

Chapter 1

(1.1) As in Example 1.3, assume outgoing plane wave fields in each region. To get \mathcal{J}_x , we need H_y , since $\hat{x} \times (\hat{z} \cdot \vec{H}) = \mathcal{J}_x$ ($\hat{x} = \hat{z}$). Then we must have E_x to get $\vec{S} = \vec{E} \times \vec{H}^* = \pm S \hat{z}$. So the form of the fields must be,

$$\begin{array}{ll} \text{for } z < 0, & \vec{E}_1 = A e^{jk_0 z} & \text{for } z > 0, & \vec{E}_2 = B e^{-jk_0 z} \\ & \vec{H}_1 = -\frac{A}{\eta_0} e^{jk_0 z} & & \vec{H}_2 = \frac{B}{\eta} e^{-jk_0 z} \end{array}$$

with $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$, $k = \omega \sqrt{\mu_0 \epsilon_0 \epsilon_r}$, $\eta_0 = \sqrt{\mu_0 / \epsilon_0}$, $\eta = \sqrt{\mu_0 / \epsilon_0 \epsilon_r}$, and A and B are unknown amplitudes to be determined.

The boundary conditions at $z=0$ are, from (1.36) and (1.37),

$$\begin{aligned} (\vec{E}_2 - \vec{E}_1) \times \hat{x} &= 0 & \Rightarrow & A = B \\ \hat{z} \times (\vec{H}_2 - \vec{H}_1) &= \mathcal{J}_x & \Rightarrow & -\left(\frac{B}{\eta} + \frac{A}{\eta_0}\right) = \mathcal{J}_0 \\ \therefore A = B &= \frac{-\mathcal{J}_0 \eta \eta_0}{\eta + \eta_0} \end{aligned}$$

1.2

$$\nabla \times \vec{E} = \hat{\rho} \left(\frac{1}{\rho} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} \right) + \hat{\phi} \left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} \right) + \hat{z} \left(\frac{1}{\rho} \frac{\partial (\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi} \right)$$

$$\begin{aligned} \nabla \times \nabla \times \vec{E} = & \hat{\rho} \left[-\frac{1}{\rho^2} \frac{\partial^2 E_\rho}{\partial \phi^2} - \frac{\partial^2 E_\rho}{\partial z^2} + \frac{\partial^2 E_z}{\partial \rho \partial z} + \frac{1}{\rho} \frac{\partial^2 E_\phi}{\partial \rho \partial \phi} + \frac{1}{\rho^2} \frac{\partial E_\phi}{\partial \phi} \right] \\ & + \hat{\phi} \left[-\frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{\rho} \frac{\partial^2 E_z}{\partial \phi \partial z} - \frac{\partial^2 E_\rho}{\partial \rho^2} - \frac{1}{\rho} \frac{\partial E_\phi}{\partial \rho} + \frac{E_\phi}{\rho^2} - \frac{1}{\rho^2} \frac{\partial E_\rho}{\partial \phi} + \frac{1}{\rho} \frac{\partial^2 E_\rho}{\partial \phi \partial \rho} \right] \\ & + \hat{z} \left[-\frac{\partial^2 E_z}{\partial \rho^2} - \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_\rho}{\partial \rho \partial z} + \frac{1}{\rho} \frac{\partial^2 E_\phi}{\partial \phi \partial z} + \frac{1}{\rho} \frac{\partial E_\rho}{\partial z} - \frac{1}{\rho} \frac{\partial E_z}{\partial \rho} \right] \end{aligned}$$

$$\begin{aligned} \nabla (\nabla \cdot \vec{E}) = & \hat{\rho} \left[\frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{\partial^2 E_z}{\partial \rho \partial z} + \frac{1}{\rho} \frac{\partial^2 E_\phi}{\partial \rho \partial \phi} + \frac{1}{\rho} \frac{\partial E_\rho}{\partial \rho} - \frac{1}{\rho^2} \frac{\partial E_\phi}{\partial \phi} - \frac{E_\rho}{\rho^2} \right] \\ & + \hat{\phi} \left[\frac{1}{\rho} \frac{\partial^2 E_z}{\partial \phi \partial z} + \frac{1}{\rho^2} \frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{1}{\rho} \frac{\partial^2 E_\rho}{\partial \rho \partial \phi} + \frac{1}{\rho^2} \frac{\partial E_\rho}{\partial \phi} \right] \\ & + \hat{z} \left[\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho} \frac{\partial^2 E_\phi}{\partial \phi \partial z} + \frac{\partial^2 E_\rho}{\partial \rho \partial z} + \frac{1}{\rho} \frac{\partial E_\rho}{\partial z} \right] \end{aligned}$$

If we apply ∇^2 to the cylindrical components of \vec{E} we get:

$$\begin{aligned} \nabla^2 \vec{E} \stackrel{?}{=} & \hat{\rho} \nabla^2 E_\rho + \hat{\phi} \nabla^2 E_\phi + \hat{z} \nabla^2 E_z \quad (\text{THIS IS NOT A VALID STEP!}) \\ = & \hat{\rho} \left[\frac{1}{\rho} \frac{\partial^2 E_\rho}{\partial \rho} + \frac{\partial^2 E_\rho}{\partial \rho^2} + \frac{1}{\rho^2} \frac{\partial^2 E_\rho}{\partial \phi^2} + \frac{\partial^2 E_\rho}{\partial z^2} \right] \\ & + \hat{\phi} \left[\frac{1}{\rho} \frac{\partial E_\phi}{\partial \rho} + \frac{\partial^2 E_\phi}{\partial \rho^2} + \frac{1}{\rho^2} \frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{\partial^2 E_\phi}{\partial z^2} \right] \\ & + \hat{z} \left[\frac{1}{\rho} \frac{\partial^2 E_z}{\partial \rho} + \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} \right] \end{aligned}$$

Note that the $\hat{\rho}$ and $\hat{\phi}$ components of $\nabla \times \nabla \times \vec{E}$ and $\nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$ do not agree. This is because $\hat{\rho}$ and $\hat{\phi}$ are not constant vectors, so $\nabla^2 \vec{E} \neq \hat{\rho} \nabla^2 E_\rho + \hat{\phi} \nabla^2 E_\phi + \hat{z} \nabla^2 E_z$. The \hat{z} components are equal.

(1.3)

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} 1 & -2j & 0 \\ 2j & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 - 8j \\ 12 + 6j \\ 24 \end{bmatrix}$$

(1.4)

$$\vec{S} = \vec{E} \times \vec{H}^* = E_0 H_0 \hat{z} \neq 0.$$

The problem here is that Poynting's theorem requires a closed surface integral for a meaningful interpretation in terms of power flow. If we calculate $\oint \vec{S} \cdot d\vec{s}$ over the closed surface of a cube bounded by the magnet faces and the capacitor plates, we will get zero, since $\hat{n} = \hat{z}$ on one side of the cube, and $\hat{n} = -\hat{z}$ on the opposite side. Since S is a constant, these terms cancel.

(1.5)

$$\text{Let } \vec{E} = A(\hat{x} - j\hat{y})e^{j(k_0 z)} + B(\hat{x} + j\hat{y})e^{j(k_0 z)},$$

where A is the amplitude of the RHP component, and B is the amplitude of the LHP component. Squaring this expression to the given linearly polarized field gives,

$$\begin{aligned} \hat{x}: & \quad A+B = E_0 \\ \hat{y}: & \quad -jA+jB = 2E_0 \end{aligned}$$

Solving for A, B gives

$$\begin{aligned} A &= (\tfrac{1}{2} + j)E_0 \\ B &= (\tfrac{1}{2} - j)E_0 \end{aligned}$$

Any linearly polarized wave can be decomposed into the sum of two circularly polarized waves.

(1.6) From eq. (1.76),

$$\vec{H} = \frac{1}{\eta_0} \hat{n} \times \vec{E} \quad , \quad \vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{S} = \vec{E} \times \vec{H}^* = \frac{1}{\eta_0} \vec{E} \times \hat{n} \times \vec{E}^*$$

$$= \frac{1}{\eta_0} [(\vec{E} \cdot \vec{E}^*) \hat{n} - (\vec{E} \cdot \hat{n}) \vec{E}^*] \quad (\text{from B.5})$$

Since $\vec{k} \cdot \vec{E}_0 = k_0 \hat{n} \cdot \vec{E}_0 = 0$ from (1.69) and (1.77), we have

$$\vec{S} = \frac{\eta_0}{\eta_0} \vec{E} \cdot \vec{E}^* = \frac{\eta_0}{\eta_0} |\vec{E}|^2 \quad \text{in } \vec{n} \text{ direction} \quad \checkmark$$

(1.7)

Writing general plane wave fields in *z*-region:

$$\vec{E}^i = \hat{x} e^{jk_0 z}$$

$$\vec{H}^i = \frac{1}{\eta_0} \hat{y} e^{jk_0 z}$$

for $z < 0$

$$\vec{E}^r = \hat{x} \Gamma e^{jk_0 z}$$

$$\vec{H}^r = -\frac{1}{\eta_0} \Gamma e^{jk_0 z}$$

for $z < 0$

$$\vec{E}^s = \hat{x} (A e^{jk_0 z} + B e^{jk_0 z})$$

$$\vec{H}^s = \frac{1}{\eta_0} (A e^{jk_0 z} - B e^{jk_0 z})$$

for $0 < z < d$

$$\vec{E}^t = \hat{x} T e^{jk_0(z-d)}$$

$$\vec{H}^t = \frac{1}{\eta_0} T e^{jk_0(z-d)}$$

for $z > d$

Now match E_x and H_y at $z=0$ and $z=d$ to obtain four equations for Γ, T, A, B :

$$z=0: \quad 1 + \Gamma = A + B \quad \frac{1}{\eta_0} (1 - \Gamma) = \frac{1}{\eta_0} (A - B)$$

$$z=d: \quad \hat{x}(-A+B) = T \quad \frac{1}{\eta_0} (-A-B) = \frac{1}{\eta_0} T \quad (\text{since } d = \lambda_0/4 \Rightarrow \Gamma = -1)$$

Solving for Γ gives

$$\Gamma = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2} \quad \checkmark$$

1.8

The incident, reflected, and transmitted fields can be written as,

$$\vec{E}^i = E_0 (\hat{x} - j\hat{y}) e^{jk_0 z} \quad \vec{H}^i = j \frac{E_0}{\eta_0} (\hat{x} - j\hat{y}) e^{jk_0 z} \quad (\text{RHCP})$$

$$\vec{E}^r = E_0 \Gamma (\hat{x} - j\hat{y}) e^{jk_0 z} \quad \vec{H}^r = -j \frac{E_0 \Gamma}{\eta_0} (\hat{x} - j\hat{y}) e^{jk_0 z} \quad (\text{LHCP})$$

$$\vec{E}^t = E_0 T (\hat{x} - j\hat{y}) e^{-jk_0 z} \quad \vec{H}^t = j \frac{E_0 T}{\eta} (\hat{x} - j\hat{y}) e^{-jk_0 z} \quad (\text{RHCP})$$

Matching fields at $z=0$ gives

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}, \quad T = \frac{2\eta}{\eta + \eta_0}$$

The Poynting vectors are: $(\hat{x} - j\hat{y}) \times (\hat{x} - j\hat{y})^* = 2j\hat{z}$

$$\text{For } z < 0: \vec{S}^- = (\vec{E}^i + \vec{E}^r) \times (\vec{H}^i + \vec{H}^r)^* = \frac{2\hat{z} |E_0|^2}{\eta_0} (1 - |\Gamma|^2 + \Gamma e^{2jk_0 z} + \Gamma^* e^{-2jk_0 z})$$

$$\text{For } z > 0: \vec{S}^+ = \vec{E}^t \times \vec{H}^{t*} = \frac{2\hat{z} |E_0|^2 |\Gamma|^2}{\eta^*} e^{-2jk_0 z} \quad \checkmark$$

At $z=0$,

$$\vec{S}^- = \frac{2\hat{z} |E_0|^2}{\eta_0} (1 - |\Gamma|^2 + \Gamma - \Gamma^*) = \frac{2\hat{z} |E_0|^2}{\eta_0} (1 + \Gamma)(1 - \Gamma^*) \checkmark$$

$$\vec{S}^+ = 2\hat{z} |E_0|^2 \frac{4\eta}{|\eta + \eta_0|^2} \quad (\text{using } T = \frac{2\eta}{\eta + \eta_0})$$

$$= \frac{2\hat{z} |E_0|^2}{\eta_0} \left(\frac{2\eta}{\eta + \eta_0} \right) \left(\frac{2\eta_0}{\eta + \eta_0} \right)^* = \frac{2\hat{z} |E_0|^2}{\eta_0} (1 + \Gamma)(1 - \Gamma^*) \quad \checkmark$$

Thus $\vec{S}^- = \vec{S}^+$ at $z=0$, and power is conserved.

1.9

From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = 2\pi j f \sqrt{\mu_0\epsilon_0} \sqrt{5-j2} = j \frac{2\pi(1000)}{300} \sqrt{5.385/-22^\circ}$$

$$= 48.5 \angle 79^\circ = 9.25 + j47.6 = \alpha + j\beta \quad (\text{nepers/m, rad/m})$$

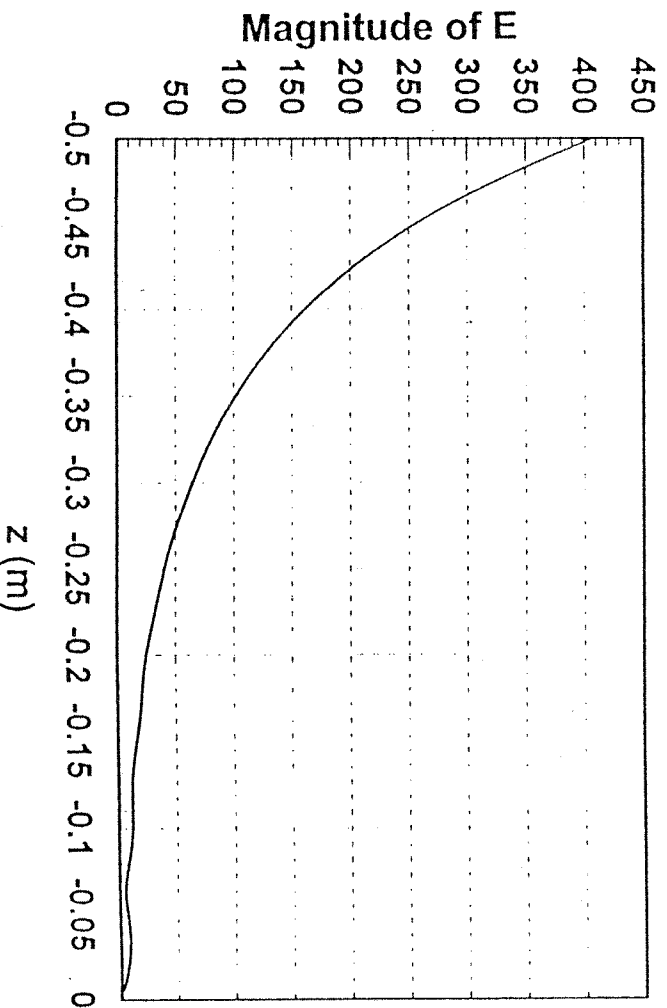
$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\sqrt{\mu_0\epsilon_0}}{j\omega\sqrt{\mu_0\epsilon}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{5-j2}} = \frac{377}{2.32 \angle -11^\circ} = 163 \angle 11^\circ \Omega$$

$$\Gamma = -1$$

For $z < 0$, $E = E^i + E^r = 4\hat{x}(e^{-\gamma z} - e^{\gamma z})$

$$|E| = 4 |e^{-\alpha z} e^{-j\beta z} - e^{\alpha z} e^{j\beta z}|$$

$|E|$ vs z is plotted below.



1.10

In this problem we are only interested in an "order-of-magnitude" estimate for the sheet thickness, so we will ignore reflections at the two interfaces between air and copper. (The more accurate solution can be saved for an exam!)

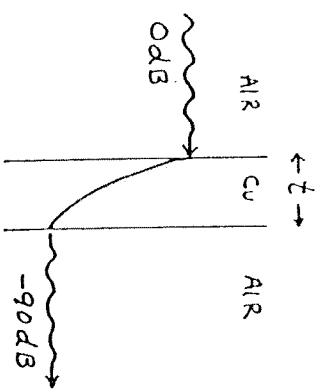
From Table 1.1, the attenuation constant is,

$$\alpha = \frac{1}{\delta_s} = \sqrt{\frac{\omega \mu \sigma}{2}} \quad . \quad \text{From Appendix F, } \sigma_{cu} = 5.8 \times 10^7 \text{ S/m.}$$

To obtain 90 dB attenuation, $20 \log e^{-\alpha t} = -90 \text{ dB}$,

$$\text{or } e^{-\alpha t} = 3.16 \times 10^{-5} \Rightarrow \alpha t = 10.4$$

F	α (1/m)	t (mm)
1 MHz	1.51×10^4	0.68
1 GHz	4.79×10^5	0.022 ✓
100 GHz	4.79×10^6	0.0022



(1.11)

From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = j\frac{2\pi(3000)}{300}\sqrt{3(-j.1)} = 5.435 + j108.964 = \alpha + j\beta \quad \text{m}^{-1}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r(-j.1)}} = 217.121\sqrt{2.855^\circ}$$

$$(a) \quad P_z = R_0 \left\{ \frac{|\bar{E}_z(z=0)|^2}{\eta^*} \right\} = 46.000 \text{ W/m}^2 \quad \checkmark$$

$$\Gamma = -1 \text{ at } z = L = 20 \text{ cm}$$

$$\bar{E}_r = \Gamma \bar{E}_z(z=L) e^{\gamma(z-L)} = -100 \hat{x} e^{-2\gamma L} e^{\gamma z}$$

$$P_r = R_0 \left\{ \frac{|\bar{E}_r(z=0)|^2}{\eta^*} \right\} = 0.595 \text{ W/m}^2 \quad \checkmark$$

$$(b) \quad \bar{E}_z = \bar{E}_i + \bar{E}_r$$

$$\bar{E}_z(z=0) = 100 \hat{x} (1 - e^{-2\gamma L}) \quad H_z(z=0) = \frac{100 \hat{y}}{\eta} (1 + e^{-2\gamma L})$$

$$P_{in} = R_0 \left\{ \bar{E}_z \times \bar{H}_z^* \cdot \hat{z} \right\} = 45.584 \text{ W/m}^2$$

But $P_i - P_r = 45.405 \text{ W/m}^2 \neq P_{in}$. This is because P_i and P_r individually are not physically meaningful in a lossy medium.

The above values were computed entirely using a FORTRAN program, with 6-digit precision. The error between $P_i - P_r$ and P_{in} is only about 0.44% - this could be made more significant if the loss were increased.

(1.12)

The current sheet will generate obliquely propagating plane waves. From (1.132)-(1.133), assume

$$\vec{E}_1 = A (\hat{x} \cos \theta_1 + \hat{y} \sin \theta_1) e^{jk_0(x \sin \theta_1 - z \cos \theta_1)}$$

$$\vec{H}_1 = \frac{-A}{\eta_0} \hat{y} e^{jk_0(x \sin \theta_1 - z \cos \theta_1)}$$

} for $z < 0$

$$\vec{E}_2 = B (\hat{x} \cos \theta_2 - \hat{y} \sin \theta_2) e^{jk(x \sin \theta_2 + z \cos \theta_2)}$$

$$\vec{H}_2 = \frac{B}{\eta} \hat{y} e^{jk(x \sin \theta_2 + z \cos \theta_2)}$$

} for $z > 0$

with $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$, $k = \sqrt{\epsilon_r} k_0$, $\eta_0 = \sqrt{\mu_0 / \epsilon_0}$, $\eta = \eta_0 / \sqrt{\epsilon_r}$.

Apply boundary conditions at $z=0$:

$$\hat{z} \times (\vec{E}_2 - \vec{E}_1) = 0 \implies A \cos \theta_1 e^{jk_0 x \sin \theta_1} - B \cos \theta_2 e^{-jk x \sin \theta_2} = 0$$

$$\hat{z} \times (\vec{H}_2 - \vec{H}_1) = J_s \implies \frac{A}{\eta_0} e^{jk_0 x \sin \theta_1} + \frac{B}{\eta} e^{-jk x \sin \theta_2} = -J_0 e^{-j\beta x}$$

For phase matching we must have $k_0 \sin \theta_1 = k \sin \theta_2 = \beta$

$\therefore \theta_1 = \sin^{-1} \beta / k_0$ ✓ $\theta_2 = \sin^{-1} \beta / k$ (must have $\beta < k_0$)

Then,

$$A \cos \theta_1 = B \cos \theta_2, \quad \frac{A}{\eta_0} + \frac{B}{\eta} = -J_0$$

$$A = \frac{-J_0 \eta_0 \cos \theta_2}{\eta \cos \theta_2 + \eta_0 \cos \theta_1}, \quad B = \frac{-J_0 \eta_0 \cos \theta_1}{\eta \cos \theta_2 + \eta_0 \cos \theta_1}$$

Check: If $\beta=0$, then $\theta_1 = \theta_2 = 0$, and $A=B = \frac{-J_0 \eta_0}{\eta + \eta_0}$,

which agrees with Problem 1.1 ✓

(1.13)

This solution is identical to the parallel polarized dielectric case of Section 1.8, except for the definitions of k_1 , k_2 , n_1 , and n_2 . Thus,

$$k_0 \sin \theta_i = k_0 \sin \theta_r = k \sin \theta_t \quad ; \quad k = k_0 \sqrt{\mu_r}$$

$$r = \frac{n \cos \theta_t - n_0 \cos \theta_i}{n \cos \theta_t + n_0 \cos \theta_i}$$

$$T = \frac{2n \cos \theta_i}{n \cos \theta_t + n_0 \cos \theta_i}$$

$$n = n_0 \sqrt{\mu_r}$$

There will be a Brewster angle if $r = 0$. This requires that,

$$n \cos \theta_t = n_0 \cos \theta_i$$

$$\sqrt{\mu_r} \sqrt{1 - \left(\frac{k_0}{k}\right)^2 \sin^2 \theta_i} = \cos \theta_t = \sqrt{1 - \sin^2 \theta_t}$$

$$\mu_r \left(1 - \frac{1}{\mu_r} \sin^2 \theta_i\right) = 1 - \sin^2 \theta_t$$

or, $\mu_r = 1$. This implies a uniform region, so there is no Brewster angle for $\mu_r \neq 1$. ✓

(1.14)

Again, this solution is similar to the perpendicular polarized case of Section 1.8, except for the definition of k_1 , k_2 , n_1 , n_2 . Thus,

$$r = \frac{n \cos \theta_i - n_0 \cos \theta_t}{n \cos \theta_i + n_0 \cos \theta_t}$$

$$T = \frac{2n \cos \theta_i}{n \cos \theta_i + n_0 \cos \theta_t}$$

A Brewster angle exists if

$$n \cos \theta_i = n_0 \cos \theta_t$$

$$\sqrt{\mu_r} \sqrt{1 - \sin^2 \theta_i} = \sqrt{1 - \frac{1}{\mu_r} \sin^2 \theta_i}$$

$$\mu_r^2 - \mu_r^2 \sin^2 \theta_i = \mu_r - \sin^2 \theta_i$$

$$\mu_r = (\mu_r + 1) \sin^2 \theta_i$$

$$\sin \theta_i = \sin \theta_t = \sqrt{\frac{\mu_r}{1 + \mu_r}} < 1 \quad \checkmark$$

Thus a Brewster angle does exist for this case.

(1.15)

$$D_x = \epsilon_0 (\epsilon_r E_x + j k E_y)$$

$$D_y = \epsilon_0 (-j k E_x + \epsilon_r E_y)$$

$$D_z = \epsilon_0 E_z$$

Then,

$$D_+ = D_x - j D_y = \epsilon_0 (\epsilon_r - k) E_x - j \epsilon_0 (\epsilon_r - k) E_y = \epsilon_0 (\epsilon_r - k) E_+$$

$$D_- = D_x + j D_y = \epsilon_0 (\epsilon_r + k) E_x + j \epsilon_0 (\epsilon_r + k) E_y = \epsilon_0 (\epsilon_r + k) E_-$$

or,

$$\begin{bmatrix} D_+ \\ D_- \\ D_z \end{bmatrix} = \begin{bmatrix} (\epsilon_r - k) & 0 & 0 \\ 0 & (\epsilon_r + k) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_+ \\ E_- \\ E_z \end{bmatrix}$$

From Maxwell's equations,

$$\nabla \times \nabla \times \vec{E} = -j \omega \mu \nabla \times \vec{H} = \omega^2 \mu [\epsilon] \vec{E}$$

$$\left. \begin{aligned} \nabla \times \vec{E} &= -j \omega \mu \vec{H} \\ \nabla \times \vec{H} &= j \omega [\epsilon] \vec{E} \end{aligned} \right\}$$

$$\nabla^2 \vec{E} + \omega^2 \mu [\epsilon] \vec{E} = 0 \quad (\text{CARTESIAN})^2$$

Expanding this wave equation gives,

$$\nabla^2 E_x + \omega^2 \mu \epsilon_0 [\epsilon_r E_x + j k E_y] = 0 \quad (1)$$

$$\nabla^2 E_y + \omega^2 \mu \epsilon_0 [-j k E_x + \epsilon_r E_y] = 0 \quad (2)$$

$$\nabla^2 E_z + k_0^2 E_z = 0 \quad (3)$$

Adding (1) + j(2) gives $\nabla^2 (\epsilon_r E_x + j k E_y) + \omega^2 \mu \epsilon_0 [(\epsilon_r + k) E_x + j(\epsilon_r + k) E_y] = 0$

$$\nabla^2 E^+ + \omega^2 \mu \epsilon_0 (\epsilon_r + k) E^+ = 0 \quad \checkmark$$

$$\therefore \beta_+ = k_0 \sqrt{\epsilon_r + k} \quad \checkmark$$

Adding (1) - j(2) gives $\nabla^2 (\epsilon_r E_x - j k E_y) + \omega^2 \mu \epsilon_0 [(\epsilon_r - k) E_x - j(\epsilon_r - k) E_y] = 0$

$$\nabla^2 E^- + \omega^2 \mu \epsilon_0 (\epsilon_r - k) E^- = 0$$

$$\therefore \beta_- = k_0 \sqrt{\epsilon_r - k} \quad \checkmark$$

Note: The wave equations for E^+ , E^- must be satisfied simultaneously. Thus, for E^+ , we must have $E^- \equiv 0$. This implies that $E_y = jE_x = jE_0$. The actual electric field is then,

$$E^+ = \hat{x} E_0 + \hat{y} E_0 = E_0 (\hat{x} + j\hat{y}) e^{-j\beta z} \quad (\text{LHCP})$$

This is a LHCP wave. Similarly for E^- , we must have $E^+ \equiv 0$:

$$E^- = \hat{x} E_0 + \hat{y} E_0 = E_0 (\hat{x} - j\hat{y}) e^{j\beta z} \quad (\text{RHCP})$$

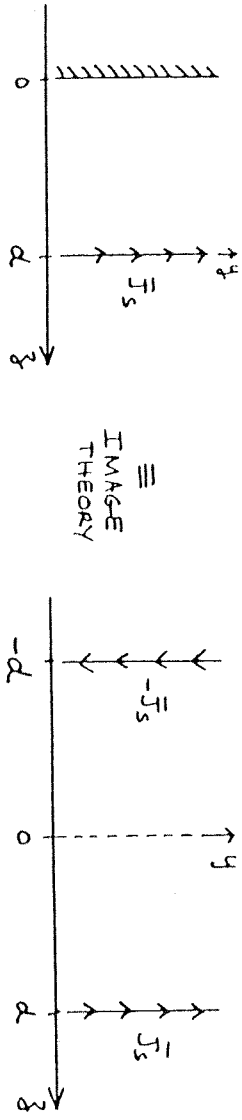
(1.16)

Comparing (1.123) and (1.131) indicates that $\vec{E}_t = R_s \vec{J}_s = R_s \hat{n} \times \vec{H}$ is the desired surface impedance relation. Applying this to the surface integral of (1.155) gives, on S ,

$$\begin{aligned} & \text{USING (B.5)} \quad \downarrow \\ & (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \hat{n} = R_s [(\hat{n} \times \vec{H}_1) \times \vec{H}_2 - (\hat{n} \times \vec{H}_2) \times \vec{H}_1] \\ & = R_s [(\cancel{\vec{H}_1} \cdot \hat{n}) \vec{H}_2 - (\cancel{\vec{H}_2} \cdot \hat{n}) \vec{H}_1 - (\vec{H}_1 \cdot \hat{n}) \vec{H}_2 + (\vec{H}_2 \cdot \hat{n}) \vec{H}_1] \\ & = 0 \end{aligned}$$

As (1.157) is obtained

(1.17)



First find the fields due to the source at $z=d$. From (1.139) -

(1.140),

For $z < d$,

$$\vec{E}_1 = A \hat{y} e^{jk_0(x \sin \theta - z \cos \theta)}$$

$$\vec{H}_1 = \frac{A}{\eta_0} (\hat{x} \cos \theta + \hat{z} \sin \theta) e^{-jk_0(x \sin \theta - z \cos \theta)}$$

For $z > d$,

$$\vec{E}_2 = B \hat{y} e^{-jk_0(x \sin \theta + z \cos \theta)}$$

$$\vec{H}_2 = \frac{B}{\eta_0} (-\hat{x} \cos \theta + \hat{z} \sin \theta) e^{jk_0(x \sin \theta + z \cos \theta)}$$

Apply boundary conditions at $z=d$:

$$\hat{z} \times [\vec{E}(d^+) - \vec{E}(d^-)] = 0 \Rightarrow A e^{jk_0 d \cos \theta} = B e^{-jk_0 d \cos \theta}$$

$$\hat{z} \times [\vec{H}(d^+) - \vec{H}(d^-)] = \vec{J}_s \Rightarrow [-B \cos \theta e^{jk_0 d \cos \theta} - A \cos \theta e^{jk_0 d \cos \theta}] \cdot e^{-jk_0 x \sin \theta} = \eta_0 J_0 e^{-j\beta x}$$

For phase matching, $k_0 \sin \theta = \beta$

$$\text{Then, } A = \frac{-\eta_0 J_0}{2 \cos \theta} e^{jk_0 d \cos \theta} \quad B = \frac{-\eta_0 J_0}{2 \cos \theta} e^{jk_0 d \cos \theta}$$

$$\vec{E} = \frac{-\eta_0 J_0 \hat{y}}{2 \cos \theta} \begin{cases} e^{jk_0[x \sin \theta - (z-d) \cos \theta]} & z < d \\ e^{jk_0[x \sin \theta + (z-d) \cos \theta]} & z > d \end{cases}$$

The fields due to the source at $z=d$ can then be found by replacing d with $-d$, and J_0 with $-J_0$:

$$\vec{E} = \frac{\eta_0 J_0 \hat{y}}{2 \cos \theta} \begin{cases} e^{-jk_0 [x \sin \theta - (z+d) \cos \theta]} & z < -d \\ e^{-jk_0 [x \sin \theta + (z+d) \cos \theta]} & z > -d \end{cases}$$

Combining these results gives the total field:

$$\vec{E} = \frac{-j \eta_0 J_0 \hat{y}}{\cos \theta} \begin{cases} e^{-jk_0 x \sin \theta} e^{-jk_0 d \sin(k_0 z \cos \theta)} & 0 < z < d \\ e^{-jk_0 x \sin \theta} e^{-jk_0 z \sin(k_0 d \cos \theta)} & z > d \end{cases}$$

CHECK: If $\beta = 0$, then $\theta = 0$ and we have,

$$\vec{E} = -j \eta_0 J_0 \hat{y} \begin{cases} e^{jk_0 d} \sin k_0 z & \text{for } 0 < z < d \\ e^{-jk_0 z} \sin k_0 d & \text{for } z > d \end{cases}$$

This agrees with the results in (1.161) - (1.162).

Chapter 2

$$2.1 \quad Y = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{(5+j6.28)(.01+j.94)} = \sqrt{(6.28/89.54^\circ)(.94/89.39^\circ)}$$

$$= 24.3 \sqrt{89.465^\circ} = 0.23 + j24.3 = \alpha + j\beta \quad \text{mS/m, rad/m} \checkmark$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{6.28/89.54^\circ}{.94/89.39^\circ}} = 25.8 \sqrt{.08} = 25.8 + j0.5 \Omega \checkmark$$

$$\cancel{R=G=0},$$

$$\alpha = 0$$

$$\beta = \omega \sqrt{LC} = 24.3 \checkmark$$

$$Z_0 = \sqrt{L/C} = 25.8 \Omega \checkmark$$

Note that β and Z_0 for the lossless case are very close to the corresponding results for the lossy (lossless) case.

2.2 Using KVL:

$$-V(\beta) + R \frac{\Delta I}{2} i(\beta) + L \frac{\Delta I}{2} \frac{\partial i(\beta)}{\partial t} + R \frac{\Delta I}{2} i(\beta + \Delta \beta) + L \frac{\Delta I}{2} \frac{\partial i(\beta + \Delta \beta)}{\partial t} + V(\beta + \Delta \beta) = 0$$

divide by $\Delta \beta$ and let $\Delta \beta \rightarrow 0$:

$$\frac{\partial V(\beta)}{\partial \beta} = -R i(\beta) - L \frac{\partial i(\beta)}{\partial t} \checkmark$$

Using KCL:

$$i(\beta) - \Delta \beta \left[G + C \frac{\partial}{\partial t} \right] [V(\beta) - \frac{\Delta I}{2} (R + L \frac{\partial}{\partial t}) i(\beta)] - i(\beta + \Delta \beta) = 0$$

divide by $\Delta \beta$ and let $\Delta \beta \rightarrow 0$:

$$\frac{\partial i(\beta)}{\partial \beta} = -G V(\beta) - C \frac{\partial V(\beta)}{\partial t} \checkmark$$

These results agree with (2.2a,b).

2.3

Ignoring fringing fields, \mathbf{E} and \mathbf{H} can be assumed

$$\text{at, } \mathbf{E}_y = \frac{-V_0}{d} \text{ V/m, } \mathbf{H}_x = \frac{V_0}{d\eta} = \frac{I_0}{W} \text{ A/m, } \eta = \sqrt{\mu/\epsilon}$$

Then $\mathbf{E} \times \mathbf{H}^* = \hat{z} |S| \checkmark$ and $I_0 = V_0 \left(\frac{W}{\eta d} \right)$.

From (2.17) - (2.20),

$$L = \frac{\mu_0}{\epsilon_0} \int_S |\mathbf{H}|^2 ds = \frac{\mu_0}{\epsilon_0} \int_{x=0}^W \int_{y=0}^d \left(\frac{I_0}{W} \right)^2 dx dy = \frac{\mu_0 d}{W} \text{ H/m } \checkmark$$

$$C = \frac{\epsilon}{V_0^2} \int_S |\mathbf{E}|^2 ds = \frac{\epsilon}{V_0^2} \int_{x=0}^W \int_{y=0}^d \left(\frac{-V_0}{d} \right)^2 dx dy = \frac{\epsilon W}{d} \text{ F/d/m } \checkmark$$

$$R = \frac{R_s}{\Gamma_0^2} \int_{C_1+C_2} |\mathbf{H}|^2 dl = \frac{2R_s}{\Gamma_0^2} \int_{x=0}^W \int_{y=0}^d \left(\frac{I_0}{W} \right)^2 dx = \frac{2R_s}{W} \text{ } \Omega/\text{m} \checkmark$$

$$G = \frac{\omega \epsilon''}{V_0^2} \int_S |\mathbf{E}|^2 ds = \frac{\omega \epsilon''}{V_0^2} \int_{x=0}^W \int_{y=0}^d \left(\frac{-V_0}{d} \right)^2 dx dy = \frac{\omega \epsilon'' W}{d} \text{ S/m } \checkmark$$

These results agree with those in Table 2.1

2.4

Assume $E_z = H_z = 0$, $\partial/\partial x = \partial/\partial y = 0$.

