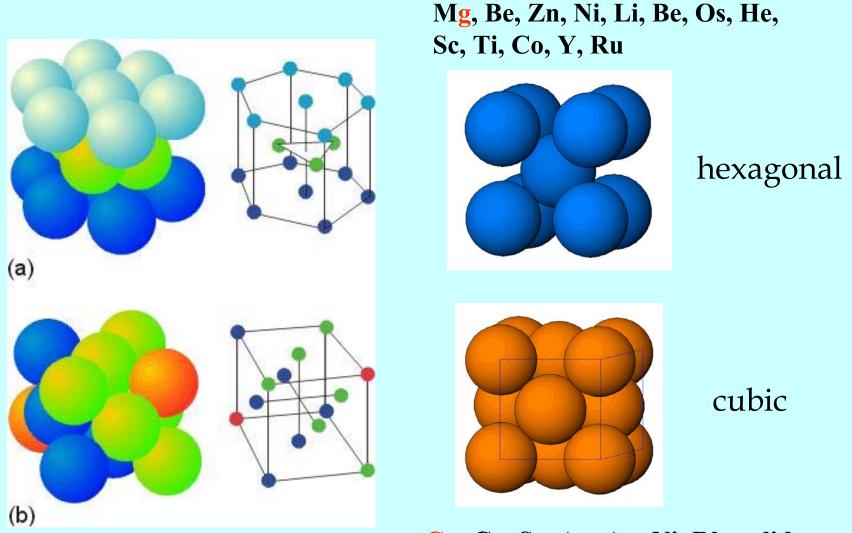




## hexagonal

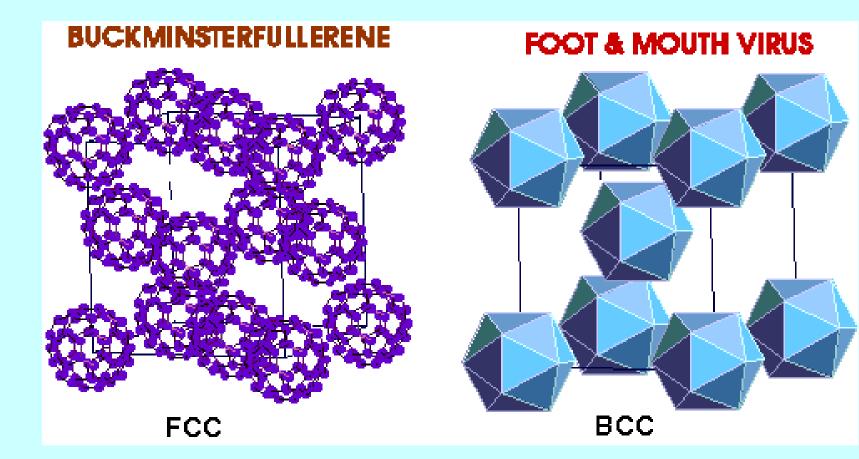
#### cubic

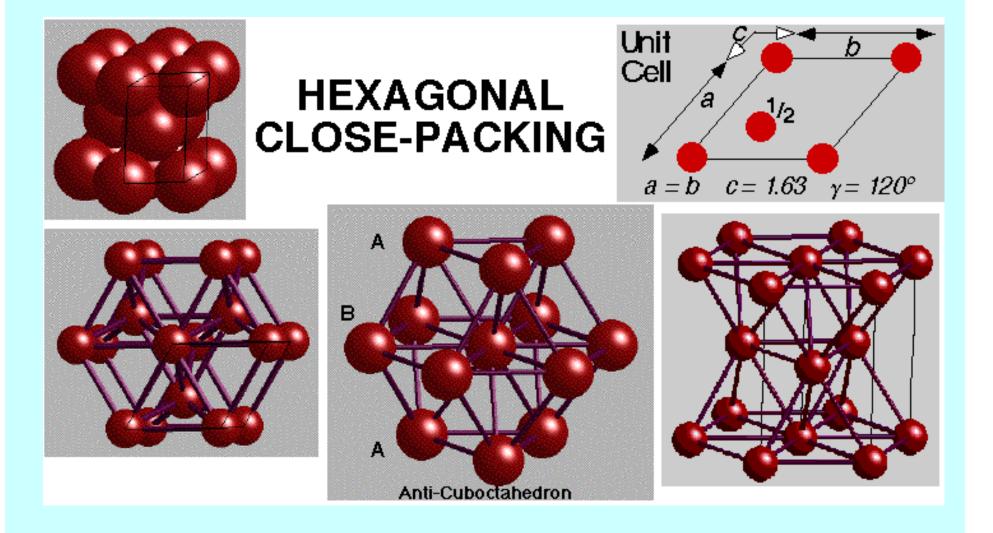


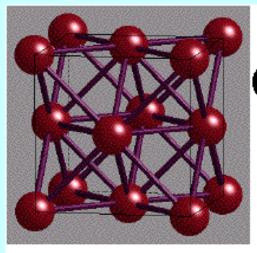


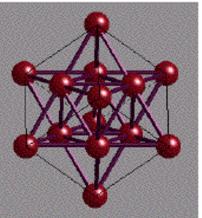
**Cu**, Ca, Sr, Ag, Au, Ni, Rh, solid Ne-Xe, F<sub>2</sub>, C<sub>60</sub>, opal (300 nm)

#### **Structures with Larger Motifs**



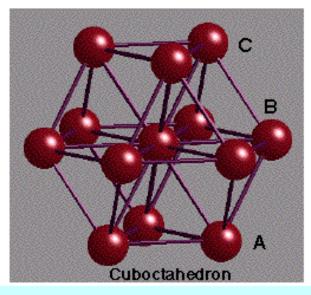


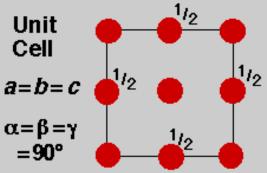


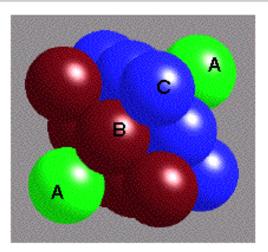


## CUBIC CLOSE-PACKING

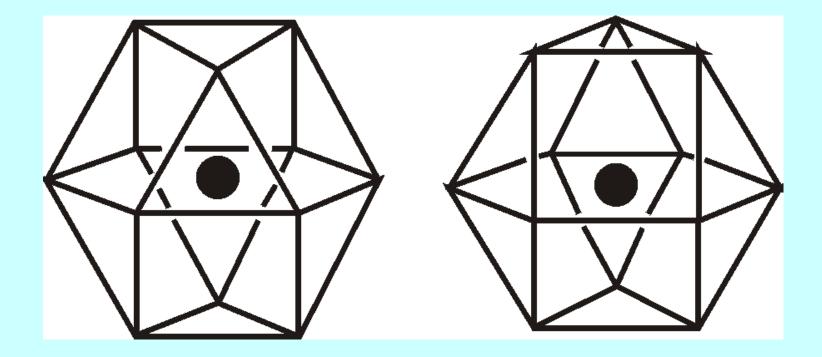
Face-Centred Cubic (FCC) Unit Cell



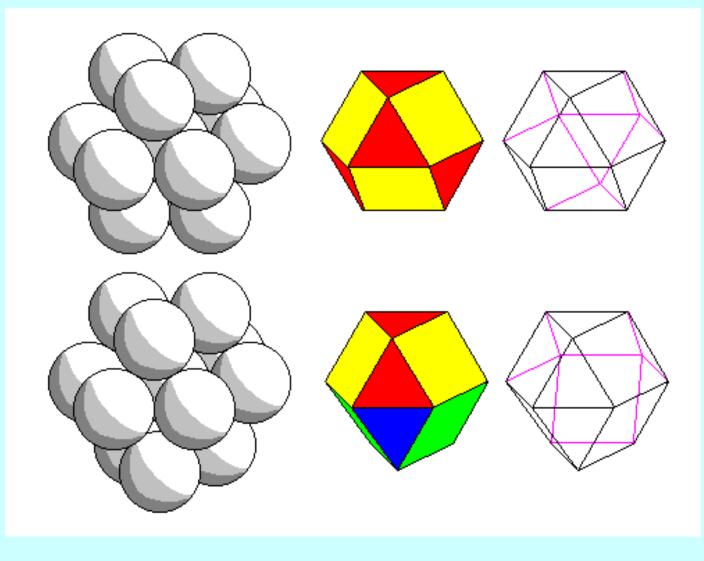




## **Coordination Polyhedrons**



## **Coordination Polyhedrons**

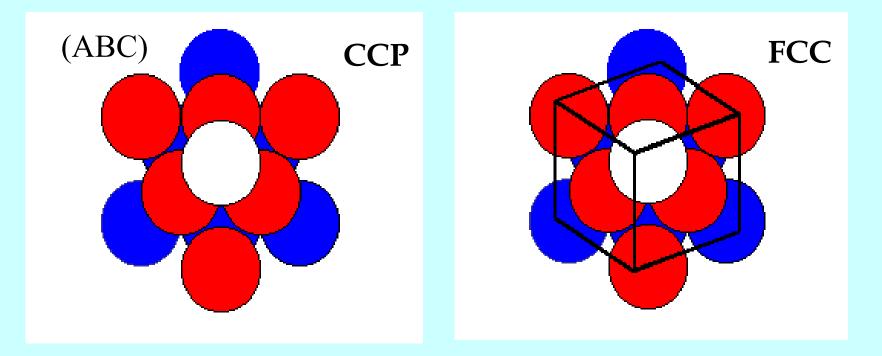


## **Space Filling**

a = lattice parameter	Atom Radius, r	Number of Atoms (lattice points), Z	Space filling
SC	a/2	1	52%
BCC	√3a/4	2	68%
FCC	√2a/4	4	74%
Diamond	√3a/8	8	34%

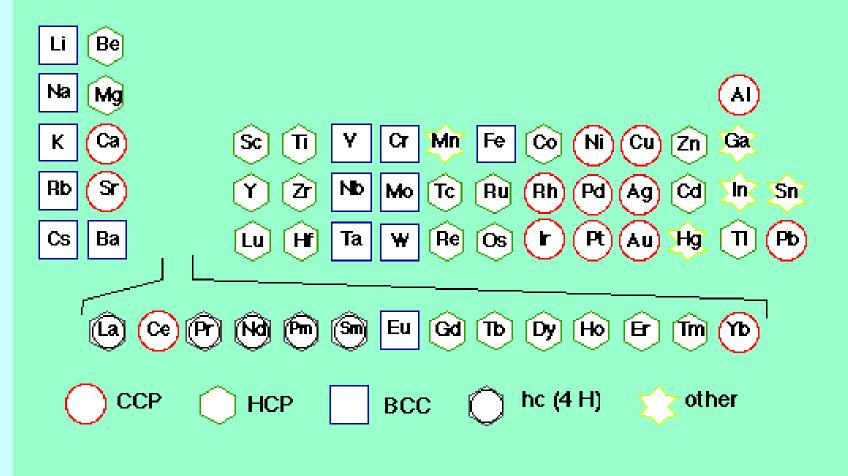
Type of Packing	Packing Efficiency	Coordination Number		
Simple cubic (sc)				
	<b>52</b> %	6		
Body-centered cubic (bcc)				
FFF	<b>68</b> %	8		
Hexagonal close-packed	<b>74</b> %	12		
(hcp) Cubic close-packed (ccp or fcc)	<b>74</b> %	12		

#### $\mathbf{CCP} = \mathbf{FCC}$

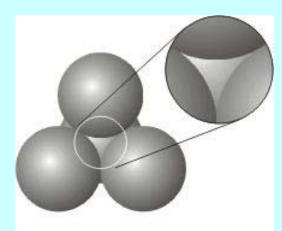


Close packed layers of CCP are oriented perpendicularly to the body diagonal of the cubic cell of FCC

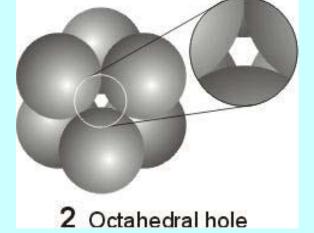
## Periodic Table of Metal Structures

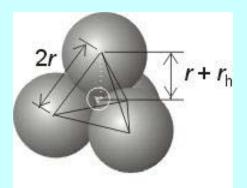


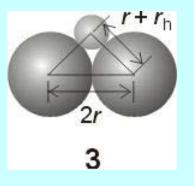
## **Two Types of Voids (Holes)**

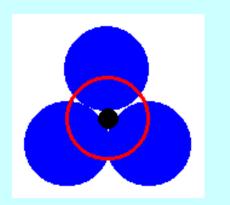


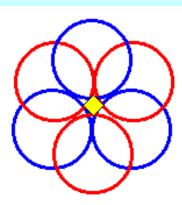
5 Tetrahedral hole

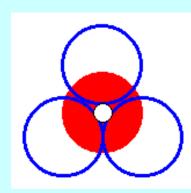












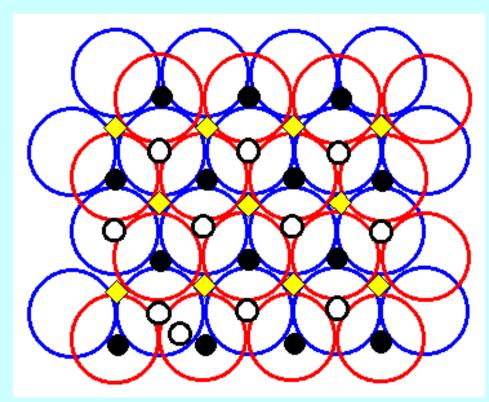
**Tetrahedral Holes T+** 

**Octahedral Holes** 

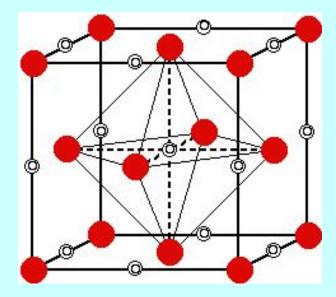
**Tetrahedral Holes T-**

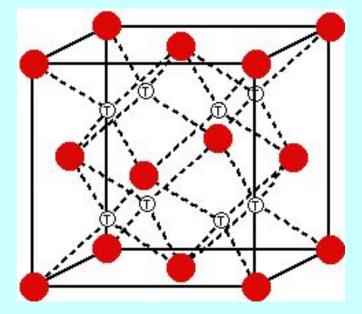
N cp atoms in lattice cell

N Octahedral Holes 2N Tetrahedral Holes



### **Two Types of Voids (Holes)**

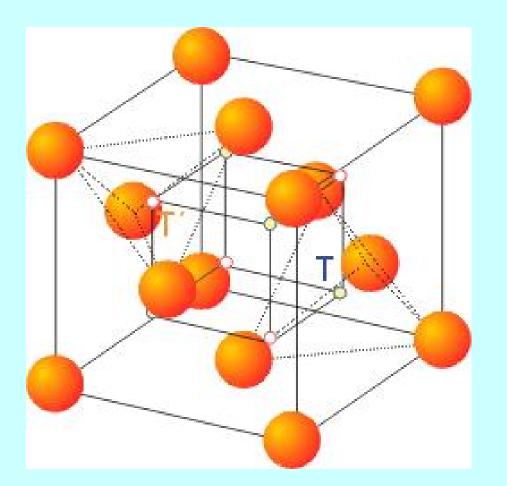




#### **Octahedral Holes**

#### **Tetrahedral Holes**

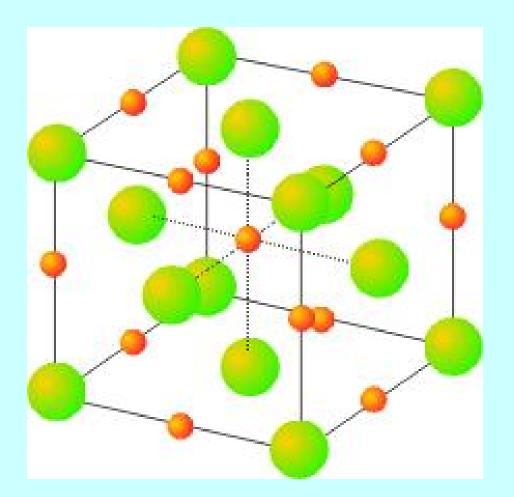
#### **Tetrahedral Holes (2N)**



 $Z = \underline{4}$ number of atoms in the cell (N)