Then Maxwell's curl equations reduce to,

$$-\frac{\partial E_y}{\partial z} = -j\omega \mu H_x \quad (1) \quad -\frac{\partial H_y}{\partial z} = j\omega \epsilon E_x \quad (3)$$

$$\frac{\partial E_x}{\partial z} = -j\omega \mu H_y \quad (2) \quad \frac{\partial H_x}{\partial z} = j\omega \epsilon E_y \quad (4)$$

Since $E_x = 0$ at $z=0$ and $z=d$, and $\partial/\partial y = 0$, we must have $E_x = 0$. Then (3) implies $H_y = 0$. So we have,

$$\frac{\partial E_y}{\partial z} = j\omega \mu H_x \quad \frac{\partial H_x}{\partial z} = j\omega \epsilon E_y$$

Now let $E_y = d V(z)$ and $H_x = \frac{1}{W} I(z)$.

Then the voltage and current are,

$$V(z) = \int_{y=0}^d E_y dy \quad I(z) = \int_{x=0}^W (\hat{y} \times \hat{z}) \cdot \hat{x} dx = - \int_{x=0}^W H_x dx$$

Then,

$$\left. \begin{aligned} \frac{\partial V(z)}{\partial z} &= -j \frac{\omega \mu d}{W} I(z) \Rightarrow L = \frac{\mu d}{W} \\ \frac{\partial I(z)}{\partial z} &= -j \frac{\omega \epsilon W}{d} V(z) \Rightarrow C = \frac{\epsilon W}{d} \end{aligned} \right\} \begin{array}{l} \text{agree with} \\ \text{Table 2.1} \end{array}$$

2.5 From Table 2.1 ($a=0.5\text{mm}$, $b=1.5\text{mm}$, $\epsilon_r=2.8$, $\tan \delta=0.005$)

$$L = \frac{\mu_0}{2\pi} \ln \frac{b}{a} = 0.219 \text{ nH/m} \quad \checkmark$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln b/a} = 142. \text{ pF/m} \quad \checkmark$$

$$R = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right) = 6.07 \text{ } \Omega/\text{m} \quad \checkmark$$

$$G = \frac{2\pi \omega \epsilon_0 \epsilon_r \tan \delta}{\ln b/a} = 0.0134 \text{ S/m} \quad \checkmark$$

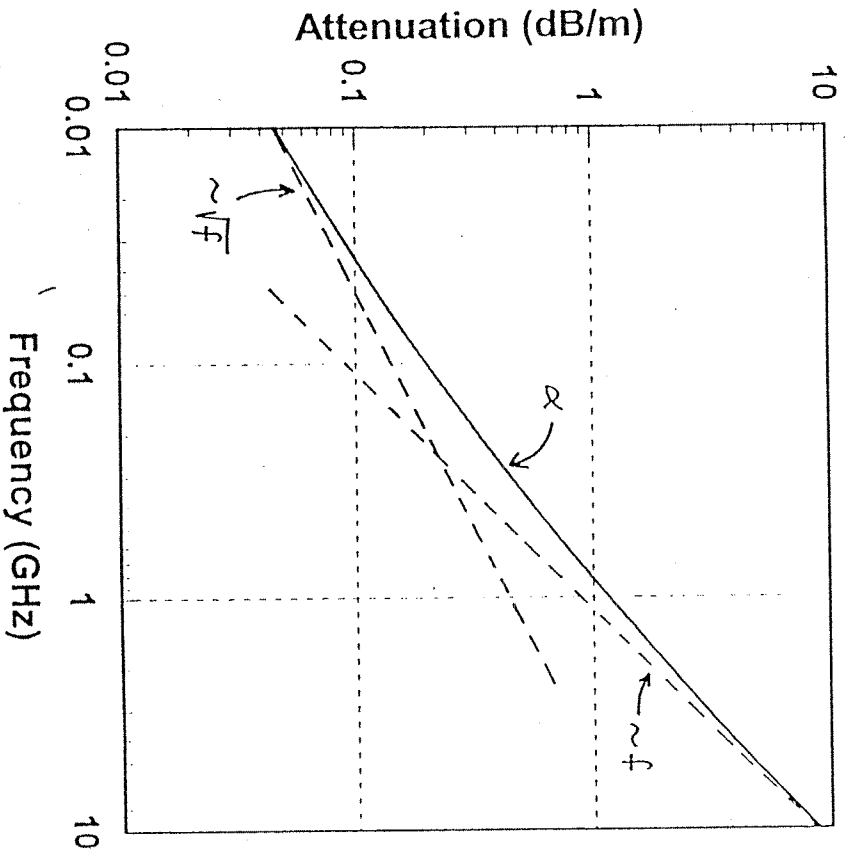
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{4128 \angle 89.916^\circ}{2.677 \angle 89.713^\circ}} = 39.3 \angle 0.10^\circ$$

$$\begin{aligned} \gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} = 105.1 \angle 89.81^\circ \quad \checkmark \\ &= 0.34 + j105.1 = \alpha + j\beta \quad (\text{np/m, rad/m}) \end{aligned}$$

2.6 Using the results from P. 2.5, with $\alpha \approx \frac{1}{2}(R/\lambda + GZ_0)$, give:

f	R_s	$R(\lambda)$	$G(s)$	α (Np/m)	α (dB/m)
10 MHz	8.24×10^{-4}	.350	4.47×10^{-5}	.0053 ✓	.046
100 MHz	2.6×10^{-3}	1.10	4.47×10^{-4}	.0228 ✓	.198 ✓
3 GHz	1.43×10^{-2}	6.07	1.34×10^{-2}	.341 ✓	2.96
10 GHz	2.61×10^{-2}	11.08	4.47×10^{-2}	1.02	8.85

(The above approximation (2.85a) is much simpler than the exact method as used in Problem 2.5, and is accurate for small loss, as shown by comparing the results at 3 GHz.) Results are plotted below:



Note that the frequency dependence is between \sqrt{f} ($R \sim \sqrt{f}$) and f ($G \sim f$), at low and high frequencies.

$$2.7 \quad Z_L = Z_L/Z_0 = 0.53 + j0.266$$

From Smith chart, $\Gamma_{in} = 0.93 - j0.7$

$$\text{So } Z_{in} = Z_0 \Gamma_{in} = 69.8 - j52.5 \Omega \quad \checkmark$$

$$\text{SWR} = 2.05, \quad \Gamma = 0.34 \angle 140^\circ \quad \checkmark$$

$$2.8 \quad |\Gamma| = \frac{S-1}{S+1} = \frac{0.5}{2.5} = 0.2$$

$$|\Gamma| = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| = \left| \frac{100 - Z_0}{100 + Z_0} \right| \quad Z_0 \text{ real.}$$

So either, $\frac{100 - Z_0}{100 + Z_0} = 0.2 \Rightarrow Z_0 = Z_L \frac{1-\Gamma}{1+\Gamma} = 100 \left(\frac{1.8}{1.2} \right) = 66.7 \Omega \quad \checkmark$

or, $\frac{100 - Z_0}{100 + Z_0} = -0.2 \Rightarrow Z_0 = Z_L \frac{1-\Gamma}{1+\Gamma} = 100 \left(\frac{1.2}{1.8} \right) = 150 \Omega \quad \checkmark$

$$2.9 \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j40}{130 + j40} = \frac{50 \angle 53^\circ}{136 \angle 17^\circ} = 0.367 \angle 36^\circ \quad \checkmark$$

$$P_{\text{LOAD}} = P_{\text{INC}} - P_{\text{REF}} = P_{\text{INC}} (1 - |\Gamma|^2) = 30 [1 - (0.367)^2] = 25.9 \text{ W} \quad \checkmark$$

$$2.10 \quad \lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{300}{3000 \sqrt{2.56}} = 6.25 \text{ cm}$$

$$L = \frac{2.0 \text{ cm}}{6.25 \text{ cm}/\lambda_g} = 0.320 \lambda_g \quad \beta L = \frac{2\pi}{\lambda_g} (0.320 \lambda_g) = 115.2^\circ$$

SMITH CHART SOLUTION:

$$Z_{in} = 18.98 - j20.55 \Omega$$

$$\Gamma_{in} = 0.62 \angle 212^\circ$$

$$\Gamma_L = 0.62 \angle 83^\circ$$

$$\text{SWR} = 4.27$$

ANALYTICAL SOLUTION:

$$\tan \beta l = -2.125$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} = 75 \frac{(37.5 + j 75) + j 75(-2.125)}{75 + j(37.5 + j 75)(-2.125)} = 18.99 - j 20.55 \Omega \checkmark$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.62 \angle 182.9^\circ \checkmark$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0.62 \angle -147.5^\circ \checkmark$$

$$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = 4.26 \checkmark$$

(2.11)

$$RL = -20 \log |\Gamma|$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = 10^{-RL/20}$$

$$|\Gamma| = \frac{SWR - 1}{SWR + 1}$$

SWR	\Gamma	RL (dB)
1.00	0.0	∞
1.01	.005	46.0
1.02	.01	40.0
1.05	.024	32.3
1.07	.0316	30.
1.10	.0476	26.4
1.20	.091	20.8
1.22	.100	20.
1.50	.200	14.
1.92	.316	10.
2.00	.333	9.5
2.50	.429	7.4

2.12

$$V_g = 15 \text{ V RMS}, \quad Z_g = 75 \Omega, \quad Z_0 = 75 \Omega, \quad Z_L = 60 - j40 \Omega, \quad \beta = 0.7 \lambda$$

$$a) \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-15 - j40}{135 - j40} = \frac{42.7 \angle -110.6^\circ}{140.8 \angle -16.5^\circ} = 0.303 \angle -94^\circ = -0.021 - j0.302$$

$$P_L = \left(\frac{V_g}{Z_0}\right)^2 \frac{1}{Z_0} (1 - |\Gamma|^2) = 0.681 \text{ W} \quad \checkmark$$

This method is actually based on $P_L = P_{in} (1 - |\Gamma|^2)$, it is the simplest method, but only applies to lossless lines.

$$b) \quad Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} = 75 \frac{60 + j190.8}{198.1 + j184.7} = 75 \frac{200 \angle 72.5^\circ}{270.8 \angle 43^\circ} \\ = 55.4 \angle 29.5^\circ = 48.2 + j27.3 \Omega$$

$$P_L = \left| \frac{V_g}{Z_g + Z_{in}} \right|^2 R_e(Z_{in}) = \left| \frac{15}{123.2 + j27.3} \right|^2 (48.2) = 0.681 \text{ W} \quad \checkmark$$

This method computes $P_L = P_{in} = |I_{in}|^2 R_{in}$, and also applies only to lossless lines.

$$c) \quad V(z) = V^+ (e^{-j\beta z} + \Gamma e^{j\beta z}) \\ V_L = V(0) = V^+ (1 + \Gamma) \quad V^+ = \frac{V_g}{2} = 7.5 \text{ V} \\ = 7.5 (1 - 0.021 - j0.302) \\ = 7.68 \angle -17^\circ$$

$$P_L = \left| \frac{V_L}{Z_L} \right|^2 R_e(Z_L) = \left(\frac{7.68}{72.1} \right)^2 (60) = 0.681 \text{ W} \quad \checkmark$$

This method computes $P_L = |I_L|^2 R_L$, and applies to lossy as well as lossless lines. Note the concept that $V^+ = V_g/2$ requires a good understanding of the transmission line equations, and only applies here because $Z_g = Z_0$.

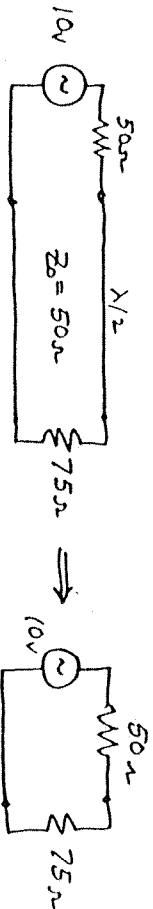
2.13

$$Z_L = jX$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$|\Gamma|^2 = \Gamma \Gamma^* = \frac{(jX - Z_0)(-jX - Z_0)}{(jX + Z_0)(-jX + Z_0)} = \frac{X^2 - j^2 Z_0 X + j Z_0 X + Z_0^2}{X^2 + Z_0^2} = 1 \quad \checkmark$$

2.14



$$\text{POWER DELIVERED BY SOURCE} = \frac{1}{2} \frac{(10)^2}{50+75} = 0.400 \text{ W} \quad \checkmark$$

$$\text{POWER DISSIPATED IN } 50\Omega \text{ LOAD} = \frac{1}{2} (50) \left(\frac{10}{50+75} \right)^2 = 0.160 \text{ W} \quad \checkmark$$

$$\text{POWER TRANSMITTED DOWN LINE} = \frac{1}{2} (75) \left(\frac{10}{50+75} \right)^2 = 0.240 \text{ W} \quad \checkmark$$

$$\text{INCIDENT POWER} = \frac{1}{2} (50) \left(\frac{10}{50+50} \right)^2 = 0.250 \text{ W} \quad \checkmark$$

$$\text{REFLECTED POWER} = P_{\text{INC}} |\Gamma|^2 = .250 \left| \frac{75-50}{75+50} \right|^2 = 0.010 \text{ W} \quad \checkmark$$

$$P_{\text{INC}} - P_{\text{REF}} = .250 - .010 = 0.240 = P_{\text{TRANS}} \quad \checkmark$$

$$P_{\text{DISS}} + P_{\text{TRANS}} = .160 + .240 = 0.400 = P_{\text{SOURCE}} \quad \checkmark$$

2.15

$$\Gamma = \frac{-20-j40}{180-j40} = \frac{44.7 \angle -116.6^\circ}{184.4 \angle -12.5^\circ} = 0.24 \angle -104^\circ = -0.058 - j0.233 \quad \checkmark$$

$$V_L = 10 \frac{80-j40}{180-j40} = 10 \frac{89.4 \angle -26^\circ}{184 \angle -12.5^\circ} = 4.86 \angle -13.5^\circ$$

$$V(z) = V^+ [e^{-j\beta z} + \Gamma e^{j\beta z}] \quad V^+ = 10 \frac{100}{100+100} = 5 \text{ V} \quad \checkmark$$

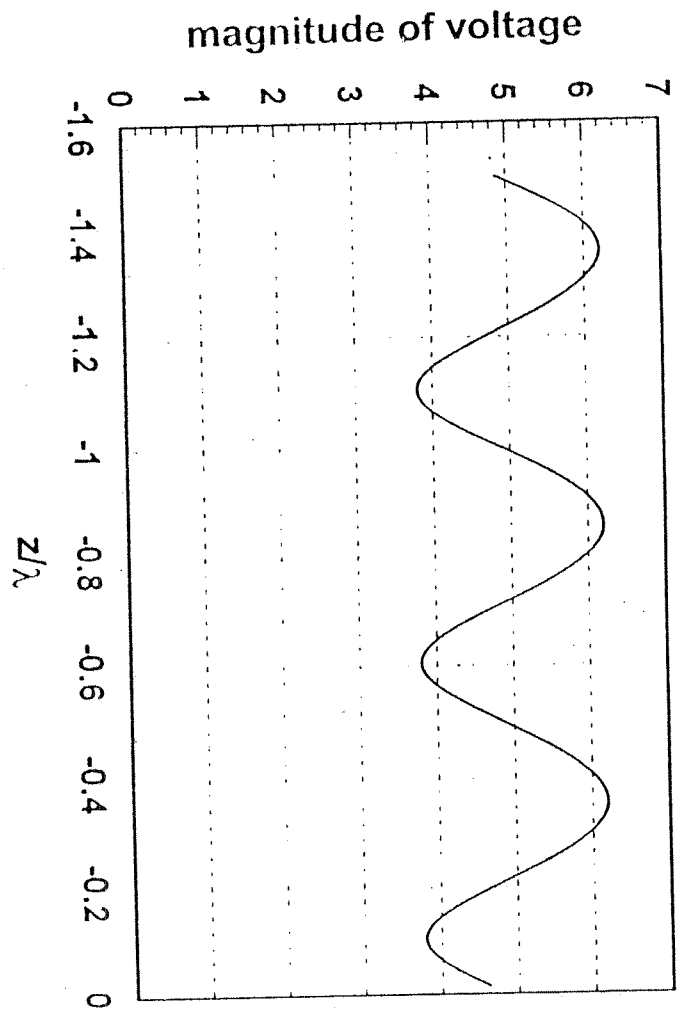
$$\text{So } V(z) = 5 [e^{-j\beta z} + \Gamma e^{j\beta z}]$$

$$V_{MAX} = 5(1+|r|) = 5(1.24) = 6.2 \text{ at } \beta = -0.355\lambda$$

$$V_{MIN} = 5(1-|r|) = 5(0.76) = 3.8 \text{ at } \beta = -0.105\lambda$$

} REPEATS EVERY $\lambda/2$

$|V(z)|$ is plotted below.



2.16

The real characteristic impedance Z_1 must satisfy,

$$100 = Z_1 \frac{(80 + j20) + jZ_1 t}{Z_1 + j(80 + j20)t}, \text{ where } t = \tan \beta l$$

Multiplying out and separating real and imaginary parts gives,

$$\text{Re: } 100Z_1 - 2000t = 80Z_1 \Rightarrow Z_1 = 100t$$

$$\text{Im: } 8000t = Z_1(20 + Z_1 t)$$

Solving for Z_1, t :

$$6 = 10t^2 \Rightarrow t = 0.775 \Rightarrow \beta l = 37.8^\circ, \underline{\underline{\lambda = 0.105\lambda}}$$

$$Z_1 = 100t = \underline{\underline{77.5\Omega}}$$

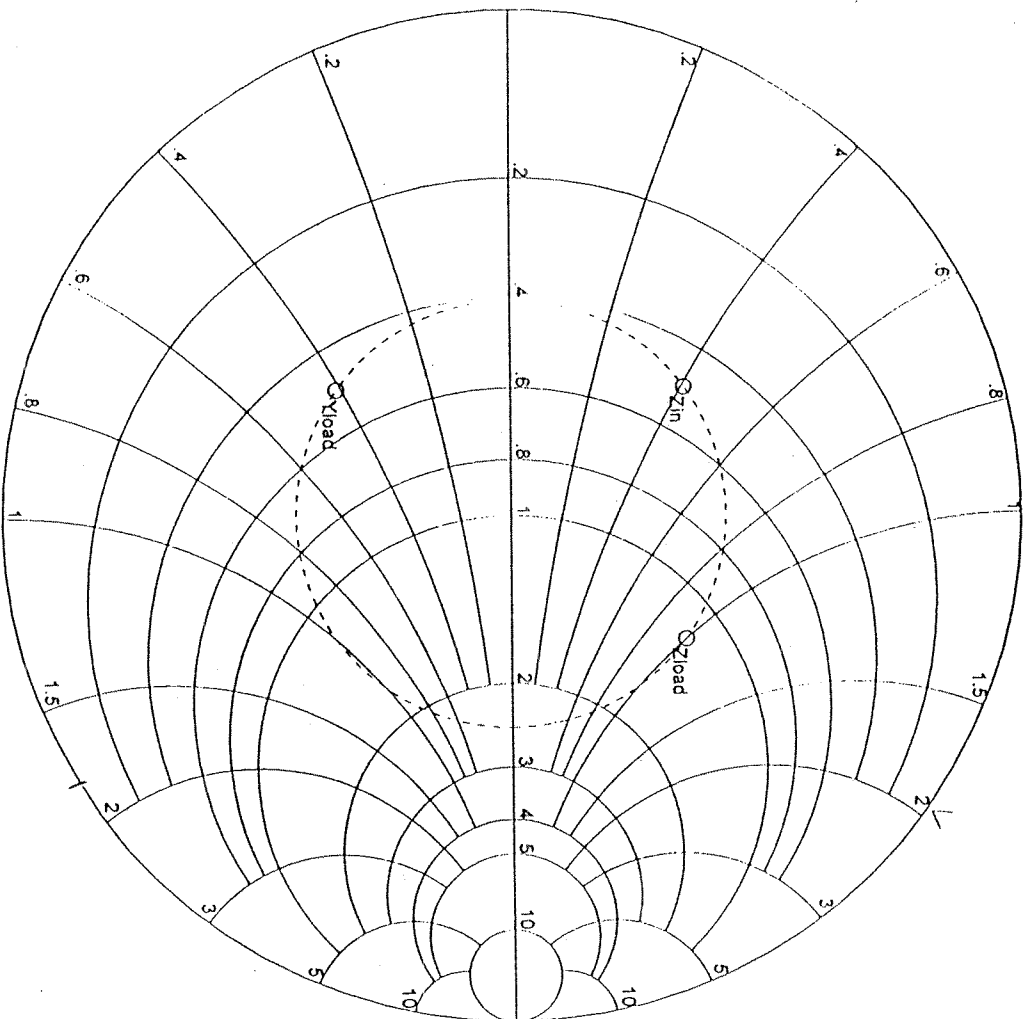
(or, $\lambda = 0.605\lambda$, etc)

2.17

$$Z_0 = 50 \Omega, Z_L = 60 + j50 \Omega, \ell = 0.4 \lambda$$

From Smith chart, ($\beta \ell = 1.2 + j1.0$)

- $\text{SWR} = 2.46$ ✓
- $\Gamma = 0.422 \angle 54^\circ$ ✓
- $Y_L = (1.492 - j.410) / 50 = 9.84 - j8.2 \text{ mS}$ ✓
- $Z_{in} = 24.5 + j20.3 \Omega$
- $\ell_{\text{MIN}} = 0.325 \lambda$
- $\ell_{\text{MAX}} = 0.075 \lambda$



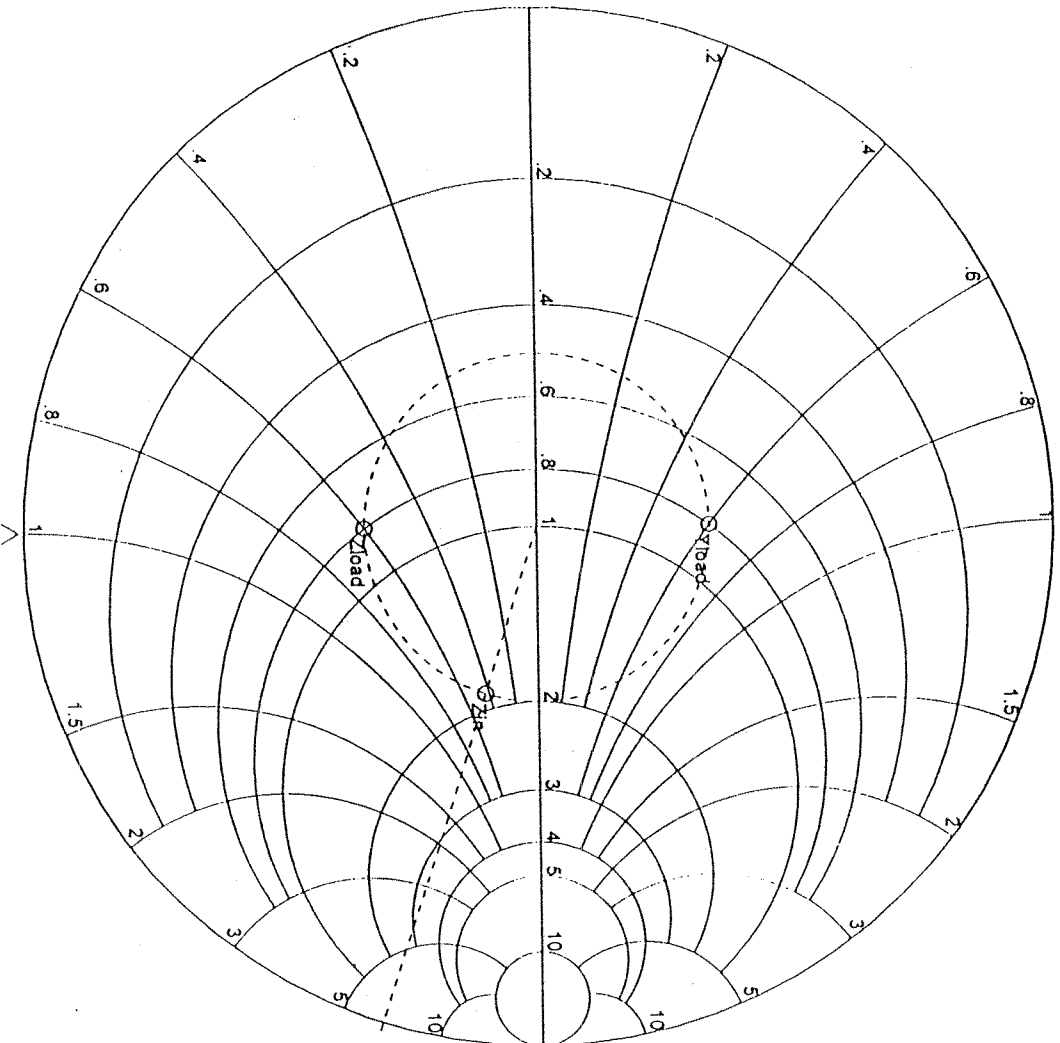
Smith Chart for Problem 2.17

2.18

$$Z_0 = 50 \Omega, \quad Z_L = 40 - j30 \Omega, \quad \lambda = 0.4 \lambda$$

From Smith chart, $(Z_L = 0.80 - j0.60)$

- $SWR = 2.00$
- $\Gamma = 0.333 \angle 270^\circ$
- $Y_L = (1.800 + j1.600) / 50 = 16.0 + j12.0 \text{ mS}$ ✓
- $Z_{in} = 93.2 - j21.6 \Omega$
- $\lambda_{min} = 0.125 \lambda$
- $\lambda_{max} = 0.375 \lambda$



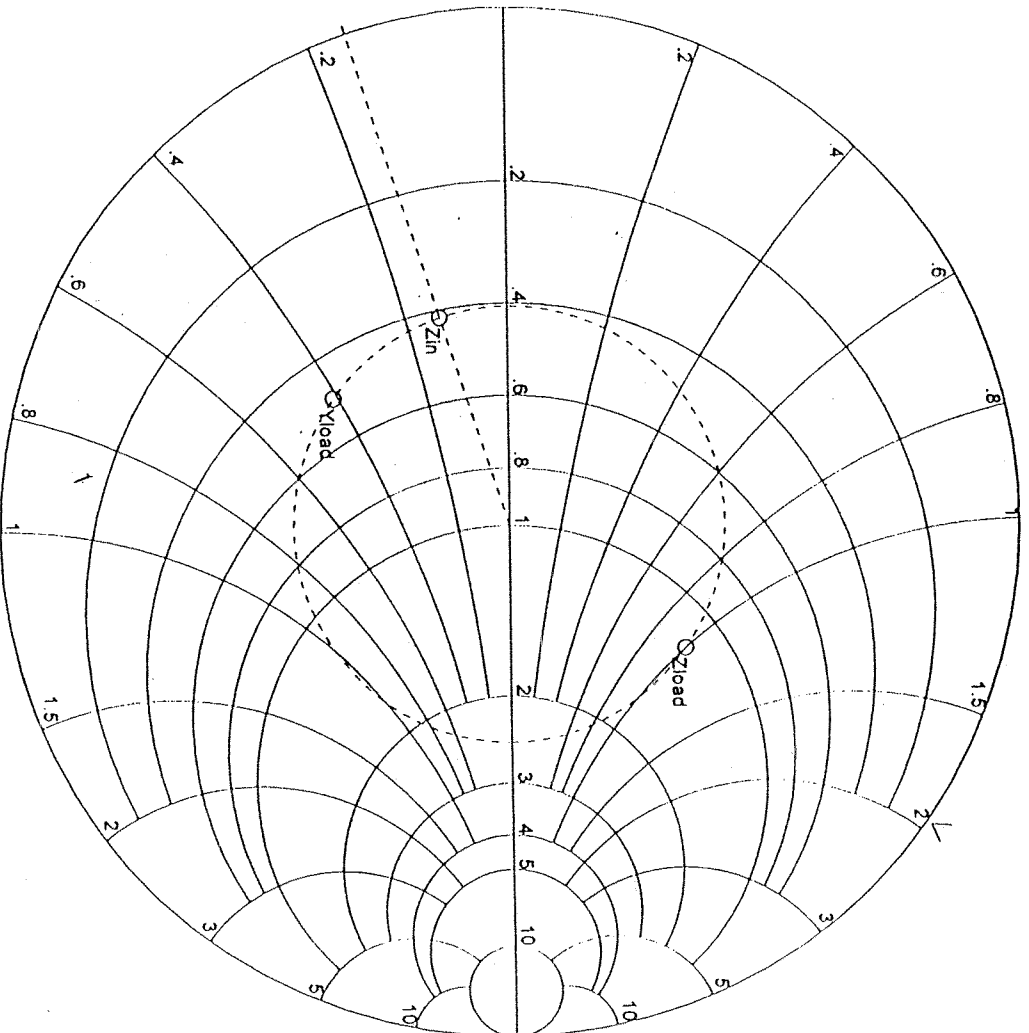
Smith Chart for Problem 2.18

2.19

$$Z_0 = 50\Omega, \quad Z_L = 60 + j50\Omega, \quad \ell = 1.8\lambda$$

From Smith chart, $(\beta\ell = 1.2 + j1.0)$

- a) $\text{SWR} = 2.46$
- b) $\Gamma = 0.422 \angle 154^\circ$
- c) $Y_L = (1.492 - j1.110) / 50 = 9.84 - j8.2 \text{ mS} \checkmark$
- d) $Z_{in} = 20.8 - j6.7\Omega$
- e) $\beta_{\text{MIN}} = 0.325\lambda$
- f) $\beta_{\text{MAX}} = 0.075\lambda$



Smith Chart for Problem 2.19

8.20

a) $\ell = 0$ or $\ell = 0.5\lambda$ ✓

b) $\ell = 0.25\lambda$ ✓

c) $\ell = 0.125\lambda$ ✓

d) $\ell = 0.406\lambda$ ✓

e) $\ell = 0.021\lambda$ ✓

These results check
with $Z_{in} = j Z_0 \tan \beta \ell$.

8.21

a) $\ell = 0.25\lambda$ ✓

b) $\ell = 0\lambda$ or 0.5λ ✓

c) $\ell = 0.375\lambda$ ✓

d) $\ell = 0.656\lambda - 0.5\lambda = 0.156\lambda$ ✓

e) $\ell = 0.271\lambda$ ✓

(add $\lambda/4$ to results of P. 8.20)
(also check with
 $Z_{in} = -j Z_0 \cot \beta \ell$)

8.22

$\lambda = 4.2$ cm. From the Smith chart, $\ell_{MIN} = 9/4.2 = 0.214\lambda$
from the load, so $Z_L = 2 - j1.9 \Rightarrow Z_L = \underline{100 - j45 \Omega}$ ✓

Analytically, using (2.58) - (2.60),

$$\Gamma = |\Gamma| e^{j\theta}, \quad |\Gamma| = \frac{2.5-1}{2.5+1} = 0.428$$

$$\theta = \pi + 2\beta \ell_{MIN} = 180 + 2(360)(.214) = -26^\circ \checkmark$$

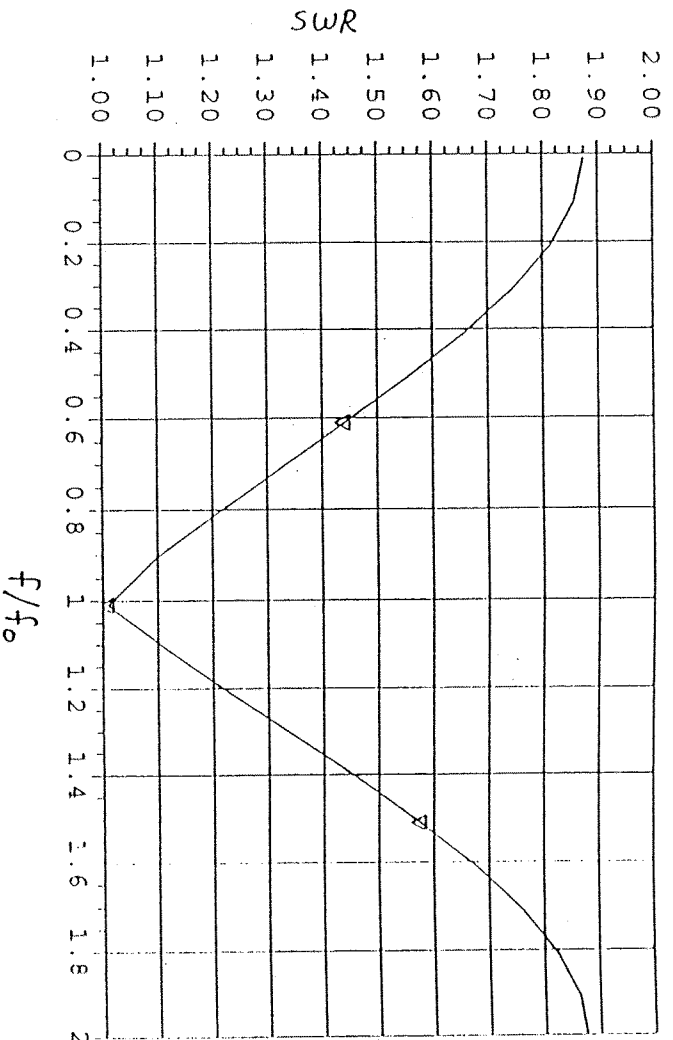
Then,

$$Z_L = \frac{1 + j428 \angle -26^\circ}{1 - j428 \angle -26^\circ} (50) = 50 \frac{1.4 \angle -7.7^\circ}{.643 \angle 17^\circ} = 109 \angle -25^\circ$$
$$= \underline{\underline{99 - j46 \Omega}}$$

2.23

$$Z_L = \sqrt{40(75)} = 54.77 \Omega$$

The VSWR is plotted vs f/f_0 below:



2.24

On the $\lambda/4$ transformer, the voltage can be expressed

as,

$$V(z) = V^+ e^{-j\beta z} + \Gamma V^+ e^{j\beta z}, \quad \Gamma = \frac{R_L - \sqrt{Z_0 R_L}}{R_L + \sqrt{Z_0 R_L}}$$

at $z = -l$,

$$V(-l) = V^+ e^{j\beta l} + \Gamma e^{-j\beta l}$$

$$V^+ = \frac{V^+}{e^{j\beta l} + \Gamma e^{-j\beta l}}, \quad V^- = \Gamma V^+$$

(assuming V^+ with a phase reference at $z = -l$.)

2.25

From (2.70),

$$V_o^+ = V_g \frac{Z_{in}}{Z_{in} + Z_g} \frac{1}{(e^{j\beta l} + \Gamma_R e^{-j\beta l})}$$

From (2.67),

$$Z_{in} = Z_0 \frac{1 + \Gamma_R e^{-2j\beta l}}{1 - \Gamma_R e^{-2j\beta l}}$$

Then,

$$\begin{aligned} \frac{Z_{in}}{Z_{in} + Z_g} &= \frac{Z_0(1 + \Gamma_R e^{-2j\beta l})}{Z_0(1 + \Gamma_R e^{-2j\beta l}) + Z_g(1 - \Gamma_R e^{-2j\beta l})} \\ &= \frac{Z_0(e^{j\beta l} + \Gamma_R e^{-j\beta l}) e^{j\beta l}}{(Z_0 + Z_g) + \Gamma_R (Z_0 - Z_g) e^{-2j\beta l}} \\ &= \frac{Z_0(e^{j\beta l} + \Gamma_R e^{-j\beta l}) e^{j\beta l}}{(Z_0 + Z_g) \left[1 + \Gamma_R \frac{Z_0 - Z_g}{Z_0 + Z_g} e^{-2j\beta l} \right]} \end{aligned}$$

Thus,

$$V_o^+ = V_g \frac{Z_0 e^{j\beta l}}{(Z_0 + Z_g)(1 - \Gamma_R \Gamma_g e^{-2j\beta l})}, \quad \text{since } \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$

2.26

$$\begin{aligned} \frac{\partial \alpha_c}{\partial a} &= \frac{R_S}{2\eta} \left[\frac{1}{a} \left(\frac{1}{\ln b/a} \right)^2 \left(\frac{1}{a} + \frac{1}{b} \right) + \frac{1}{\ln b/a} \left(-\frac{1}{a^2} \right) \right] = 0 \\ a \left(\frac{1}{a} + \frac{1}{b} \right) &= \ln b/a \\ (1 + b/a) &= b/a \quad \ln b/a \end{aligned}$$

If $x = b/a$, then $1 + x = x \ln x$.