 $N = \underline{8}$ number of tetrahedral holes (2N)

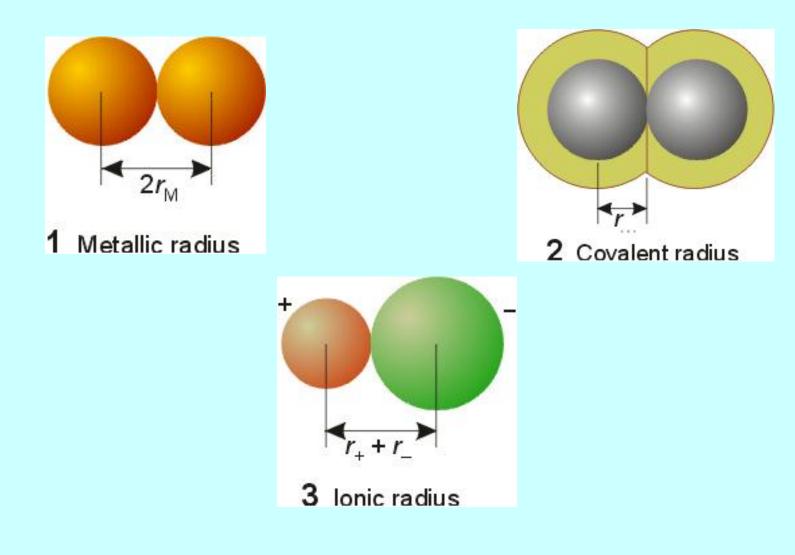
#### **Octahedral Holes (N)**



Z = 4number of atoms in the cell (N)

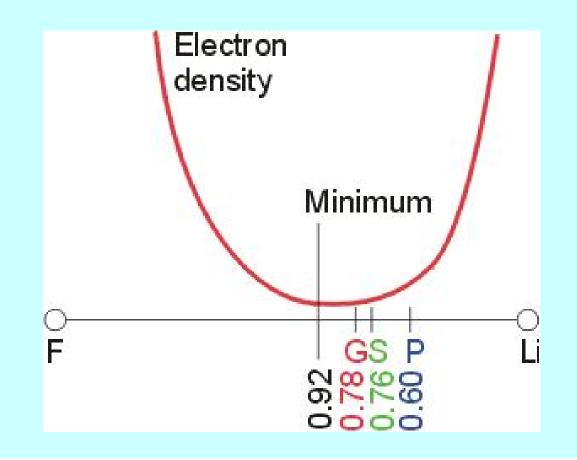
N = 4number of octahedral holes (N)

## **Different Types of Radii**

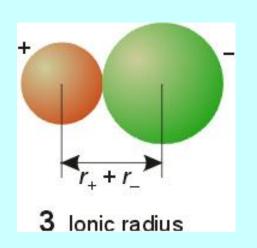


## Variation of the electron density along the Li – F axis in LiF

- **P** Pauling radius
- G Goldschmidt radius
- S Shannon radius.



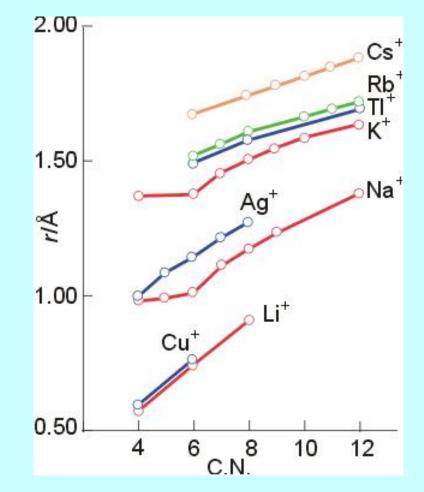
#### Variation of ionic radii with coordination number



The radius of one ion was fixed to a reasonable value

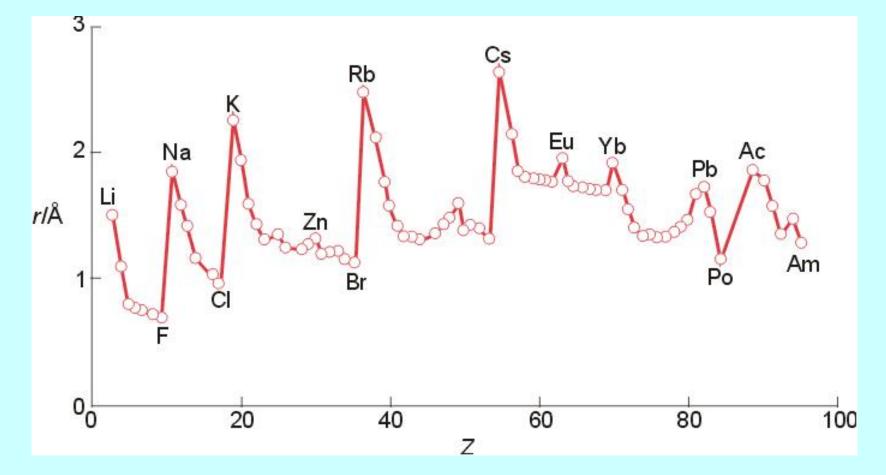
 $(r(O^{2-}) = 140 \text{ pm})$  (Linus Pauling)

That value is then used to compile a set of self consistent values for all other ions.



## Variation of atomic radii

through the Periodic table

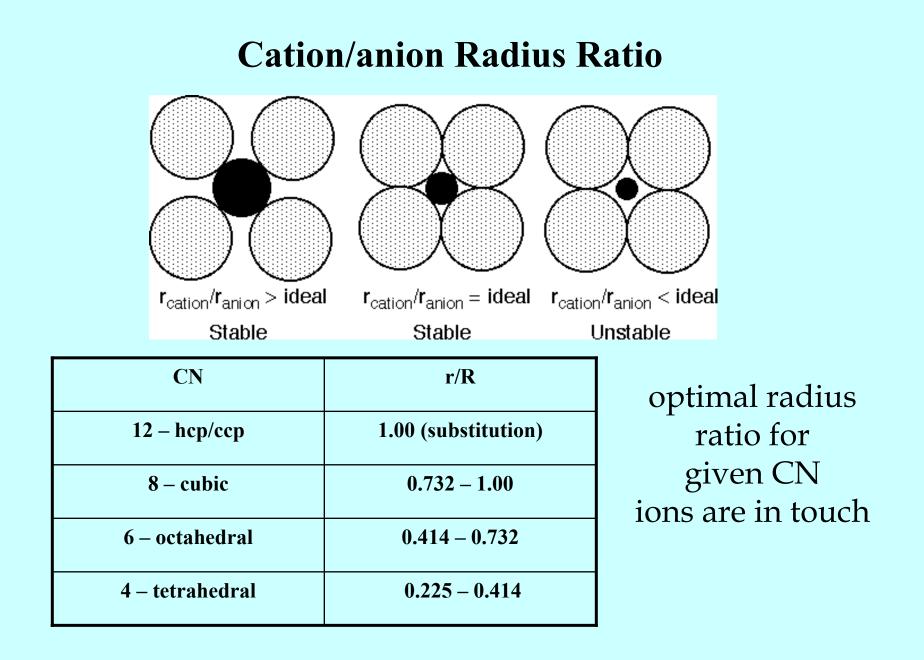


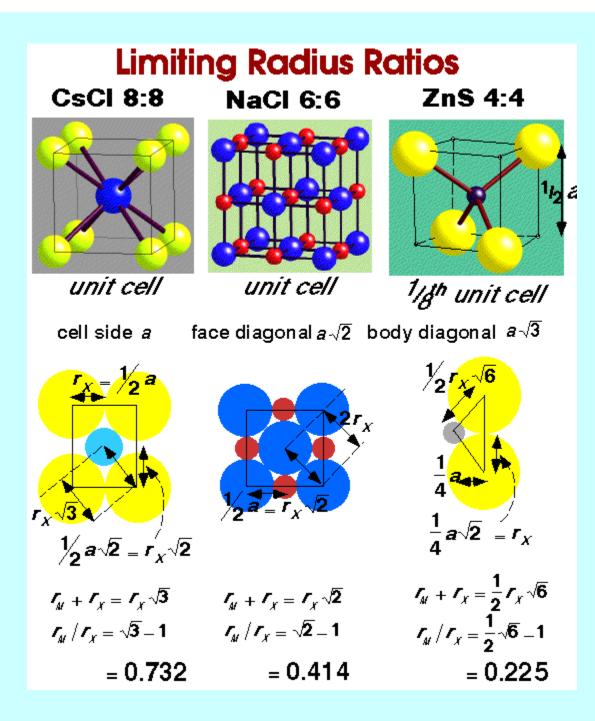
#### General trends for ionic radii

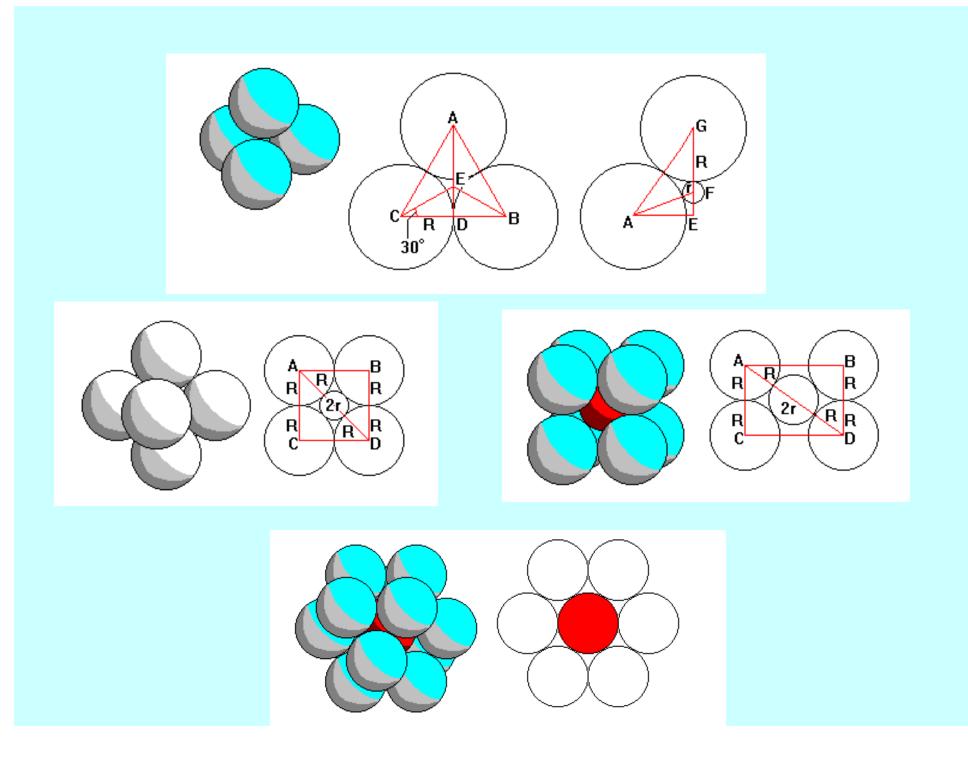
1. Ionic radii increase down a group.

(Lanthanide contraction restricts the increase of heavy ions)

- 2. Radii of equal charge ions decrease across a period
- 3. Ionic radii increase with increasing coordination number the higher the CN the bigger the ion
- 4. The ionic radius of a given atom decreases with increasing charge (r(Fe<sup>2+</sup>) > r(Fe<sup>3+</sup>))
- 5. Cations are usually the smaller ions in a cation/anion combination (exceptions: r(Cs<sup>+</sup>) > r(F<sup>-</sup>))
- 6. Frequently used for rationalization of structures: "radius ratio" r(cation)/r(anion) (< 1)

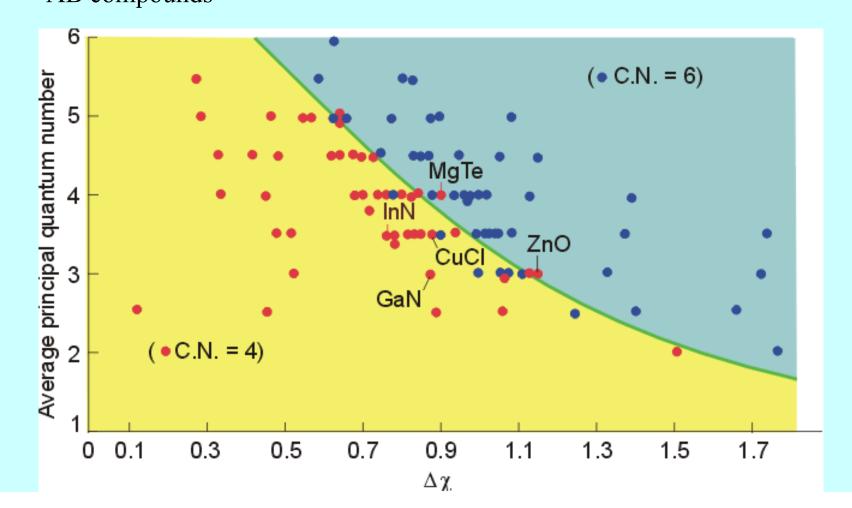






#### **Structure Map**

Dependence of the structure type (coordination number) on the electronegativity difference and the average principal quantum number (size and polarizability) AB compounds



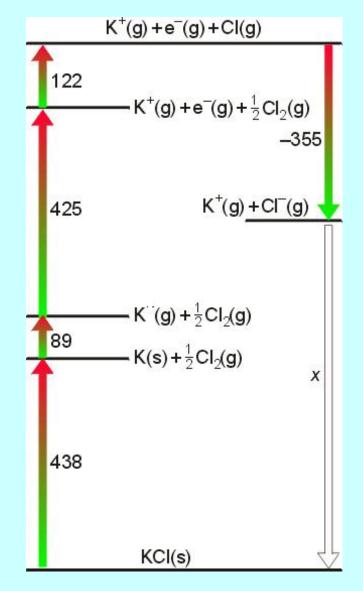
#### **Lattice Enthalpy**

The <u>lattice enthalpy</u> change  $\Delta H_L^0$  is the standard molar enthalpy change for the following process:

$$M^+_{(gas)} + X^-_{(gas)} \rightarrow MX_{(solid)} \qquad \Delta H^0_L$$

Because the formation of a solid from a "gas of ions" is always <u>exothermic</u> lattice enthalpies (defined in this way) are usually negative. If entropy considerations are neglected the <u>most stable crystal structure</u> of a given compound is the one with the <u>highest lattice enthalpy</u>.

# Lattice enthalpies can be determined by a thermodynamic cycle $\rightarrow$ Born-Haber cycle



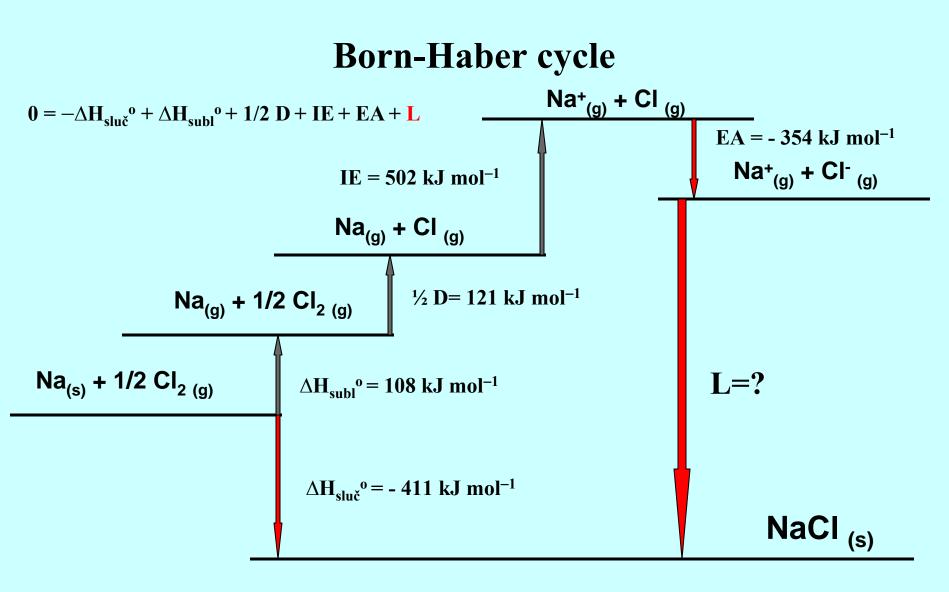
A Born-Haber cycle for KCl

(all enthalpies: kJ mol<sup>-1</sup> for <u>normal</u> <u>conditions</u>  $\rightarrow$  standard enthalpies)

standard enthalpies of

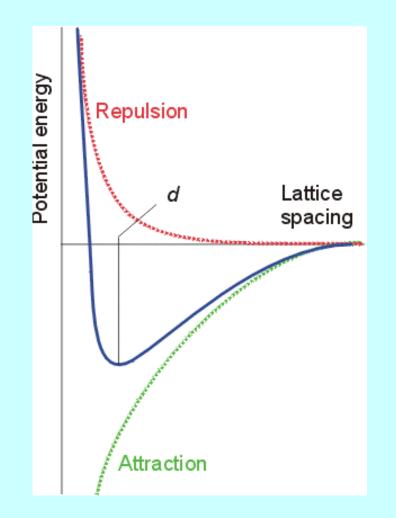
- formation: 438

- sublimation: +89 (K)
- ionization: + 425 (K)
- atomization: +244 (Cl<sub>2</sub>)
- electron affinity: -355 (Cl)
- lattice enthalpy: x



0 = 411 + 108 + 121 + 502 + (-354) + L  $L = -788 \text{ kJ mol}^{-1}$ all enthalpies: kJ mol<sup>-1</sup> for normal conditions  $\rightarrow$  standard enthalpies

#### **Lattice Enthalpy**



$$L = E_{coul} + E_{rep}$$

One ion pair  $E_{coul} = (1/4\pi\epsilon_0) z_A z_B / d$ 

 $E_{rep} = B / d^n$ 

n = Born exponent (experimental measurement of compressibilty)

#### **Lattice Enthalpy**

1 mol of ions

 $E_{coul} = N_A (e^2 / 4 \pi \epsilon_0) (z_A z_B / d) A$ 

$$E_{rep} = N_A B / d^n$$

$$L = N_A A \frac{Z_A Z_B e^2}{4\pi\varepsilon_0 d} + N_A \frac{B}{d^n}$$

 $L = E_{coul} + E_{rep}$ 

Find minimum dL/d(d) = 0

#### **Calculation of lattice enthalpies**

Coulombic contributions to lattice enthalpies

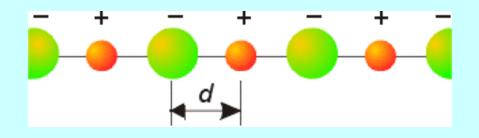
$$V_{AB} = -A \begin{pmatrix} z_{+}z_{-}e^{2} \\ 4\pi\varepsilon_{0}r_{AB} \end{pmatrix} N$$
Coulomb potential of an ion pair

V<sub>AB</sub>: Coulomb potential (electrostatic potential)

- A: <u>Madelung constant</u> (depends on structure type)
- N: Avogadro constant
- z: charge number
- e: elementary charge
- $\boldsymbol{\varepsilon}_{o}$ : dielectric constant (vacuum permittivity)
- $r_{AB}$ : shortest distance between cation and anion

#### **Madelung Constant**

Count all interactions in the crystal lattice

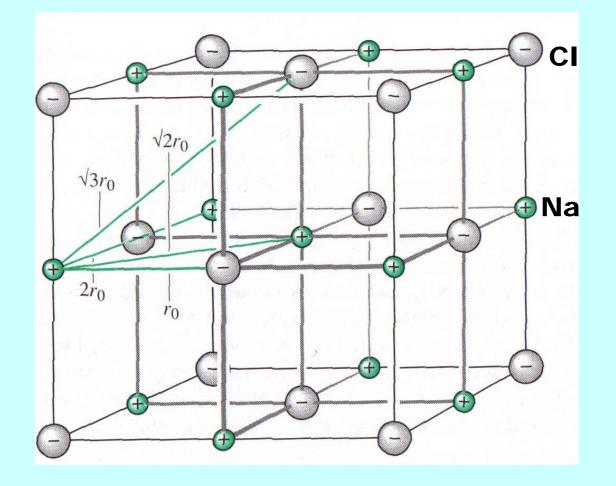


 $E_{\text{coul}} = (e^2 / 4 \pi \epsilon_0)^* (z_A z_B / d)^* [+2(1/1) - 2(1/2) + 2(1/3) - 2(1/4) + \dots]$ 

 $E_{coul} = (e^2 / 4 \pi \epsilon_0)^* (z_A z_B / d)^* (2 \ln 2)$ 

Madelung constant A (for linear chain of ions) = sum of convergent series

#### **Calculation of the Madelung constant**



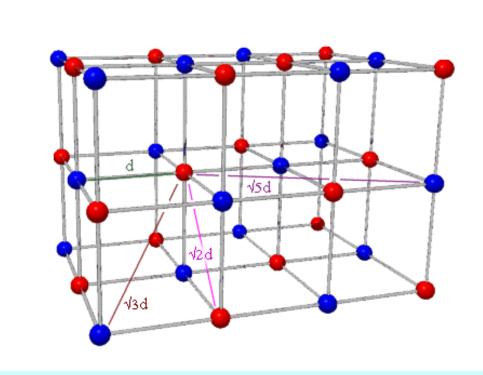
3D ionic solids: Coulomb attraction and repulsion

Madelung constants: CsCl: 1.763 NaCl: 1.748 ZnS: 1.641 (wurtzite) ZnS: 1.638 (sphalerite) ion pair: 1.0000 (!)

 $A = 6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - \frac{6}{2} + \frac{24}{\sqrt{5}} \dots = 1.748... \text{ (NaCl)}$ 

(infinite summation)

#### **Madelung constant for NaCl**



 $E_{\text{coul}} = (e^2 / 4 \pi \epsilon_0) * (z_A z_B / d) * [6(1/1) - 12(1/\sqrt{2}) + 8(1/\sqrt{3}) - 6(1/\sqrt{4}) + 24(1/\sqrt{5}) \dots]$ 

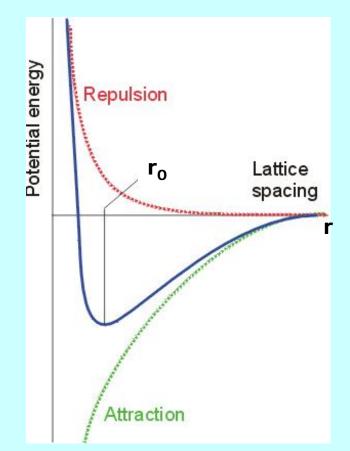
convergent series

 $E_{coul} = (e^2 / 4 \pi \epsilon_0) * (z_A z_B / d) * A$ 

#### **Madelung Constants for other Structural Types**

Structural Type	A	
NaCl	1.74756	
CsCl	1.76267	
CaF <sub>2</sub>	2.519	
ZnS Sfalerite	1.63805	
ZnS Wurtzite	1.64132	

## Born repulsion $\mathbf{V}_{\text{Born}}$



## **Repulsion arising from overlap of electron clouds**

Because the electron density of atoms decreases exponentially towards zero at large distances from the nucleus the Born repulsion shows the same behavior

approximation:

$$V_{Born} = \frac{B}{r^n}$$

B and n are constants for a given atom type; n can be derived from compressibility measurements (~8)

### **Total lattice enthalpy from Coulomb interaction and Born repulsion**

$$\Delta H_L^0 = Min.(V_{AB} + V_{Born})$$

(set first derivative of the sum to zero)

$$\Delta H_{L}^{0} = -A \frac{z_{+} z_{-} e^{2}}{4\pi\varepsilon_{0} r_{0}} N(1 - \frac{1}{n})$$

Measured (calculated) lattice enthalpies (kJ mol<sup>-1</sup>): NaCl: -772 (-757); CsCl: -652 (-623) (measured from Born Haber cycle)

# The Kapustinskii equation

Kapustinskii found that if the Madelung constant for a given structure is divided by **the number of ions in one formula unit** (v) the resulting values are almost constant:

Structure	Madel. const.(A)	<b>A/</b> <sub>V</sub>	Coordination
CsCl	1.763	0.88	8:8
NaCl	1.748	0.87	6:6
CaF <sub>2</sub>	2.519	0.84	8:4
$\alpha$ -Al <sub>2</sub> O <sub>3</sub>	4.172	0.83	6:4

→ general lattice energy equation that can be applied to any crystal regardless of the crystal structure

$$\Delta H_{L}^{0} = -\frac{1.079 \cdot 10^{5} v \cdot z_{+} z_{-}}{r_{+} \cdot r_{-}}$$

Most important advantage of the Kapustinski equation

- → it is possible to apply the equation for lattice calculations of crystals with polyatomic ions (e.g. KNO<sub>3</sub>, (NH<sub>4</sub>)<sub>2</sub>SO<sub>4</sub>...).
- → a set of <u>"thermochemical radii" was derived</u> for further calculations of lattice enthalpies

Ion	рт	Ion	рт	Ion	рт
NH <sub>4</sub> <sup>+</sup>	151	$ClO_4^-$	226	$MnO_4^{2-}$	215
$Me_4N^+$	215	CN-	177	$O_2^{2-}$	144
PH <sub>4</sub> <sup>+</sup>	171	CNS <sup>-</sup>	199	OH-	119
AlCl <sub>4</sub>	281	$CO_3^{2-}$	164	$PtF_6^{2-}$	282
BF <sub>4</sub>	218	$IO_3^-$	108	$PtCl_6^{2-}$	299
BH <sub>4</sub>	179	$N_3^-$	181	$PtBr_6^{2-}$	328
BrO <sub>3</sub>	140	NCO <sup>-</sup>	189	$PtI_6^{2-}$	328
CH <sub>3</sub> COO <sup>-</sup>	148	$NO_2^-$	178	$SO_4^{2-}$	244
$ClO_3^-$	157	$NO_3^{-}$	165	$SeO_4^{2-}$	235

 Table 1.13
 Thermochemical radii of polyatomic ions\*

\* J.E. Huheey (1983) Inorganic Chemistry, 3rd edn, Harper and Row, London, based on data from H.D.B. Jenkins and K.P. Thakur (1979) J. Chem. Ed., 56, 576.