If $\frac{\partial \alpha_c}{\partial b}$ is taken, the same result is obtained if $x = a/b$.

Now solve the equation for x :

Using interval-halving method:

x	$x \ln x - x - 1$
1	-2.0
2	-1.6
3	-1.704
4	.545
3.5	-.115
3.6	.011
3.55	-.052
→ 3.59	-.01

For $x = \frac{b}{a} = 3.59$,

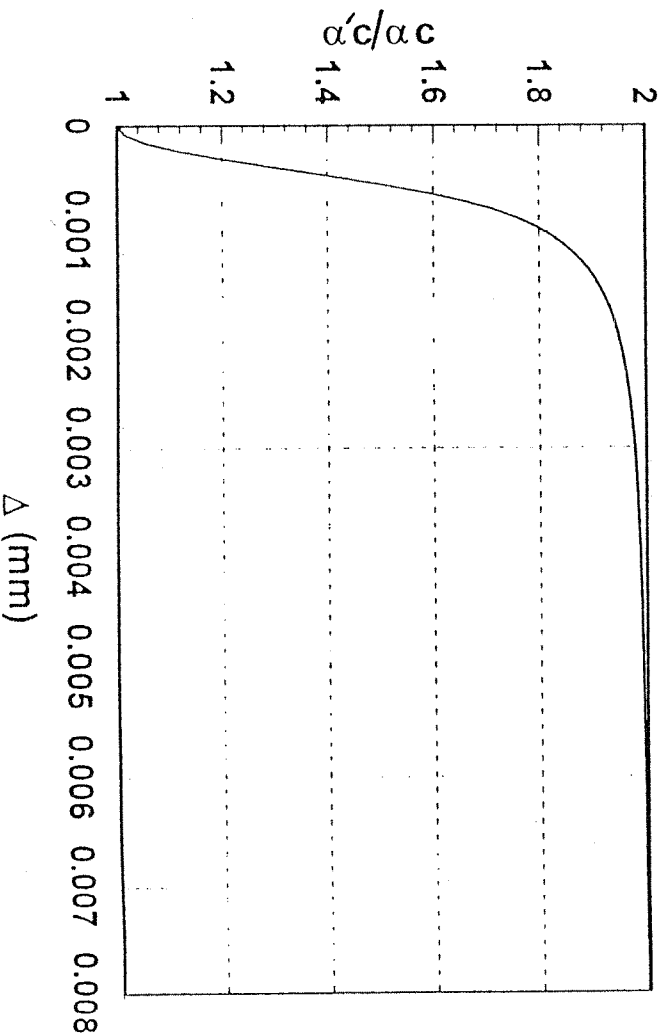
$$Z_0 = \frac{\eta}{2\pi} \ln \frac{b}{a} = \frac{377}{1\sqrt{\epsilon_r}} \ln(3.59) = \frac{76.7}{1\sqrt{\epsilon_r}} \approx 77 \Omega \text{ for } \epsilon_r = 1,$$

Thus, for an air dielectric, minimum attenuation occurs for a characteristic impedance near 77Ω .

(2.27) The skin depth of copper at 10 GHz is $\delta_s = 6.60 \times 10^{-7} \text{ m}$.

Then, compute $\frac{\alpha'_c}{\alpha_c} = 1 + \frac{2}{\pi} \tan^{-1} 1.4 \left(\frac{\Delta}{\delta_s} \right)^2$ (2.107)

The results are plotted below.



2.28 Since the generator is matched to the line,
 $V_0^+ = \frac{V_g}{2} e^{-\gamma l}$ (phase reference at $z=0$)

$$\alpha = 0.5 \text{ dB}/\lambda = 0.0576 \text{ nepers}/\lambda$$

$$\gamma l = (\alpha + j\beta)l = 0.1325 + j108^\circ \quad \checkmark$$

$$\text{Thus } |V_0^+| = \frac{10}{2} e^{-\alpha l} = 4.38 \text{ V.}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = 0.333, \quad \Gamma(l) = \Gamma e^{-2\gamma l}$$

From (2.92) - (2.94) we then have,

$$P_{in} = \frac{|V_0^+|^2}{2Z_0} [1 - |\Gamma(l)|^2] e^{2\alpha l} = \frac{(4.38)^2}{100} [e^{2(0.1325)} - (0.333)^2 e^{-2(0.1325)}]$$

$$= \underline{0.2337 \text{ W}} \quad (\text{power delivered to line})$$

$$P_L = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2) = \frac{(4.38)^2}{100} [1 - (0.333)^2] = \underline{0.1706 \text{ W}} \quad (\text{power to load})$$

$$P_{loss} = P_{in} - P_L = 0.2337 - 0.1706 = \underline{0.0631 \text{ W}}$$

The input impedance is,

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} = 50 \frac{100 + 50(.845 + j2.19)}{50 + 100(.845 + j2.19)} = 32.5 - j12.4 \Omega$$

The input current is,

$$I_{in} = \frac{V_g}{R_g + Z_{in}} = \frac{10}{82.5 - j12.4} = \underline{0.1199 \angle 8.5^\circ \text{ A}}$$

The generator power is,

$$P_s = \frac{1}{2} V_g |I_{in}| = 5(0.1199) = \underline{0.600 \text{ W}}$$

Power lost in R_g is,

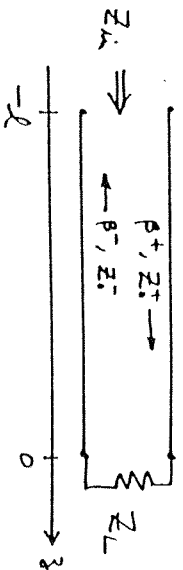
$$P_{R_g} = \frac{1}{2} |I_{in}|^2 R_g = \frac{1}{2} (0.1199)^2 (50) = \underline{0.3594 \text{ W}}$$

CHECK:

$$P_L + P_{loss} + P_{R_g} = 0.1706 + 0.0631 + 0.3594 = 0.5931 \text{ W} \approx P_s \quad \checkmark$$

$$P_{in} + P_{R_g} = 0.2337 + 0.3594 = 0.5931 \text{ W} \approx P_s \quad \checkmark$$

2.29



$$V(z) = V_0^+ e^{j\beta^+ z} + V_0^- e^{j\beta^- z}$$

$$I(z) = \frac{V_0^+}{Z_0^+} e^{j\beta^+ z} - \frac{V_0^-}{Z_0^-} e^{j\beta^- z}$$

at $z=0$ (load),

$$V(0) = V_0^+ + V_0^-$$

$$I(0) = \frac{V_0^+}{Z_0^+} - \frac{V_0^-}{Z_0^-}$$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+/Z_0^+ - V_0^-/Z_0^-} = \frac{1 + V_0^-/V_0^+}{\frac{1}{Z_0^+} - \frac{V_0^-}{V_0^+} \frac{1}{Z_0^-}}$$

as usual, let $\Gamma(0) = V_0^-/V_0^+$. Then,

$$Z_L \left(\frac{1}{Z_0^+} - \Gamma \frac{1}{Z_0^-} \right) = 1 + \Gamma$$

$$\frac{Z_L}{Z_0^+} - 1 = \Gamma \left(1 + \frac{Z_0^+}{Z_0^-} \right)$$

$$\Gamma = \Gamma(0) = \frac{Z_L - Z_0^-}{Z_L + Z_0^+} \quad (\text{at load})$$

The input impedance is,

$$\begin{aligned} Z_{in} &= \frac{V(-l)}{I(-l)} = \frac{V_0^+ [e^{j\beta^+ l} + \Gamma e^{-j\beta^- l}]}{V_0^+ [\frac{1}{Z_0^+} e^{j\beta^+ l} - \Gamma \frac{1}{Z_0^-} e^{j\beta^- l}]} \\ &= \frac{(Z_L + Z_0^+) e^{j\beta^+ l} + (Z_L - Z_0^-) e^{-j\beta^- l}}{\frac{1}{Z_0^+} (Z_L + Z_0^+) e^{j\beta^+ l} - \frac{1}{Z_0^-} (Z_L - Z_0^-) e^{-j\beta^- l}} \end{aligned}$$

This result does not simplify much further. From (2.42),

$$\Gamma(-l) = \Gamma(0) e^{-j(\beta^+ + \beta^-)l} \quad (\text{reflection coefficient at the input})$$

Chapter 3

3.1 $k_c^2 = k^2 - \beta^2$

H_x : multiply (3.3a) by $\omega \epsilon$, multiply (3.4b) by β , and add:

$$\omega \epsilon \frac{\partial E_z}{\partial y} - j\beta^2 H_x - \beta \frac{\partial H_z}{\partial x} = -j\omega^2 \mu \epsilon H_x$$

$$H_x = \frac{k_c}{k_c^2} \left[\omega \epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x} \right] \checkmark$$

H_y : multiply (3.3b) by $-\omega \epsilon$, multiply (3.4a) by β , and add:

$$\omega \epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} + j\beta^2 H_y = j\omega^2 \mu \epsilon H_y$$

$$H_y = \frac{j}{k_c^2} \left[\omega \epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} \right] \checkmark$$

E_x : multiply (3.3b) by $-\beta$, multiply (3.4a) by $\omega \mu$, and add:

$$j\beta^2 E_x + \beta \frac{\partial E_z}{\partial x} + \omega \mu \frac{\partial H_z}{\partial y} = j\omega^2 \mu \epsilon E_x$$

$$E_x = \frac{j}{k_c^2} \left[\beta \frac{\partial E_z}{\partial x} + \omega \mu \frac{\partial H_z}{\partial y} \right] \checkmark$$

E_y : multiply (3.3a) by β , multiply (3.4b) by $\omega \mu$, and add:

$$\beta \frac{\partial E_z}{\partial y} + j\beta^2 E_y - \omega \mu \frac{\partial H_z}{\partial x} = j\omega^2 \mu \epsilon E_y$$

$$E_y = \frac{j}{k_c^2} \left[\beta \frac{\partial E_z}{\partial y} - \omega \mu \frac{\partial H_z}{\partial x} \right] \checkmark$$

3.2 From (3.66) - (3.67),

$$H_z = B_n \cos \frac{n\pi y}{d} e^{j\beta z}$$

$$H_y = \frac{j\beta}{k_c} B_n \sin \frac{n\pi y}{d} e^{j\beta z}$$

From (3.71), $P_o = \frac{\omega \mu d W \beta}{4 k_c^2} |B_n|^2$ for $n > 0$, β real.

From (2.97), the power lost in both plates is,

$$P_L = 2 \left(\frac{R_s}{2} \right) \int_s |\vec{H}_t|^2 ds = R_s \int_{z=0}^1 \int_{x=0}^w [|H_z(y=0)|^2 + |H_z(y=w)|^2] dx dz$$

$$= R_s W |B_w|^2$$

Then $\alpha_c = \frac{P_L}{2P_0} = \frac{2R_s k_c^2}{k d \eta \beta}$ (agree with (3.72)) ✓

3.3 From Appendix I, $a = 1.07$ cm, $b = 0.43$ cm.

$$f_{c10} = \frac{c}{2a} = 14.02 \text{ GHz} \checkmark$$

$$f_{c20} = \frac{c}{2b} = 28.04 \text{ GHz} \checkmark$$

$$f_{c01} = \frac{c}{2b} = 34.88 \text{ GHz} \checkmark$$

LOWEST
ORDER
MODES

The bandwidth covered by f_{c10} and f_{c20} is 100%.

The recommended operating range for K-band guide is 18.0 - 26.5 GHz, or a bandwidth of 47%. The reduction in bandwidth is 53%.

3.4 $\sigma = 2.56 \times 10^7$ S/m for brass, so $R_s = \sqrt{\frac{\omega \mu}{2\sigma}} = 0.0555 \Omega$

$$k = \sqrt{\epsilon_r} k_0 = \sqrt{2.6} \frac{2\pi (20,000)}{300} = 675, \text{ m}^{-1}$$

$$\beta = \sqrt{k^2 - k_0^2} = 608.3 \text{ m}^{-1} \checkmark$$

$$\eta = \frac{377}{\sqrt{\epsilon_r}} = 234 \Omega \checkmark$$

From (3.29) the attenuation due to dielectric loss is,

$$\alpha_d = \frac{k^2 \tan \delta}{2\beta} = \frac{(675)^2 (.001)}{2(608.3)} = 0.375 \text{ nepers/m}$$

From (3.96) the attenuation due to conductor loss is,

$$\alpha_c = \frac{R_s}{2\beta k \eta} (2b\pi^2 + a^3 k^2) = \frac{.0555 [2(.043)^2 \pi^2 + (.107)^3 (675)^2]}{(.0107)^3 (.0043)(608.3)(675)(234)} = 0.0705 \text{ nepers/m} \checkmark$$

Note that the attenuation due to dielectric loss is much greater than the conductor loss, even for a relatively low-loss dielectric material.

The total attenuation is

$$\alpha = \alpha_c + \alpha_d = 0.446 \text{ nepers/m}$$

$$\text{In dB/m, } \alpha = 0.446(8.686) = 3.87 \text{ dB/m} \quad \checkmark \text{ (agrees with P4AD)}$$

3.5

In the section of guide of width $a/2$, the TE_{10} mode is below cutoff (evanescent), with an attenuation constant α :

$$k = \frac{2\pi(12,000)}{300} = 251.3 \text{ m}^{-1} \quad \checkmark$$

$$\alpha = \sqrt{\left(\frac{\pi}{a/2}\right)^2 - k^2} = \sqrt{\left(\frac{2\pi}{0.02286}\right)^2 - (251.3)^2} = 111.3 \text{ nepers/m} \quad \checkmark$$

To obtain 100 dB attenuation (ignoring reflections),

$$-100 \text{ dB} = 20 \log e^{-\alpha L}$$

$$10^{-5} = e^{-\alpha L}$$

$$L = \frac{11.5}{111.3} = 0.103 \text{ m} = \underline{10.3 \text{ cm}} \quad \checkmark$$

3.6 The TE₁₀ H-fields from (3.89) are:

$$H_x = \frac{j\beta_0 A}{\pi} \sin \frac{\pi x}{a} e^{-j\beta_0 z}$$

$$H_y = 0$$

$$H_z = A \cos \frac{\pi x}{a} e^{-j\beta_0 z}$$

$\vec{J}_s = \hat{n} \times \vec{H}$, so the surface currents are,

$$\text{ON BOTTOM WALL: } \hat{n} = \hat{y}; \quad \vec{J}_s = -\hat{z} \frac{j\beta_0 A}{\pi} \sin \frac{\pi x}{a} e^{-j\beta_0 z} + \hat{x} A \cos \frac{\pi x}{a} e^{-j\beta_0 z} \quad \checkmark$$

$$\text{ON TOP WALL: } \hat{n} = -\hat{y}; \quad \vec{J}_s = \hat{z} \frac{j\beta_0 A}{\pi} \sin \frac{\pi x}{a} e^{-j\beta_0 z} - \hat{x} A \cos \frac{\pi x}{a} e^{-j\beta_0 z} \quad \checkmark$$

$$\text{ON LEFT SIDE WALL: } \hat{n} = \hat{x}, x=0; \quad \vec{J}_s = -\hat{y} A e^{-j\beta_0 z}$$

$$\text{ON RIGHT SIDE WALL: } \hat{n} = -\hat{x}, x=a; \quad \vec{J}_s = \hat{y} A e^{-j\beta_0 z}$$

Note that the top and bottom currents are the negative of each other, as are the left and right side wall currents.

Along the centerline of the top or bottom (broad) walls, $x=a/2$, so the surface currents can be reduced to,

$$\vec{J}_s = \pm \hat{z} \frac{j\beta_0 A}{\pi} e^{-j\beta_0 z},$$

which shows that current flows in only in the longitudinal direction. Thus a narrow longitudinal slot will not break any current lines, and will have a negligible effect on the operation of the waveguide.

3.7

From (3.101),

$$\begin{aligned} \mathbf{E} \times \mathbf{H}^* \cdot \hat{\mathbf{z}} &= E_x H_y^* - E_y H_x^* \\ &= \frac{\omega \epsilon \beta m^2 \pi^2}{a^2 k_z^4} |B|^2 \cos^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b} \\ &\quad + \frac{\omega \epsilon \beta n^2 \pi^2}{b^2 k_z^4} |B|^2 \sin^2 \frac{m\pi x}{a} \cos^2 \frac{n\pi y}{b} \end{aligned}$$

So the power flows down the guide is,

$$P_o = \frac{1}{2} \int_{x=0}^a \int_{y=0}^b \mathbf{E} \times \mathbf{H}^* \cdot \hat{\mathbf{z}} \, dx \, dy = \frac{\omega \epsilon \beta \pi^2 |B|^2}{2 k_z^4} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \frac{a b}{4} = \frac{\omega \epsilon \beta a b}{8 k_z^2} |B|^2$$

The power loss in the walls is,

$$\begin{aligned} P_L &= \frac{R_s}{2} \int_c \int_s |\mathbf{H}_\pm|^2 \, ds = R_s \left\{ \int_{x=0}^a \int_{y=0}^b |H_x(y=0)|^2 \, dx + \int_{y=0}^b |H_y(x=0)|^2 \, dy \right\} \\ &= R_s \left\{ \frac{\omega^2 \epsilon^2 n^2 \pi^2}{b^2 k_z^4} |B|^2 \frac{a}{2} + \frac{\omega^2 \epsilon^2 m^2 \pi^2}{a^2 k_z^4} |B|^2 \frac{b}{2} \right\} \\ &= R_s \frac{\omega^2 \epsilon^2 \pi^2}{2 k_z^4} |B|^2 \left(\frac{n^2 a}{b^2} + \frac{m^2 b}{a^2} \right) \end{aligned}$$

So the attenuation is,

$$\begin{aligned} \alpha_c &= \frac{P_L}{P_o} = \frac{R_s \omega^2 \epsilon^2 \pi^2 y k_z^2}{2 k_z^4 \omega \epsilon \beta a b} \left(\frac{n^2 a}{b^2} + \frac{m^2 b}{a^2} \right) \\ &= \frac{2 R_s k \pi^2}{k_z^2 \beta \eta} \left(\frac{n^2}{b^3} + \frac{m^2}{a^3} \right) \quad \text{neper/m} \checkmark \end{aligned}$$

3.8 From (3.109), the propagation constant is a solution of,

$$k_a \tan k_d z + k_d \tan k_a (a-z) = 0,$$

where

$$\beta = \sqrt{k_0^2 - k_a^2} = \sqrt{\epsilon_r k_0^2 - k_a^2} \quad (3.106)$$

Since $\beta = 0$ at cutoff, we have that $k_a = k_0$, and $k_d = \sqrt{\epsilon_r} k_0$. Thus we must find the root of the following equation:

$$f(k_0) = k_0 \tan \sqrt{\epsilon_r} k_0 z + \sqrt{\epsilon_r} k_0 \tan k_0 z = 0 \quad (\text{since } z = a/2)$$

We know that $k_e = k_0$ must be between k_c of the empty guide, and k_c for the completely filled guide:

$$k_c (\text{EMPTY}) = \frac{\pi}{a} = 137. \text{ m}^{-1}$$

$$k_c (\text{FILLED}) = \frac{\pi}{\sqrt{\epsilon_r} a} = 92. \text{ m}^{-1}$$

k_0	$f(k_0)$
95	-1366
100	-362
105	-44
110	171
106	2.3
105.9	-2.25
✓ → 105.95	.017

This result is accurate to at least four figures, and agrees with a result given in the Waveguide Handbook.

The cutoff frequency is,

$$f_c = \frac{k_c^2}{2\pi} = 5.06 \text{ GHz}$$

3.9 The lowest order mode will have an H_3 component which is even in x , and no variation in y . Thus, h_3 can be written as,

$$h_3(x, y) = \begin{cases} A \cos k_d x & \text{for } |x| < W/2 & (k_e = k_d) \\ B e^{-k_a |x|} & \text{for } |x| > W/2 & (k_e = jk_a) \end{cases}$$

where k_d and k_a are the cutoff wavenumbers in the dielectric and air regions, respectively, satisfying

$$\beta = \sqrt{\epsilon_r k_0^2 - k_d^2} = \sqrt{k_0^2 - k_a^2} \quad (\text{phase matching})$$

Next, we need e_y , from (3.19d):

$$e_y(x, y) = \frac{j\omega\mu}{k_0^2} \frac{\partial h_3}{\partial x} = \begin{cases} -\frac{j\omega\mu A}{k_d} \sin k_d x & \text{for } |x| < W/2 \\ \frac{j\omega\mu B}{k_a} e^{-k_a x} & \text{for } x > W/2 \end{cases}$$

Matching h_3 and e_y at $x=W/2$ gives,

$$A \cos k_d W/2 = B e^{-k_a W/2}$$

$$-\frac{A}{k_d} \sin k_d W/2 = \frac{B}{k_a} e^{-k_a W/2}$$

Setting the determinant of these equations to zero gives,

$$k_a \tan k_d W/2 + k_d = 0.$$

A TEM mode cannot exist by itself because of the impossibility of phase matching at $x=W/2$. (For a TEM mode, $\beta = k$ in both regions, which is not possible.)

3.10

$$h_z(y) = \begin{cases} A \cos k_d y & \text{for } 0 < y < t & (k_e = k_d) \\ B \cos k_a (d-y) & \text{for } t < y < d & (k_e = k_a) \end{cases}$$

where k_a and k_e are the cutoff wavenumbers in the dielectric and air regions, satisfying,

$$\beta = \sqrt{\epsilon_r k_0^2 - k_d^2} = \sqrt{k_a^2 - k_e^2}$$

Also, h_z has been chosen to satisfy $E_x = 0$ at $y = 0, d$.
(since $E_x \sim \partial h_z / \partial y$). From (3.19),

$$E_x(y) = \frac{-j\omega\mu}{k_e^2} \frac{\partial h_z}{\partial y} = \begin{cases} \frac{j\omega\mu A}{k_d} \sin k_d y & \text{for } 0 < y < t \\ \frac{-j\omega\mu B}{k_a} \sin k_a (d-y) & \text{for } t < y < d \end{cases}$$

Enforcing continuity of h_z and E_x at $y = t$ yields,

$$A \cos k_d t - B \cos k_a (d-t) = 0$$

$$\frac{A}{k_d} \sin k_d t + \frac{B}{k_a} \sin k_a (d-t) = 0$$

Setting the determinant of this set of equations to zero gives,

$$k_d \tan k_a (d-t) + k_a \tan k_d t = 0 \quad \checkmark$$

(This result agrees with a result on p. 160 of Harrington)

The TEM mode cannot exist on this structure.

3.11

Maxwell's curl equations are,
 $\nabla \times \vec{E} = -j\omega \mu \vec{H}$, $\nabla \times \vec{H} = j\omega \epsilon \vec{E}$

The ρ and ϕ components in cylindrical form are,

$$\frac{1}{\rho} \frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega \mu H_\rho \quad \frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega \epsilon E_\rho$$

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = -j\omega \mu H_\phi \quad \frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} = j\omega \epsilon E_\phi$$

Now assume $\vec{E}(\rho, \phi, z) = \underline{E}(\rho, \phi) e^{j\beta z}$
 $\vec{H}(\rho, \phi, z) = \underline{H}(\rho, \phi) e^{j\beta z}$

Then $\partial/\partial z \rightarrow -j\beta$, and the above equations reduce to:

$$\frac{1}{\rho} \frac{\partial E_\phi}{\partial \phi} + j\beta E_\phi = -j\omega \mu H_\rho \quad (1) \quad \frac{1}{\rho} \frac{\partial H_z}{\partial \phi} + j\beta H_\phi = j\omega \epsilon E_\rho \quad (3)$$

$$-j\beta E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega \mu H_\phi \quad (2) \quad -j\beta H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega \epsilon E_\phi \quad (4)$$

Multiply (2) by $-\beta$, multiply (3) by $\omega \mu$, and add:

$$j\beta^2 E_\rho + \beta \frac{\partial E_z}{\partial \rho} + \omega \mu \frac{\partial H_z}{\partial \phi} = j\omega^2 \mu \epsilon E_\rho$$

$$E_\rho = \frac{-j}{k_c^2} \left[\beta \frac{\partial E_z}{\partial \rho} + \omega \mu \frac{\partial H_z}{\partial \phi} \right]$$

Multiply (1) by β , multiply (4) by $\omega \mu$, and add:

$$\frac{\beta}{\rho} \frac{\partial E_\phi}{\partial \phi} + j\beta^2 E_\phi - \omega \mu \frac{\partial H_z}{\partial \rho} = j\omega^2 \mu \epsilon E_\phi$$

$$E_\phi = \frac{-j}{k_c^2} \left[\frac{\beta}{\rho} \frac{\partial E_z}{\partial \phi} - \omega \mu \frac{\partial H_z}{\partial \rho} \right]$$

Multiply (1) by $\omega \epsilon$, multiply (4) by β , and add:

$$\omega \epsilon \frac{\partial E_z}{\partial \phi} - j\beta^2 H_\rho - \beta \frac{\partial H_z}{\partial \rho} = -j\omega^2 \mu \epsilon H_\rho$$

$$H_\rho = \frac{j}{k_c^2} \left[\omega \epsilon \frac{\partial E_z}{\partial \phi} - \beta \frac{\partial H_z}{\partial \rho} \right]$$

$$\omega \in \frac{\partial E_z}{\partial \rho} + \beta \frac{\partial H_z}{\partial \phi} + j\beta^2 H_\phi = j\omega^2 \mu \in H_\phi$$

$$H_\phi = \frac{j}{k_c^2} \left[\omega \in \frac{\partial E_z}{\partial \rho} + \beta \frac{\partial H_z}{\partial \phi} \right]$$

with $k_c^2 = k^2 - \beta^2$.

These results agree with those of (3.110). ✓

3.12 Let $A=1, B=0$ in (3.141). Then the transverse fields are,

$$E_\rho = \frac{j\beta}{k_c} \sin n\phi J_n'(k_c \rho) e^{j\beta z}$$

$$E_\phi = \frac{j\beta n}{k_c^2 \rho} \cos n\phi J_n(k_c \rho) e^{j\beta z}$$

$$H_\rho = \frac{j\omega \epsilon n}{k_c^2 \rho} \cos n\phi J_n(k_c \rho) e^{j\beta z}$$

$$H_\phi = -\frac{j\omega \epsilon}{k_c} \sin n\phi J_n'(k_c \rho) e^{j\beta z}$$

$$\mathbf{E} \times \mathbf{H}^* \cdot \hat{z} = \epsilon \rho H_\phi^* - E_\phi H_\rho^*$$

The power flows down the guide is, for $n > 0$,

$$P_0 = \frac{1}{2} \int_{\rho=0}^a \int_{\phi=0}^{2\pi} \left[\frac{\beta \omega \epsilon}{k_c^2} \sin^2 n\phi J_n'^2(k_c \rho) + \frac{\beta \omega \epsilon n^2}{k_c^4 \rho^2} \cos^2 n\phi J_n^2(k_c \rho) \right] \rho d\phi d\rho$$

$$= \frac{\beta \omega \epsilon \pi}{2 k_c^2} \int_{\rho=0}^a \left[J_n'^2(k_c \rho) + \frac{n^2}{k_c^2 \rho^2} J_n^2(k_c \rho) \right] \rho d\rho$$

$$\text{let } x = k_c \rho \\ dx = k_c d\rho \\ k_c a = \nu_{nm}$$

$$= \frac{\beta \omega \epsilon \pi}{2 k_c^4} \int_{x=0}^{\nu_{nm}} \left[J_n'^2(x) + \frac{n^2}{x^2} J_n^2(x) \right] x dx = \frac{\beta \omega \epsilon \pi}{4 k_c^4} \nu_{nm}^2 J_n'^2(\nu_{nm}) \quad (\text{SEE C.16})$$

The power lost in the conducting wall is,

$$P_L = \frac{R_s}{2} \int_{\phi=0}^{2\pi} \int_{\rho=0}^a |H_\phi(\rho=a)|^2 a d\phi dz = \frac{a R_s}{2} \frac{\omega^2 \epsilon^2}{k_c^2} J_n'^2(\nu_{nm}) \int_{\phi=0}^{2\pi} \sin^2 n\phi d\phi \\ = \frac{a R_s \omega^2 \epsilon^2 \pi}{2 k_c^2} J_n'^2(\nu_{nm})$$

The attenuation is then,

$$\alpha_c = \frac{P_L}{2P_0} = \frac{a R_s \omega^2 \epsilon^2 \pi^2 k_c^4}{4 k_c^2 \beta \omega \epsilon \pi \nu_{nm}^2} = \frac{a R_s \omega \epsilon k_c^2}{\beta \nu_{nm}^2} = \frac{k R_s}{\beta \eta a} \text{ nepers/m} \quad \checkmark$$

3.13 From Figure 3.13, the first four modes to propagate are the TE_{11} , TM_{01} , TE_{21} , and TE_{01} (TM_{11} has same f_c as TE_{01}).

Using (3.127) and (3.140):

$$TE_{11}: f_c = \frac{\eta' c}{2\pi a} = \frac{1.841(300)}{2\pi(1.002)} = 10,988 \text{ MHz} \quad \checkmark$$

$$TM_{01}: f_c = \frac{\eta' c}{2\pi a} = \frac{2.405(300)}{2\pi(1.008)} = 14,354 \text{ MHz} \quad \checkmark$$

$$TE_{21}: f_c = \frac{\eta' c}{2\pi a} = \frac{3.054(300)}{2\pi(1.008)} = 18,227 \text{ MHz} \quad \checkmark$$

$$TE_{01}: f_c = \frac{\eta' c}{2\pi a} = \frac{3.832(300)}{2\pi(1.008)} = 22,871 \text{ MHz} \quad \checkmark$$

3.14 From (3.153), $\vec{E}(\rho, \phi) = \frac{V_0 \ln b/\rho}{\ln b/a}$

From (3.13) and Appendix,

$$\vec{E}(\rho, \phi) = -\nabla_{\perp} \vec{E}(\rho, \phi) = -\left(\hat{\rho} \frac{\partial E}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial E}{\partial \phi}\right) = \frac{V_0 \hat{\rho}}{\rho \ln b/a}$$

Then, $\vec{E}(\rho, \phi, z) = \vec{E}(\rho, \phi) e^{j\beta z} = \frac{V_0 \hat{\rho} e^{j\beta z}}{\rho \ln b/a} \quad \checkmark \quad (4.155) \text{ of 1st Ed.}$

From (3.18), $\vec{H}(\rho, \phi) = \frac{1}{\eta} \hat{z} \times \vec{E}(\rho, \phi) = \frac{V_0 \hat{\phi}}{\eta \rho \ln b/a}$

Then, $\vec{H}(\rho, \phi, z) = \frac{V_0 \hat{\phi} e^{j\beta z}}{\eta \rho \ln b/a} \quad \checkmark \quad (4.157) \text{ of 1st Ed.}$

The potential between the two conductors is,

$$V_{ab} = \int_{\rho=a}^{\rho=b} E(\rho, \phi, z) d\rho = V_0 e^{j\beta z} \quad \checkmark \quad (4.158) \text{ of 1st Ed.}$$

The current on the inner conductor is,

$$I_a = \int_{\phi=0}^{2\pi} H_{\phi}(a, \phi, z) a d\phi = \frac{2\pi V_0 e^{j\beta z}}{\eta \ln b/a} \quad \checkmark \quad (4.159) \text{ of 1st Ed.}$$

The characteristic impedance is,

$$Z_0 = \frac{V_{ab}}{I_a} = \frac{\eta \ln b/a}{2\pi} \quad \checkmark \quad (4.162) \text{ of 1st Ed.}$$

3.15 The solution is similar to the TE mode case for the coax, but with E_z in place of H_z :

$$E_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) [C J_n(k_c \rho) + D Y_n(k_c \rho)]$$

Then the boundary condition that $E_z = 0$ at $\rho = a$ and at $\rho = b$ yields two equations:

$$C J_n(k_c a) + D Y_n(k_c a) = 0$$

$$C J_n(k_c b) + D Y_n(k_c b) = 0$$

$$\text{or, } J_n(k_c a) Y_n(k_c b) = J_n(k_c b) Y_n(k_c a)$$

For the TM₀₁ mode, $n=0$. Let $x = k_c a$. Then for $b = 2a$, we have that $k_c b = 2x$, and so the above equation can be written as,

$$f(x) = J_0(x) Y_0(2x) - J_0(2x) Y_0(x) = 0$$

We know that k_c should be greater than k_c for a circular waveguide of radius b , for which $k_{c01} = 78.1/b = 2.405/a$, which implies that $x = 1.2$. So we can begin the root search at $x = 1.2$. Using a table of Bessel functions give the following results in only a few minutes:

x	$J_0(x)$	$Y_0(x)$	$J_0(2x)$	$Y_0(2x)$	$f(x)$
1.2	.671	.228	.003	.510	.342
1.5	.512	.382	-.260	.377	.292
2.0	.224	.510	-.397	-.017	.198
3.1	-.292	.343	.202	-.248	.003
3.2	-.326	.307	.243	-.200	-.011

Linear interpolation between $x = 3.1$ and 3.2 gives a more accurate value for the root:

$$f(x) \approx .003 + \frac{.003 - (-0.011)}{3.1 - 3.2} (x - 3.1)$$

$$\approx .437 - .14x = 0$$

$$x = \frac{.437}{.14} = \underline{\underline{3.12}} = k_c a$$

3.16 From (3.175),

$$\mathbf{E} \times \mathbf{H}^* \cdot \hat{\mathbf{z}} = -E_y H_x^* = \begin{cases} \frac{\omega \mu_0 \beta |B|^2}{k_c^2} \sin^2 k_c x & \text{for } 0 \leq x \leq d \\ \frac{\omega \mu_0 \beta |B|^2}{h^2} \cos^2 k_c d e^{-2h(x-d)} & \text{for } d \leq x < \infty \end{cases}$$

The power flow is,

$$\begin{aligned} P_0 &= \frac{1}{2} \int_{x=0}^{\infty} \int_{y=0}^{\infty} \mathbf{E} \times \mathbf{H}^* \cdot \hat{\mathbf{z}} dy dx \\ &= \frac{\omega \mu_0 \beta |B|^2}{2 k_c^2} \int_{x=0}^d \sin^2 k_c x dx + \frac{\omega \mu_0 \beta |B|^2}{2 h^2} \cos^2 k_c d \int_{x=d}^{\infty} e^{-2h(x-d)} dx \\ &= \frac{\omega \mu_0 \beta |B|^2}{2} \left[\frac{x}{2} - \frac{\sin 2k_c x}{4k_c} \right]_0^d + \frac{\cos^2 k_c d}{h^2} \left(\frac{e^{-2h(x-d)}}{-2h} \right) \Big|_d^{\infty} \\ &= \frac{\omega \mu_0 \beta |B|^2}{2} \left[\frac{1}{2} \left(\frac{d}{2} - \frac{\sin 2k_c d}{4k_c} \right) + \frac{\cos^2 k_c d}{2h^3} \right] \end{aligned}$$

The power loss is,

$$\begin{aligned} P_L &= \frac{R_s}{2} \int_S |H_T|^2 ds = \frac{R_s}{2} \int_{y=0}^1 \int_{z=0}^1 \left[|H_x(x=0)|^2 + |H_z(x=0)|^2 \right] dz dy \\ &= \frac{R_s}{2} |B|^2 \end{aligned}$$

So the attenuation is,

$$\begin{aligned} \alpha_c &= \frac{P_L}{P_0} = \frac{2 R_s}{4 \omega \mu_0 \beta \left[\frac{1}{2} \left(\frac{d}{2} - \frac{\sin 2k_c d}{4k_c} \right) + \frac{\cos^2 k_c d}{2h^3} \right]} \\ &= \frac{R_s}{k_0 \eta_0 \beta \left[\frac{d}{4k_c} - \frac{\sin 2k_c d}{2k_c^3} + \frac{\cos^2 k_c d}{h^3} \right]} \end{aligned}$$

3.17 Following the derivation in Section 3.6 for the TM surface wave of a dielectric slab:

$$k_c^2 = \mu_r k_0^2 - \beta^2 \quad \text{for } 0 \leq y \leq d$$

$$h^2 = \beta^2 - k_0^2 \quad \text{for } y \geq d$$

Then,

$$E_z(x, y) = \begin{cases} A \sin k_c y & \text{for } 0 \leq y \leq d \\ B e^{-hy} & \text{for } y > d \end{cases}$$

This form of E_z is selected to satisfy $E_z = 0$ at $y = 0$, and to have exponential decay for $y \rightarrow \infty$ (radiation condition). Next, we need H_x ($H_y = E_x = H_z = 0$); From (3.23a),

$$H_x = \frac{j\omega\epsilon_0}{k_c^2} \frac{\partial E_z}{\partial y} = \begin{cases} \frac{j\omega\epsilon_0}{k_c} A \cos k_c y & \text{for } 0 \leq y \leq d \\ \frac{j\omega\epsilon_0}{h} B e^{-hy} & \text{for } y > d \end{cases}$$

At $y = d$:

$$E_z \text{ continuous} \Rightarrow A \sin k_c d = B e^{-hd}$$

$$H_x \text{ continuous} \Rightarrow \frac{A}{k_c} \cos k_c d = \frac{B}{h} e^{-hd}$$

or, $h \cos k_c d = k_c \sin k_c d$

$$h = k_c \tan k_c d \quad \checkmark$$

and,

$$h^2 + k_c^2 = (\mu_r - 1) k_0^2 \quad \checkmark$$

These two equations must be solved simultaneously to find h and k_c .