# Lattice Enthalpy

$$L = N_A M \frac{Z_A Z_B e^2}{4\pi\varepsilon_0 d} \left(1 + \frac{1}{n}\right)$$

El. config.	n
He	5
Ne	7
Ar	9
Kr	10
Xe	12

Born – Mayer

Porn I anda

$$L = N_A M \frac{Z_A Z_B e^2}{4\pi\varepsilon_0 d} \left(1 - \frac{d^*}{d}\right)$$

 $d^* = 0.345 \text{ Å}$ 

### **Lattice Enthalpy**

Kapustinski

M/v je přibližně konstantní pro všechny typy struktur v = počet iontů ve vzorcové jednotce

M je nahrazena 0.87 v, není nutno znát strukturu

$$L = 1210v \frac{Z_A Z_B}{d} \left( 1 - \frac{0,345}{d} \right)$$

#### Kapustinski

structure	М	CN	stoichm	MIV
CsCl	1.763	(8,8)	AB	0.882
NaCl	1.748	(6,6)	AB	0.874
ZnS sfalerite	1.638	(4,4)	AB	0.819
ZnS wurtzite	1.641	(4,4)	AB	0.821
CaF <sub>2</sub> fluorite	2.519	(8,4)	AB <sub>2</sub>	0.840
TiO <sub>2</sub> rutile	2.408	(6,3)	AB <sub>2</sub>	0.803
Cdl <sub>2</sub>	2.355	(6,3)	AB <sub>2</sub>	0.785
Al <sub>2</sub> O <sub>3</sub>	4.172	(6,4)	A <sub>2</sub> B <sub>3</sub>	0.834

v = the number of ions in one formula unit

### Lattice Enthalpy of NaCl

Born – Lande calculation $L = -765 \text{ kJ mol}^{-1}$ Only ionic contribution

Experimental Born – Haber cycle  $L = -788 \text{ kJ mol}^{-1}$ 

Lattice Enthalpy consists of ionic and covalent contribution

#### Applications of lattice enthalpy calculations:

- $\rightarrow$  thermal stabilities of ionic solids
- $\rightarrow$  stabilities of oxidation states of cations
- $\rightarrow$  Solubility of salts in water
- $\rightarrow$  calculations of electron affinity data
- → lattice enthalpies and stabilities of ,,non existent" compounds

# **Pauling's Rules**

Five principles which could be used to determine the structures of complex ionic/covalent crystals

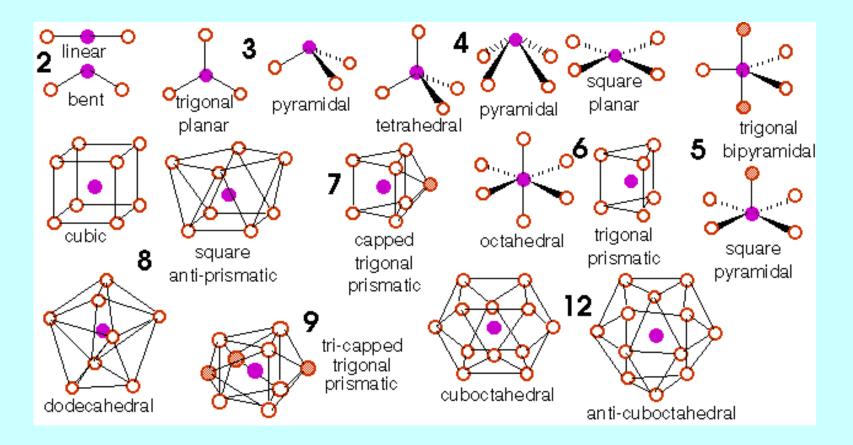
#### Pauling's Rule no. 1 Coordination Polyhedra

A coordinated polyhedron of anions is formed about each cation.

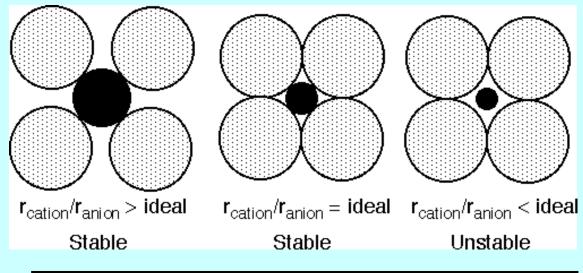
Cation-Anion distance is determined by sums of ionic radii.

Cation coordination environment is determined by radius ratio.

### **Coordination Polyhedra**



# **Cation/Anion Radius Ratio**



CN	r/R
12 – hcp/ccp	1.00 (substitution)
8 – cubic	0.732 - 1.00
6 – octahedral	0.414 - 0.732
4 – tetrahedral	0.225 - 0.414

# Ionic Radii

R.D. Shannon and C.T. Prewitt, Acta Cryst. B25, 925-945 (1969) R.D. Shannon, Acta Cryst. A32, 751-767 (1976)

As the coordination number (CN) increases, the Ionic Radius increases

Sr <sup>2+</sup>			
CN	Radius, Å		
6	1.32		
8	1.40	As the o	xidation state increases, cations get smaller
9	1.45	(6-fold c	oordination, in Å)
10	1.50		
12	1.58	Mn2+	0.810
		Mn3+	0.785
		Mn4+	0.670
		Ti2+	1.000
		Ti3+	0.810
		Ti4+	0.745

# Ionic Radii

The radius increases down a group in the periodic table. The exception - 4d/5d series in the transition metals - the lanthanide contraction

#### (6-fold coordination, in Å)

Al3+	0.675	
Ga3+	0.760	
ln3+	0.940	
TI3+	1.025	Right to left across the periodic table the radius decreases
Ti4+ Zr4+	0.745 0.86	(6 coordinate radii, in Å)
Hf4+	0.85	La3+ 1.172
11147	0.00	Nd3+ 1.123
		Gd3+ 1.078
		Lu3+ 1.001

# **Pauling's Rules**

#### Pauling's Rule no. 2 Bond Strength

The bond valence sum of each ion equals its oxidation state.

The valence of an ion ( $V_i$ , equal to the oxidation state of the ion) is equal to a sum of the valences of its bonds ( $s_{ii}$ ).

In a stable ionic structure the charge on an ion is balanced by the sum of electrostatic bond strengths  $(s_{ii})$  to the ions in its coordination polyhedron.

TiO<sub>2</sub> (Rutile) Ti - oxidation state of +4, coordinated to 6 oxygens. V<sub>Ti</sub> = 4 = 6 (s<sub>ij</sub>)  $s_{ij} = 2/3$ 

The bond valence of oxygen, coordinated by 3 Ti atoms  $Vo = 3 (s_{ij}) = 3 (-2/3) = -2$ 

Each bond has a valence of  $s_{ij}$  with respect to the cation and  $-s_{ij}$  with respect to the anion.

# **Bond Strength**

Correlation of the valence of a bond  $s_{ii}$  with the bond distance  $d_{ii}$ .

$$s_{ij} = \exp{\frac{R_{ij} - d_{ij}}{b}}$$

b = 0.37,  $R_{ij}$  is determined empirically from structures where bond distances and ideal valences are accurately known.

Tables of R<sub>ij</sub> values for given bonding pairs (i.e. Nb-O, Cr-N, Mg-F, etc.) have been calculated, just as tables of ionic radii are available.

Use of the bond valence concept

A) To check experimentally determined structures for correctness, or bonding instabilities

B) To predict new structures

C) To locate light atoms such as hydrogen or Li ion, which are hard to find experimentally

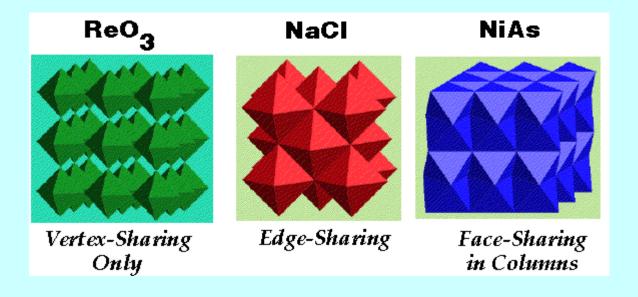
D) To determine ordering of ions which are hard to differentiate experimentally, such as  $AI^{3+}$  and  $Si^{4+}$ , or  $O^{2-}$  and  $F^{-}$ 

# **Pauling's Rules**

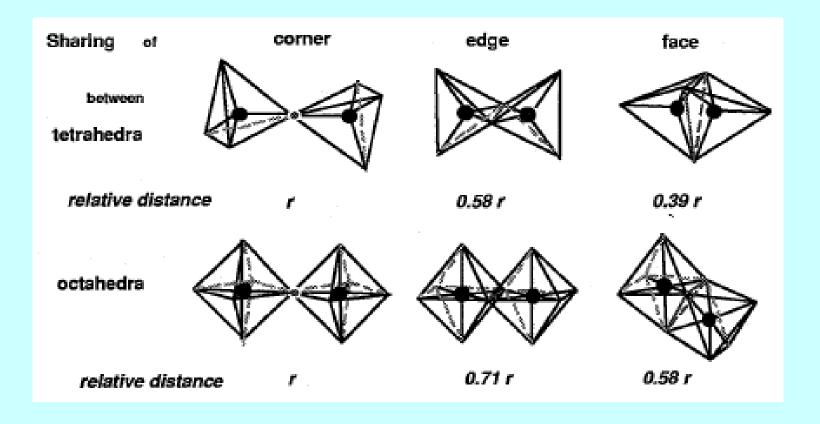
#### Pauling's Rule no. 3 Polyhedral Linking

The presence of shared edges, and particularly shared faces decreases the stability of a structure. This is particularly true for cations with large valences and small coordination number.

Avoid shared polyhedral edges and/or faces.



#### **Polyhedral Linking**



The Coulombic interactions - maximize the cation-anion interactions (attractive), and minimize the anion-anion and cation-cation interactions (repulsive).

The cation-anion interactions are maximized by increasing the coordination number and decreasing the cation-anion distance. If ions too close - electronelectron repulsions.

The cation-cation distances as a function of the cation-anion distance (M-X)

Polyhedron/Sharing	Corner	Edge	Face
2 Tetrahedra	2 M-X	1.16 MX	0.67 MX
2 Octahedra	2 M-X	1.41 MX	1.16 MX

The cation-cation distance decreases, (the Coulomb repulsion increases) as the

•degree of sharing increases (corner < edge < face)

•CN decreases (cubic < octahedral < tetrahedral)

•cation oxidation state increases (this leads to a stronger Coulomb repulsion)

# **Pauling's Rules**

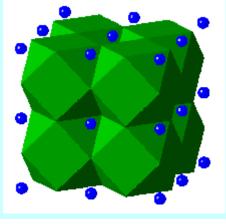
#### Pauling's Rule no. 4 Cation Evasion

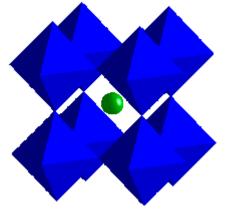
In a crystal containing different cations those with large valence and small coord. number tend not to share anions.

Perovskite, CaTiO<sub>3</sub>

 $Ca^{II}$  12-coordinate  $CaO_{12}$  cuboctahedra share FACES

Ti<sup>IV</sup> 6-coordinate TiO<sub>6</sub> octahedra share only VERTICES





# **Pauling's Rules**

#### Pauling's Rule no. 5 Environmental Homogeneity the rule of parsimony

The number of chemically different coordination environments for a given ion tends to be small.

Once the optimal chemical environment for an ion is found, if possible all ions of that type should have the same environment.

### Characteristic Structures of Solids = Structure Types

Rock salt NaCl LiCl, KBr, AgCl, MgO, TiO, FeO, SnAs, UC, TiN, ...

Fluorite <u>CaF</u><sub>2</sub> BaCl<sub>2</sub>, K<sub>2</sub>O, PbO<sub>2</sub> ...

Lithium bismutide <u>Li<sub>3</sub>Bi</u>

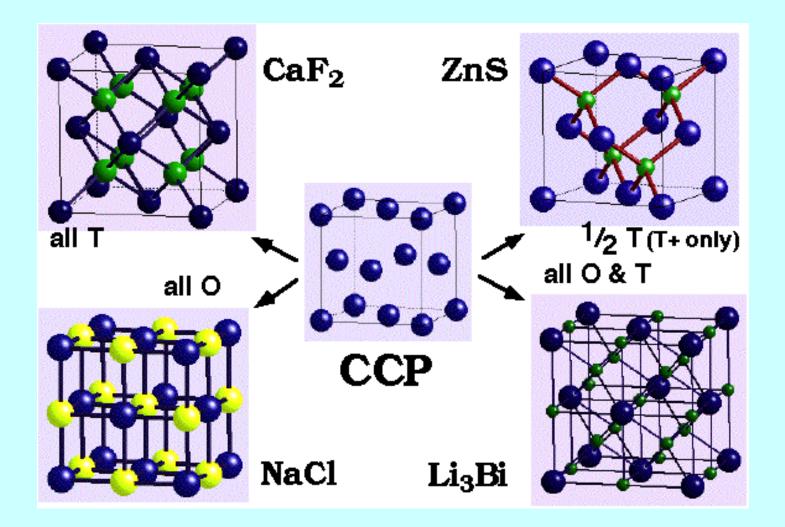
Sphalerite (zinc blende) ZnS CuCl, HgS, GaAs ...

Nickel arsenide NiAs FeS, PtSn, CoS ...