3.1.8 TM_{0m} mode. $H_z = 0$ $E_z(\rho, \phi, z) = E_3(\rho, \phi) e^{-j\beta z}$

(No TEM mode can be supported by this line because of the impossibility of phase matching at $\rho = b$)

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_0^2\right) E_3(\rho, \phi) = 0$$

$\partial/\partial \phi = 0$ for $n=0$ modes $\Rightarrow E_\phi = H_\rho = 0$.

$$E_3(\rho, \phi) = \begin{cases} A J_0(k_\rho \rho) + B Y_0(k_\rho \rho) & \text{for } a \leq \rho \leq b \\ C J_0(k_a \rho) + D Y_0(k_a \rho) & \text{for } b \leq \rho \leq c \end{cases}$$

where $\beta^2 = \epsilon_r k_0^2 - k_\rho^2 = k_0^2 - k_a^2$.

The boundary conditions are:

$$E_z = 0 \text{ at } \rho = a \text{ and } \rho = c,$$

$$E_z \text{ and } H_\phi \text{ are continuous at } \rho = b.$$

From (3.110d),

$$H_\phi = \frac{j\omega \epsilon}{k_\rho^2} \frac{\partial E_z}{\partial \rho},$$

So we get the following four equations:

$$A J_0'(k_\rho a) + B Y_0'(k_\rho a) = 0$$

$$C J_0'(k_a c) + D Y_0'(k_a c) = 0$$

$$A J_0(k_\rho b) + B Y_0(k_\rho b) = C J_0(k_a b) + D Y_0(k_a b)$$

$$\epsilon_r k_\rho [A J_0'(k_\rho b) + B Y_0'(k_\rho b)] = k_a [C J_0'(k_a b) + D Y_0'(k_a b)]$$

k_a and k_ρ can be expressed in terms of β , and β can be found so that the determinant of the above system of equations vanishes. This is as far as we can go without actual values for a, b, c , and ϵ_r .

3.19 To use (3.180) we compute $\sqrt{\epsilon_r Z_0} = \sqrt{2.2} (100) = 148.3 > 120$.

$$\text{and, } x = \frac{30\pi}{\sqrt{\epsilon_r Z_0}} = .441 = 0.195$$

$$\text{Then } w/b = .85 - \sqrt{.6 - x} = 0.2136$$

$$w = .2136 (.316 \text{ cm}) = \underline{0.0675 \text{ cm}} \quad \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_r} f} = \frac{300}{\sqrt{2.2} (4000)} = \underline{5.06 \text{ cm}} \quad \checkmark$$

3.20 First try $w/d < 2$:

$$\text{From (3.197), } A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} (.23 + .11/\epsilon_r) = 2.21 \quad \checkmark$$

$$\text{Then, } w/d = \frac{8eA}{e^{2A} - 2} = 0.90 < 2 \quad \text{OK} \quad \checkmark$$

$$w = .9 (.158 \text{ cm}) = \underline{0.142 \text{ cm}} \quad \checkmark$$

From (3.195) the effective permittivity is,

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + d/w}} = \underline{1.758} \quad \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_e} f} = \frac{300}{\sqrt{1.758} (4000)} = \underline{5.656 \text{ cm}} \quad \checkmark$$

$$(3.21) \quad k_0 = \frac{2\pi f}{c} = 104.7 \text{ m}^{-1} \quad ; \quad R_S = \sqrt{\frac{Wd}{Z_0}} = \sqrt{\frac{2\pi (5 \times 10^9)(4\pi \times 10^{-7})}{2(5.813 \times 10^7)}} = 0.0182 \checkmark$$

MICROSTRIP CASE:

First try $W/d > 2$: From (3.197), $B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}} = 8.0$

$$W/d = \frac{2}{\pi} [B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r} \{ \ln(B - 1) + 0.39 - \frac{0.6}{\epsilon_r} \}] = 3.09 > 2 \checkmark$$

$$W = 3.09 (1.6 \text{ cm}) = \underline{0.494 \text{ cm}}$$

$$\text{From (3.195), } \epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}} = 1.87' \Rightarrow \lambda_g = \frac{c}{\sqrt{\epsilon_e} f} = \underline{4.38 \text{ cm}} \checkmark$$

From (3.198),

$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_e - 1)}{2 \sqrt{\epsilon_e} (\epsilon_r - 1)} \tan \delta = \underline{0.061 \text{ nepers/m}} \checkmark$$

From (3.199),

$$\alpha_c = \frac{R_S}{Z_0 W} = \underline{0.073 \text{ nepers/m}} \checkmark$$

Total MS attenuation:

$$\alpha_{MS} = (0.061 + 0.073) \left(\frac{2\pi d}{W} \right) (16.9) (0.0438 \frac{\text{m}}{\lambda_g}) \left(\frac{8.686 \text{ dB}}{\text{neper}} \right) = \underline{0.82 \text{ dB}} \checkmark$$

STRIPLINE CASE:

From (3.180), $\sqrt{\epsilon_r} Z_0 = \sqrt{2} Z_0 = 74 < 120$. $\chi = \frac{30\pi}{\sqrt{\epsilon_r} Z_0} = 0.441 = 0.833$

$$W/b = \chi = 0.833 \Rightarrow W = 0.833 (1.32 \text{ cm}) = \underline{0.267 \text{ cm}} \checkmark$$

$$\lambda_g = \frac{c}{\sqrt{\epsilon_r} f} = \underline{4.045 \text{ cm}} \quad A = 1 + \frac{2W}{b - Z} + \frac{1}{\pi} \frac{b + Z}{b - Z} \ln \left(\frac{2b - Z}{Z} \right) = 4.73$$

From (3.181),

$$\alpha_c = \frac{2.7 \times 10^{-3} R_S \epsilon_r Z_0}{30\pi b} A = \underline{0.084 \text{ nepers/m}} \checkmark$$

$$\text{From (3.30), } \alpha_d = \frac{k \tan \delta}{2} = \frac{\sqrt{1.2} (104.7) (0.001)}{2} = \underline{0.078 \text{ nepers/m}} \checkmark$$

Total S.L. attenuation:

$$\alpha_{SL} = (0.084 + 0.078) \left(\frac{2\pi d}{W} \right) (16.9) (0.04045 \frac{\text{m}}{\lambda_g}) \left(\frac{8.686 \text{ dB}}{\text{neper}} \right) = \underline{0.91 \text{ dB}} \checkmark$$

Thus the microstrip line should be used.

3.2.2

$$h_z(x, y, z) = h_z(x, y) e^{-j\beta z} \quad ; h_z \text{ real, } \beta \text{ real.}$$

From (3.19),

$$H_x = \frac{-j\beta}{k_z^2} \frac{\partial h_z}{\partial x} e^{-j\beta z}$$

$$H_y = \frac{j\beta}{k_z^2} \frac{\partial h_z}{\partial y} e^{-j\beta z}$$

$$E_x = \frac{j\omega\mu}{k_z^2} \frac{\partial h_z}{\partial y} e^{-j\beta z}$$

$$E_y = \frac{j\omega\mu}{k_z^2} \frac{\partial h_z}{\partial x} e^{-j\beta z}$$

$$\nabla \times \vec{H}^* = (E_x H_z^* - E_y H_x^*) \hat{z} - E_x H_z^* \hat{y} + E_y H_z^* \hat{x}$$

$$= \frac{\omega\mu\beta}{k_z^2} \left[\left(\frac{\partial h_z}{\partial y} \right)^2 \hat{z} + \frac{j\omega\mu}{k_z^2} \left(\frac{\partial h_z}{\partial y} \hat{y} + \frac{\partial h_z}{\partial x} \hat{x} \right) h_z \right]$$

So if h_z is real (or a real function times a complex constant), there is real power flow only in the z -direction.

3.2.3

Write the incident, reflected, and transmitted TE₁₀ fields as follows:

$$E_y^i = E_0 \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$E_y^r = E_0 \Gamma \sin \frac{\pi x}{a} e^{j\beta z}$$

$$H_x^i = \frac{-E_0}{Z_a} \sin \frac{\pi x}{a} e^{j\beta z}$$

$$H_x^r = \frac{E_0 \Gamma}{Z_a} \sin \frac{\pi x}{a} e^{j\beta z}$$

$$E_y^t = E_0 T \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$H_x^t = \frac{-E_0 T}{Z_d} \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$\text{where } \beta_a = \sqrt{k_0^2 - (\pi/a)^2}$$

$$Z_a = k_0 \eta_0 / \beta_a$$

$$\beta_d = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$$

$$Z_d = k_0 \eta_0 / \beta_d$$

Match fields at $z=0$ to obtain:

$$1 + \Gamma = T$$

(E_y continuous)

$$\frac{1}{Z_a} (-1 + \Gamma) = \frac{-T}{Z_d}$$

(H_x continuous)

Solving for Γ gives,

$$\Gamma = \frac{Z_d - Z_a}{Z_d + Z_a}$$

which agrees with the transmission line theory result if Z_{TE} is used as Z_0 in each region. ✓

3.24 $Z_{TM} = \eta \beta/k = \eta_0 \beta/k_0 \sqrt{\epsilon_r}$

for $0 < x < d$, $Z_a = \eta_0 k_{xa}/k_0$

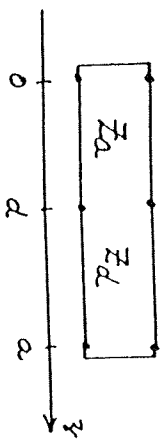
for $d < x < a$, $Z_d = \eta_0 k_{xd}/\sqrt{\epsilon_r} k_0$

$$\beta = \sqrt{\epsilon_r k_0^2 - k_{xd}^2 - (\pi/b)^2} = \sqrt{k_0^2 - k_{xa}^2 - (\pi/b)^2}$$

Applying (3.215):

$$Z_a \tan \beta a d + Z_d \tan \beta d (a-b) = 0$$

The m-th root of this equation applies to the TM_m mode.

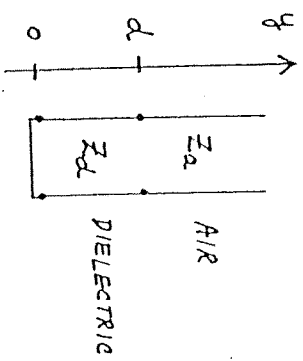


3.25

$$Z_a = k_{ya} \eta_0/k_0 = -j h \eta_0/k_0$$

$$Z_d = k_{yd} \eta/k = k_{yd} \eta_0/k_0$$

$$\beta = \sqrt{\mu_r k_0^2 - k_{yd}^2} = \sqrt{k_0^2 - k_{ya}^2} = \sqrt{k_0^2 + h^2}$$



Applying (3.215):

$$Z_a + j Z_d \tan k_{yd} d = 0$$

$$h = k_{yd} \tan k_{yd} d = 0$$

This agrees with the solution to Problem 3.17, with

$$k_0 = k_{yd}. \quad \checkmark$$

B.26 For X-band guide, $a = 2.286 \text{ cm}$.

$$k_c = \frac{2\pi f \sqrt{\epsilon_r}}{c} = \frac{2\pi (9500) \sqrt{2.08}}{300} = 287. \text{ m}^{-1} \quad \checkmark$$

$$\beta = \sqrt{k_c^2 - (\pi/a)^2} = 252. \text{ m}^{-1} \quad \checkmark$$

$$\text{speed of light in Teflon} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.08}} = \underline{2.08 \times 10^8 \text{ m/sec}} \quad \checkmark$$

$$\text{phase velocity} = v_p = \frac{\omega}{\beta} = \frac{2\pi (9.5 \times 10^9)}{252} = \underline{2.37 \times 10^8 \text{ m/sec}} \quad \checkmark$$

From (3.234),

$$\begin{aligned} \text{group velocity} = v_g &= \left(\frac{d\beta}{d\omega} \right)^{-1} = \left(\frac{d\beta}{dk} \frac{dk}{d\omega} \right)^{-1} = \left(\frac{k}{\beta} \sqrt{\epsilon_r} \right)^{-1} \\ &= \frac{\beta}{k \sqrt{\epsilon_r}} = \frac{252 (2.08 \times 10^8)}{287.} = \underline{1.83 \times 10^8 \text{ m/sec}} \end{aligned}$$

Note that $v_g < \frac{c}{\sqrt{\epsilon_r}} < v_p$.

B.27

$$P_{\text{MAX}} = c a^2 \ln \frac{b}{a}$$

$$\frac{dP_{\text{MAX}}}{da} = 2a \ln \frac{b}{a} - \frac{a^2}{a} = 0$$

$$2 \ln \frac{b}{a} - 1 = 0$$

$$2 \ln x = 1$$

$$\ln x = 0.5$$

$$x = 1.65$$

$$Z_0 = \frac{377}{2\pi} \ln \frac{b}{a} = \frac{120\pi}{2\pi} \left(\frac{1}{2} \right) = \underline{30 \Omega}$$

4.1 (This problem is essentially the same as P. 3.23)

Write the fields of incident, reflected, and transmitted TE₁₀ modes in each region:

$$E_y^i = E_0 \sin \frac{\pi x}{a} e^{-j\beta_0 z}$$

$$E_y^r = E_0 \Gamma \sin \frac{\pi x}{a} e^{j\beta_0 z}$$

$$H_x^i = \frac{-E_0}{Z_0} \sin \frac{\pi x}{a} e^{-j\beta_0 z}$$

$$H_x^r = \frac{E_0 \Gamma}{Z_0} \sin \frac{\pi x}{a} e^{j\beta_0 z}$$

$$E_y^t = E_0 T \sin \frac{\pi x}{a} e^{-j\beta_2 z}$$

$$H_x^t = \frac{-E_0 T}{Z_2} \sin \frac{\pi x}{a} e^{-j\beta_2 z}$$

$$\text{where } \beta_0 = \sqrt{k_0^2 - (\pi/a)^2}$$

$$Z_0 = \eta_0 / \beta_0$$

$$\beta_2 = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$$

$$Z_2 = \eta_0 / \beta_2$$

Match fields at $z=0$:

$$1 + \Gamma = T$$

$$\frac{1}{Z_2} (-1 + \Gamma) = \frac{-T}{Z_2}$$

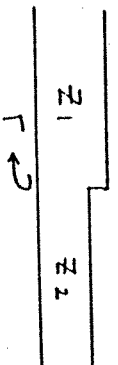
Solving gives,

$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0}$$

$$\text{As in Example 4.2, } \Gamma = -0.629 \checkmark$$

4.2 Using a transmission line analogy gives,

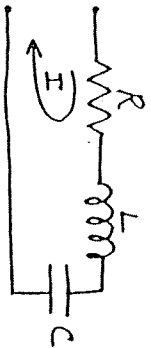
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



where $Z_1 = \eta_0 / \beta_1$, $Z_2 = \eta_0 / \beta_2$.

But $\beta_1 = \beta_2 = \sqrt{k_0^2 - (\pi/a)^2}$ in both regions, since only the height (b) of the guide changes. Thus, $\Gamma = 0$ from above. This is obviously not correct, as E_y should be zero for $b/2 < y < b$. Higher order TE_n modes must be considered, in a mode matching procedure. This will result in a solution where $\Gamma \neq 0$. Consideration of only the dominant mode is not adequate.

4.3



$$P_R = \frac{1}{2} |I|^2 R \quad \Rightarrow \quad R = \frac{P_R}{\frac{1}{2} |I|^2}$$

$$W_m = \frac{1}{4} L |I|^2 \quad \Rightarrow \quad L = \frac{2W_m}{\frac{1}{2} |I|^2}$$

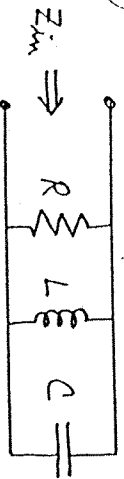
$$W_e = \frac{1}{4} C |V_e|^2 = \frac{1}{4\omega^2 C} |I|^2 \quad \Rightarrow \quad \frac{1}{\omega^2 C} = \frac{2W_e}{\frac{1}{2} |I|^2}$$

The input impedance is,

$$Z_{in} = R + j(\omega L + \frac{1}{\omega C}) = \frac{P_R + 2j\omega(W_m - W_e)}{\frac{1}{2} |I|^2} \quad \checkmark$$

in agreement with (4.17)

4.4



$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{j\omega C}} = \frac{1}{\frac{1}{R} + j\omega(C - \frac{1}{\omega^2 L})}$$

$$Z(-\omega) = \frac{1}{\frac{1}{R} - j\omega(C - \frac{1}{\omega^2 L})} = Z^*(\omega) \quad \checkmark$$

$$\begin{aligned} \textcircled{4.5} \quad P_{in} &= \frac{1}{2} [V]^T [I]^* = \frac{1}{2} [V]^T [Y] [V]^* \\ &= \frac{1}{2} \sum_{n=1}^N \sum_{m=1}^N V_m Y_{mn}^* V_n^* \end{aligned}$$

If lossless, $\text{Re}\{P_{in}\} = 0$. Since the V_n 's are independent, we first let all $V_m = 0$, except for V_n . Then,

$$\begin{aligned} P_{in} &= \frac{1}{2} V_n Y_{nn}^* V_n^* = \pm |V_n|^2 Y_{nn}^* \\ \therefore \text{Re}\{Y_{nn}^*\} &= \text{Re}\{Y_{nn}\} = 0 \quad \checkmark \end{aligned}$$

Now let all port voltages be zero except for V_m and V_n . Then,

$$P_{in} = \frac{1}{2} V_m Y_{mn}^* V_n^* + \frac{1}{2} V_n Y_{nm}^* V_m^*$$

$$\text{So,} \quad \text{Re}\{V_m Y_{mn}^* V_n^* + V_n Y_{nm}^* V_m^*\} = 0$$

If $Y_{mn} = Y_{nm}$ (reciprocal), then

$$\text{Re}\{Y_{mn}^* (V_m V_n^* + V_n V_m^*)\} = \text{Re}\{Y_{mn}^* [(V_m V_n^* + (V_m V_n^*)^*)]\} = 0$$

Since $A+A^*$ is real, we must have $\text{Re}\{Y_{mn}\} = 0 \quad \checkmark$

$\textcircled{4.6}$ Let $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, and show that Z_{ij} 's can be found

such that $P_{in} = 0$, but not all Z_{ij} 's are pure imaginary.

$$\begin{aligned} P_{in} &= \frac{1}{2} [I]^T [Z]^* [I]^* = \frac{1}{2} (I_1 Z_{11} I_1^* + I_1 Z_{21} I_2^* + I_2 Z_{12} I_1^* + I_2 Z_{22} I_2^*) \\ &= \frac{1}{2} (Z_{11} |I_1|^2 + Z_{22} |I_2|^2 + Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*) \end{aligned}$$

To be lossless, we must have $\text{Re}\{Z_{11}\} = \text{Re}\{Z_{22}\} = 0$.

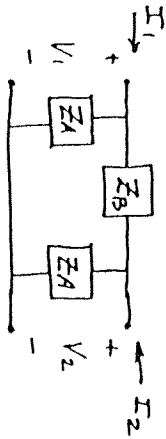
Also, $\text{Re}\{Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*\} = 0$.

This will occur if $Z_{12} = -Z_{21}^*$ (since $\text{Re}\{A-A^*\} = 0$).

For example, if $Z_{12} = a + jb$, then $Z_{21} = -a + jb$.

Thus, $[Z]$ is not symmetric, and the answer is NO.

4.7



From (4.28),

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{V_1}{I_1 \left(\frac{2Z_A + Z_B}{Z_A (Z_A + Z_B)} \right)} = \frac{Z_A (Z_A + Z_B)}{2Z_A + Z_B} \quad \checkmark \quad (\text{BY SYMMETRY})$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{I_1 Z_{11} \left(\frac{Z_A}{Z_A + Z_B} \right)}{I_1} = \frac{Z_A^2}{2Z_A + Z_B} \quad \checkmark \quad (\text{BY RECIPROCALITY})$$

From (4.29),

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{I_1}{I_1 \left(\frac{Z_A Z_B}{Z_A + Z_B} \right)} = \frac{Z_A + Z_B}{Z_A Z_B} \quad \checkmark \quad (\text{BY SYMMETRY})$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{-V_1 / Z_B}{V_1} = \frac{-1}{Z_B} \quad \checkmark \quad (\text{BY RECIPROCALITY})$$

CHECK: $[Z][Y] = [U]$?

$$Z_{11} Y_{11} + Z_{12} Y_{21} = \frac{(Z_A + Z_B)^2}{Z_B (2Z_A + Z_B)} - \frac{Z_A^2}{Z_B (2Z_A + Z_B)} = \frac{2Z_A Z_B + Z_B^2}{Z_B (2Z_A + Z_B)} = 1 \quad \checkmark$$

$$Z_{11} Y_{12} + Z_{12} Y_{22} = \frac{-Z_A (Z_A + Z_B)}{Z_B (2Z_A + Z_B)} + \frac{Z_A (Z_A + Z_B)}{Z_B (2Z_A + Z_B)} = 0 \quad \checkmark$$

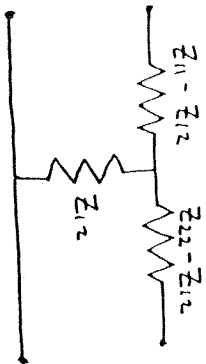
Similarly for the T-network. The results are,

$$Z_{11} = Z_{22} = \frac{Y_A + Y_B}{Y_A Y_B} \quad \checkmark \quad Z_{12} = Z_{21} = \frac{-1}{Y_B} \quad \checkmark$$

$$Y_{11} = Y_{22} = \frac{Y_A (Y_A + Y_B)}{2Y_A + Y_B} \quad \checkmark \quad Y_{12} = Y_{21} = \frac{Y_A^2}{2Y_A + Y_B} \quad \checkmark$$

4.8

Model the two-port as below:



Then, $Z_{sc}^{(1)} = Z_{11} - Z_{12} + \frac{Z_{12}(Z_{22} - Z_{12})}{Z_{22}} = Z_{11} - Z_{12}^2/Z_{22}$

$$Z_{sc}^{(2)} = Z_{22} - Z_{12}^2/Z_{11}$$

$$Z_{oc}^{(1)} = Z_{11} \quad \checkmark$$

$$Z_{oc}^{(2)} = Z_{22} \quad \checkmark$$

From the first equation,

$$Z_{12}^2 = -(Z_{sc}^{(1)} - Z_{11})Z_{22} = (Z_{oc}^{(1)} - Z_{sc}^{(1)})Z_{oc}^{(2)}$$

4.9

From (4.58), $V_m = V_m^+ + V_m^-$
 $Z_0 I_m = V_m^+ - V_m^-$

Solving for V_m^+ , V_m^- :

$$V_m^+ = (V_m + Z_0 I_m)/2$$

$$V_m^- = (V_m - Z_0 I_m)/2$$

at port 1:

$$V_1^+ = \frac{1}{2} [10 \angle 0^\circ + 50 (.1 \angle 30^\circ)] = 7.27 \angle 9.9^\circ$$

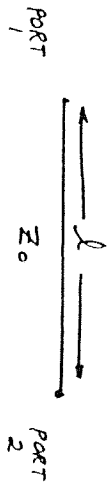
$$V_1^- = \frac{1}{2} [10 \angle 0^\circ - 50 (.1 \angle 30^\circ)] = 3.10 \angle -23.8^\circ$$

at port 2:

$$V_2^+ = \frac{1}{2} [12 \angle 90^\circ + 50 (.15 \angle 120^\circ)] = 9.44 \angle 101.5^\circ \quad \checkmark$$

$$V_2^- = \frac{1}{2} [12 \angle 90^\circ - 50 (.15 \angle 120^\circ)] = 3.33 \angle 55.7^\circ \quad \checkmark$$

4.10 a)



$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} = 0$$

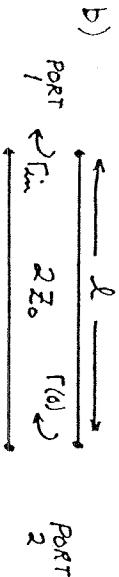
$$S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+ = 0} = 0$$

$$S_{12} = \left. \frac{V_1^-}{V_2^+} \right|_{V_1^+ = 0} = e^{j\beta l}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0} = e^{j\beta l}$$

$$[S] = \begin{bmatrix} 0 & e^{j\beta l} \\ e^{j\beta l} & 0 \end{bmatrix}$$

unitary ✓



$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} = \Gamma_{in} = S_{22}$$

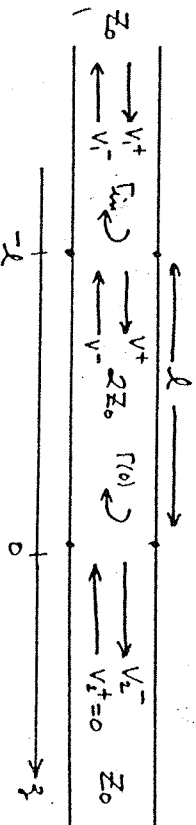
$$\Gamma(0) = \frac{Z_0 - 2Z_0}{Z_0 + 2Z_0} = -\frac{1}{3}$$

$$Z_{in} = 2Z_0 \frac{1 + \Gamma(0) e^{-2j\beta l}}{1 - \Gamma(0) e^{-2j\beta l}} = 2Z_0 \frac{1 - \frac{1}{3} e^{-3j\beta l}}{1 + \frac{1}{3} e^{-3j\beta l}}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{2(1 - \frac{1}{3} e^{-3j\beta l}) - (1 + \frac{1}{3} e^{-3j\beta l})}{2(1 - \frac{1}{3} e^{-3j\beta l}) + (1 + \frac{1}{3} e^{-3j\beta l})} = \frac{1 - e^{-3j\beta l}}{3 - \frac{1}{3} e^{-3j\beta l}}$$

CHECK: if $l = \lambda/2$, $\Gamma_{in} = 0$ ✓

For $S_{21} = S_{12}$, consider the following circuit:



On the $2Z_0$ line we have,

$$V_1 = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{in}) = V^+ + e^{j\beta l} - \frac{1}{3} V^+ e^{-j\beta l}$$

$$V_2 = V_2^+ + V_2^- = V_2^- = V^+ (1 - \frac{1}{3})$$

Thus,
$$V_2^- = \frac{2}{3} V^+ = \frac{2}{3} V_1^+ (1 + \Gamma_{in})$$

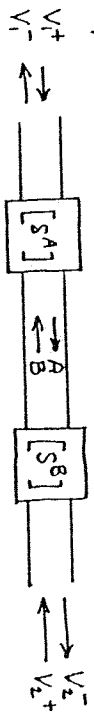
$$S_{21} = \frac{V_2^-}{V_1^+} \Big|_{V_2^+ = 0} = \frac{\frac{2}{3} (1 + \Gamma_{in})}{e^{j\beta l} - \frac{1}{3} e^{j\beta l}} = \frac{8}{3} \frac{e^{j\beta l}}{3 - \frac{1}{3} e^{2j\beta l}}$$

CHECK: if $\ell = \lambda$, $\Gamma_{in} = \Gamma_{out} = 0$, $\Gamma_{in} = 0$, $S_{11} = 0$, $S_{21} = 1$ ✓

if $\ell = \lambda/2$, $\Gamma_{in} = 0$, $S_{11} = 0$, $S_{21} = -1$ ✓

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 &= \left| \frac{1 - e^{-2j\beta l}}{3 - \frac{1}{3} e^{2j\beta l}} \right|^2 + \frac{64}{9} \frac{1}{|3 - \frac{1}{3} e^{2j\beta l}|^2} \\ &= \frac{|1 - e^{-2j\beta l}|^2 + \frac{64}{9}}{|3 - \frac{1}{3} e^{2j\beta l}|^2} = \frac{1 - e^{-2j\beta l} - e^{2j\beta l} + 1 + \frac{64}{9}}{9 - e^{2j\beta l} - e^{-2j\beta l} + \frac{1}{9}} = 1 \quad \checkmark \quad (\text{UNITARY}) \end{aligned}$$

4.11 Define wave amplitudes as shown:



Then,
$$\begin{bmatrix} V_1^- \\ A \end{bmatrix} = \begin{bmatrix} S_A^1 \\ B \end{bmatrix} \begin{bmatrix} V_1^+ \\ B \end{bmatrix} \quad \begin{bmatrix} B \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_B^1 \\ A \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^+ \end{bmatrix}$$

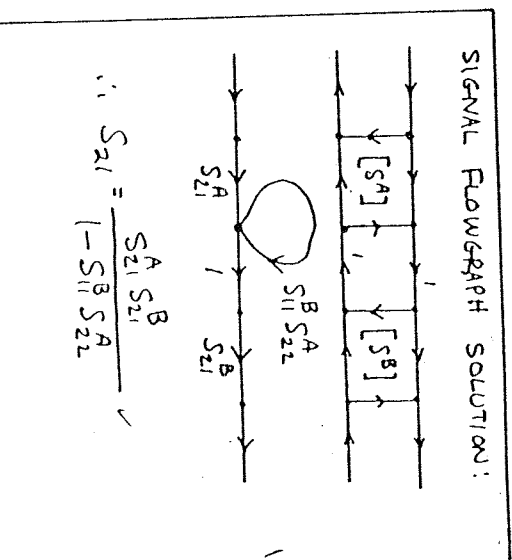
$S_{21} = \frac{V_2^-}{V_1^+} \Big|_{V_2^+ = 0}$. When $V_2^+ = 0$, we have $B = S_{11}^B A$, $V_2^- = S_{21}^B A$.

Then, $A = S_{21}^A V_1^+ + S_{22}^A B = S_{21}^A V_1^+ + S_{22}^A S_{11}^B A$

$$\frac{V_2^-}{S_{21}^B} = S_{21}^A V_1^+ + S_{22}^A S_{11}^B \frac{V_2^-}{S_{21}^B}$$

$$V_2^- \left(\frac{1 - S_{22}^A S_{11}^B}{S_{21}^B} \right) = S_{21}^A V_1^+$$

So,
$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B} \quad \checkmark$$



4.12

a)

$$[S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{bmatrix}$$

$S_{12} = S_{21}$ since reciprocal

$[S]$ is unitary if lossless, so

$$1^{st} \text{ Row: } |S_{11}|^2 + |S_{21}|^2 = 1$$

(or 1st col)

$$|S_{21}|^2 = 1 - |S_{11}|^2 \quad \checkmark$$

b)

$$[S] = \begin{bmatrix} S_{11} & S_{21} \\ 0 & S_{22} \end{bmatrix}$$

$S_{12} \neq S_{21}$ since nonreciprocal

$$1^{st} \text{ Row: } |S_{11}|^2 + |S_{21}|^2 = 1$$

$$1^{st} \text{ Col: } |S_{11}|^2 = 1$$

$$\therefore |S_{21}| = 0$$

4.13

a matched, reciprocal, 3-port network has an $[S]$ matrix of the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

If the network is lossless, then $[S]$ must be unitary:

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad (1)$$

$$S_{13} S_{23}^* = 0 \quad (4)$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad (2)$$

$$S_{12} S_{13}^* = 0 \quad (5)$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \quad (3)$$

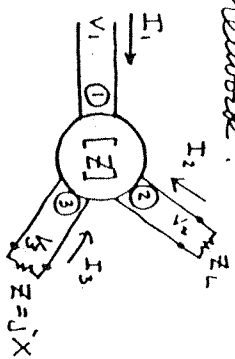
$$S_{12} S_{23}^* = 0 \quad (6)$$

To show that a contradiction exists, assume that $S_{12} = 0$, in order to satisfy (5) and (6). Then from (1), $|S_{13}|^2 = 1$, and from (3), $|S_{23}| = 0$. But then (2) will be contradicted. Similarly, a contradiction will follow if we let $S_{13} = 0$, or $S_{23} = 0$.

a circulator is an example of a nonreciprocal, lossless, matched 3-port network.

4.14 For this problem it is easiest to use the Z-matrix for a loadable reciprocal 3-port network:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} jX_{11} & jX_{12} & jX_{13} \\ jX_{12} & jX_{22} & jX_{23} \\ jX_{13} & jX_{23} & jX_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$



If we terminate port 3 in a reactance jX , then $V_3 = -jXI_3$. Then we must find jX so that $V_2 = 0$ for $V_1 \neq 0$. If $V_2 = 0$, then $I_2 = 0$:

$$V_3 = jX_{13}I_1 + jX_{33}I_3 = -jXI_3$$

$$I_3 = \frac{-X_{13}I_1}{X_{33} + X}$$

$$V_2 = jX_{12}I_1 + jX_{23}I_3 = \left(jX_{12} - \frac{jX_{23}X_{13}}{X_{33} + X} \right) I_1 = 0$$

So,

$$X_{12}X_{33} + XX_{12} - X_{13}X_{23} = 0$$

$$X = \frac{X_{13}X_{23} - X_{12}X_{33}}{X_{12}} \quad \checkmark$$

CHECK: The input impedance at Port 1 is,

$$\begin{aligned} Z_{in}^{(1)} &= \frac{V_1}{I_1} = \frac{jX_{11}I_1 + jX_{13}I_3}{I_1} = jX_{11} + jX_{13} \left(\frac{-X_{13}}{X_{33} + X} \right) \\ &= j \left(X_{11} - \frac{X_{13}^2}{X_{33} + X} \right) \quad \text{which is pure imaginary} \quad \checkmark \end{aligned}$$

4.15

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

Assume the network is fed at port 1, so $V_1^+ \neq 0$. Port 2 is terminated in a matched load, so $V_2^+ = 0$. Port 3 is terminated in a reactive load, so $V_3^+ = e^{j\phi} V_3^-$. We must find $e^{j\phi}$ so that $V_1^- / V_1^+ = 0$.

$$V_3^- = S_{13} V_1^+ + S_{33} V_3^+ = e^{j\phi} V_3^+$$

$$V_3^+ = \frac{S_{13} V_1^+}{e^{j\phi} - S_{33}}$$

$$V_1^- = S_{11} V_1^+ + S_{13} V_3^+ = S_{11} V_1^+ + \frac{S_{13}^2 V_1^+}{e^{j\phi} - S_{33}}$$

$$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{13}^2}{e^{j\phi} - S_{33}} = 0 \Rightarrow e^{j\phi} = S_{33} - \frac{S_{13}^2}{S_{11}} \quad \checkmark$$

We should also verify that this quantity has unit magnitude:

$$|S_{33} - S_{13}^2/S_{11}|^2 = \frac{|S_{11}|^2 |S_{33}|^2 - |S_{13}|^4 - S_{11}^* S_{33}^* S_{13}^2 - S_{11} S_{33} S_{13}^{*2}}{|S_{11}|^2}$$

The unitary properties of [S] lead to four equations:

$$\begin{aligned} |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 &= 1 & S_{11} S_{12}^* + S_{12} S_{11}^* + |S_{13}|^2 &= 0 \\ 2|S_{13}|^2 + |S_{33}|^2 &= 1 & S_{12} S_{13}^* + S_{11} S_{13}^* + S_{13} S_{33}^* &= 0 \end{aligned}$$

Eliminating S_{12} from the two equations on the right yields,

$$-2|S_{11}|^2 - \frac{S_{11} S_{13}^* S_{33}}{S_{13}} - \frac{S_{11}^* S_{13} S_{33}^*}{S_{13}^*} + |S_{13}|^2 = 0$$

$$\text{or, } -S_{11} S_{13}^{*2} S_{33} - S_{11}^* S_{13}^2 S_{33}^* = 2|S_{11}|^2 |S_{13}|^2 - |S_{13}|^4$$

$$\text{Then, } |S_{33} - S_{13}^2/S_{11}|^2 = \frac{|S_{11}|^2 |S_{33}|^2 + |S_{13}|^4 + 2|S_{11}|^2 |S_{13}|^2 - |S_{13}|^4}{|S_{11}|^2}$$

$$= |S_{33}|^2 + 2|S_{13}|^2 = 1 \quad \checkmark$$

4/16

a) To be lossless, $[S]$ must be unitary:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = (1)^2 + (.6)^2 + (.6)^2 = 0.73 \neq 1$$

Thus the network is not lossless

b) To be reciprocal, $[S]$ must be symmetric:

Since $S_{13} \neq S_{31}$, the network is not reciprocal.

c) When all other ports are matched, $\Gamma = S_{11}$

$$\text{Thus, } R_L = -20 \log |\Gamma| = -20 \log(.1) = \underline{20 \text{ dB}} \text{ at port 1. } \checkmark$$

d) When all other ports are matched, the insertion loss from port 2 to port 4 is,

$$IL = -20 \log |S_{42}| = -20 \log(.6) = \underline{4.4 \text{ dB}}. \text{ Phase delay} = \underline{45^\circ} \checkmark$$

e) For a short circuit at port 3, and matched loads at other ports, we have,

$$V_2^+ = V_4^+ = 0, \quad V_3^+ = -V_3^-$$

$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+ = S_{11}V_1^+ - S_{13}V_3^-$$

$$V_3^- = S_{31}V_1^+ + S_{33}V_3^+ = S_{31}V_1^+ - S_{33}V_3^-$$

Solving the second equation for V_3^- :

$$V_3^- = \frac{S_{31}V_1^+}{1 + S_{33}}$$

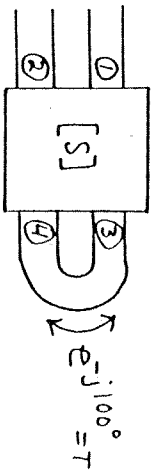
Then,

$$\Gamma^{(1)} = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{13}S_{31}}{1 + S_{33}} \quad \checkmark$$

$$= .1j - \frac{(.6 \angle 45^\circ)(.6 \angle -45^\circ)}{1 + 0} = .1j - .36 = 0.374 \angle 164^\circ \quad \checkmark$$

(verified with supercomputer)

4.17



$$V_4^+ = T V_3^-$$

$$V_3^+ = T V_4^-$$

Assume feed at port 1, port 2 matched. Then $V_2^+ = 0$. Also, $S_{12} = S_{13} = S_{21} = S_{24} = S_{31} = S_{34} = S_{42} = S_{43} = 0$.

DIRECT SOLUTION:

$$V_2^- = S_{23} V_3^+ = S_{23} T V_4^-$$

$$V_3^- = S_{33} V_3^+$$

$$V_4^- = S_{41} V_1^+ + S_{44} V_4^+ = S_{41} V_1^+ + S_{44} T V_3^-$$

$$= S_{41} V_1^+ + S_{44} S_{33} T V_3^+$$

$$= S_{41} V_1^+ + S_{44} S_{33} T^2 V_4^-$$

Solve for V_4^- :

$$V_4^- = \frac{S_{41} V_1^+}{1 - S_{44} S_{33} T^2}$$

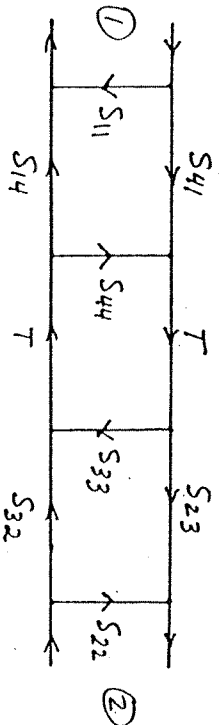
Then,

$$T_{21} = \frac{V_2^-}{V_1^+} = \frac{S_{23} S_{41} T}{1 - S_{33} S_{44} T^2} = \frac{(.7 \angle -45^\circ)(.8)(1 \angle -100^\circ)}{1 - (.7 \angle 45^\circ)(.6 \angle 90^\circ)(1 \angle -200^\circ)}$$

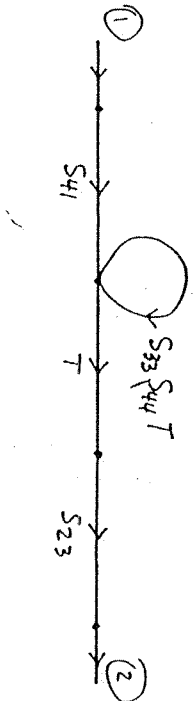
$$= 0.453 \angle 0.4^\circ$$

$$IL = -20 \log(.453) = 6.88 \text{ dB}$$

SIGNAL FLOWGRAPH SOLUTION:

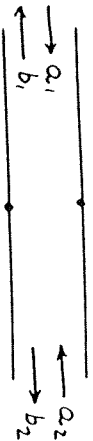


(SPLITTING RULE AND SELF-LOOP RULE)



$$T_{21} = \frac{S_{41} S_{23} T}{1 - S_{33} S_{44} T^2}$$

4.18



From (4.61),

$$S_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \quad \checkmark \quad (\text{reflection coefficient at port 1 when port 2 is matched})$$

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} = \frac{2Z_{01}}{Z_{01} + Z_{02}} \quad \checkmark \quad (\text{transmission coefficient from port 2 to port 1})$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} = \frac{2Z_{02}}{Z_{01} + Z_{02}} \quad \checkmark \quad (\text{transmission coefficient from port 1 to port 2})$$

$$S_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} = \frac{Z_{01} - Z_{02}}{Z_{01} + Z_{02}} \quad \checkmark \quad (\text{reflection coefficient at port 2 when port 1 is matched})$$

4.19

Using Table 4.2 for S to Z transformation:

$$(1 + S_{11}) = 1.4765 \angle 28.3^\circ \quad (1 + S_{22}) = 1.4765 \angle -28.3^\circ$$

$$(1 - S_{11}) = .9899 \angle -45^\circ \quad (1 - S_{22}) = .9899 \angle 45^\circ$$

$$S_{12} S_{21} = -0.36 \quad (1 - S_{11})(1 - S_{22}) - S_{12} S_{21} = 1.340$$

Then,

$$Z_{11} = \frac{50(1.4765 \angle 28.3^\circ)(.9899 \angle 45^\circ) - .36}{1.340} = 2.24 + j52.2 \Omega \quad \checkmark$$

$$Z_{12} = Z_{21} = 50 \frac{2(.6 \angle 90^\circ)}{1.340} = j44.8 \Omega \quad \checkmark$$

$$Z_{22} = \frac{50(.9899 \angle -45^\circ)(1.4765 \angle -28.3^\circ) - .36}{1.340} = 2.24 - j52.2 \Omega \quad \checkmark$$

(verified with supercompact)

4.20

From (4.62),

$$S_{2j}' = \frac{\sqrt{Z_{0j}}}{\sqrt{Z_{0i}}} S_{ij}$$

So,

$$S_{11}' = S_{11} \quad \checkmark$$

$$S_{12}' = \sqrt{\frac{Z_{02}}{Z_{01}}} S_{12} \quad \checkmark$$

$$S_{21}' = \sqrt{\frac{Z_{01}}{Z_{02}}} S_{21} \quad \checkmark$$

$$S_{22}' = S_{22} \quad \checkmark$$

4.21

From Table 4.1 the ABCD parameters for a transmission line section are,

$$A = D = \cos \beta l, \quad B = j Z_0 \sin \beta l, \quad C = j Y_0 \sin \beta l.$$

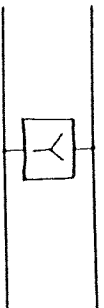
Now use Table 4.2 to convert to Z-parameters:

$$Z_{11} = \frac{A}{C} = \frac{\cos \beta l}{j Y_0 \sin \beta l} = -j Z_0 \cot \beta l \quad \checkmark$$

$$Z_{12} = Z_{21} = \frac{1}{C} = -j Z_0 \csc \beta l \quad \checkmark$$

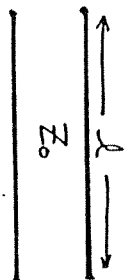
$$Z_{22} = \frac{D}{C} = \frac{\cos \beta l}{j Y_0 \sin \beta l} = -j Z_0 \cot \beta l \quad \checkmark$$

4.22



$$A = \frac{V_1}{V_2} \Big|_{I_2=0} = 1 \checkmark \quad C = \frac{I_1}{I_2} \Big|_{I_2=0} = Y \checkmark$$

$$B = \frac{V_1}{I_2} \Big|_{V_2=0} = 0 \checkmark \quad D = \frac{I_1}{I_2} \Big|_{V_2=0} = 1 \checkmark$$



$$\text{for } I_2=0, \quad V_1 = V^+ (e^{j\beta l} + e^{-j\beta l}) = V^+ 2 \cos \beta l$$

$$V_2 = 2V^+ = V_1 / \cos \beta l$$

$$\text{So, } A = \frac{V_1}{V_2} \Big|_{I_2=0} = \cos \beta l \checkmark$$

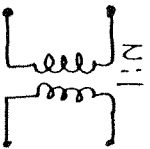
$$C = \frac{I_1}{I_2} \Big|_{I_2=0} = \frac{V_1}{Z_{in} V_2} = \frac{\cos \beta l}{j Z_0 \cot \beta l} = j Y_0 \sin \beta l \checkmark$$

$$\text{for } V_2=0, \quad V_1 = V^+ (e^{j\beta l} - e^{-j\beta l}) = V^+ 2j \sin \beta l$$

$$I_2 = \frac{2V^+}{Z_0}$$

$$\text{So, } B = \frac{V_1}{I_2} \Big|_{V_2=0} = j Z_0 \sin \beta l \checkmark$$

$$D = \frac{I_1}{I_2} \Big|_{V_2=0} = \frac{V_1}{Z_{in} I_2} = \frac{B}{Z_{in}} = \frac{j Z_0 \sin \beta l}{j Z_0 \tan \beta l} = \cos \beta l \checkmark$$



$$A = \frac{V_1}{V_2} \Big|_{I_2=0} = \frac{N V_2}{V_2} = N \checkmark$$

$$B = \frac{V_1}{I_2} \Big|_{V_2=0} = 0 \checkmark$$

$$C = \frac{I_1}{I_2} \Big|_{I_2=0} = 0 \checkmark$$

$$D = \frac{I_1}{I_2} \Big|_{V_2=0} = \frac{1}{N} \checkmark$$

4.23

NOTE: Difference in sign for Z and ABCD.

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = \frac{V_1}{V_2} \left. \frac{V_2}{I_1} \right|_{I_2=0} = A/C \checkmark$$

for $I_1=0$,

$$V_1 = AV_2 - BI_2$$

$$0 = CV_2 - DI_2 \Rightarrow V_2 = DI_2/C$$

$$V_1 = \left(\frac{AD}{C} - B\right) I_2$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = \frac{AD-BC}{C} \checkmark$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = 1/C \checkmark$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = D/C \checkmark$$

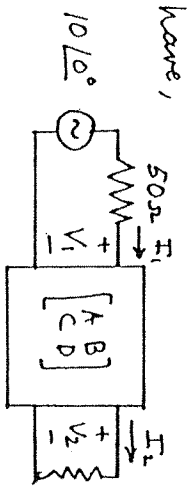
4.24

Using Table 4.1, the ABCD matrix of the cascade of four components is,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 40+j30 \\ 0 & 1 \end{bmatrix}}_{R+jX} \underbrace{\begin{bmatrix} 3 & 0 \\ 0 & 1/3 \end{bmatrix}}_{\text{TRANSF. T-LINE}} \underbrace{\begin{bmatrix} 0 & j75 \\ j/75 & 0 \end{bmatrix}}_{\text{T-LINE}} = \begin{bmatrix} (-.1333+j.1778) & j225 \\ j/225 & 0 \end{bmatrix}$$

CHECK: $AD-BC=1 \checkmark$

Then we have,



$$V_1 = AV_2 + BI_2 = (A+B/60) V_2$$

$$I_1 = CV_2 + DI_2 = (C+D/60) V_2$$

$$I_2 = V_2/60.$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{(A+B/60)}{(C+D/60)} = \frac{(-.1333+j.1778+j225/60)}{j/225} = 883.8 + j30.0 \Omega \checkmark$$

$$V_L = V_2 = \frac{I_1}{(C+D/60)} = \frac{V_g}{(C+D/60)(Z_{in}+50)} = \frac{10(-j225)}{733.8+j30} = 2.41 \angle -92^\circ$$

68

4.25

DIRECT CALCULATION:

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = \frac{\frac{V_1}{2 + Y_1 Y}}{\frac{V_1}{2 + Y_1 Y}} = 1 + YZ$$

$$B = \left. \frac{V_1}{I_2} \right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = \frac{I_1}{I_1/Y} = Y$$

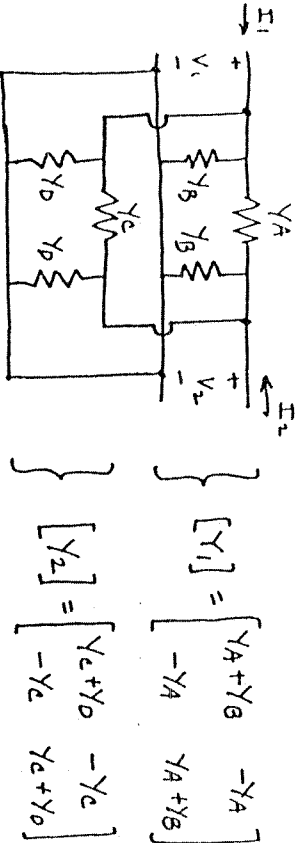
$$D = \left. \frac{I_1}{I_2} \right|_{V_2=0} = 1$$

CHECK: $AD - BC = 1 + YZ - ZY = 1 \checkmark$

CALCULATION USING CASCADE: (From Table 4.1)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 + ZY & Z \\ Y & 1 \end{bmatrix} \checkmark$$

4.26



$$\left. \begin{matrix} \text{ } \\ \text{ } \end{matrix} \right\} [Y_1] = \begin{bmatrix} Y_A + Y_B & -Y_A \\ -Y_A & Y_A + Y_B \end{bmatrix}$$

$$\left. \begin{matrix} \text{ } \\ \text{ } \end{matrix} \right\} [Y_2] = \begin{bmatrix} Y_C + Y_D & -Y_C \\ -Y_C & Y_C + Y_D \end{bmatrix}$$

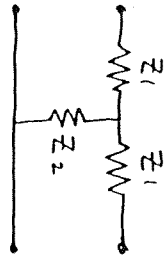
Adding $[Y]$ matrices gives:

$$[Y] = [Y_1] + [Y_2] = \begin{bmatrix} Y_A + Y_B + Y_C + Y_D & -Y_A - Y_C \\ -Y_A - Y_C & Y_A + Y_B + Y_C + Y_D \end{bmatrix}$$

By direct calculation, we obtain similar results:

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = Y_A + Y_B + Y_C + Y_D \quad \checkmark \quad Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} = -(Y_A + Y_C) \quad \checkmark$$

Now apply to bridged-T network (Example 5.7 of 1st edition)



$$[Z_A] = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{bmatrix}$$

$$[Y_A] = \frac{1}{D} \begin{bmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_1 + Z_2 \end{bmatrix} \quad \checkmark$$

$$D = (Z_1 + Z_2)^2 - Z_2^2 = Z_1^2 + 2Z_1Z_2 \quad \checkmark$$



$$[Y_B] = \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix} \quad \checkmark$$

$$[Y_{tot}] = [Y_A] + [Y_B] = \begin{bmatrix} \frac{1}{Z_3} + \frac{Z_1 + Z_2}{D} & -(\frac{1}{Z_3} + \frac{Z_2}{D}) \\ -(\frac{1}{Z_3} + \frac{Z_2}{D}) & \frac{1}{Z_3} + \frac{Z_1 + Z_2}{D} \end{bmatrix} \quad \checkmark$$

4.27

$$V_1 = AV_2 - BI_2$$

$$V_n = V_n^+ + V_n^-$$

$$I_1 = CV_2 - DI_2$$

$$I_n = (V_n^+ - V_n^-)/Z_0$$

So,

$$V_1^+ + V_1^- = A(V_2^+ + V_2^-) - B(V_2^+ - V_2^-)/Z_0$$

$$V_1^+ - V_1^- = C(V_2^+ + V_2^-)Z_0 - D(V_2^+ - V_2^-)$$

For $V_2^- = 0$,

$$V_1^+ + V_1^- = (A + B/Z_0)V_2^+ \quad \text{--- (1)}$$

$$V_1^+ - V_1^- = (CZ_0 + D)V_2^+ \quad \text{--- (2)}$$

eliminate V_2^+ :

$$V_1^+ + V_1^- = \frac{A + B/Z_0}{CZ_0 + D} (V_1^+ - V_1^-) \dots$$

$$V_1^- (CZ_0 + D + A + B/Z_0) = V_1^+ (A + B/Z_0 - CZ_0 - D)$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+ = 0} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \quad \checkmark$$

eliminate V_1^- :

$$2V_1^+ = (A + B/Z_0 + CZ_0 + D)V_2^+$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+ = 0} = \frac{2}{A + B/Z_0 + CZ_0 + D} \quad \checkmark$$

for $V_1^+ = 0$ the above set reduces to,

$$V_1^- = (A - B/Z_0) V_2^+ + (A + B/Z_0) V_2^-$$

$$-V_1^- = (CZ_0 - D) V_2^+ + (CZ_0 + D) V_2^-$$

eliminate V_1^- :

$$(A - B/Z_0 + CZ_0 - D) V_2^+ + (A + B/Z_0 + CZ_0 + D) V_2^- = 0$$

$$S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+ = 0} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \quad \checkmark$$

eliminate V_2^- :

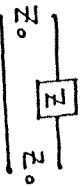
$$\frac{V_1^-}{A + B/Z_0} - \frac{A - B/Z_0}{A + B/Z_0} V_2^+ = \frac{-V_1^-}{CZ_0 + D} - \frac{CZ_0 - D}{CZ_0 + D} V_2^+$$

$$V_1^- \left(\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D} \right) = V_2^+ \left(\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D} \right)$$

$$S_{12} = \left. \frac{V_1^-}{V_2^+} \right|_{V_1^+ = 0} = \frac{\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}}{\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \quad \checkmark$$

These results agree with Table 4.2.

4.28



From Table 4.1,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

convert to [S] using Table 4.2:

$$S_{11} = \frac{1 + Z/Z_0 - 1}{1 + Z/Z_0 + 1} = \frac{Z}{2Z_0 + Z} \quad ; \quad S_{12} = \frac{2}{1 + Z/Z_0 + 1} = \frac{2Z_0}{2Z_0 + Z}$$

$$1 - S_{11} = \frac{2Z_0}{2Z_0 + Z} = S_{12} \quad \checkmark$$



From Table 4.1,

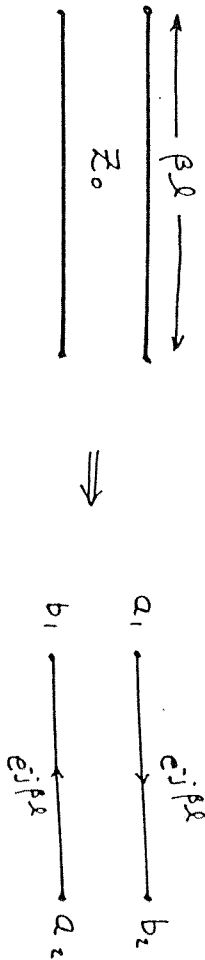
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$$

convert to [S]:

$$S_{11} = \frac{1 - Z_0/Z - 1}{1 + Z_0/Z + 1} = \frac{-Z_0}{2Z + Z_0} \quad ; \quad S_{12} = \frac{2}{1 + Z_0/Z + 1} = \frac{2Z}{2Z + Z_0}$$

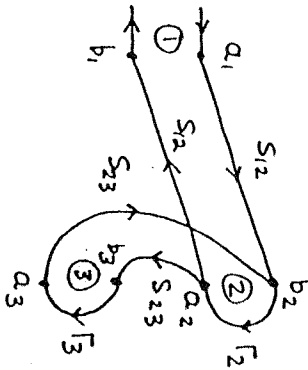
$$1 + S_{11} = \frac{2Z}{2Z + Z_0} = S_{12} \quad \checkmark$$

4.29



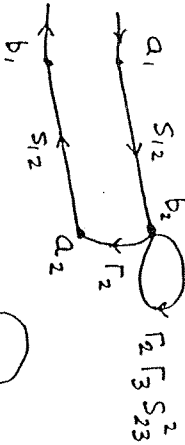
4.30

The signal flowgraph is as follows:



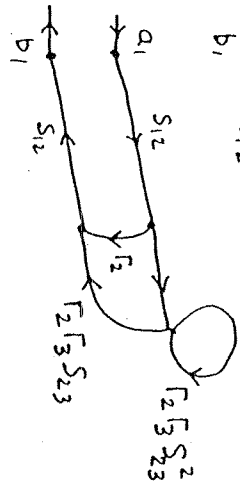
$$\text{but } \Gamma_{in} = \frac{b_1}{a_1}$$

Using the reduction rules:



$$\therefore b_2 = a_1 \frac{S_{12}}{1 - \Gamma_2 \Gamma_3 S_{23}^2} \quad \checkmark$$

$$b_1 = a_1 \frac{S_{12} \Gamma_2}{1 - \Gamma_2 \Gamma_3 S_{23}^2} \quad \checkmark$$



$$\therefore b_3 = b_2 \frac{\Gamma_2 S_{23}}{1 - \Gamma_2 \Gamma_3 S_{23}^2}$$

$$\begin{aligned} \text{Then, } \frac{P_2}{P_1} &= \frac{b_2^2 - a_2^2}{a_1^2 - b_1^2} = \frac{b_2^2 (1 - |\Gamma_2|^2)}{a_1^2 (1 - |\Gamma_{in}|^2)} = \frac{|S_{12}|^2 (1 - |\Gamma_2|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 \left(1 - \frac{|S_{12}^2 \Gamma_2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}\right)} \\ &= \frac{|S_{12}|^2 (1 - |\Gamma_2|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 - |S_{12}^2 \Gamma_2|^2} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \frac{P_3}{P_1} &= \frac{b_3^2 - a_3^2}{a_1^2 - b_1^2} = \frac{b_3^2 (1 - |\Gamma_3|^2)}{a_1^2 (1 - |\Gamma_{in}|^2)} = \frac{|S_{12}|^2 |\Gamma_2 S_{23}|^2 (1 - |\Gamma_3|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^4 \left(1 - \frac{|S_{12}^2 \Gamma_2|^2}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2}\right)} \\ &= \frac{|S_{12}|^2 |\Gamma_2|^2 |S_{23}|^2 (1 - |\Gamma_3|^2)}{|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 (|1 - \Gamma_2 \Gamma_3 S_{23}^2|^2 - |S_{12}^2 \Gamma_2|^2)} \quad \checkmark \end{aligned}$$

4.31

The complex reflected power can be computed using

(4.88):

$$\begin{aligned}
 P_r &= \int_S \mathbf{E}^T \mathbf{X} \bar{\mathbf{H}}^T \mathbf{r}^* \cdot \hat{\mathbf{z}} \, ds = - \int_{x=0}^a \int_{y=0}^b \mathbf{E}_y^T \mathbf{H}_x^T \mathbf{r}^* \, dx \, dy \\
 &= -b \int_{x=0}^a \left[\sum_n A_n \sin \frac{n\pi x}{a} e^{j\beta_n^a z} \right] \left[\sum_m \frac{A_m^*}{Z_m} \sin \frac{m\pi x}{a} e^{-j\beta_m^{a*} z} \right] dx \\
 &= \frac{-ab}{2} \sum_{n=1}^{\infty} \frac{|A_n|^2}{Z_n^*} e^{j(\beta_n^a - \beta_n^{a*})z}
 \end{aligned}$$

The only propagating mode is the $n=1$ (TE₁₀) mode, so β_1^a is real, and β_n^a is imaginary for $n>1$. Let $\alpha_n = j\beta_n = \sqrt{(n\pi/a)^2 - k_0^2}$ for $n>1$. Then $Z_1^a = k_0 \eta_0 / \beta_1^a$, and $Z_n^a = k_0 \eta_0 / \beta_n^a = j k_0 \eta_0 / \alpha_n$ for $n>1$.

$$\text{Then } P_r = \frac{-ab}{2} \left[\frac{|A_1|^2 \beta_1^a}{k_0 \eta_0} - j \sum_{n=2}^{\infty} \frac{|A_n|^2 \alpha_n}{k_0 \eta_0} e^{2\alpha_n z} \right] \text{ for } z < 0.$$

So we see that $\text{Im}\{P_r\} > 0$, indicating an inductive load.

4.32

This problem can be solved by setting up two equations for A_1 and A_2 using (4.97). But a general computer program had been written for Section 4.6, so this was used to obtain the results that $A_1 = 0.071 \angle 150^\circ$, and $A_2 = 0.29 \angle -13^\circ$. The computer program listing is shown on the following page.

```

C MOD: ANALYSIS OF ASYMMETRIC H-PLANE STEP
COMPLEX AM=1.001, QM(100,100), PM=1.001, ZRC, ZNA
COMPLEX DET, X0Z0, XN, BTNA, BTKC
DIMENSION L1(100), MM(100)
PI=3.14159265
Z0=377.
write(6,*) ' Enter lambda/a, c/a, N:'
100 READ(6,*) A, C, NE
XLM=1.
a=1./A
c=C*A
XK0=2.*PI/XLM
C FILE ARRAYS
BTNA=SQRT(XK0*XK0-(PI/A)**2)
Z1A=XK0*Z0/BTNA
DO 40 M=1,NE
DO 10 N=1,NE
BTNA=CSORT((1.,0.)*XK0*XK0-(N*PI/A)**2)
IF(AIMAG(BTNA).GT.0.) BTNA=-BTNA
ZNA=XK0*Z0/BTNA
QM(N,N)=(0.,0.)
IF(M.EQ.N) QM(N,N)=A/2.
DO 50 K=1,NE
BTKC=CSORT((1.,0.)*XK0*XK0-(K*PI/C)**2)
IF(AIMAG(BTKC).GT.0.) BTKC=-BTKC
ZKC=XK0*Z0/BTKC
QM(M,N)=QM(N,N)-BTKC*2./C*XI(K,M,A,C)*XI(K,N,A,C)/ZNA
CONTINUE
50 CONTINUE
PM(N)=(0.,0.)
IF(M.EQ.1) PM(N)=-A/2.
DO 60 K=1,NE
BTKC=CSORT((1.,0.)*XK0*XK0-(K*PI/C)**2)
IF(AIMAG(BTKC).GT.0.) BTKC=-BTKC
ZKC=XK0*Z0/BTKC
PM(N)=PM(N)-BTKC*2./C*XI(K,M,A,C)*XI(K,1,A,C)/Z1A
60 CONTINUE
C INVERT MATRIX AND COMPUTE A VECTOR (standard matrix inversion)
40 CONTINUE
CALL OMINV(QM,DET,L1,MM,100,NE)
DO 20 M=1,NE
AM(M)=(0.,C.)
DO 20 N=1,NE
AM(N)=AM(N)-QM(M,N)*PM(N)
write(6,*) ' A vector (mag, phase):'
DO 30 N=1,NE
AMP=CABS(AM(N))
PHS=180.*ATAN2(AMAG(AM(N)),REAL(AM(N)))/PI
write(6,*) N,AMP,PHS
30 COMPUTE REACTANCE
X0Z0=(0.,-1.)*(1.+AM(1))/(1.-AM(1))
XLMG=2.*PI/B**1A
XN=X0Z0*XLMG/12.*A)
WRITE(6,*) ' c/a=',C/A,' lambda/a=',XLM/A,' Xn=',XN
GOTO 100
200 CALL EXIT
END
FUNCTION XI(M,N,A,C)
PI=3.14159265
ARGM=PI*(M/C-N/A)
ARGP=PI*(M/C+N/A)
XIP=SIN(ARGP-C)/(2.*ARGP)
XIM=C/2.
IF(ABS(ARGM).GT.1.E-6) XIM=SIN(ARGM*C)/(2.*ARGM)
XI=XIM-XIP
RETURN
END

```

4.33

This solution is essentially the same as the analysis in Section 4.6. Let $d = (a - a)/2$

$$E_y^i = \sin \frac{\pi x}{a} e^{j\beta^a z}$$

$$H_x^i = \frac{-1}{Z_0} \sin \frac{\pi x}{a} e^{j\beta^a z}$$

$$E_y^r = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} A_n \sin \frac{n\pi x}{a} e^{j\beta_n^a z}$$

$$H_x^r = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{A_n}{Z_0} \sin \frac{n\pi x}{a} e^{j\beta_n^a z}$$

$$E_y^t = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} B_n \sin \frac{n\pi}{c} (x-d) e^{j\beta_n^a z}$$

$$H_x^t = - \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{B_n}{Z_0} \sin \frac{n\pi}{c} (x-d) e^{j\beta_n^a z}$$

$$\text{where } \beta_n^a = \sqrt{k_0^2 - (n\pi/a)^2}$$

$$\beta_n^c = \sqrt{k_0^2 - (n\pi/c)^2}$$

The solution has the same form as (4.97):

$$\frac{a}{2} A_m + \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{k=1 \\ \text{odd}}}^{\infty} \frac{2 Z_0^i I_{km} I_{kn}}{c Z_0^a} A_n = \sum_{\substack{k=1 \\ \text{odd}}}^{\infty} \frac{2 Z_0^i I_{km} I_{k1}}{c Z_0^a} - \frac{a}{2} S_{m1}$$

for $m = 1, 3, 5, \dots$,
and,

$$I_{mn} = \int_{x=d}^{d+c} \sin \frac{m\pi}{c} (x-d) \sin \frac{n\pi x}{a} dx$$

$$S_{mn} = \begin{cases} 1 & \text{for } m=n \\ 0 & \text{for } m \neq n \end{cases}$$

4.34

From (4.110) the source current is,

$$\vec{J}_s = \hat{x} \frac{2B^+ m\pi}{a} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} + \hat{y} \frac{2B^+ n\pi}{b} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$$

From Table 3.2, the transverse fields for \pm traveling TM_{mn} modes are,

$$E_x = \frac{\mp j\beta m\pi}{k_c^2 a} C^\pm \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{\mp j\beta z}$$

$$E_y = \frac{\mp j\beta n\pi}{k_c^2 b} C^\pm \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H_x = \frac{j\omega \epsilon m\pi}{k_c^2 b} C^\pm \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H_y = \frac{-j\omega \epsilon n\pi}{k_c^2 a} C^\pm \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{\mp j\beta z}$$

where C^\pm are the unknown amplitudes. At $z=0$, E_z is continuous, so $C^+ = -C^-$. Also, $\hat{z} \times (\vec{H}^+ - \vec{H}^-) = \vec{J}_s$,
 or $H_y^+ - H_y^- = J_{sx}$ and $-H_x^+ + H_x^- = J_{sy}$. So,

$$J_{sx}: \quad \frac{-j\omega \epsilon m\pi}{k_c^2 a} (C^+ - C^-) = 2B^+ \frac{m\pi}{a} \implies C^+ - C^- = \frac{k_c^2 B^+}{-j\omega \epsilon}$$

$$J_{sy}: \quad \frac{j\omega \epsilon n\pi}{k_c^2 b} (-C^+ + C^-) = 2B^+ \frac{n\pi}{a} \implies C^+ - C^- = \frac{k_c^2 B^+}{-j\omega \epsilon} \quad \checkmark$$

Since these fields satisfy Maxwell's equations and the boundary conditions, they must form the unique solution.

4.35

From (4.114), the source current is,

$$\bar{M}_s = -\hat{x} 2B_{mn} \frac{\mu\epsilon}{b} \sin \frac{\mu\pi x}{a} \cos \frac{\mu\pi y}{b} + \hat{y} 2B_{mn} \frac{\mu\epsilon}{a} \cos \frac{\mu\pi x}{a} \sin \frac{\mu\pi y}{b}$$

From Table 3.2, the transverse TM_{mn} fields for \pm traveling waves can be written as,

$$E_x^\pm = \frac{-\mu\pi}{a} C^\pm \cos \frac{\mu\pi x}{a} \sin \frac{\mu\pi y}{b} e^{\pm j\beta z}$$

$$E_y^\pm = \frac{-\mu\pi}{b} C^\pm \sin \frac{\mu\pi x}{a} \cos \frac{\mu\pi y}{b} e^{\pm j\beta z}$$

$$H_x^\pm = \pm \frac{\omega\epsilon}{\beta} \frac{\mu\pi}{b} C^\pm \sin \frac{\mu\pi x}{a} \cos \frac{\mu\pi y}{b} e^{\pm j\beta z}$$

$$H_y^\pm = \mp \frac{\omega\epsilon}{\beta} \frac{\mu\pi}{a} C^\pm \cos \frac{\mu\pi x}{a} \sin \frac{\mu\pi y}{b} e^{\pm j\beta z}$$

where C^\pm is the unknown amplitude. At $z=0$, \bar{H}_z is continuous, so $C^+ = -C^-$. At $z=0$, $(\bar{E}^+ - \bar{E}^-) \times \hat{z} = \bar{M}_s$, so we have,

$$-E_x^+ + E_x^- = M_{sy} \Rightarrow C^+ - C^- = 2B_{mn} \Rightarrow C^+ = -C^- = B_{mn}^+$$

$$E_y^+ - E_y^- = M_{sx} \Rightarrow -C^+ + C^- = -2B_{mn}^+ \Rightarrow C^+ = -C^- = B_{mn}^+ \checkmark$$

Since these fields satisfy Maxwell's equations and the relevant boundary conditions, they must form the unique solution.

4.36

Following Example 4.8:

$$\vec{J}(x, y, z) = I(y) \delta(x-a/2) \delta(z) \quad \text{for } 0 < y < d.$$

$$\vec{E}_1 = \hat{y} \sin \frac{\pi x}{a}, \quad \vec{h}_1 = \frac{-\hat{x}}{Z_1} \sin \frac{\pi x}{a}, \quad Z_1 = \eta_0 N_0 / \beta_1$$

From (4.119),

$$P_1 = \frac{ab}{Z_1}$$

From (4.118),

$$A_1^+ = \frac{-1}{P_1} \int_{-r}^r \sin \frac{\pi x}{a} e^{j\beta_1 z} I(y) \delta(x-a/2) \delta(z) dx dy dz$$

$$= \frac{-I_0}{P_1} \int_{y=0}^d \frac{\sin k(a-y)}{\sin kd} dy = \frac{-I_0}{P_1 \sin kd} \int_0^d \sin kw dw$$

(let $w = d-y$)

$$= \frac{I_0 Z_1 (\cos kd - 1)}{\eta_0 ab \sin kd}$$

The total power flow in the TE₁₀ mode is,

$$P = \frac{ab |A_1^+|^2}{2Z_1},$$

for both + and - traveling waves, since $|A_1^+| = |A_1^-|$.