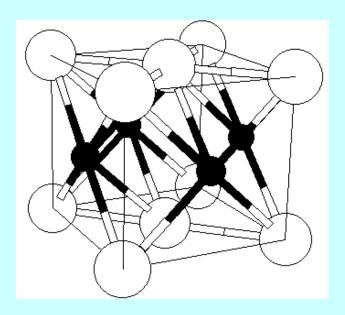
Wurtzite ZnS ZnO, MnS, SiC

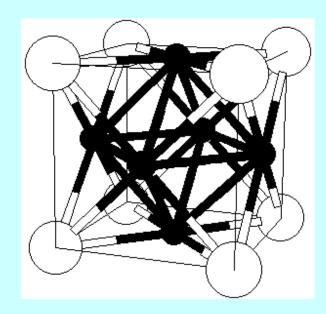
**Rhenium diboride ReB**<sub>2</sub>

## **Structure Types Derived from CCP = FCC**



# **Structure Types Derived from CCP = FCC**





# **Structure Types Derived from CCP = FCC**

Anions/cell (= 4)	Oct. (Max 4)	<b>Tet. (Max 8)</b>	Stoichiometry	Compound
4	100% = 4	0	$\mathbf{M}_4 \mathbf{X}_4 = \mathbf{M} \mathbf{X}$	NaCl
				(6:6 coord.)
4	0	100% = 8	$\mathbf{M}_{8}\mathbf{X}_{4}=\mathbf{M}_{2}\mathbf{X}$	Li <sub>2</sub> O
				(4:8 coord.)
4	0	<b>50%</b> = 4	$M_4X_4 = MX$	ZnS, sfalerite
				(4:4 coord.)
4	<b>50%</b> = 2	0	$\mathbf{M}_2\mathbf{X}_4 = \mathbf{M}\mathbf{X}_2$	CdCl <sub>2</sub>
4	100% = 4	100% = 8	$\mathbf{M}_{12}\mathbf{X}_4 = \mathbf{M}_3\mathbf{X}$	Li <sub>3</sub> Bi
4	50% = 2	12.5% = 1	$M_3X_4$	MgAl <sub>2</sub> O <sub>4</sub> ,
spinel				

# **Comparison between structures** with filled octahedral and tetrahedral holes

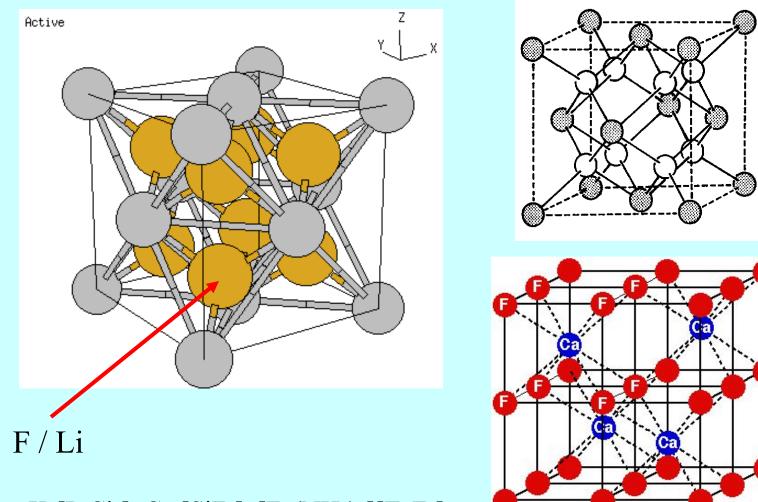
fcc(ccp)	hcp
NaCl	NiAs
CaF₂	ReB <sub>2</sub>
Li <sub>3</sub> Bi	(Na <sub>3</sub> As) (!) problem
sphalerite (ZnS)	wurtzite (ZnS)
CdCl <sub>2</sub>	Cdl <sub>2</sub>
	CaF <sub>2</sub> Li <sub>3</sub> Bi sphalerite (ZnS)

### Fluorite CaF<sub>2</sub> and antifluorite Li<sub>2</sub>O

Fluorite structure = a face-centered cubic array (FCC) of **cations** = cubic close packing (CCP) of cations with all tetrahedral holes filled by anions = a simple cubic (SC) array of anions.

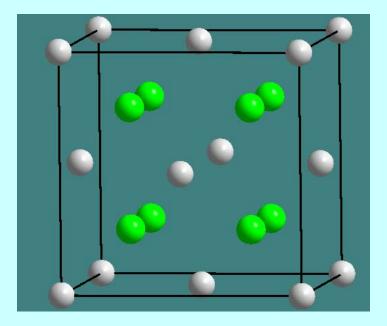
Antifluorite structure = a face-centred cubic (FCC) array of **anions** = cubic close packing (CCP) of anions, with cations in all of the tetrahedral holes (the reverse of the fluorite structure).

# Fluorite (CaF<sub>2</sub>, antifluorite Li<sub>2</sub>O)



 $K_2[PtCl_6], Cs_2[SiF_6], [Fe(NH_3)_6][TaF_6]_2$ 

#### Fluorite structures (CaF<sub>2</sub>, antifluorite Li<sub>2</sub>O)



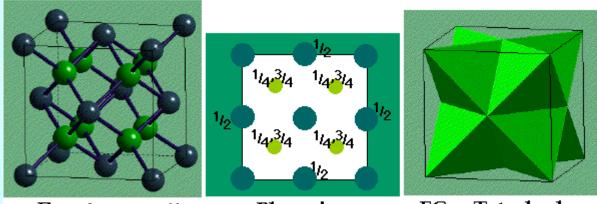
Oxides: Na<sub>2</sub>O, K<sub>2</sub>O, UO<sub>2</sub>, ZrO<sub>2</sub>, ThO<sub>2</sub>

alkali metal sulfides, selenides and tellurides

 $K_2[PtCl_6], (NH_4)_2[PtCl_6],$   $Cs_2[SiF_6],$  $[Fe(NH_3)_6][TaF_6]_2.$ 

 $CaF_{2}, SrF_{2}, SrCl_{2}, BaF_{2}, BaCl_{2}, CdF_{2}, HgF_{2}, EuF_{2}, \beta-PbF_{2}, PbO_{2}$  $Li_{2}O, Li_{2}S, Li_{2}Se, Li_{2}Te, Na_{2}O, Na_{2}S, Na_{2}Se, Na_{2}Te, K_{2}O, K_{2}S$ 

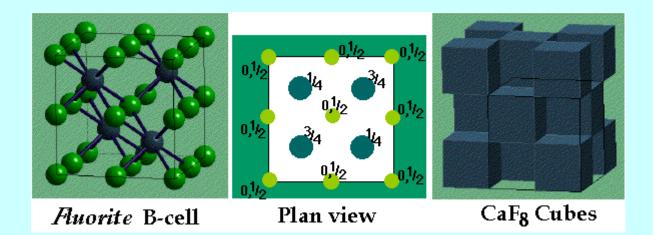
### Fluorite structures (CaF<sub>2</sub>, antifluorite Li<sub>2</sub>O)



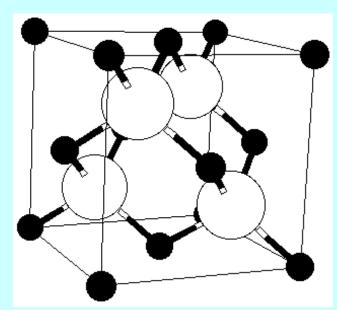
*Fluorite* A-cell

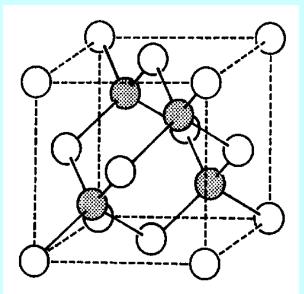
#### Plan view

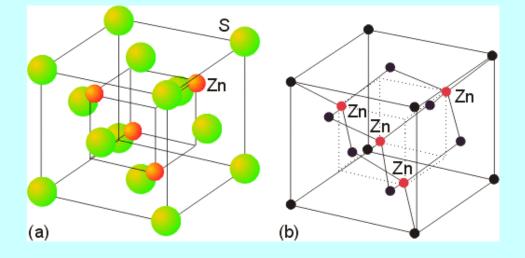
FCa<sub>4</sub> Tetrahedra



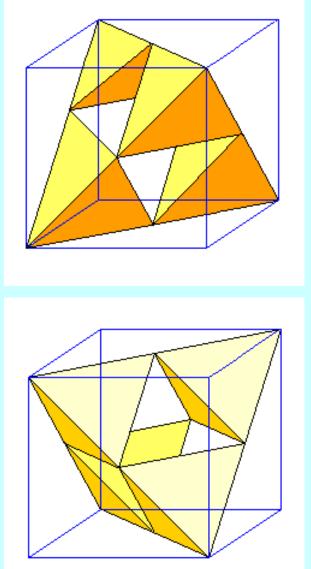
### Sphalerite (zincblende, ZnS)



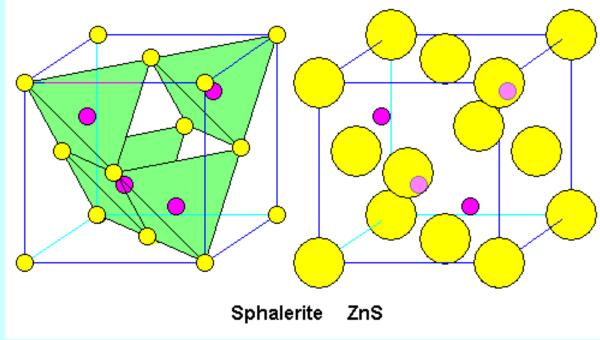


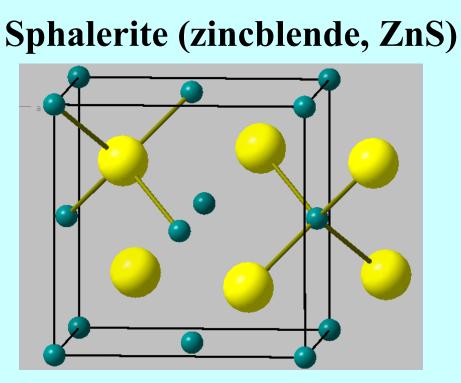


Cubic close packing of anions with 1/2 tetrahedral holes filled by cations



# Sphalerite (zincblende, ZnS)





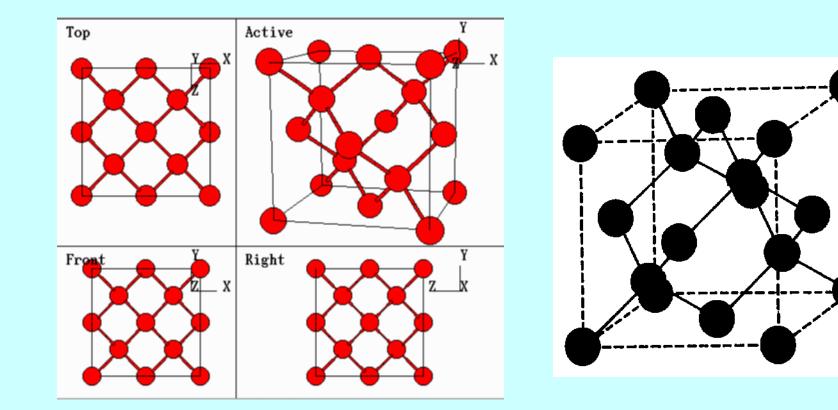
13-15 compounds: BP, BAs, AIP, AlAs, GaAs, GaP, GaSb, AlSb, InP, InAs, InSb

12-16 compounds: BeS, BeSe, BeTe, β-MnS (red), β-MnSe, β-CdS, CdSe, CdTe, HgS, HgSe, HgTe, ZnSe, ZnTe

Halogenides: AgI, CuF, CuCl, CuBr, CuI, NH4F

Borides: PB, AsB Carbides: β-SiC Nitrides: BN

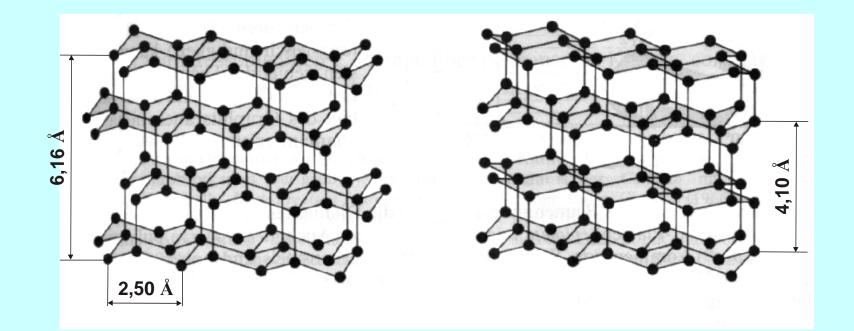
## Diamond



## Diamond

#### cubic

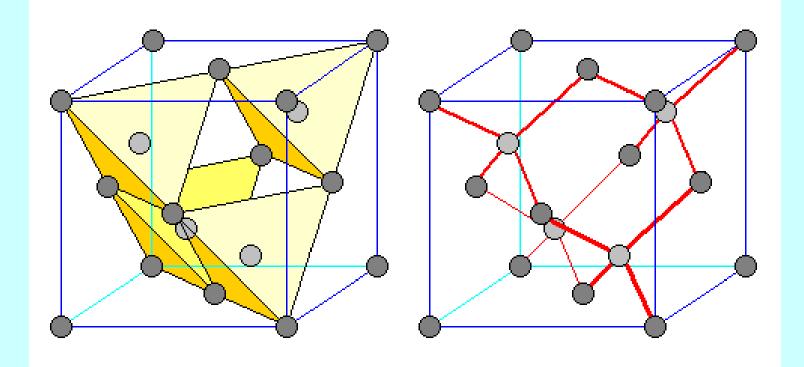
#### hexagonal



#### SiO<sub>2</sub> cristobalite

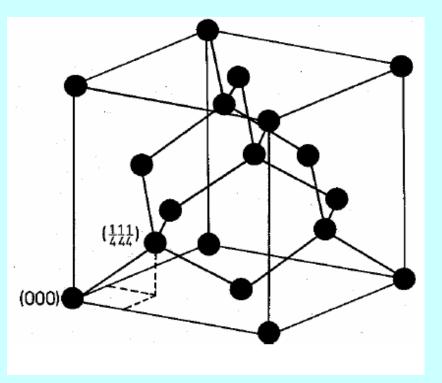
SiO<sub>2</sub> tridymite ice

# **Cubic Diamond**



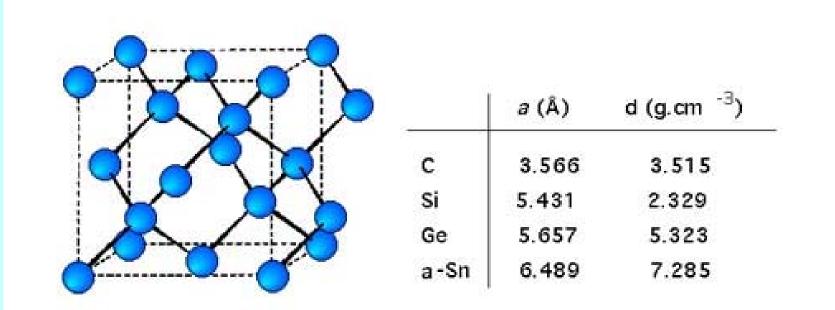
### **Diamond Structure**

#### C, Si, Ge, $\alpha$ -Sn

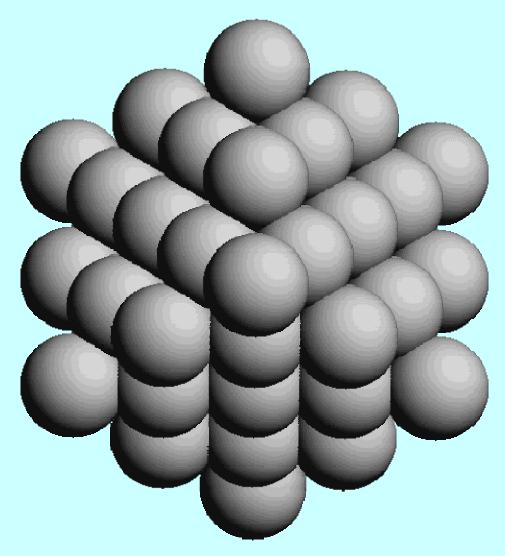


- Add 4 atoms to a FCC
- Tetrahedral bond arrangement
- Each atom has 4 nearest neighbors and 12 next nearest neighbors

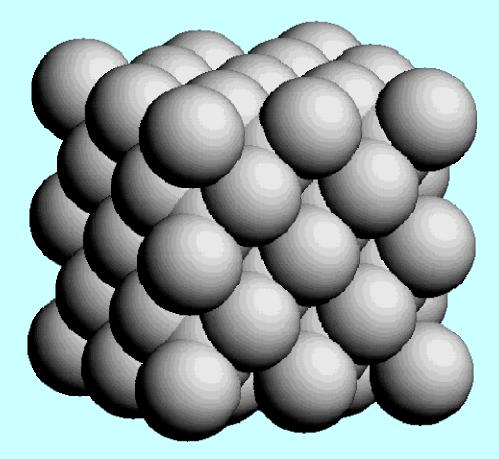
## **Elements of the 14th Group**



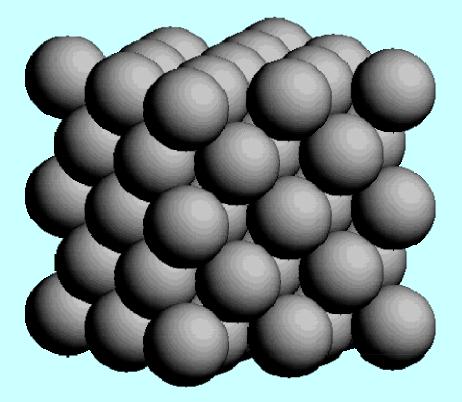
## Diamond Lattice (111) Hard Sphere Model



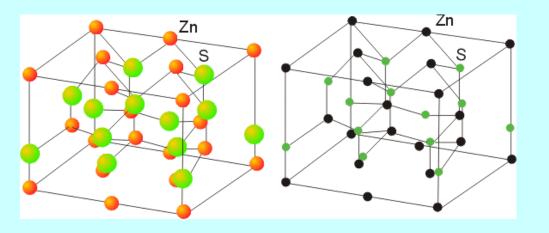
## Diamond Lattice (111) Hard Sphere Model



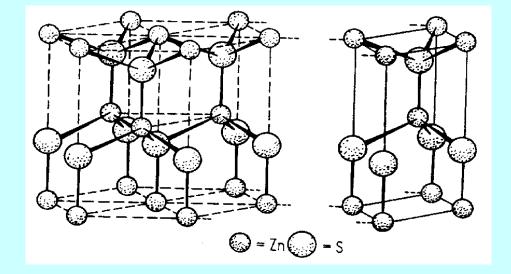
#### Face Centered Cubic Lattice (111) Hard Sphere Model



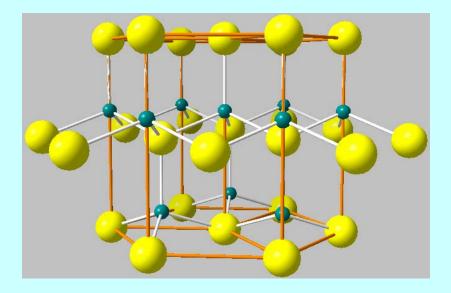
#### Wurzite, ZnS



Hexagonal close packing of anions with 1/2 tetrahedral holes filled by cations

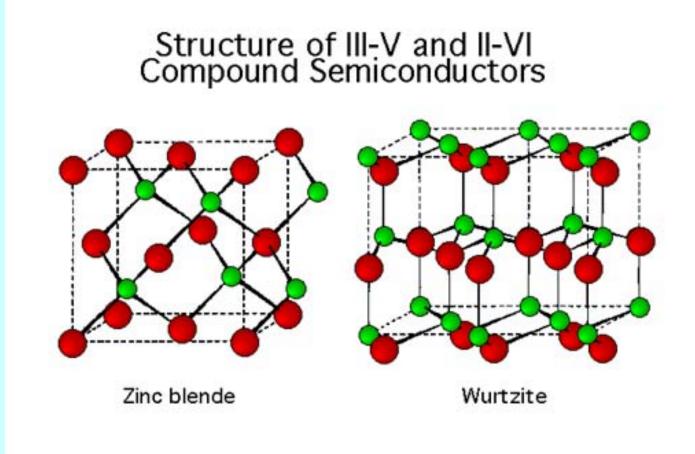


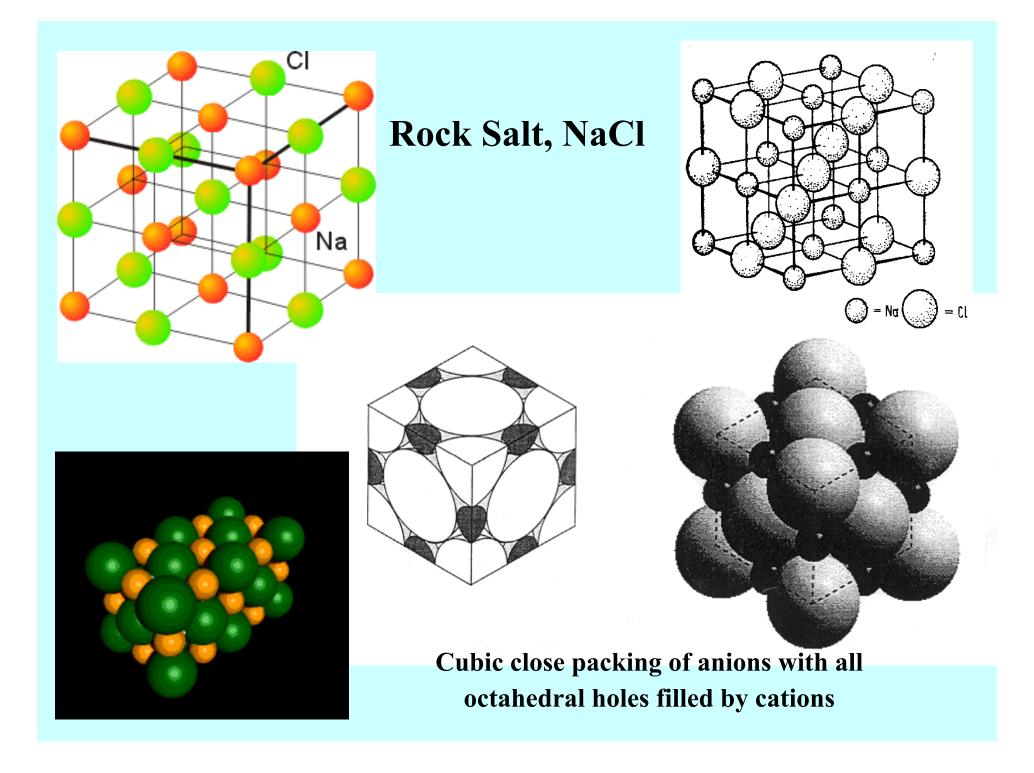
## Wurzite, ZnS



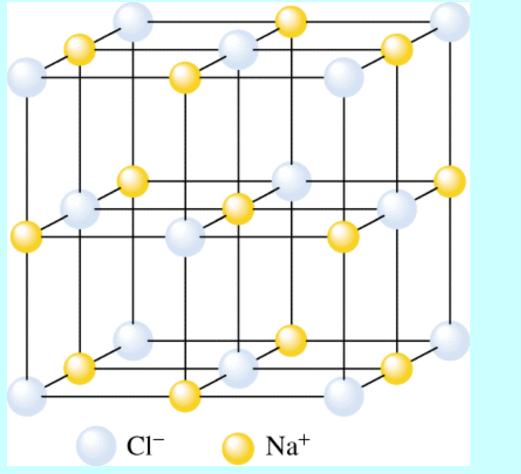
#### ZnO, ZnS, ZnSe, ZnTe, BeO, CdS, CdSe, MnS, AgI, AlN

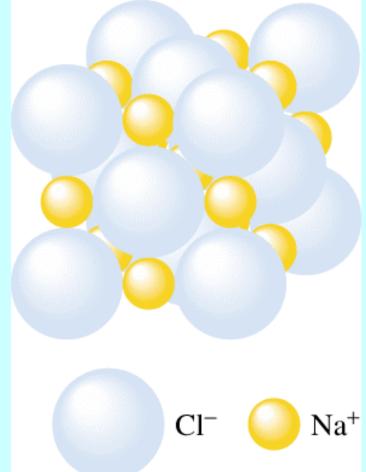
## Semiconductors of 13-15 and 12-16 type



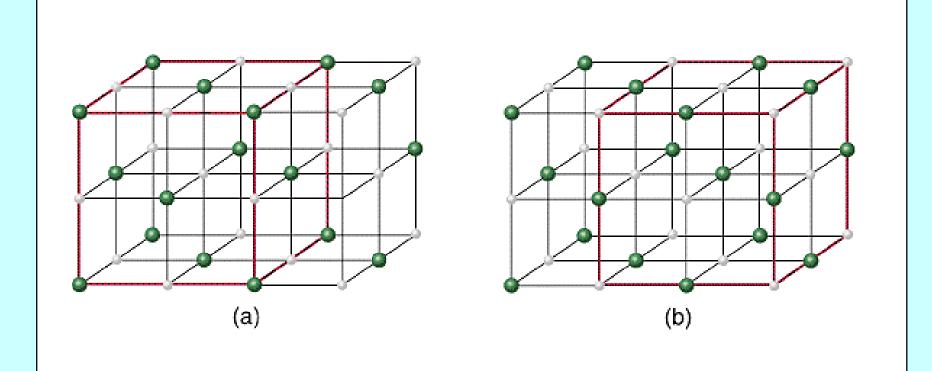


## **Rock Salt, NaCl**



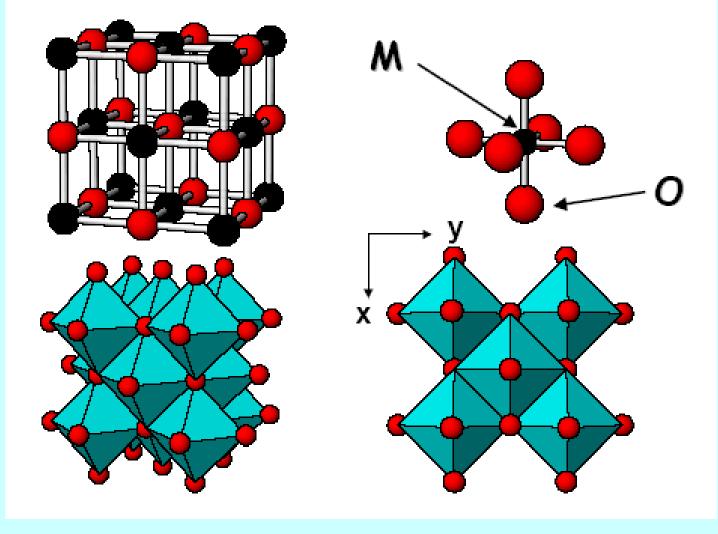


## **Rock Salt, NaCl**

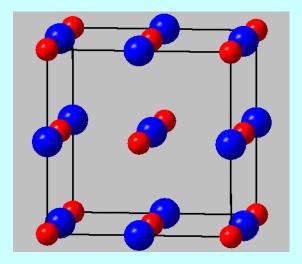


Anion and cation sublattices

# Rock Salt Crystal Structure



## **Rock salt structures (NaCl)**



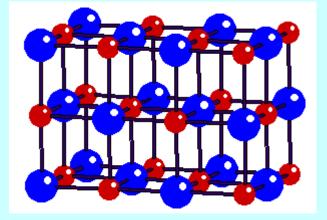
Hydrides: LiH, NaH, KH, NH<sub>4</sub>BH<sub>4</sub> – H<sub>2</sub> storage material

**Borides: ZrB, HfB** 

Carbides: TiC, ZrC, VC, UC

Nitrides: ScN, TiN, UN, CrN, VN, ZrN

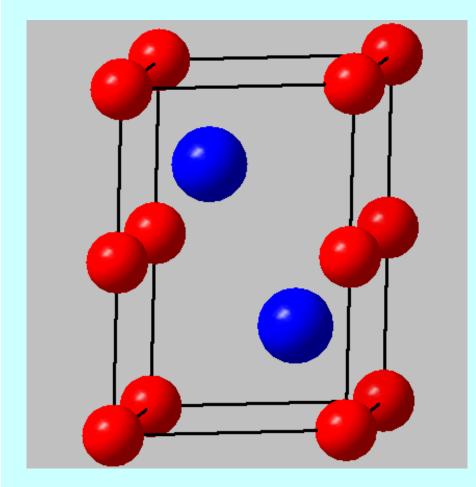
Oxides: MgO, CaO, SrO, BaO, TiO, VO, MnO, FeO, CoO, NiO Chalcogenides: MgS, CaS, SrS, BaS, α-MnS, MgSe, CaSe, SrSe, BaSe, CaTe



Halides: LiF, LiCl, LiBr, LiI, NaF, NaBr, NaI, KF, KCl, KBr, KI, RbF, RbCl, RbBr, AgCl, AgF, AgBr

Intermetallics: SnAs Other FeS<sub>2</sub> (pyrite), CaC<sub>2</sub>, NaO<sub>2</sub>

#### NiAs - type



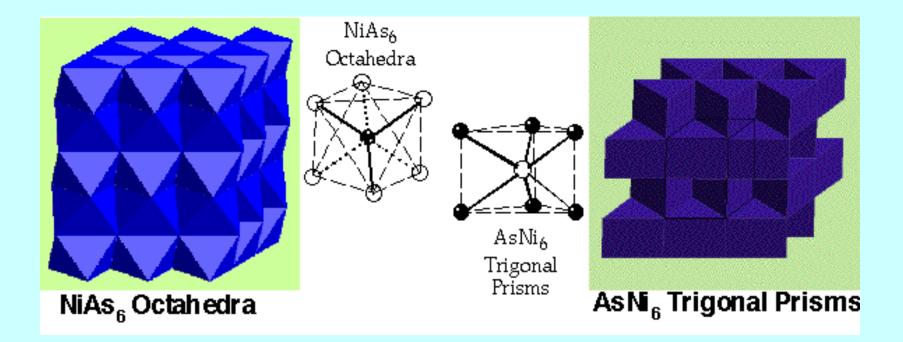
Hexagonal close packing of anions with all octahedral holes filled by cations