Then the radiation resistance is,

$$R_{\text{rad}} = \frac{2P}{I_0^2} = \frac{ab |A_1^+|^2}{I_0^2 Z_1} = \frac{Z_1}{ab} \frac{(1 - \cos kd)^2}{\eta_0^2 \sin^2 kd}$$

$$= \frac{Z_1}{\eta_0^2 ab} \frac{(2 \sin^2 \frac{kd}{2})^2}{4 \sin^2 \frac{kd}{2} \cos^2 \frac{kd}{2}} = \frac{Z_1}{\eta_0^2 ab} \tan^2 \frac{kd}{2} \quad \checkmark$$

4.37

Following Example 4.8:

$$\bar{J}(x, y, z) = \hat{z} S(x, z) [S(x - a/4) - S(x - 3a/4)] \hat{y} \quad \text{for } 0 < y < b$$

From Table 3.2,

$$TE_{10}: \quad \bar{E}_1 = \hat{y} \sin \frac{\pi x}{a}$$

$$\bar{h}_1 = \frac{-\hat{x}}{z_1} \sin \frac{\pi x}{a}$$

$$z_1 = k_0 \eta_0 / \beta_1$$

$$TE_{20}: \quad \bar{E}_2 = \hat{y} \sin \frac{2\pi x}{a}$$

$$\bar{h}_2 = \frac{-\hat{x}}{z_2} \sin \frac{2\pi x}{a}$$

$$z_2 = k_0 \eta_0 / \beta_2$$

$$P_1 = ab/z_1$$

$$\beta_1 = \sqrt{k_0^2 - (\pi/a)^2}$$

$$P_2 = ab/z_2$$

$$\beta_2 = \sqrt{k_0^2 - (2\pi/a)^2}$$

From (4.118):

$$A_1^+ = \frac{-1}{P_1} \int_V \bar{E}_1 \cdot \bar{J} dV = -\frac{Ib}{P_1} (\sin \frac{\pi}{4} - \sin \frac{3\pi}{4}) = 0 \quad \checkmark$$

$$A_2^+ = \frac{-1}{P_2} \int_V \bar{E}_2 \cdot \bar{J} dV = -\frac{Ib}{P_2} (\sin \frac{\pi}{2} - \sin \frac{3\pi}{2}) = \frac{-2Ib}{a}$$

Since the excitation has an odd symmetry about the center of the guide, it will only excite modes that have an electric field with an odd symmetry about $x=a/2$. This implies the TEM mode, for m even, will be excited. The TE₁₀ mode is not excited.

4.38

as in Example 4.9,

$$\bar{P}_m = \hat{z} I_0 \pi r_0^2 S(x) S(y-b/a) S(z)$$

$$\bar{M} = j \omega \mu_0 \bar{P}_m$$

$$= \hat{z} j \omega \mu_0 I_0 \pi r_0^2 S(x) S(y-b/a) S(z) \quad \text{V/m}^2$$

For the TE₁₀ mode,

$$\bar{E}_1 = \hat{y} A \sin \frac{\pi x}{a}$$

$$\bar{h}_1 = \frac{-\hat{x}}{Z_1} A \sin \frac{\pi x}{a}$$

$$h_{31} = \frac{\hat{z} \pi}{k_0 \eta_0 a} \cos \frac{\pi x}{a}$$

and $Z_1 = k_0 \eta_0 / \beta_1$, $P_1 = ab/Z_1$.

From (4.122),

$$A_1^+ = \frac{1}{P_1} \int_V (-\bar{h}_1 + \hat{z} h_{31}) \cdot \bar{M} e^{j\beta_1 z} dV$$

$$= \frac{Z_1}{ab} \int_V h_{31} M dV = \frac{-\pi^2 Z_1 I_0 r_0^2}{a^2 b} = A^-$$

Stronger coupling can usually be obtained by coupling to transverse field components - compare with the coefficient A^+ in Example 4.9, which is greater by a factor of $\beta_1 a / \pi$.

4.39

FIRST SOLUTION: (all fields and currents are TE₁₀)

$$E_y = B \sin \frac{\pi x}{a} [e^{j\beta z} - e^{-j\beta z}] = -2j B \sin \frac{\pi x}{a} \sin \beta z \quad 0 < z < d$$

$$H_x = \frac{B}{Z_1} \sin \frac{\pi x}{a} [-e^{j\beta z} - e^{-j\beta z}] = \frac{-2B}{Z_1} \sin \frac{\pi x}{a} \cos \beta z \quad 0 < z < d$$

This satisfies $E_y = 0$ at $z = 0$.

$$E_y = C \sin \frac{\pi x}{a} e^{j\beta(z-d)} \quad z > d$$

$$H_x = \frac{C}{Z_1} \sin \frac{\pi x}{a} e^{-j\beta(z-d)} \quad z > d$$

at $z = d$, E_y is continuous, so

$$-2j B \sin \beta d = C$$

at $z = d$, $\hat{z} \times (\vec{H}^+ - \vec{H}^-) = \vec{J}_s$, or

$$\frac{C}{Z_1} + \frac{2B}{Z_1} \cos \beta d = \frac{2VA}{a}$$

Solving for B, C :

$$B = \frac{\pi Z_1 A}{a} e^{-j\beta d} \quad , \quad C = \frac{\pi Z_1 A}{a} (e^{-2j\beta d} - 1)$$

SECOND SOLUTION: (using (4.105) and (4.106b)):

E_y due to J_{sy} at $z = d$:

$$E_y^{\pm} = \frac{-\pi Z_1 A}{a} \sin \frac{\pi x}{a} e^{\mp j\beta(z-d)}$$

E_y due to $-J_{sy}$ at $z = -d$:

$$E_y^{\pm} = \frac{\pi Z_1 A}{a} \sin \frac{\pi x}{a} e^{\mp j\beta(z+d)}$$

For $0 < z < d$,

$$E_y = \frac{\pi Z_1 A}{a} \sin \frac{\pi x}{a} [e^{j\beta(z+d)} - e^{j\beta(z-d)}] = \frac{-2j\pi Z_1 A}{a} e^{j\beta d} \sin \frac{\pi x}{a} \sin \beta z \quad \checkmark$$

For $z > d$,

$$E_y = \frac{\pi Z_1 A}{a} \sin \frac{\pi x}{a} [e^{j\beta(z+d)} - e^{-j\beta(z-d)}] = \frac{-2j\pi Z_1 A}{a} \sin \beta d \sin \frac{\pi x}{a} e^{-j\beta z} \quad \checkmark$$

These results agree with those from the first solution.

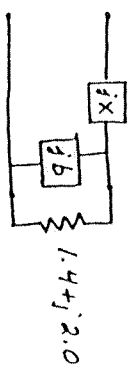
Chapter 5

5.1 (Smith chart solutions)

a) $Z_L = 1.4 + j2.0$

inside $1+jx$ circle

SOLN #1: $b_1 = -0.10$ ✓
 $x_1 = -1.7$ ✓

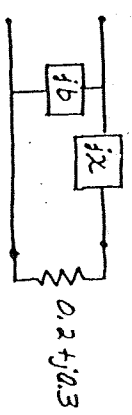


SOLN #2: $b_2 = 0.78$ ✓
 $x_2 = 1.7$ ✓

b) $Z_L = 0.2 + j0.3$

outside $1+jx$ circle

SOLN #1: $x_1 = 0.10$ ✓
 $b_1 = 2.0$ ✓

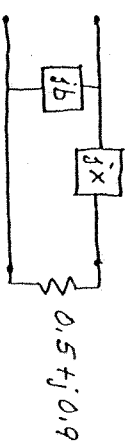


SOLN #2: $x_2 = -0.70$ ✓
 $b_2 = -2.0$ ✓

c) $Z_L = 0.5 + j0.9$

outside $1+jx$ circle

SOLN #1: $x_1 = -0.40$ ✓
 $b_1 = 0.96$ ✓

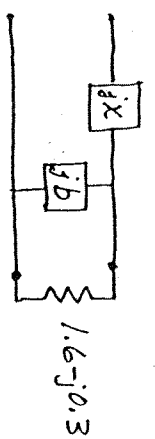


SOLN #2: $x_2 = -1.4$ ✓
 $b_2 = -0.96$ ✓

d) $Z_L = 1.6 - j0.3$

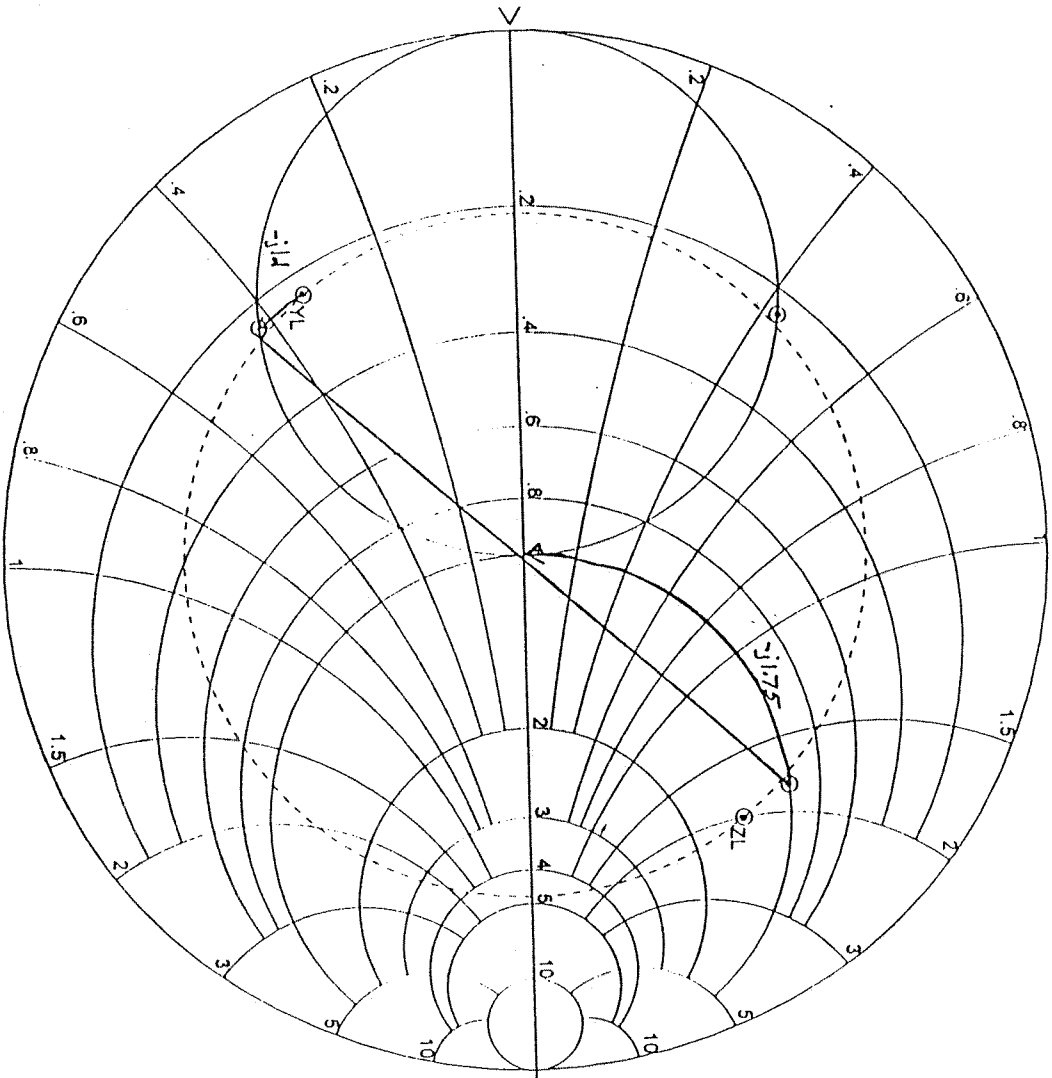
inside $1+jx$ circle

SOLN #1: $b_1 = 0.38$ ✓
 $x_1 = 0.80$ ✓



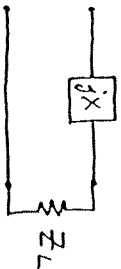
SOLN #2: $b_2 = -0.62$ ✓
 $x_2 = -0.80$ ✓

(The Smith chart for 5.1a is shown on the following page.)



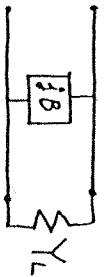
Smith chart for Problem 5.1a

5.2
a)



也就是說，每 $R_L(Z_L)$ 時，才可用單一元件互
成匹配
matching possible if $Z_L = Z_0 - jX$ ✓

b)



matching possible if $Y_L = \frac{1}{Z_0} - jB$ ✓

5.3

(analytical solution)

From (5.9),

$$t = \frac{160 \pm \sqrt{200[(100-200)^2 + (160)^2]}}{200-100} = 1.6 \pm 2.67$$

So, $t_1 = 4.27$, $t_2 = -1.07$

Then from (5.10) the possible stub positions are,

$$\begin{aligned} d_1 &= \frac{\lambda}{2\pi} \tan^{-1} t_1 = 0.213\lambda \quad \checkmark \\ d_2 &= \frac{\lambda}{2\pi} (\pi + \tan^{-1} t_2) = 0.370\lambda \quad \checkmark \end{aligned}$$

From (5.8b) the susceptances at the stub locations are,

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0 [R_L^2 + (X_L + Z_0 t)^2]}$$

So, $B_1 = 0.0133$, $B_2 = -0.0134$,

Then from (5.11a) the open-circuited stub lengths are,

$$L_1 = \frac{\lambda}{2\pi} \tan^{-1}(B_1 Z_0) = 0.353\lambda \quad \checkmark \quad (X/2 \text{ added to get } L_1 > 0)$$

$$L_2 = \frac{\lambda}{2\pi} \tan^{-1}(B_2 Z_0) = 0.148\lambda \quad \checkmark$$

5.4

For short-circuited stubs, see B_1, B_2 from

Problem 5.2 with (5.11b):

$$l_1 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_1} = 0.103 \lambda$$

$$l_2 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_2} = 0.398 \lambda$$

(These lengths are $\lambda/4$ shorter or longer than the open-circuited stub lengths.)

5.5

(Smith chart solution)

The normalized load impedance is $Z_L = 0.40 - j1.2$. To intersect the $1+jx$ circle, we must move back from the load either of the following distances:

$$d_1 = 0.19 + (0.5 - 0.355) = 0.335 \lambda \checkmark$$

$$\text{or, } d_2 = 0.31 + (0.5 - 0.355) = 0.455 \lambda \checkmark$$

The reactances necessary for matching are then,

$$X_{S1} = -2.1 \Omega$$

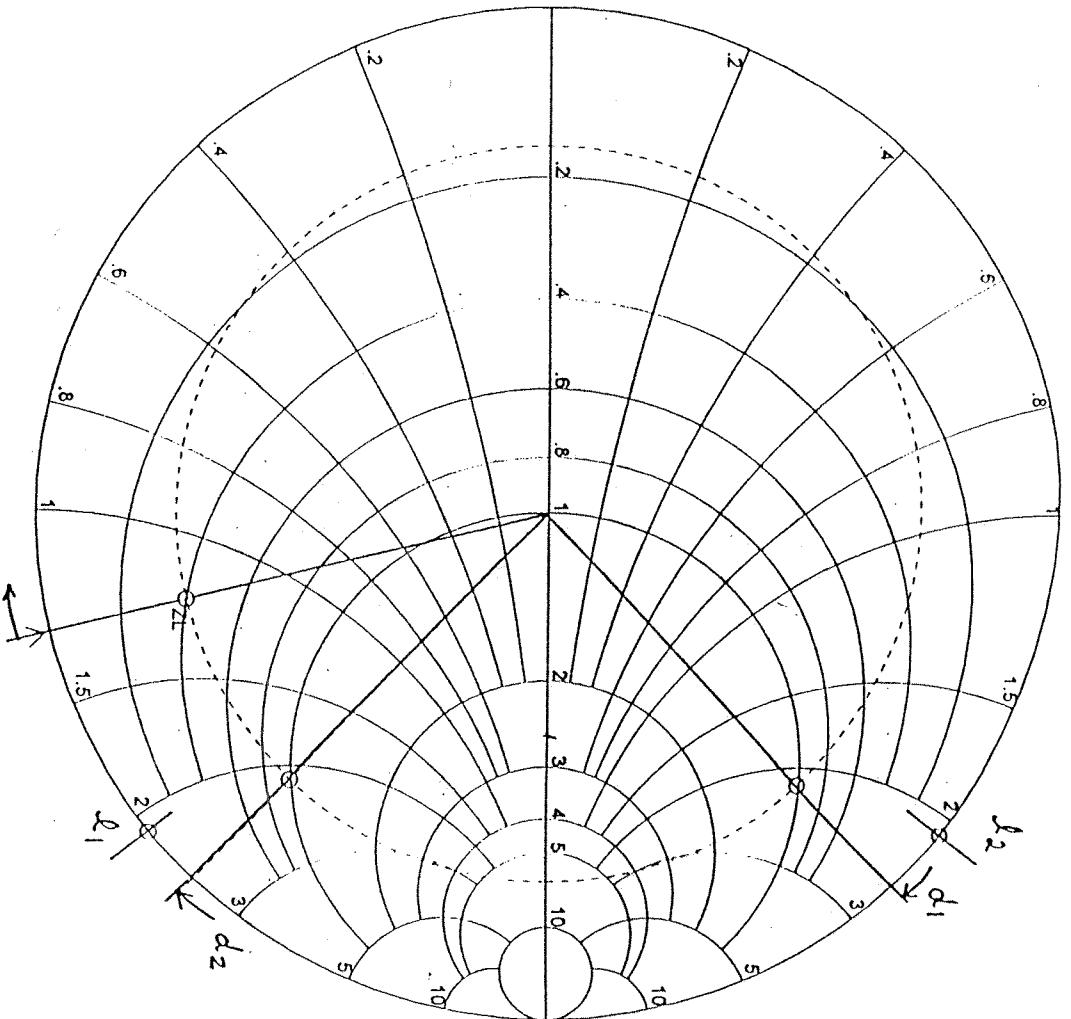
$$X_{S2} = 2.1 \Omega$$

Then the open-circuited stub lengths are,

$$l_1 = 0.32 - 0.25 = 0.07 \lambda \checkmark$$

$$l_2 = 0.25 + 0.18 = 0.43 \lambda \checkmark$$

(The Smith chart for this problem is shown on the following page.)

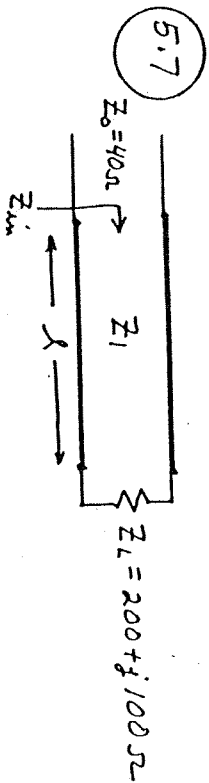


Smith chart for Problem 5.5

5.6 The stub lengths for short-circuited stubs are $\lambda/4$ longer or shorter than the above lengths:

$$l_1 = 0.07 + 0.25 = 0.32 \lambda$$

$$l_2 = 0.43 - 0.25 = 0.18 \lambda$$



To match this load, we must find Z_1 and l so that $Z_{in} = Z_0 = 40 \Omega$:

$$Z_{in} = 40 = Z_1 \frac{(200 + j100) + jZ_1 l}{Z_1 + j(200 + j100)l}, \text{ with } l = \tan \beta l.$$

$$(40Z_1 - 4000l) + j8000l = 200Z_1 + j(100 + Z_1 l)Z_1$$

Equating real and imaginary parts gives two equations for the two unknowns, Z_1 and l : (if they exist!)

$$\text{Re: } 40Z_1 - 4000l = 200Z_1 \Rightarrow Z_1 = -25l$$

$$\text{Im: } 8000l = Z_1(100 + Z_1 l)$$

$$8000l = -25l(100 - 25l^2)$$

$$l = \pm \sqrt{16.8} = \pm 4.10 \quad (\text{use } -4.10 \text{ so that } Z_1 > 0) \checkmark$$

$$\text{Then, } \beta l = \tan^{-1}(-4.10) = -76.3^\circ \cong 104^\circ \Rightarrow \underline{l = 0.288 \lambda}$$

The characteristic impedance is then,

$$Z_1 = -25(-4.10) = 102.5 \Omega \quad \checkmark$$

(Note: Not all load impedances can be matched in this way - a good exam problem to determine which impedances can be matched using this technique!)

5.8 From (2.91) the impedance of a terminated lossy line is,

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}, \quad \gamma l = \alpha l + j\beta l$$

For $Z_L = \infty$ (o.c.), the normalized input admittance is,

$$Y_{in} = \tanh \gamma l = \frac{\tanh \alpha l + j \tanh \beta l}{1 + j \tanh \alpha l \tanh \beta l}$$

The normalized input susceptance is,

$$B_{in} = \frac{\tanh \beta l (1 - \tanh^2 \alpha l)}{1 + \tanh^2 \alpha l \tanh^2 \beta l} \quad \text{at this point, we could find max. } B_{in} \text{ by calculating } \frac{dB_{in}}{d\alpha l}$$

Since maximum susceptance for a lossless line is obtained for $\beta l = \pi/2$, we expect βl to be close to $\pi/2$ for the lossy case. So let $\beta l = \pi/2 + \Delta$, where Δ is small. Also, αl is small, so we have $\tanh \alpha l \approx \alpha l$, and $\tanh \beta l = -\cot \Delta \approx -1/\Delta$.

$$\text{Then, } B_{in} \approx \frac{\frac{-1}{\Delta} (1 - \alpha^2 l^2)}{1 + \alpha^2 l^2 / \Delta^2} \approx \frac{-1}{\Delta + \alpha^2 l^2 / \Delta}$$

To maximize B_{in} , we can minimize $\Delta + \alpha^2 l^2 / \Delta$ with respect to l :

$$\frac{d}{d\Delta} (\Delta + \alpha^2 l^2 / \Delta) = \frac{d\Delta}{d\Delta} + \frac{2\alpha^2 l^2}{\Delta^2} + \alpha^2 l^2 \left(\frac{-1}{\Delta^2} \right) \frac{d\Delta}{d\Delta} = 0$$

$$\text{or, since } \frac{d\Delta}{d\Delta} = \beta, \quad \Delta = \beta l - \frac{\pi}{2}$$

$$\beta + \frac{2\alpha^2 l^2}{\Delta} - \frac{\alpha^2 l^2}{\Delta^2} \beta = 0$$

since $\Delta = \beta l - \pi/2$, we have,

$$l^2 \beta (\alpha^2 + \beta^2) - \pi l (\alpha^2 + \beta^2) + \beta \frac{\pi^2}{4} = 0$$

Solve for l :

$$\begin{aligned} \rho &= \frac{\pi(\alpha^2 + \beta^2) \pm \sqrt{\pi^2(\alpha^2 + \beta^2)^2 - \beta^2 \pi^2(\alpha^2 + \beta^2)}}{2\beta(\alpha^2 + \beta^2)} \\ &= \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta\sqrt{\alpha^2 + \beta^2}} \approx \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta^2} \quad (\text{since } \alpha^2 \ll \beta^2) \end{aligned}$$

Then, $\Delta = \beta l - \pi/2 \approx \frac{\pi\alpha}{2\beta} \approx \alpha l$ (since $\beta \approx \pi/2l$)

The corresponding value of b_{in} is,

$$b_{in}^{MAX} = \frac{\pm 1}{\alpha l + \rho} = \frac{\pm 1}{2\alpha l} = \frac{\pm 2}{\alpha \lambda} \quad (\text{since } l \approx \lambda/4)$$

For $\alpha = 0.01$ nepers/ λ , $b_{in}^{MAX} = \frac{\pm 2}{.01} = \pm 200$

(This check with direct calculation of y_{in} vs. l .)

The reactance of a short-circuited line is the dual case of the above problem, so $x_{in}^{MAX} = \pm 200$.

5.9

Smith chart solution

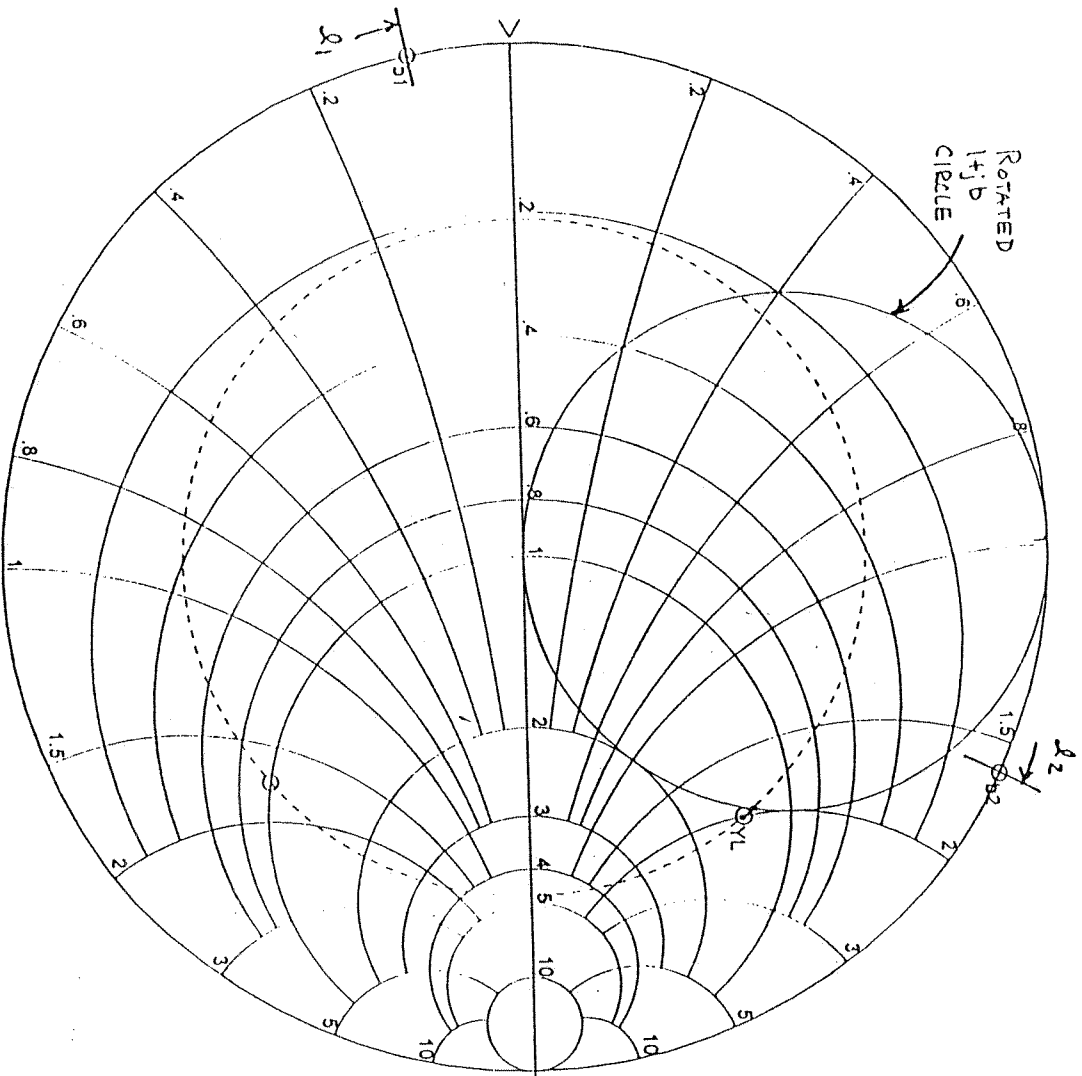
1. Plot $Y_L = 1.4 + j2.0$ (ADMITTANCE CHART)
2. Plot rotated $1+jb$ circle
3. add a stub susceptance of $-j0.1$ to Y_L to move to rotated $1+jb$ circle
4. move $\lambda/8$ towards generator, to $1+jb$ circle
5. add a stub susceptance of $+j1.6$ to move to center of chart.
6. The open-circuited stub lengths are,
 $l_1 = 0.484\lambda$ ✓
 $l_2 = 0.161\lambda$ ✓

(Analytically, we obtain $l_1 = 0.487\lambda$, $l_2 = 0.163\lambda$)

The alternative matching solution has,

$$l_1' = 0.326\lambda \quad \checkmark$$
$$l_2' = 0.053\lambda \quad \checkmark$$

The Smith chart for the first solution is shown on the following page.



Smith chart for Problem 5.9

5.10

Analytic solution

Let $t = \tan \beta d = \tan 135^\circ = -1.0$

From (5.22) the first stub susceptance is,

$$b_1 = -b_L + \frac{1 \pm \sqrt{(1+t^2)g_L - g_L^2 t^2}}{t} = -3.92 \text{ or } -2.08 \checkmark$$

From (5.23) the second stub susceptance is,

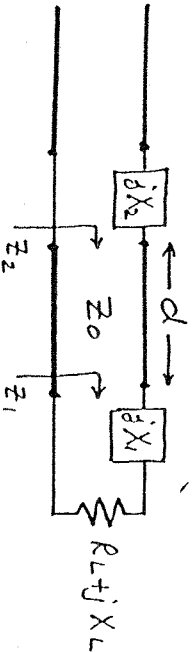
$$b_2 = \frac{\pm \sqrt{(1+t^2)g_L - g_L^2 t^2} + g_L}{g_L t} = -1.65 \text{ or } -3.45 \checkmark$$

From (5.24b) the S.C. stub lengths are,

$$l_1 = 0.04\lambda, \text{ or } 0.071\lambda \quad \checkmark$$

$$l_2 = 0.087\lambda, \text{ or } 0.045\lambda \quad \checkmark$$

5.11



$$Z_1 = R_L + j(X_L + X_1)$$

$$Z_2 = Z_0 \frac{R_L + j(X_L + X_1 + Z_0 t)}{Z_0 + j t (R_L + j X_L + X_1)} = Z_0 \quad t = \tan \beta d$$

Solving for R_L :

$$R_L = Z_0 \frac{1+t^2}{2t^2} \left[1 \pm \sqrt{\frac{1-4t^2(Z_0 - X_L t - X_1 t)^2}{Z_0(1+t^2)^2}} \right]$$

So we must have,

$$0 \leq R_L \leq Z_0 \frac{1+t^2}{2t^2} = \frac{Z_0}{\sin^2 \beta d}$$

The first stub reactance is,

$$X_1 = -X_L + \frac{Z_0 \pm \sqrt{(1+t^2)R_L Z_0 - R_L^2 t^2}}{t}$$

The second stub reactance is,

$$X_2 = \frac{\pm Z_0 \sqrt{Z_0 R_L (1+t^2) - R_L^2 t^2} + R_L Z_0}{R_L t}$$

The stub lengths are given by,

$$l_{0c} = \frac{1}{2\pi} \tan^{-1} \left(\frac{Z_0}{X} \right), \quad l_{sc} = \frac{1}{2\pi} \tan^{-1} \left(\frac{X}{Z_0} \right)$$

5.12

Using the Smith chart

($Z_0 = 100 \Omega$)

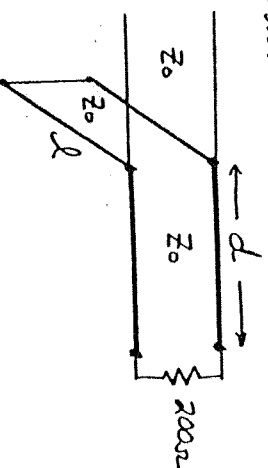
a) A single short-circuited shunt stub:

at f_0 , $d_1 = 0.152\lambda$ ✓ $d_2 = 0.348\lambda$ ✓

$b_1 = -0.7$ $b_2 = +0.7$

$l_1 = 0.153\lambda$ ✓ $l_2 = 0.347\lambda$ ✓

$|\Gamma_1| = 0$ $|\Gamma_2| = 0$



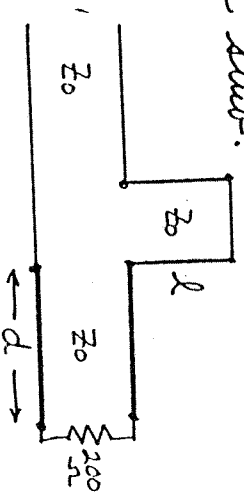
b) A single short-circuited series stub:

at f_0 , $d_1 = 0.098\lambda$ ✓ $d_2 = 0.402\lambda$ ✓

$x_1 = 0.7$ $x_2 = -0.7$

$l_1 = 0.097\lambda$ ✓ $l_2 = 0.403\lambda$ ✓

$|\Gamma_1| = 0$ $|\Gamma_2| = 0$



c) A double short-circuited shunt stub: (let $d = \lambda/8$)

at f_0 ,

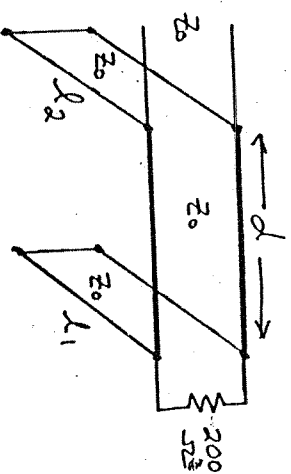
$b_1 = 0.14$ $b'_1 = 1.85$

$l_1 = 0.272\lambda$ ✓ $l'_1 = 0.421\lambda$ ✓

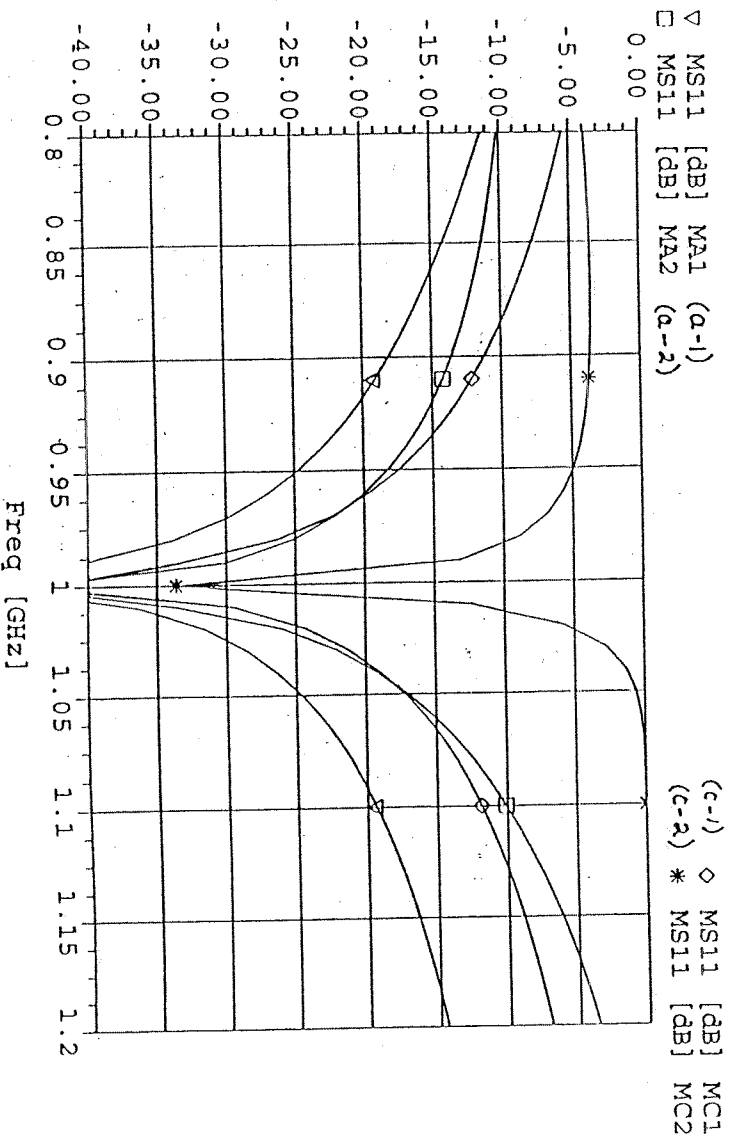
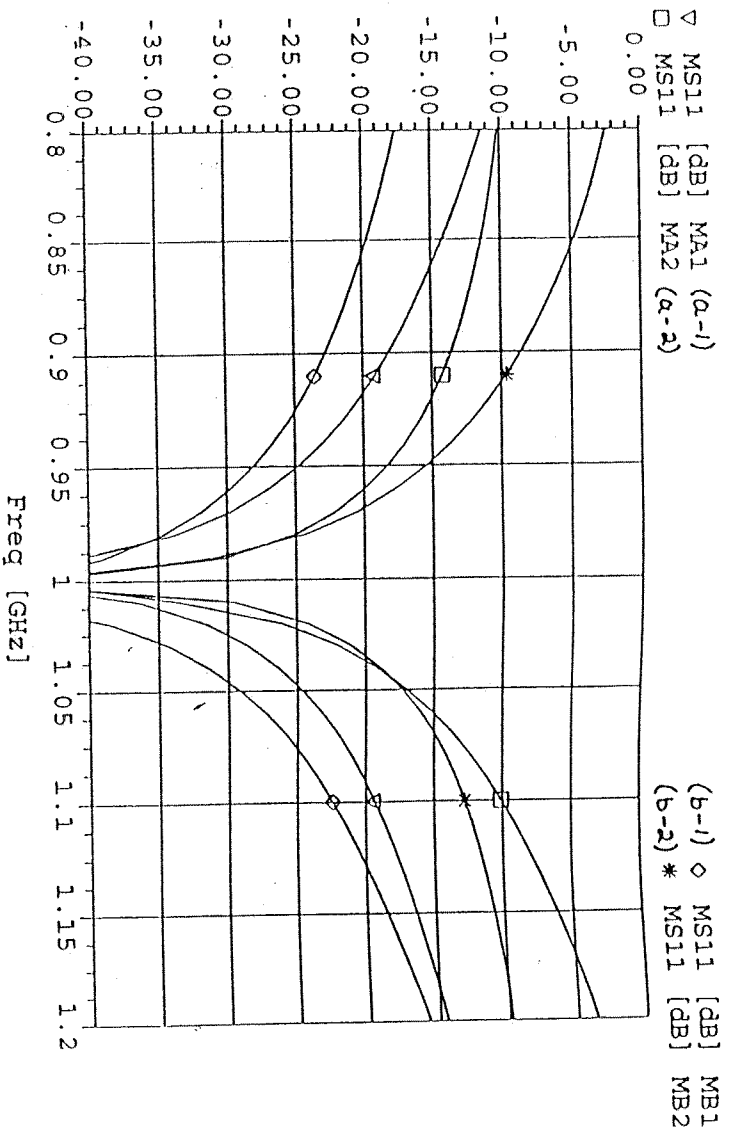
$b_2 = -0.73$ $b'_2 = 2.75$

$l_2 = 0.15\lambda$ ✓ $l'_2 = 0.444\lambda$ ✓

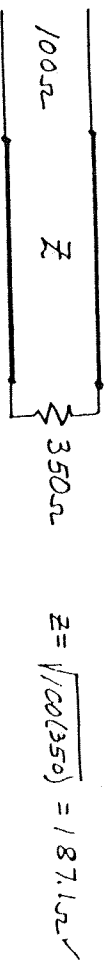
$|\Gamma_1| = 0$ $|\Gamma'_1| = 0$



Plots of return loss vs. f/f for these six solutions are shown on the following page. (only 4 curves could be plotted on graph). These results show that the tuner of solution (b-1), the series stub tuner, gives the best bandwidth. This is probably because the stub length and line length are shortest for this case, giving the smallest frequency variation.