NiS, NiAs, NiSb, NiSe, NiSn, NiTe, FeS, FeSe, FeTe, FeSb, PtSn, CoS, CoSe, CoTe, CoSb, CrSe, CrTe, CoSb,

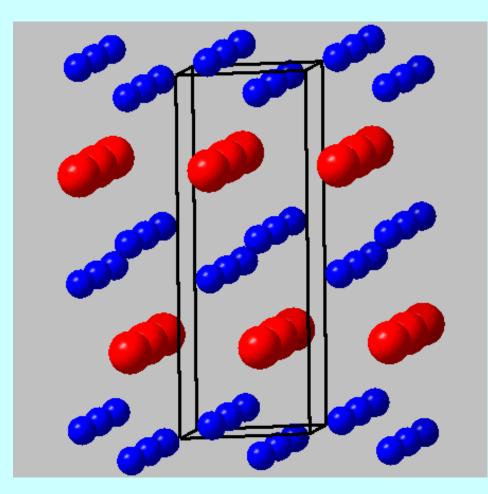
PtB (anti-NiAs structure)

## NiAs - type

#### Hexagonal close packing of anions with all octahedral holes filled by cations

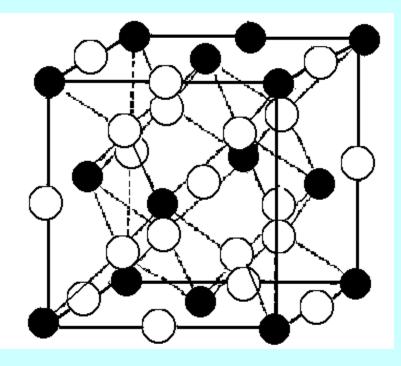


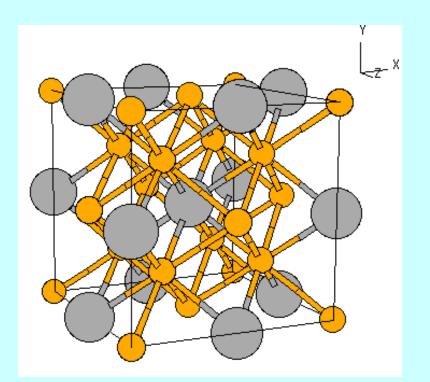
## **ReB<sub>2</sub> - type**



#### Hexagonal close packing of anions with all tetrahedral holes filled by cations

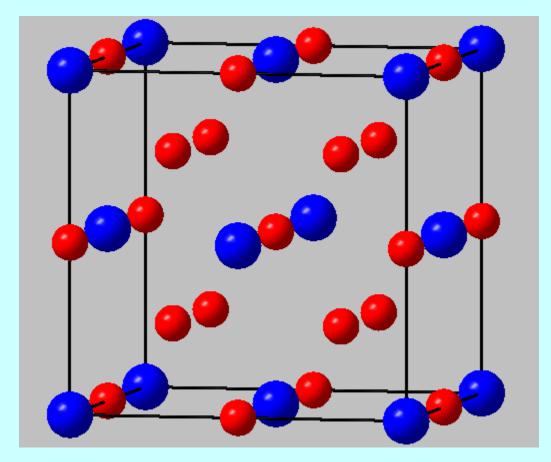
# Li<sub>3</sub>Bi - type (anti BiF<sub>3</sub>)





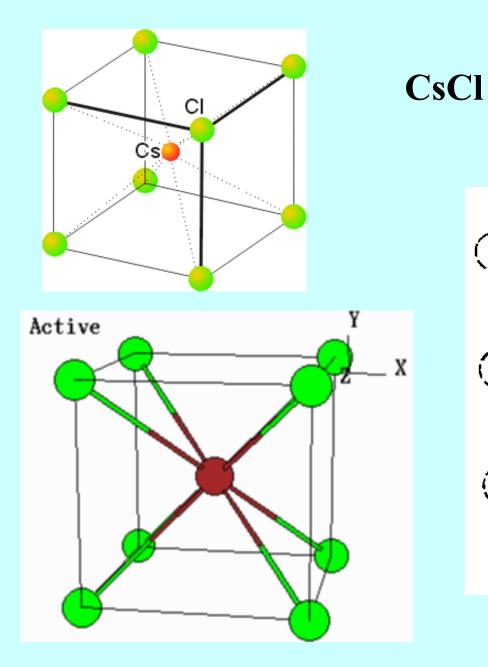
 $[Cr(NH_3)_6]Cl_3, K_3[Fe(CN)_6]$ bcc

## Li<sub>3</sub>Bi - type (anti BiF<sub>3</sub>)

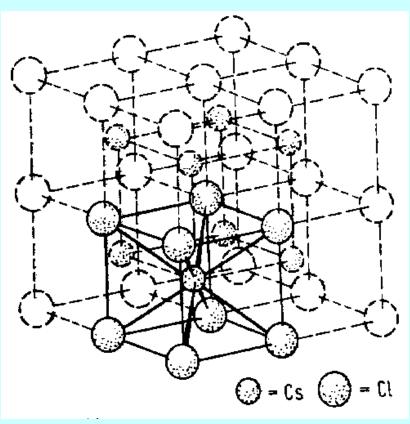


Fe<sub>3</sub>Al [Cr(NH<sub>3</sub>)<sub>6</sub>]Cl<sub>3</sub> K<sub>3</sub>[Fe(CN)<sub>6</sub>]

Cubic close packing of anions with all tetrahedral and octahedral holes filled by cations

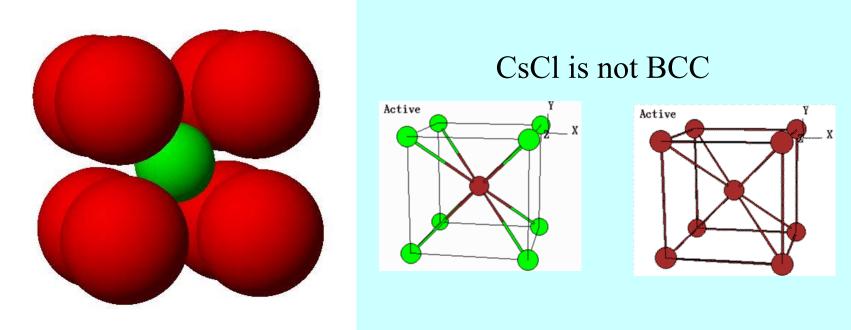


Primitive cubic packing of anions with all cubic holes filled by cations



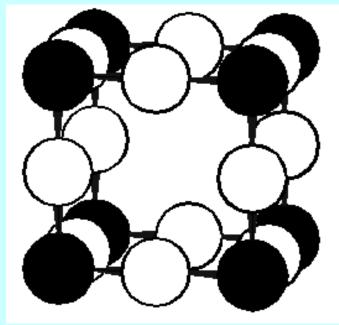
Primitive cubic packing of CsCl<sub>8</sub> cubes sharing all faces

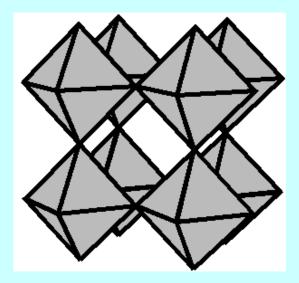
## CsCl



#### CsBr, CsI, CsCN, NH<sub>4</sub>Cl, NH<sub>4</sub>Br, TlCl, TlBr, TlI, CuZn, CuPd, LiHg







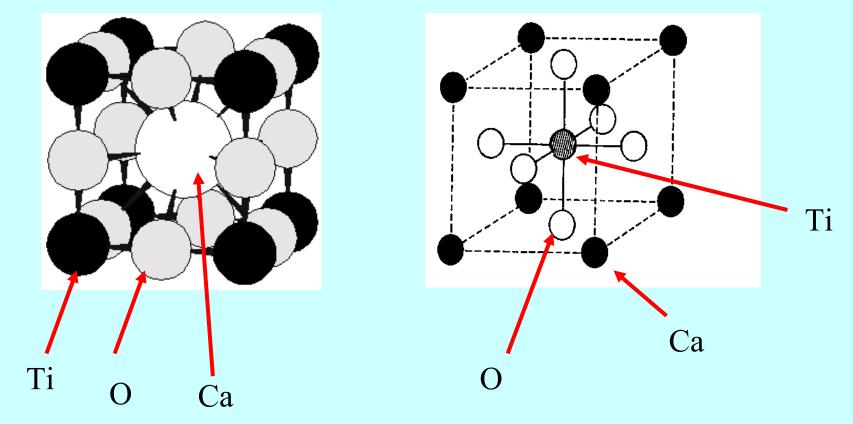
SC of ReO<sub>6</sub> octahedra

# NaCl structure with 3/4 of cations removed and 1/4 of anions removed

UO<sub>3</sub>, MoF<sub>3</sub>, NbF<sub>3</sub>, TaF<sub>3</sub>, Cu<sub>3</sub>N

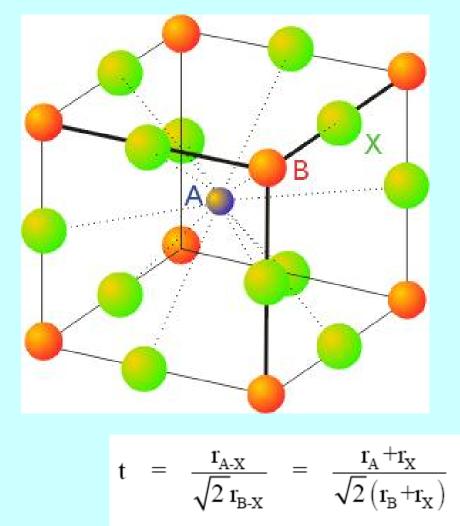
## Perovskite, CaTiO<sub>3</sub>

#### Two equvivalent views of the unit cell of perovskite



Cubic "close packing" of Ca and O with 1/4 octahedral holes filled by Ti cations

#### **Perovskite structure CaTiO**<sub>3</sub>



 $TiO_6$  – octahedra

 $CaO_{12}$  – cuboctahedra

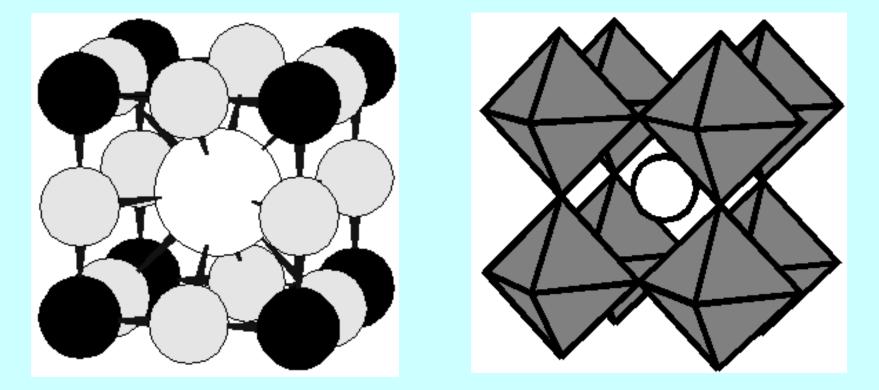
(Ca<sup>2+</sup> and O<sup>2-</sup> form a cubic close packing)

preferred structure of piezoelectric, ferroelectric and superconducting materials

**Goldschmidt's tolerance factor** 

## Perovskite, CaTiO<sub>3</sub>

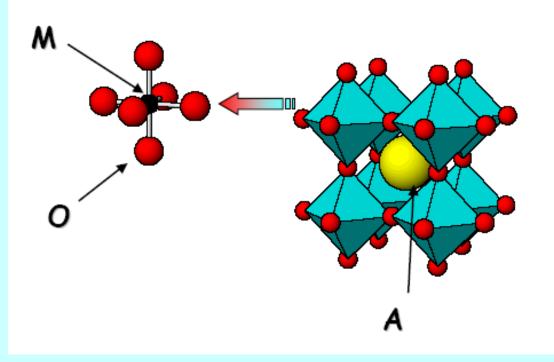
Cubic "close packing" of A and X with 1/4 octahedral holes filled by B cations



**Similarity to CsCl** 

## Perovskite, CaTiO<sub>3</sub>

#### Perovskite Crystal Structure

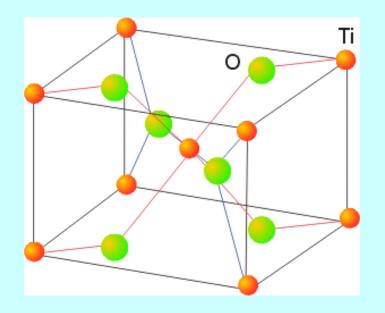


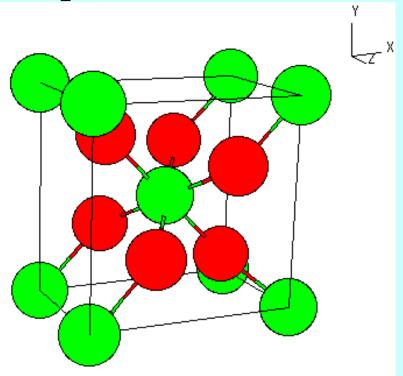
MgSiO<sub>3</sub>, CaSiO<sub>3</sub>

KNbO<sub>3</sub>, KTaO<sub>3</sub>, KIO<sub>3</sub>, NaNbO<sub>3</sub>, NaWO<sub>3</sub>, LaCoO<sub>3</sub>, LaCrO<sub>3</sub>, LaFeO<sub>3</sub>, LaGaO<sub>3</sub>, LaVO<sub>3</sub>, SrTiO<sub>3</sub>, SrZrO<sub>3</sub>, SrFeO<sub>3</sub>