5.13



an SWR of 2 corresponds to a reflection coefficient magnitude of $\Gamma_M = \frac{S-1}{S+1} = 1/3$

Then from (5.33) the bandwidth is,

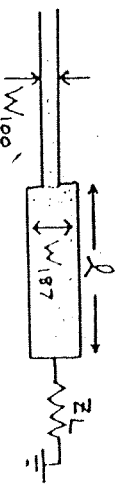
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{\Gamma_M}{\sqrt{1 - \Gamma_M^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|} \right] = 71\%$$

MICROSTRIP LAYOUT:

$\epsilon_r = 2.2$, $d = 0.159$ cm, $f = 4$ GHz

First try $w/d < 2$:

for W_{100} , $A_{100} = 2.213$, $W_{100}/d = 0.896 < 2$ (OK), $W_{100} = 0.142$ cm
 for W_{197} , $A_{197} = 4.047$, $W_{197}/d = 0.140 < 2$ (OK), $W_{197} = 0.0222$ cm



From (3.195), ϵ_e for W_{197} is $\epsilon_e = 1.66$.

Then the physical length of the $\lambda/4$ transformer is,

$$l = \frac{\lambda g}{4} = \frac{c}{4\sqrt{\epsilon_e} f} = 1.455$$
 cm ✓

5.14

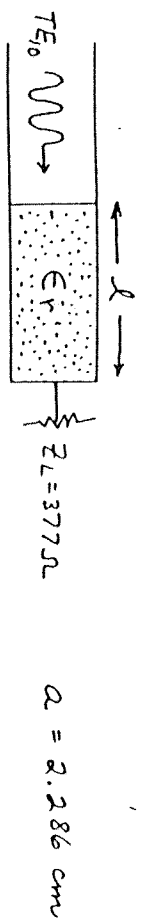
From (5.34) and (5.36), the partial reflection coefficients

are,

$$\Gamma_1 = \frac{Z_L - Z_1}{Z_L + Z_1} = \frac{150 - 100}{150 + 100} = 0.2 \quad ; \quad \Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2} = \frac{225 - 150}{225 + 150} = 0.2$$

Since the approximate expression for Γ in (5.42) is identical to the numerator for the exact expression in (5.41), the greatest error will occur when the denominator of (5.41) departs from unity to the greatest extent. This occurs for $\theta = 0$ or 180° . Then (5.41) gives the exact Γ at 0.384, while (5.42) gives the approximate $\Gamma = 0.4$. Thus the error is about 4%.

5.15



$$k_0 = \frac{2\pi f}{c} = 209.4 \text{ m}^{-1}$$

In the air-filled guide,

$$\beta_a = \sqrt{k_0^2 - (\pi/a)^2} = 158.0 \text{ m}^{-1} \quad \checkmark$$

$$Z_a = \frac{k_0 \eta_0}{\beta_a} = \frac{(209.4)(377)}{158} = 499.6 \Omega \quad \checkmark$$

So the matching section impedance must be,

$$Z_M = \sqrt{Z_a Z_L} = \sqrt{(499.6)(377)} = 434.0 \Omega$$

$$= \frac{k_m \eta_0}{\beta_m} = \frac{k_0 \eta_0}{\beta_m}$$

So the propagation constant of the matching section must be,

$$\beta_m = \frac{k_0 \eta_0}{Z_M} = \frac{(209.4)(377)}{434} = 181.9 \text{ m}^{-1}$$

$$= \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$$

Solving for ϵ_r :

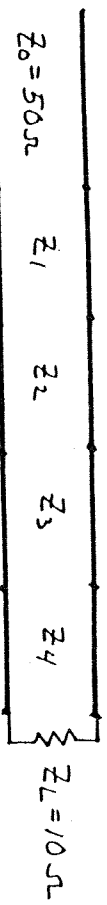
$$\epsilon_r = \frac{\beta_m^2 + (\pi/0.2286)^2}{(209.4)^2} = 1.185 \quad \checkmark$$

The physical length of the matching section is,

$$l = \frac{\lambda_g}{4} = \frac{2\pi}{4\beta_m} = \frac{\pi}{2\beta_m} = 0.86 \text{ cm} \quad \checkmark$$

(Note that this type of matching is not possible if $Z_L \gg Z_a$.)

5.16



Z_0/Z_L is not in Table 5.1, so we will use (5.53):

$$n=0: \ln \frac{Z_1}{Z_0} = 2^{-4} C_0^4 \ln \frac{10}{50} = -0.10059 \quad \Rightarrow Z_1 = 45.2 \Omega \quad \checkmark$$

$$n=1: \ln \frac{Z_2}{Z_1} = 2^{-4} C_1^4 \ln \frac{10}{50} = -0.40236 \quad \Rightarrow Z_2 = 30.2 \Omega \quad \checkmark$$

$$n=2: \ln \frac{Z_3}{Z_2} = 2^{-4} C_2^4 \ln \frac{10}{50} = -0.60354 \quad \Rightarrow Z_3 = 16.5 \Omega \quad \checkmark$$

$$n=3: \ln \frac{Z_4}{Z_3} = 2^{-4} C_3^4 \ln \frac{10}{50} = -0.40236 \quad \Rightarrow Z_4 = 11.1 \Omega \quad \checkmark$$

$$\text{CHECK: } \ln \frac{Z_5}{Z_4} = 2^{-4} C_4^4 \ln \frac{10}{50} = -0.10059 \quad \Rightarrow Z_5 = 10.04 \Omega = Z_L \quad \checkmark$$

If we interpolate between $Z_1/Z_0 = 4$ and $Z_1/Z_0 = 5$ in Table 5.1, we obtain $Z_1 = 45.1\Omega$, $Z_2 = 30.0\Omega$, $Z_3 = 16.5\Omega$, and $Z_4 = 11.1\Omega$ (after interchanging source and load ends). These results are close to the first solution, from (5.53).

From (5.55) and Example 5.6, the bandwidth is,

$$A = \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0.0503$$

(Note: this approximation actually gives better results than the "exact" value of A from $2^{-N} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$.)

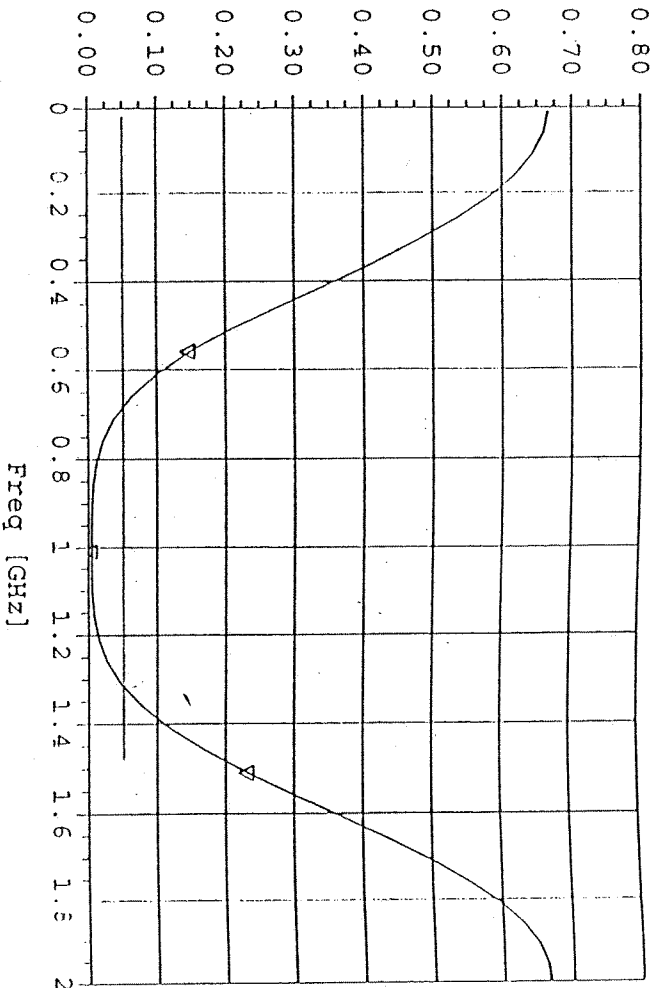
$$\text{Then, } \frac{\Delta F}{F_0} = 2 - \frac{4}{\pi} \cos^{-1} \left[\frac{1}{2} \left| \frac{\Gamma_{in}}{A} \right|^N \right] = 67\%$$

The calculated input reflection coefficient magnitude for this solution is plotted vs frequency on the following page. The bandwidth from this graph is about,

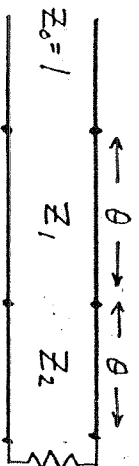
$$\frac{\Delta F}{F_0} = \frac{1.32 - 0.68}{1} \times 100 = 64\%,$$

which is close to the above calculation.

▽ MS11 [mag]



5.17



From (5.50) the derived input reflection coefficient response is ($N=2$):

$$\Gamma(\theta) = 2A(1 + \cos 2\theta)$$

From the above circuit, we have that $\Gamma(0) = \frac{R-1}{R+1} = 0.2$, so $A = 0.2/4 = 0.05$.

Now we calculate the input reflection coefficient of the above circuit using ABCD matrices and conversion to s-parameters:

$$\begin{aligned} \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} \cos \theta & jZ_1 \sin \theta \\ jY_1 \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & jZ_2 \sin \theta \\ jY_2 \sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \theta - Z_1 Y_2 \sin^2 \theta & j(Z_1 + Z_2) \cos \theta \sin \theta \\ j(Y_1 + Y_2) \sin \theta \cos \theta & \cos^2 \theta - Y_1 Z_2 \sin^2 \theta \end{bmatrix} \end{aligned}$$

Using Table 4.2 to convert to S-parameter gives the input reflection coefficient as,

$$\Gamma(\theta) = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{A+B-C-D}{S} + \frac{4[\Gamma_L/S^2]}{1 - \frac{-A+B-C+D}{S}\Gamma_L}$$

$$= \frac{(A+B-C-D)[S - \Gamma_L(-A+B-C+D)] + 4\Gamma_L}{S[S - \Gamma_L(-A+B-C+D)]}$$

where $S = A+B+C+D$, $\Gamma_L = \frac{R-1}{R+1}$.

This result can be equated to $2\pi(1 + \cos 2\theta)$, and solved for Z_1 and Z_2 , but this is a very lengthy procedure. Instead, we will first evaluate both expressions at $\theta = 90^\circ$;

$$\Gamma(90^\circ) = 0, \text{ and } \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\theta=90^\circ} = \begin{bmatrix} -Z_1\gamma_2 & 0 \\ 0 & -\gamma_1 Z_2 \end{bmatrix}$$

So $\Gamma(\theta)$ reduces to the following equation:

$$(-Z_1\gamma_2 + \gamma_1 Z_2) [-(\gamma_1 Z_2 + Z_1\gamma_2) - \Gamma_L(Z_1\gamma_2 - \gamma_1 Z_2)] + 4\Gamma_L = 0$$

$$(Z_1^2\gamma_2^2 - \gamma_1^2 Z_2^2) + \Gamma_L(Z_1^2\gamma_2^2 + \gamma_1^2 Z_2^2 + 2) = 0$$

$$(Z_1^4 - Z_2^4) + \Gamma_L(Z_1^4 + Z_2^4 + 2Z_1^2 Z_2^2) = 0$$

$$(Z_1^2 - Z_2^2) + \Gamma_L(Z_1^2 + Z_2^2) = 0$$

$$Z_2^2 = Z_1^2 \frac{1 + \Gamma_L}{1 - \Gamma_L} = Z_1^2 R \Rightarrow \underline{Z_2 = Z_1 \sqrt{R}} \quad (\text{for } Z_0 = A)$$

Another equation is harder to find, so we will make use of the fact that the transformer will be symmetric:

$$\frac{Z_1^{-1}}{Z_1+1} = \frac{R - Z_2}{R + Z_2} = \frac{R/Z_2 - 1}{R/Z_2 + 1}$$

Thus, $\frac{R}{Z_2} = Z_1$ or $\underline{Z_1 = R^{1/4}}$ (for $Z_0 = 1$)

If $R = 1.5$, these results reduce to,

$$Z_1 = (1.5)^{1/4} = 1.1067 \quad \checkmark$$

$$Z_2 = 1.1067 \sqrt{1.5} = 1.3554 \quad \checkmark$$

which agree with Table 5.1

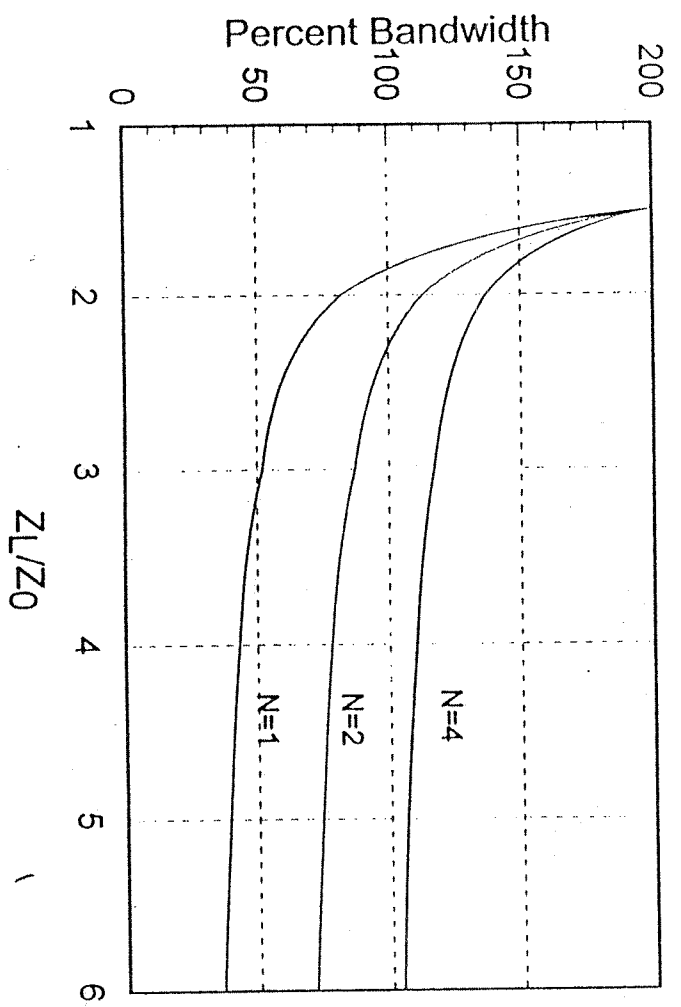
5.18

From (5.55), the fractional bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{A} \cos^{-1} \left[\frac{1}{2} \left(\frac{Z_L}{A} \right)^N \right], \text{ with } A = 2^{-N} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|$$

$\frac{Z_L}{Z_0}$	N=1		N=2		N=4	
	A	$\Delta f/f_0 - \%$	A	$\Delta f/f_0 - \%$	A	$\Delta f/f_0 - \%$
1.5	0.1000	200	0.0500	200	0.0125	200
2.0	0.1667	82	0.0833	113	0.0208	137
3.0	0.2500	52	0.1250	87	0.0313	117
4.0	0.3000	43	0.1500	78	0.0375	110
5.0	0.3333	39	0.1667	74	0.0417	106
6.0	0.3571	36	0.1786	71	0.0446	104

This data is plotted in the graph below.



5.19

$$x \doteq \sec \theta_m \cos \theta$$

(5.60a) ✓

$$n=1: T_1(x) = x = \sec \theta_m \cos \theta$$

$$n=2: T_2(x) = 2x^2 - 1 = 2 \sec^2 \theta_m \cos^2 \theta - 1$$

$$= 2 \sec^2 \theta_m \left(\frac{1}{2} + \frac{1}{2} \cos 2\theta \right) - 1$$

$$= \sec^2 \theta_m (1 + \cos 2\theta) - 1$$

(5.60b) ✓

$$n=3: T_3(x) = 4x^3 - 3x = 4 \sec^3 \theta_m \cos^3 \theta - 3 \sec \theta_m \cos \theta$$

$$= \sec^3 \theta_m (3 \cos \theta + \cos 3\theta) - 3 \sec \theta_m \cos \theta$$

(5.60c) ✓

$$n=4: T_4(x) = 8x^4 - 8x^2 + 1$$

$$= 8 \sec^4 \theta_m \cos^4 \theta - 8 \sec^2 \theta_m \cos^2 \theta + 1$$

$$= \sec^4 \theta_m (3 + 4 \cos 2\theta + \cos 4\theta) - 4 \sec^2 \theta_m (1 + \cos 2\theta) + 1$$

(5.60d) ✓

$$\textcircled{5.20} \quad S_m = 1.2 \Rightarrow |T_m| = \frac{S_m - 1}{S_m + 1} = 0.091 = A$$

From (5.61),

$$\begin{aligned} \Gamma(\theta) &= 2 e^{j4\theta} \left[\Gamma_0 \cos 4\theta + \Gamma_1 \cos 2\theta + \frac{1}{2} \Gamma_2 \right] \\ &= A e^{j4\theta} \Gamma_4 (A \sec \theta_m \cos \theta) \\ &= A e^{j4\theta} \left[A \sec^4 \theta_m (\cos 4\theta + 4 \cos 2\theta + 3) - 4 A \sec^2 \theta_m (\cos 2\theta + 1) + 1 \right] \end{aligned}$$

From (5.63),

$$\begin{aligned} A \sec \theta_m &= \cosh \left[\frac{1}{N} \cosh^{-1} \left(\left| \frac{Z_m Z_L / Z_0}{Z \Gamma_m} \right| \right) \right] \\ &= \cosh \left[\frac{1}{4} \cosh^{-1} \left(\frac{1}{2(0.091)} \ln \frac{60}{40} \right) \right] = 1.0655 \end{aligned}$$

$$\text{so, } \theta_m = \cos^{-1}(1/1.0655) = 20^\circ$$

Equating $\cos 4\theta$ terms:

$$2 \Gamma_0 = A \sec^4 \theta_m \Rightarrow \Gamma_0 = 0.0586 = \Gamma_4$$

Equating $\cos 2\theta$ terms:

$$2 \Gamma_1 = A (4 \sec^4 \theta_m - 4 \sec^2 \theta_m) \Rightarrow \Gamma_1 = 0.02795 = \Gamma_3$$

Equating constant terms:

$$\Gamma_2 = A (3 \sec^4 \theta_m - 4 \sec^2 \theta_m + 1) = 0.0296$$

Then the characteristic impedances are,

$$Z_1 = Z_0 \frac{1 + \Gamma_0}{1 - \Gamma_0} = 44.98 \Omega \quad \checkmark$$

$$Z_2 = Z_1 \frac{1 + \Gamma_1}{1 - \Gamma_1} = 47.57 \Omega \quad \checkmark$$

$$Z_3 = Z_2 \frac{1 + \Gamma_2}{1 - \Gamma_2} = 50.47 \Omega \quad \checkmark$$

$$Z_4 = Z_3 \frac{1 + \Gamma_3}{1 - \Gamma_3} = 53.37 \Omega \quad \checkmark$$

$$\text{CHECK: } Z_5 = Z_4 \frac{1 + \Gamma_4}{1 - \Gamma_4} = 60.01 \Omega = Z_L \quad \checkmark$$

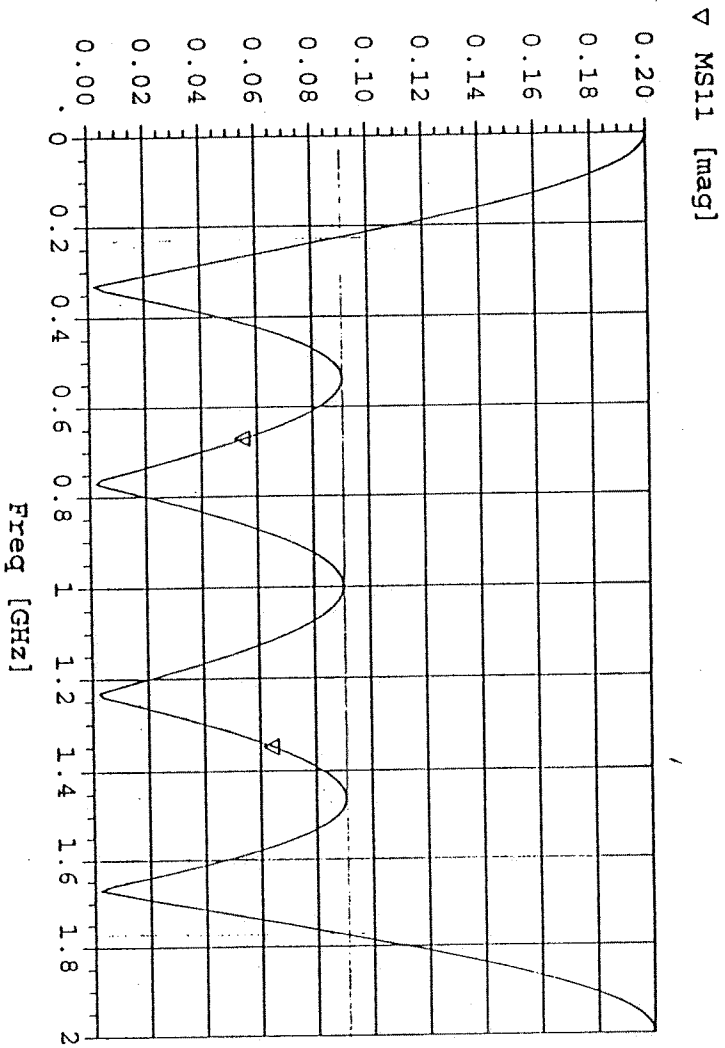
From (5.64) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4 \theta_m}{\pi} = 2 - \frac{4(20)}{180} = 15.6\% \quad \checkmark$$

The calculated reflection coefficient magnitude versus frequency is shown below. The bandwidth from this graph gives a value of approximately,

$$\frac{\Delta f}{f_0} = \frac{1.78 - .23}{1} \times 100 = 155\%$$

in close agreement with the above calculation.



5.21

From (5.61) and (5.60b),

$$\Gamma(\theta) = A e^{-j\theta} T_2 (\text{see } \theta_m \cos \theta) = A e^{-j\theta} [\text{see } \theta_m (1 + \cos 2\theta) - 1]$$

$$\Gamma(0) = A T_2 (\text{see } \theta_m) = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| = \frac{R-1}{R+1} = 0.2 \quad ; \quad A = \Gamma_m = 0.05$$

As in Problem 5.17, we will evaluate $\Gamma(\theta)$ for $\theta = 90^\circ$.

Then $\Gamma(90^\circ) = \Gamma_m$. Also, as in Problem 5.17, from symmetry we have that $Z_1 Z_2 = R$. Then,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -Z_1^2/R & 0 \\ 0 & -R/Z_1^2 \end{bmatrix} \quad (Z_0 = 1)$$

$$\begin{aligned} \Gamma(90^\circ) = \Gamma_m &= \frac{(-Z_1^2/R + R/Z_1^2) \left[-\frac{(Z_1^2/R + R/Z_1^2) - \Gamma_Q (Z_1^2/R - R/Z_1^2)}{-(Z_1^2/R + R/Z_1^2) - \Gamma_Q (Z_1^2/R - R/Z_1^2)} \right] + 4\Gamma_Q}{-(Z_1^2/R + R/Z_1^2) \left[-\frac{(R^2 - Z_1^4) \left[-(Z_1^4 + R^2) - \Gamma_Q (Z_1^4 - R^2) \right] + 4\Gamma_Q R^2 Z_1^4}{(Z_1^4 + R^2) \left[(Z_1^4 + R^2) + \Gamma_Q (Z_1^4 - R^2) \right]} \right]} \end{aligned}$$

$$\begin{aligned} \Gamma_m (Z_1^4 + R^2)^2 + \Gamma_m \Gamma_Q (Z_1^4 + R^2) (Z_1^4 - R^2) &= -(R^2 - Z_1^4) (Z_1^4 + R^2) + \Gamma_Q (Z_1^4 - R^2)^2 + 4\Gamma_Q R^2 Z_1^4 \\ Z_1^8 (\Gamma_m - 1) (\Gamma_Q + 1) + 2Z_1^4 R^2 (\Gamma_m - \Gamma_Q) - R^4 (\Gamma_m + 1) (\Gamma_Q - 1) &= 0 \end{aligned}$$

For $\Gamma_m = 0.05$, $\Gamma_Q = 0.2$, $R = 1.5$:

$$-1.140 Z_1^8 - 0.6750 Z_1^4 + 4.2525 = 0$$

$$Z_1^4 = \frac{0.675 \pm 4.455}{-2.280} = 1.65789 \Rightarrow Z_1 = 1.1347 Z_0 \checkmark$$

$$Z_2 = R/Z_1 = 1.3219 Z_0 \checkmark$$

These results agree with Table 5.2.

5.22

$$|\Gamma(\theta)| = A(0.1 + \cos^2 \theta), \quad 0 < \theta < \pi$$

From (5.46a), for $N=2$,

$$|\Gamma(\theta)| = 2(\Gamma_0 \cos 2\theta + \frac{1}{2}\Gamma_1) = A(0.1 + \cos^2 \theta) \\ = A(0.6 + 0.5 \cos 2\theta)$$

When $\theta=0$, $|\Gamma(0)| = 1.1A = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1.5j}{1.5+1} = 0.2 \Rightarrow A = 0.182$

Equating coefficients of $\cos 2\theta$:

$$2\Gamma_0 = 0.5A \Rightarrow \Gamma_0 = 0.0455$$

Equating constant terms:

$$\Gamma_1 = 0.6A = 0.109$$

So the characteristic impedances are,

$$Z_1 = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0} = 1.095 Z_0$$

$$Z_2 = Z_1 \frac{1+\Gamma_1}{1-\Gamma_1} = 1.363 Z_0$$

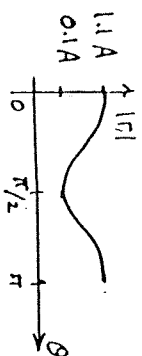
CHECK: at $\theta = \pi/2$, the input impedance to the transformer

will be, $Z_{in} = \frac{Z_1^2}{(Z_2^2/Z_L)} = \frac{Z_1 Z_0 Z_1^2}{Z_2^2} = 0.968 Z_0$

So the input reflection coefficient is,

$$\Gamma_{in} = \left| \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \right| = 0.016$$

which is reasonably close to $|\Gamma(\pi/2)| = 0.1A = 0.018$



5.23

$$\frac{d(\ln z/z_0)}{dz} = A \sin \frac{\pi z}{L}$$

$$\ln(z/z_0) = B - \frac{LA}{\pi} \cos \frac{\pi z}{L} \quad \text{From (5.17)}$$

$$z(z) = C e^{-\frac{LA}{\pi} \cos \frac{\pi z}{L}}$$

$$z(0) = z_0 = C e^{-LA/\pi}, \quad z(L) = z_1 = C e^{+LA/\pi}$$

Solve for C, A to get,

$$C = \sqrt{z_0 z_1}$$

$$A = \frac{\pi}{2L} \ln(z_1/z_0) \quad \checkmark$$

From (5.17),

$$\Gamma(\theta) = \frac{1}{2} \int_{z=0}^L e^{-2j\beta z} \frac{d}{dz} (\ln z/z_0) dz$$

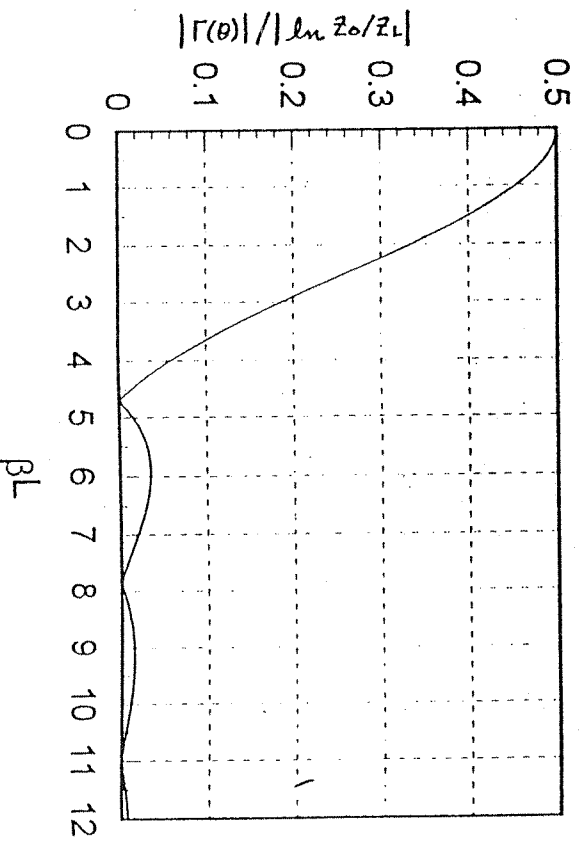
$$= \frac{1}{2} \int_{z=0}^L A \sin \frac{\pi z}{L} e^{-2j\beta z} dz$$

$$= \frac{A}{2} \frac{e^{-2j\beta z} \left[-2j\beta \sin \frac{\pi z}{L} - \frac{\pi}{L} \cos \frac{\pi z}{L} \right]}{(\pi/L)^2 - 4\beta^2} \Big|_0^L$$

$$= \frac{\pi A}{2L} \frac{e^{-j\beta L} (e^{j\beta L} + e^{j\beta L})}{(\pi/L)^2 - 4\beta^2}$$

$$\text{So, } |\Gamma(\theta)| = \frac{\pi^2}{2} \left| \ln \frac{z_1}{z_0} \right| \frac{\cos \beta L}{\pi^2 - (2\beta L)^2} \quad \checkmark$$

This result is plotted as shown:



5.24

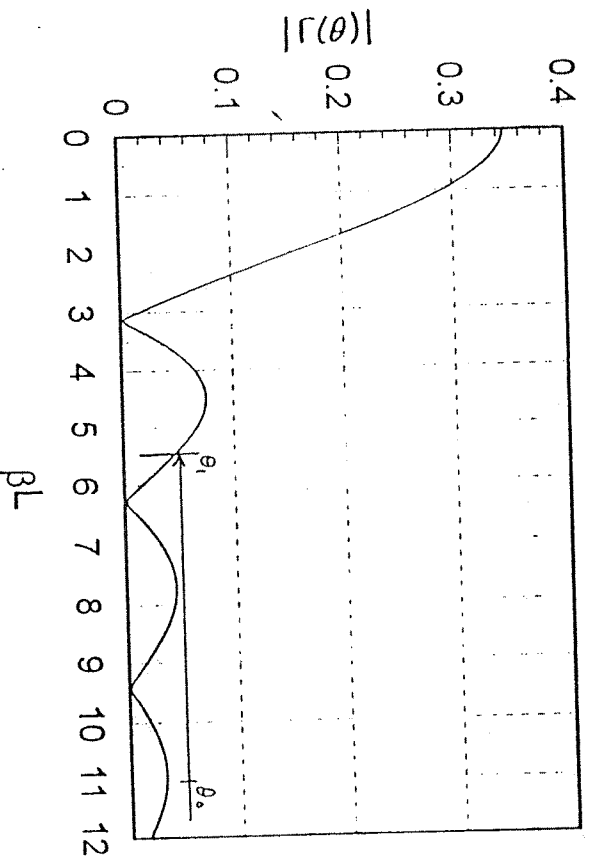
From (5.68), $Z(\beta) = Z_0 e^{-\alpha \beta}$ for $0 < \beta < L$.

$$\alpha = \frac{1}{L} \ln \frac{Z_L}{Z_0} = \frac{0.693}{L}$$

From (5.70),

$$|T(\theta)| = \frac{1}{2} \left| \ln \frac{Z_L}{Z_0} \right| \left| \frac{\sin \beta L}{\beta L} \right| = 0.346 \left| \frac{\sin \beta L}{\beta L} \right| \checkmark$$

This result is plotted in the graph shown below:



We see that the lower frequency limit for $|T| \leq 0.05$ is $\theta_1 = 5.5$. To obtain 100% bandwidth, we must have,

$$\frac{\theta_2 - \theta_1}{(\theta_1 + \theta_2)/2} = 1, \text{ or } \theta_2 = 3\theta_1 = 16.5$$

Then at the center frequency,

$$\theta_0 = \frac{\theta_1 + \theta_2}{2} = 11.0 = \beta L$$

So,

$$L = \frac{11 \lambda_0}{2\pi} = 1.75 \lambda_0 \checkmark$$

From (5.64), θ_m for a Chebyshev transformer with 100% bandwidth is,

$$\frac{\Delta F}{F_0} = 2 - \frac{4\theta_m}{\pi} = 1 \Rightarrow \theta_m = \pi/4.$$

Then from (5.63),

$$\text{All } \theta_m = \cosh \left[N \cosh^{-1} \left(\frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \right) \right]$$

$$1.414 = \cosh \left[N (2.5846) \right] \Rightarrow N = 2.93 \Rightarrow \underline{N=3}$$

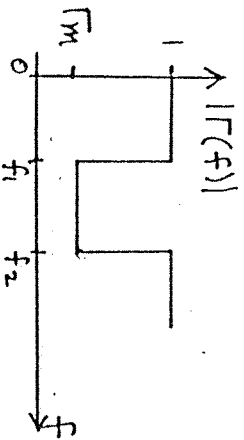
So $N=3$ sections would be required, for a length of $3\lambda/4$ at the center frequency. This is much shorter than the exponential taper matching section.