ThTaN<sub>3</sub>, BaTaO<sub>2</sub>N

## Rutile, TiO<sub>2</sub>





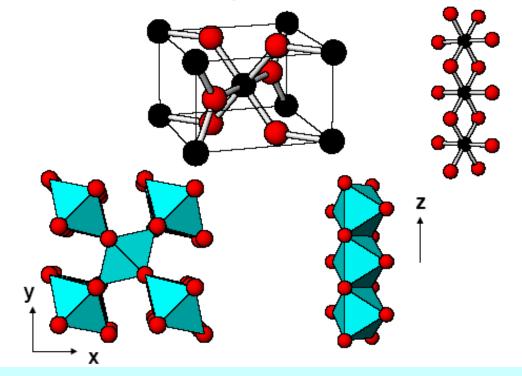
 $CN - stoichiometry Rule A_x B_y$ 

**<u>Distorted</u>** hexagonal close packing of anions with 1/2 octahedral holes filled by cations (giving a tetragonal lattice)

CN(A) / CN(B) = y / x

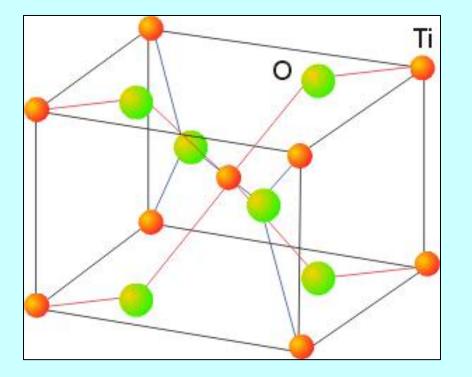
## Rutile, TiO<sub>2</sub>

#### Rutile Crystal Structure



GeO<sub>2</sub>, CrO<sub>2</sub>, IrO<sub>2</sub>, MoO<sub>2</sub>, NbO<sub>2</sub>, β-MnO<sub>2</sub>, OsO<sub>2</sub>, VO<sub>2</sub> (>340K), RuO<sub>2</sub>, CoF<sub>2</sub>, FeF<sub>2</sub>, MgF<sub>2</sub>, MnF<sub>2</sub>

## The rutile structure: TiO<sub>2</sub>

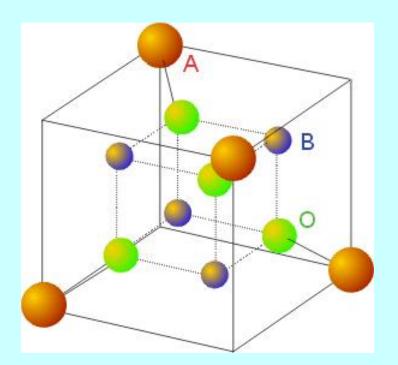


 $TiO_6$  – octahedra

OTi<sub>3</sub> – trigonal planar

(alternative to CaF<sub>2</sub> for highly charged smaller cations)

#### The spinel structure: MgAl<sub>2</sub>O<sub>4</sub>



fcc array of O<sup>2-</sup> ions,  $A^{2+}$  occupies 1/8 of the tetrahedral and  $B^{3+}$  1/2 of the octahedral holes

 $\rightarrow$  <u>normal spinel</u>: AB<sub>2</sub>O<sub>4</sub>

 $\rightarrow \frac{\text{inverse spinel}}{B[AB]O_4 (Fe_3O_4)}:$ Fe<sup>3+</sup>[Fe<sup>2+</sup>Fe<sup>3+</sup>]O<sub>4</sub>

→ basis structure for several <u>magnetic materials</u>

## Spinel

 $AB_2X_4$  Spinel normal: Cubic close packing of anions with 1/2 octahedral holes filled by B cations and 1/8 tetrahedral holes by A cations

MgAl<sub>2</sub>O<sub>4</sub>, CoAl<sub>2</sub>O<sub>4</sub>, MgTi<sub>2</sub>O<sub>4</sub>, Fe<sub>2</sub>GeO<sub>4</sub>, NiAl<sub>2</sub>O<sub>4</sub>, MnCr<sub>2</sub>O<sub>4</sub>

 $AB_2X_4$  Spinel inverse: As for spinel but A cations and 1/2 of B cations interchanged

MgFe<sub>2</sub>O<sub>4</sub>, NiFe<sub>2</sub>O<sub>4</sub>, MgIn<sub>2</sub>O<sub>4</sub>, MgIn<sub>2</sub>S<sub>4</sub>, Mg<sub>2</sub>TiO<sub>4</sub>, Zn<sub>2</sub>TiO<sub>4</sub>, Zn<sub>2</sub>SnO<sub>4</sub>, FeCo<sub>2</sub>O<sub>4</sub>.

## Garnets



Naturally occuring garnets  $A_3B_2Si_3O_{12} = A_3B_2(SiO_4)_3$ 

 $A_3$  = divalent cation (Mg, Fe, Mn or Ca) dodecahedral  $B_2$  = trivalent (Al, Fe<sup>3+</sup>, Ti, or Cr) octahedral  $Si_3$  = tetravalent, tetrahedral Since Ca is much larger in radius than the other divalent cations, there are two series of garnets: one with calcium and one without: pyralspite contain Al (pyrope, almandine, spessartine) ugrandite contain Ca (uvarovite, grossular, andradite)

Synthetic garnets A<sub>3</sub>B<sub>5</sub>O<sub>12</sub>

A<sub>3</sub> = trivalent cations, large size (Y, La,...) B<sub>5</sub> = trivalent (Al, Fe<sup>3+</sup>, Ti, or Cr) 2B octahedral, 3B tetrahedral Y<sub>3</sub>Al<sub>5</sub>O<sub>12</sub> Y<sub>3</sub>Fe<sub>5</sub>O<sub>12</sub>

## Garnets

Pyrope	$Mg_3Al_2(SiO_4)_3$
Almandine	$Fe_3Al_2(SiO_4)_3$
Spessartine	$Mn_3Al_2(SiO_4)_3$
Uvarovite	$Ca_3Cr_2(SiO4)3$
Grossular	$Ca_3Al_2(SiO_4)_3$
Andradite	$Ca_3Fe_2(SiO_4)_3$

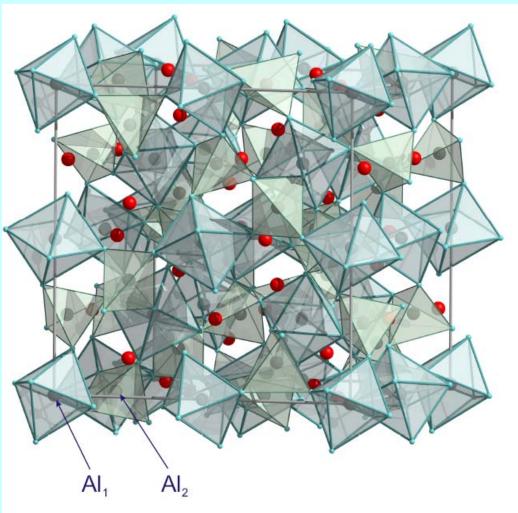
## Garnets

#### Garnet Y<sub>3</sub>Al<sub>5</sub>O<sub>12</sub>

Y<sub>3</sub> = red - dodecahedral trivalent cations, large size

Al<sub>5</sub> = blue 2 octahedral 3 tetrahedral

**O**<sub>12</sub>



#### **Layered Structures**

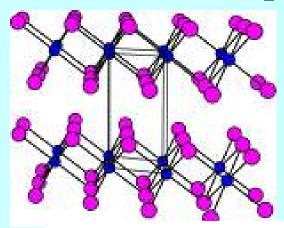
CdI<sub>2</sub> Hexagonal close packing of anions with 1/2 octahedral holes filled by cations

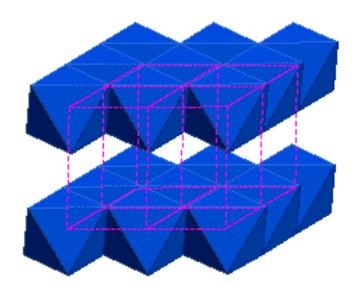
CoI<sub>2</sub>, FeI<sub>2</sub>, MgI<sub>2</sub>, MnI<sub>2</sub>, PbI<sub>2</sub>, ThI<sub>2</sub>, TiI<sub>2</sub>, TmI<sub>2</sub>, VI<sub>2</sub>, YbI<sub>2</sub>, ZnI<sub>2</sub>, VBr<sub>2</sub>, TiBr<sub>2</sub>, MnBr<sub>2</sub>, FeBr<sub>2</sub>, CoBr<sub>2</sub>, TiCl<sub>2</sub>, TiS<sub>2</sub>., TaS<sub>2</sub>.

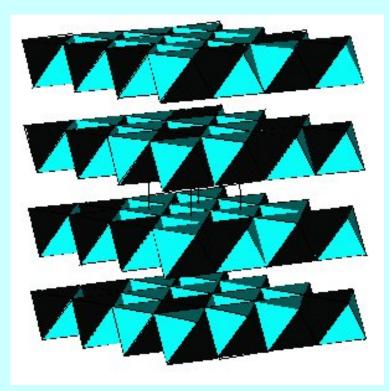
CdCl<sub>2</sub> Cubic close packing of anions with 1/2 octahedral holes filled by cations

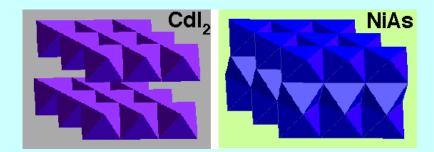
CdCl<sub>2</sub>, CdBr<sub>2</sub>, CoCl<sub>2</sub>, FeCl<sub>2</sub>, MgCl<sub>2</sub>, MnCl<sub>2</sub>, NiCl<sub>2</sub>, NiI<sub>2</sub>, ZnBr<sub>2</sub>, ZnI<sub>2</sub>, Cs<sub>2</sub>O\* (anti-CdCl<sub>2</sub> structure)

# CdI<sub>2</sub> Hexagonal Close Packing

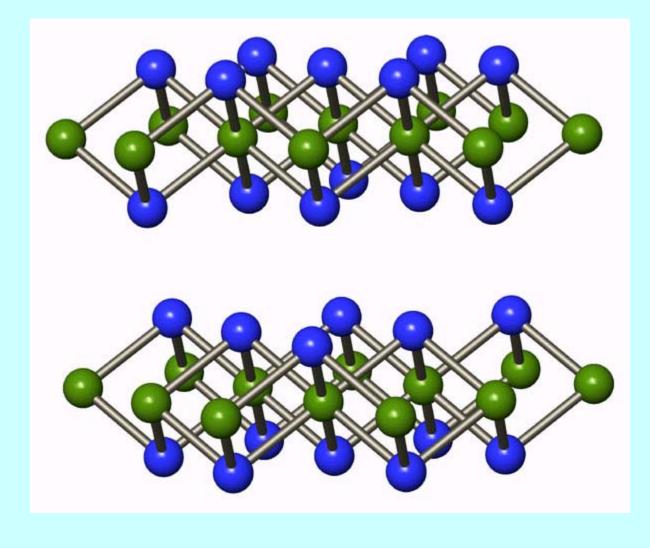




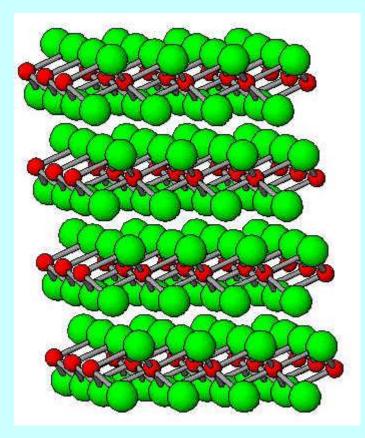


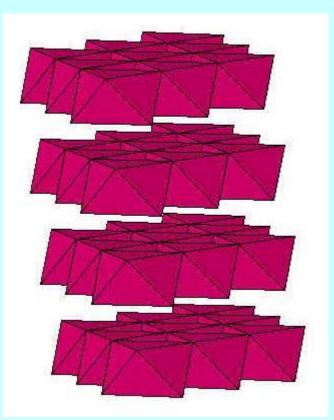


# CdCl<sub>2</sub> Cubic Close Packing

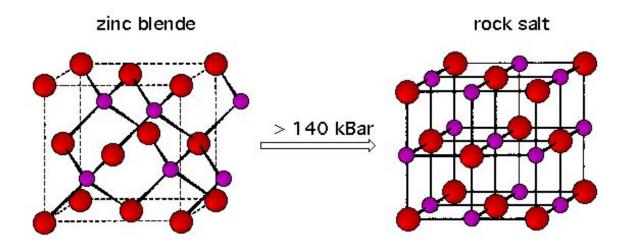


# CdCl<sub>2</sub> Cubic close packing





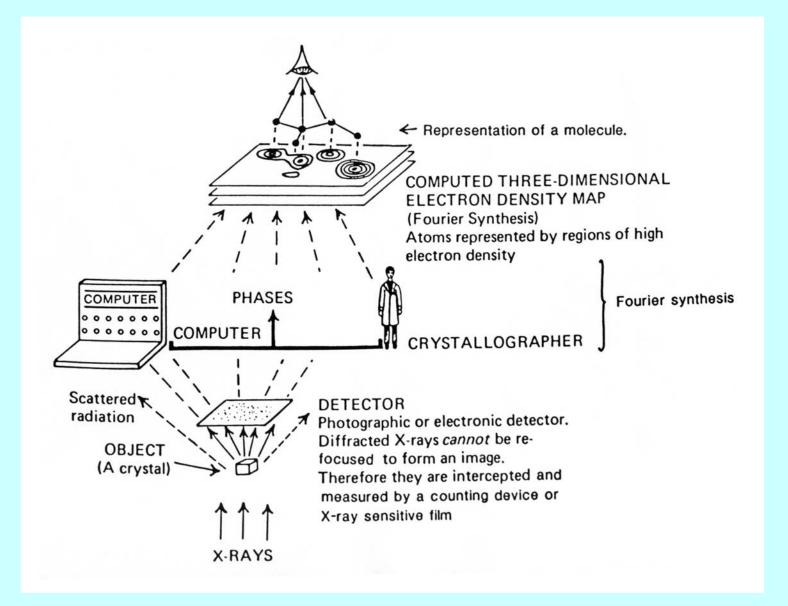
## **High Pressure Transformations**



- high pressure phases
- •higher density
- higher coodination number
- •higher symmetry
- transition to from nonmetal to metal
- •band mixing
- •longer bonds

Pressure/Coordination Number Rule: increasing pressure – higher CN Pressure/Distance Paradox: increasing pressure – longer bonds

#### X-ray structure analysis with single crystals



#### Principle of a four circle X-ray diffractometer for single crystal structure analysis