(5.25)

From Figure 5.22, the Bode-Fano limit for a parallel RC load is,

$$\int_0^{\infty} \ln \left| \frac{F(\omega)}{F(0)} \right| d\omega \leq \frac{\pi}{RC}$$

The optimum reflection coefficient magnitude response will be as shown:



So, as in (5.80),

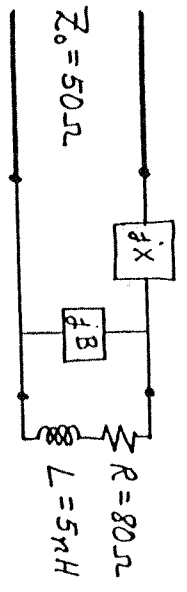
$$\ln \frac{1}{\Gamma_m} \leq \frac{\pi}{\Delta\omega RC} = \frac{\pi}{2\pi(10^{-2}) \times 10^9 (100)(1.5 \times 10^{-12})} = 0.417$$

or,

$$\Gamma_m \geq 0.659 \Rightarrow RL < 3.6 \text{ dB.}$$

5.26

L-match network solution:

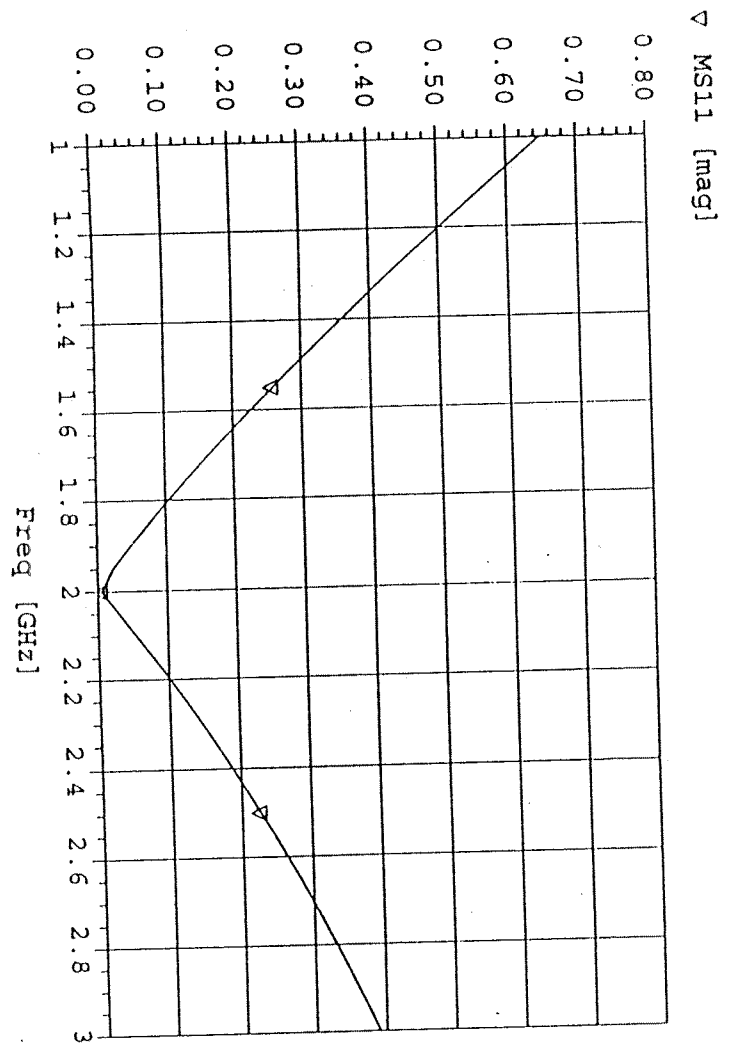


At $f = 2\text{GHz}$, $Z_L = 80 + j63\Omega$, $Z_L = 1.6 + j1.26$ (INSIDE $1 + jx$)
 a Smith chart solution gives,

$jB = -j1.8 \Rightarrow$ INDUCTOR with $L = 22.1nH$. ✓

$jx = -j1.25 \Rightarrow$ CAPACITOR with $C = 1.27pF$. ✓

The input reflection coefficient magnitude is plotted below, where it is seen that the bandwidth for $|r| < 0.1$ is 20%.



Bode - Fano limit:

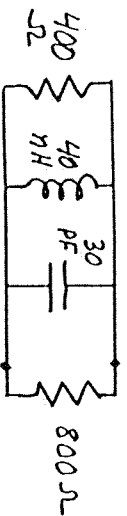
From Figure 5.22d, the Bode-Fano criteria gives a bandwidth limit of

$$\Delta\omega = \frac{\pi R}{L} \frac{1}{\ln \sqrt{M}} = 2.18 \times 10^{10} = \omega_2 - \omega_1$$

$$\frac{\Delta F}{f_0} = \frac{f_2 - f_1}{f_0} = \frac{2.18 \times 10^{10}}{2\pi (2 \times 10^9)} = 174\%$$

This is considerably more than the bandwidth of the L-section match.

6.1



From Table 6.1 for a parallel RLC circuit:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(40 \times 10^{-3})(30 \times 10^{-12})}} = 145 \text{ MHz}$$

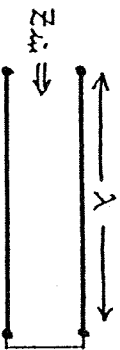
$$Q = \omega_0 RC = 2\pi f_0 RC = 2\pi (145 \times 10^6)(400)(30 \times 10^{-12}) = 10.9 \checkmark$$

$$Q_c = \frac{R_L}{\omega_0 L} = \frac{R_L}{R} Q = \frac{800}{400} (10.9) = 21.8 \checkmark$$

Then the loaded Q is, from (6.23),

$$Q_L = \frac{1}{\frac{1}{Q_c} + \frac{1}{Q}} = 7.27 \checkmark$$

6.2



$$l = \lambda = \frac{2\pi V\beta}{\omega_0} \text{ for } \omega = \omega_0$$

This circuit has a series-type resonance, like the short-circuited $\lambda/2$ resonator. Thus, let

$$\beta l = \frac{\omega_0 l}{V\beta} + \frac{\Delta\omega l}{V\beta} = 2\pi \left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then from (6.24) the input impedance is,

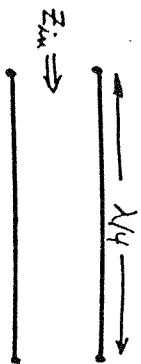
$$Z_{in} \approx Z_0 \frac{\alpha l + j2\pi \frac{\Delta\omega}{\omega_0}}{1 + j2\pi \frac{\Delta\omega}{\omega_0}} \approx Z_0 (\alpha l + j2\pi \frac{\Delta\omega}{\omega_0}) = R + jL \Delta\omega$$

$$\text{Thus } R = Z_0 \alpha l, \quad L = \frac{\pi Z_0}{\omega_0}$$

And,

$$Q = \frac{\omega_0 L}{R} = \frac{\pi Z_0}{Z_0 \alpha l} = \frac{\pi}{\alpha l} = \frac{\beta}{2\alpha} \quad (\text{since } l = \lambda = \frac{2\pi}{\beta} \text{ at } \omega_0.)$$

6.3

 $Z_{in} \Rightarrow$

$$l = \frac{\lambda}{4} = \frac{\pi v_p}{2\omega_0} \text{ for } \omega = \omega_0$$

This circuit has a series-type resonance, like the short-circuited $\lambda/2$ line. So let,

$$\beta l = \frac{\omega_0 l}{v_p} + \frac{\Delta\omega l}{v_p} = \frac{\pi}{2} \left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then, $\tan \beta l = \tan \frac{\pi}{2} \left(1 + \frac{\Delta\omega}{\omega_0}\right) = -\cot \frac{\Delta\omega \pi}{2\omega_0} \approx \frac{-2\omega_0}{\pi \Delta\omega}$

The input impedance is,

$$Z_{in} = Z_0 \frac{1 + j \tan \beta l \tanh \alpha l}{\tanh \alpha l + j \tan \beta l} \approx \frac{1 - j \frac{2\omega_0}{\pi \Delta\omega}}{\alpha l - j \frac{2\omega_0}{\pi \Delta\omega}} \alpha l$$

$$\approx Z_0 \frac{\alpha l + j \frac{\pi \Delta\omega}{2\omega_0}}{1 + j \frac{\pi \Delta\omega}{2\omega_0}} \alpha l \approx Z_0 \left(\alpha l + j \frac{\pi \Delta\omega}{2\omega_0} \right) = R + jL \Delta\omega$$

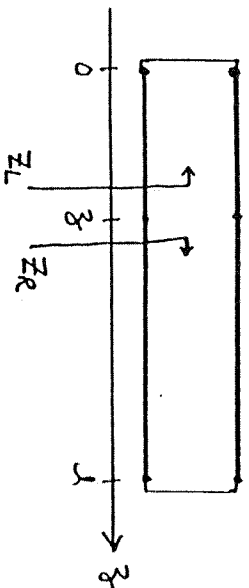
$$\therefore R = Z_0 \alpha l \quad \checkmark, \quad L = \frac{\pi Z_0}{4\omega_0} \quad \checkmark$$

Then,

$$Q = \frac{\omega_0 L}{R} = \frac{\pi}{4\alpha l} = \frac{\beta}{2\alpha} \quad \checkmark$$

$$\text{(since } l = \frac{\lambda}{4} = \frac{\pi}{2\beta} \text{ at resonance)}$$

6.4

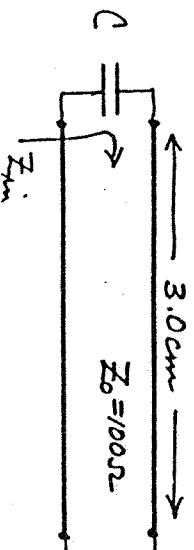


$$\beta l = \pi$$

$$Z_L = j Z_0 \tan \beta l$$

$$Z_R = j Z_0 \tan \beta(l - \beta l) = j Z_0 \tan(\pi - \beta l) = -j Z_0 \tan \beta l = Z_L^* \quad \checkmark$$

6.5



$$f_0 = 6 \text{ GHz} \quad \checkmark$$

$$\beta = \frac{2\pi f}{c} = 125.7 \text{ m}^{-1} \text{ for an air-filled line}$$

$$\beta l = (125.7)(0.03) = 216^\circ \quad \checkmark$$

$$Z_{in} = j Z_0 \tan \beta l = j(100) \tan 216^\circ = j 72.6 \Omega = j \omega L \quad \checkmark$$

To achieve resonance we must have,

$$Z_{in} = (jX_C)^* = \frac{Z}{\omega C}$$

$$\text{So, } C = \frac{1}{\omega X_{in}} = 0.365 \text{ pF} \quad \checkmark$$

The equivalent circuit at 6 GHz, with the shunt resistor, is as follows:



$$R = 10,000 \Omega$$

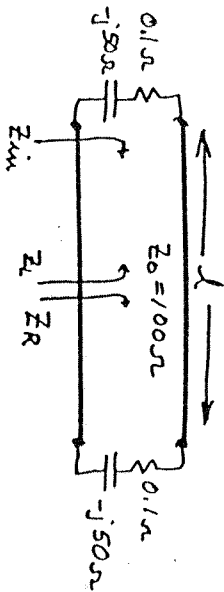
$$C = 0.365 \text{ pF}$$

$$L = \frac{X_{in}}{\omega} = \frac{72.6}{2\pi(6 \times 10^9)} = 1.93 \text{ nH} \quad \checkmark$$

So the Q is,

$$Q = \omega RC = 2\pi(6 \times 10^9)(10,000)(0.365 \times 10^{-12}) = 138 \quad \checkmark$$

6.6



Since the network is symmetrical, at the midpoint of the line we must have, $Z_L = Z_R^*$, or $\tan \beta z = 0$:

Let $z = \tan \beta l/2$ and $Z_L = R_L + jX_L$. ($R_L = 0.1, X_L = -50$)

$$Z_R = Z_0 \frac{Z_L + jZ_0 z}{Z_0 + jZ_L z} = Z_0 \frac{R_L + j(X_L + Z_0 z)}{(Z_0 - X_L z) + jR_L z}$$

$$= Z_0 \frac{R_L(Z_0 - X_L z) + R_L z(X_L + Z_0 z) + j(X_L + Z_0 z)(Z_0 - X_L z) - jR_L^2 z}{(Z_0 - X_L z)^2 + (R_L z)^2}$$

$$\text{Im} \{Z_R\} = 0 \Rightarrow$$

$$(X_L + Z_0 z)(Z_0 - X_L z) - R_L^2 z = 0$$

$$-X_L Z_0 z^2 + (Z_0^2 - X_L^2 - R_L^2)z + Z_0 X_L = 0$$

$$5000 z^2 + 7500 z - 5000 = 0$$

$$z^2 + 1.5z - 1 = 0$$

$$z = \frac{-1.5 \pm \sqrt{(1.5)^2 + 4}}{2} = -0.75 \pm 1.25 = \begin{cases} 0.50 \Rightarrow \beta l = 53.1^\circ \\ -2.00 \Rightarrow \beta l = -126.9^\circ = 53.1^\circ \end{cases}$$

So,

$$l = \frac{53.1^\circ}{360^\circ} \lambda = 0.148 \lambda \quad \tan \beta l = 1.332$$

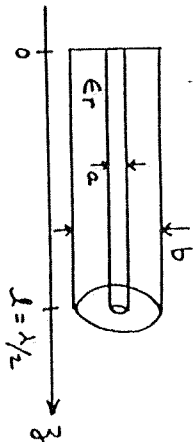
CHECK:

$$Z_{in} = 100 \frac{(0.1 - j50) + j133.2}{100 + j(1j50)(1.332)} = 0.1 + j50 \Omega \quad \checkmark$$



$$Q = \frac{W_L}{R} = \frac{X_L}{R} = \frac{50}{0.2} = 250.$$

6.7



$$\beta l = \pi$$

$$\beta = \pi/l$$

$$\eta = \sqrt{\mu_0/\epsilon}$$

From Section 2.2 the TEM fields of a coaxial line are,

$$\mathbf{E}^\pm = \hat{\rho} \frac{V_0}{\rho \ln b/a} e^{\mp j\beta z} \quad , \quad \mathbf{H}^\pm = \pm \hat{\phi} \frac{V_0}{\eta \ln b/a} e^{\mp j\beta z}$$

$E_\rho = 0$ at $z=0$ in the resonator, as the standing wave fields can be written as,

$$E_\rho = \frac{V_0}{\rho \ln b/a} [e^{-j\beta z} - e^{j\beta z}] = \frac{-2jV_0}{\rho \ln b/a} \sin \beta z$$

$$H_\phi = \frac{V_0}{\eta \ln b/a} [e^{-j\beta z} + e^{j\beta z}] = \frac{2V_0}{\eta \ln b/a} \cos \beta z$$

From (1.84) and (1.86) the time-average stored electric and magnetic energies are,

$$W_e = \frac{\epsilon}{4} \int_V |\mathbf{E}|^2 dV = \frac{\epsilon}{4} \int_{\rho=a}^b \int_{\phi=0}^{2\pi} \int_{z=0}^l \left(\frac{2V_0}{\rho \ln b/a} \right)^2 \sin^2 \frac{\pi z}{l} \rho d\phi dz d\rho$$

$$= \frac{\pi \epsilon V_0^2}{\ln b/a}$$

$$W_m = \frac{\mu_0}{4} \int_V |\mathbf{H}|^2 dV = \frac{\mu_0}{4} \int_{\rho=a}^b \int_{\phi=0}^{2\pi} \int_{z=0}^l \left(\frac{2V_0}{\eta \ln b/a} \right)^2 \cos^2 \frac{\pi z}{l} \rho d\phi dz d\rho$$

$$= \frac{\pi \mu_0 V_0^2}{\eta^2 \ln b/a} = \frac{\pi \epsilon V_0^2}{\ln b/a} = W_e \quad \checkmark$$

$$6.8 \quad Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R + j(\omega L - \frac{1}{\omega C})} = \frac{1}{\frac{R}{Z_0^2} + j\omega(\frac{L}{Z_0^2} - \frac{1}{\omega^2 C Z_0^2})}$$

The input impedance of a parallel RLC circuit is,

$$Z_{in} = \frac{1}{\frac{1}{R'} + \frac{1}{j\omega L} + j\omega C'} = \frac{1}{R' + j\omega(C' - \frac{1}{\omega^2 L'})}$$

Thus the original circuit acts as a parallel RLC resonator with $R' = Z_0^2/R$, $C' = L/Z_0^2$, $L' = CZ_0^2$.

(This is the basis for using $\lambda/4$ lines as impedance and admittance inverters.)

6.9

From (6.40),

$$F_{101} = \frac{a}{2\pi} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = \frac{3 \times 10^8}{2\pi} \sqrt{\left(\frac{\pi}{.05}\right)^2 + \left(\frac{\pi}{.05}\right)^2} = 4.24 \text{ GHz} \quad (TE_{101}) \checkmark$$

$$F_{102} = \frac{c}{2\pi} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{2\pi}{d}\right)^2} = \frac{3 \times 10^8}{2\pi} \sqrt{\left(\frac{\pi}{.05}\right)^2 + \left(\frac{2\pi}{.05}\right)^2} = 6.71 \text{ GHz} \quad (TE_{102}) \checkmark$$

at 4.24 GHz, $R_S = \sqrt{\frac{\omega \mu_0}{2\sigma}} = 0.0165 \Omega$; $k = 88.8 \text{ m}^{-1}$ ✓

at 6.71 GHz, $R_S = \sqrt{\frac{\omega \mu_0}{2\sigma}} = 0.0207 \Omega$; $k = 140.5 \text{ m}^{-1}$ ✓

From (6.46), with $a=b=d$,

$$(2a^2a^3b + 2bd^3 + a^2a^3d + ad^3) = 3a^4(1+a^2)$$

So,

$$Q_{101} = \frac{k^3 a^7 \eta_0}{2\pi^2 R_S} \frac{1}{3a^4(1+1)} = \frac{k^3 a^3 \eta_0}{12\pi^2 R_S} = 16,886. \quad (TE_{101}) \checkmark$$

$$Q_{102} = \frac{k^3 a^7 \eta_0}{2\pi^2 R_S} \frac{1}{3a^4(1+4)} = \frac{k^3 a^3 \eta_0}{30\pi^2 R_S} = 21,325. \quad (TE_{102}) \checkmark$$

6.10

From Table 3.2, the magnetic fields of the TM_{11} waveguide mode are,

$$H_x^{\pm} = \frac{B^{\pm}}{b} \sin \frac{\pi x}{a} \cos \frac{\pi y}{b} e^{\mp j\beta z}$$

$$H_y^{\pm} = \frac{B^{\pm}}{a} \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} e^{\mp j\beta z}$$

To have current maxima at $z=0$, the cavity fields must be,

$$H_x = \frac{A}{b} \sin \frac{\pi x}{a} \cos \frac{\pi y}{b} \cos \frac{\pi z}{d}$$

$$H_y = \frac{A}{a} \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} \cos \frac{\pi z}{d}$$

The stored magnetic energy is,

$$W_m = \frac{\mu_0}{4} \int_V |H|^2 dV = \frac{\mu_0}{4} A^2 \frac{a}{2} \frac{b}{2} \frac{d}{2} \left(\frac{1}{b^2} + \frac{1}{a^2} \right) = \frac{ab d \mu_0 A^2}{32} \left(\frac{1}{a^2} + \frac{1}{b^2} \right)$$

The power lost in the walls is,

$$P_R = \frac{R_s}{2} \int_S |H_t|^2 dS = R_s \left\{ \int_{x=0}^a \int_{z=0}^d |H_x(y=0)|^2 dz dx + \int_{y=0}^b \int_{z=0}^d |H_y(x=0)|^2 dy dz + \int_{x=0}^a \int_{y=0}^b [|H_x(z=0)|^2 + |H_y(z=0)|^2] dx dy \right\}$$

$$= \frac{A^2 R_s}{4} \frac{a^3 d + b^3 d + a^3 b + a b^3}{a^2 b^2}$$

Then,

$$Q = \frac{\omega_0 (W_e + W_m)}{P_R} = \frac{2 \omega_0 W_m}{P_R} = \frac{k_0 \mu_0}{4 R_s} \frac{a b d (a^2 + b^2)}{(a^3 d + b^3 d + a^3 b + a b^3)} \quad \checkmark$$

6.11

From Section 3.3 the transverse fields of the TE_{10} mode in the two regions can be written as,

$$E_y = \begin{cases} A \sin \frac{\pi x}{a} \sin \beta_a z & \text{for } 0 < z < d-t \\ B \sin \frac{\pi x}{a} \sin \beta_d (d-z) & \text{for } d-t < z < d \end{cases}$$

$$H_x = \begin{cases} -j \frac{A}{Z_a} \sin \frac{\pi x}{a} \cos \beta_a z & \text{for } 0 < z < d-t \\ -j \frac{B}{Z_d} \sin \frac{\pi x}{a} \cos \beta_d (d-z) & \text{for } d-t < z < d \end{cases}$$

where $\beta_a = \sqrt{k_0^2 - (\pi/a)^2}$, $\beta_d = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2}$

$$Z_a = \eta_0 / \beta_a \quad , \quad Z_d = \eta / \beta_d = \eta_0 / \beta_d$$

Continuity of E_y, H_x at $z = d-t$:

E_y: $A \sin \beta_a (d-t) = B \sin \beta_d t$

H_x: $\frac{A}{Z_a} \cos \beta_a (d-t) = \frac{B}{Z_d} \cos \beta_d t$

Divide to obtain:

$$Z_a \tan \beta_a (d-t) = Z_d \tan \beta_d t$$

$$\beta_d \tan \beta_a (d-t) = \beta_a \tan \beta_d t$$

This equation can be solved for k_0 . β_a and β_d are functions of k_0 as given above.

6.12

TM modes: $(\nabla^2 + k^2)E_z = 0$

Let $E_z(x, y, z) = X(x)Y(y)Z(z)$.

Substitute into wave equation and divide by XYZ :

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \frac{1}{Y} \frac{d^2 Y}{dy^2} + \frac{1}{Z} \frac{d^2 Z}{dz^2} + k^2 = 0$$

By the separation of variables argument,

$$\frac{1}{X} \frac{d^2 X}{dx^2} = -k_x^2 \quad \Rightarrow X(x) = A \cos k_x x + B \sin k_x x$$

$$\frac{1}{Y} \frac{d^2 Y}{dy^2} = -k_y^2 \quad \Rightarrow Y(y) = C \cos k_y y + D \sin k_y y$$

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} = -k_z^2 \quad \Rightarrow Z(z) = E \cos k_z z + F \sin k_z z$$

with $k^2 = k_x^2 + k_y^2 + k_z^2$.

Now, $E_z = 0$ for $x=0, a$ and $y=0, b$. Therefore,

$A = C = 0$ and $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$. To enforce the

remaining boundary conditions, we need E_x or E_y :

From Maxwell's equations,

$$E_x = \frac{1}{k^2 - k_z^2} \frac{\partial^2 E_z}{\partial x \partial z} = \frac{1}{k^2 - k_z^2} (B k_x \cos k_x x)(D \sin k_y y) \cdot (-k_z E \sin k_z z + k_z F \cos k_z z)$$

For $E_x = 0$ at $z=0, d$ we must have $F=0$, and $k_z = \frac{p\pi}{d}$.

Thus, $k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$,

which determines the resonant frequencies. The solution for TE modes is similar.

6.13

From Table 3.5 the fields of the TM_{nm} mode are ($\beta=0$):

$$E_z = A \sin n\phi J_n(k_c \rho)$$

$$H_\phi = \frac{j\omega\epsilon n}{k_c^2 \rho} A \cos n\phi J_n(k_c \rho)$$

$$H_\rho = -\frac{j\omega\epsilon}{k_c} A \sin n\phi J_n'(k_c \rho), \quad k_c = \gamma_{nm}/a = k$$

The stored electric energy is,

$$\begin{aligned} W_e &= \frac{\epsilon}{4} \int_V |\mathbf{E}|^2 dV = \frac{A^2 \epsilon}{4} \int_0^a \int_0^{2\pi} \int_0^{2\pi} \sin^2 n\phi J_n^2(k_c \rho) \rho d\rho d\phi dz \\ &= \frac{A^2 \epsilon}{4} \pi a \frac{a^2}{2} J_n'^2(\gamma_{nm}) = \frac{A^2 a^2 \pi d \epsilon}{8} J_n'^2(\gamma_{nm}) \quad (\text{using C.14}) \end{aligned}$$

The power loss due to finite conductivity is,

$$\begin{aligned} P_d &= \frac{R_s}{2} \int_S |\mathbf{H}_\phi|^2 dS \\ &= \frac{R_s}{2} \left\{ \int_{\phi=0}^{2\pi} \int_{z=0}^d |H_\phi(\rho=a)|^2 a d\phi dz + 2 \int_{\phi=0}^{2\pi} \int_{z=0}^a [|H_\rho|^2 + |H_\phi|^2] \rho d\rho dz \right\} \\ &= \frac{A^2 R_s}{2} \left\{ \frac{\pi a d}{\eta^2} J_n'^2(\gamma_{nm}) + \frac{2\pi}{\eta^2} \frac{\gamma_{nm}^2}{2k_c^2} J_n'^2(\gamma_{nm}) \right\} \\ &= \frac{A^2 R_s \pi}{2 \eta^2} (a d + a^2) J_n'^2(\gamma_{nm}) \end{aligned}$$

$$\text{Then, } Q_c = \frac{2\omega W_e}{P_d} = \frac{\omega d \pi d \epsilon (2\eta^2)}{4 R_s \pi a (d+a)} = \frac{a d k \eta}{2 R_s (d+a)} \quad \checkmark$$

The power lost in the dielectric is,

$$P_d = \frac{\omega \epsilon''}{2} \int_V |\mathbf{E}|^2 dV = \frac{\omega \epsilon''}{2} \tan \delta \int_V |\mathbf{E}|^2 dV = \frac{2k W_e}{\eta \epsilon''} \tan \delta$$

$$\text{So, } Q_d = \frac{2\omega W_e}{P_d} = \frac{1}{\tan \delta} \quad \checkmark \quad (\text{as in (6.48)})$$

6.14 From Figure 6.10, maximum Q for the TE_{111} mode occurs for $2a/d \approx 1.7$. From (6.53a) the resonant frequency of the TE_{111} mode is,

$$f_{111} = \frac{c}{2\pi} \sqrt{\left(\frac{\pi_1'}{a}\right)^2 + \left(\frac{\pi}{a}\right)^2} = \frac{c}{2\pi} \sqrt{\left(\frac{1.841}{a}\right)^2 + \left(\frac{1.7\pi}{2a}\right)^2} = \frac{1.55 \times 10^8}{a} \\ = 7 \times 10^9.$$

Thus, $a = 2.21 \text{ cm}$ ✓

$$d = \frac{2a}{1.7} = 2.60 \text{ cm} \checkmark$$

$\sqrt{\epsilon_0} = 6.17 \times 10^7 \text{ S/m}$; $k = 146.6 \text{ m}^{-1}$; $ka = 3.24$

$$R_S = \sqrt{\frac{\omega \mu_0}{2\sigma}} = 0.0215 \Omega \checkmark$$

$$\beta = \frac{\pi}{a} = 120.8 \text{ m}^{-1} \quad \rho_{11}' = 1.841$$

From (6.57) the Q is

$$Q_c = \frac{(ka)^3 \eta a d \left[1 - \left(\frac{1}{\rho_{11}'}\right)^2\right]}{4 \left(\rho_{11}'\right)^2 R_S \left\{ \frac{a d}{2} \left[1 + \left(\frac{R_S a}{\rho_{11}'}\right)^2\right] + \left(\frac{R_S a}{\rho_{11}'}\right)^2 \left(1 - \frac{1}{\rho_{11}'^2}\right) \right\}} \\ = 15,336.$$

6.15

Choose coordinate system so that $b < a < d$.
Then the dominant resonant mode is the TE_{101} mode:

$$f_{101} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{d}\right)^2} = 5.2 \text{ GHz}$$

$$\text{or, } \frac{1}{a^2} + \frac{1}{d^2} = \left(\frac{2f_{101}}{c}\right)^2 = (34.7)^2$$

The next two higher modes must be either the TM_{110} ,
 TE_{102} , or TE_{011} modes:

$$\left(\frac{2f_{110}}{c}\right)^2 = \frac{1}{a^2} + \frac{1}{b^2} = (34.7)^2 + \frac{1}{d^2}$$

$$\left(\frac{2f_{102}}{c}\right)^2 = \frac{1}{a^2} + \frac{4}{d^2} = (34.7)^2 + \frac{3}{d^2}$$

$$\left(\frac{2f_{011}}{c}\right)^2 = \frac{1}{b^2} + \frac{1}{d^2}$$

Since $d > a$, $f_{011} < f_{110}$

Try $f_{011} = 6.5 \text{ GHz}$; $f_{110} = 7.2 \text{ GHz}$

Then we have, $\frac{1}{b^2} - \frac{1}{d^2} = 1100$.

$$\frac{1}{b^2} + \frac{1}{d^2} = 1878.$$

Solving gives,

$$b = 2.60 \text{ cm } \checkmark$$

$$d = 5.00 \text{ cm } \checkmark$$

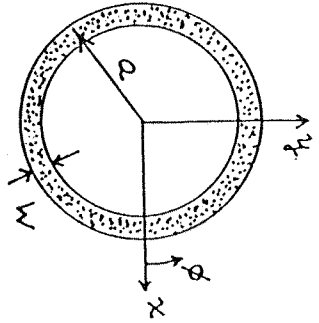
$$a = 3.53 \text{ cm } \checkmark$$

CHECK:

$$b < a < d \quad \text{OK } \checkmark$$

$$f_{102} = 7.35 \text{ GHz} > f_{110} = 7.2 \text{ GHz } \checkmark$$

6.16



$$e^{\pm j\beta a\phi} = e^{\pm jn\phi}, \quad n=1, 2, 3, \dots$$

FOR PERIODICITY

So, $\beta a = \frac{2\pi a}{\lambda_g} = \frac{2\pi a \sqrt{\epsilon_r} f}{c} = n$

$$f = \frac{nc}{2\pi a \sqrt{\epsilon_r}}; \quad n=1, 2, 3, \dots$$

(The ring circumference is $2\pi a = n\lambda_g$)

The above result assumes $a \gg w$, so that curvature effects can be neglected. This type of resonator is most often coupled using a gap feed to a microstrip line.

6.17

For TM_{nm0} modes we have $H_z = 0$ and $\frac{\partial z}{\partial z} = 0$. The wave equation for E_z is,

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \rho^2 \frac{\partial^2}{\partial \phi^2} + k^2 \right) E_z = 0$$

(from 4.134)
($\beta = k$)

The general solution is,
 $E_z = (A_n \cos n\phi + B_n \sin n\phi) J_n(k\rho)$ (finite at $\rho=0$)

Since the choice of $\sin n\phi$ or $\cos n\phi$ (or any combination) depends only on the choice of the $\phi=0$ reference, we can let $B_n = 0$.

Then,

$$E_z = A_n \cos n\phi J_n(k\rho)$$

We can find H_ϕ from (4.110d):

$$H_\phi = \frac{-j\omega\epsilon}{k^2} \frac{\partial E_z}{\partial \rho} = \frac{-j\omega\epsilon}{k} A_n \cos n\phi J_n'(k\rho)$$

For $H_\phi = 0$ at $\rho = a$ we require $J_n'(ka) = 0$, or $ka = p'_{nm}$.

So the resonant frequency is,

$$f_{nm0} = \frac{ck}{2\pi \sqrt{\epsilon_r}} = \frac{c p'_{nm}}{2\pi a \sqrt{\epsilon_r}}$$

and,

$$f_{110} = \frac{c p'_{11}}{2\pi a \sqrt{\epsilon_r}} = \frac{1.8412c}{2\pi a \sqrt{\epsilon_r}} \quad \checkmark$$

This solution neglects the effect of fringing fields.

6.18

From (6.70), $\tan \beta/2 = \alpha/\beta$,

with

$$\alpha = \sqrt{\left(\frac{2.405}{a}\right)^2 - k_0^2}$$

$$\beta = \sqrt{\epsilon_r k_0^2 - (2.405/a)^2}$$

The value of k_0 at resonance must lie between

$$k_0 = \frac{2.405}{a} = 602, \text{ and } k_0 = \frac{2.405}{a\sqrt{\epsilon_r}} = 100.$$

We carry out a trial-and-error numerical search as follows:

k_0	α	β	$\tan \beta/2 = \alpha/\beta$
110	592	275	-1.8
120	590	399	-1.02
150	583	672	.008
145	584	631	-.12
→ 149	583	664	-.0018

Thus, the resonant frequency is,

$$f_0 = \frac{ck_0}{2\pi} = 7.11 \text{ GHz} \checkmark$$

(measured value is 7.8 GHz)

6.19

Following the analysis of Section 6.5, for TE₀₁s mode:

$$H_z = H_0 J_0(k_c \rho) e^{\pm j\beta z}$$

$$E_\phi = \frac{j\omega\mu_0 H_0}{k_c} J_0'(k_c \rho) e^{\pm j\beta z} = A J_0'(k_c \rho) e^{\pm j\beta z}$$

$$H_\rho = \frac{\mp j\beta H_0}{k_c} J_0'(k_c \rho) e^{\pm j\beta z} = \frac{\mp A}{Z_T E} J_0'(k_c \rho) e^{\pm j\beta z}$$

$$\text{for } |z| < L/2, \beta = \sqrt{\epsilon_r k_0^2 - k_c^2} = \sqrt{(\epsilon_r / a)^2 - k_0^2} \quad ; \quad Z_{TE} = \frac{\omega\mu_0}{\beta} = Z_d$$

$$\text{for } |z| > L/2, j\beta = \alpha = \sqrt{k_0^2 - k_c^2} = \sqrt{(a_0/a)^2 - k_0^2} \quad ; \quad Z_{TE} = \frac{j\omega\mu_0}{\alpha} = Z_a$$

So the standing wave fields can be written as,

$$E_\phi = \begin{cases} A J_0'(k_c \rho) [e^{j\beta z} - e^{-j\beta z}] = -2jA J_0'(k_c \rho) \sin\beta z & \text{for } |z| < L/2 \\ B J_0'(k_c \rho) e^{-\alpha z} & \text{for } z > L/2 \end{cases}$$

$$H_\rho = \begin{cases} \frac{A}{Z_d} J_0'(k_c \rho) [e^{j\beta z} + e^{-j\beta z}] = \frac{2A}{Z_d} J_0'(k_c \rho) \cos\beta z & \text{for } |z| < L/2 \\ \frac{B}{Z_a} J_0'(k_c \rho) e^{-\alpha z} & \text{for } z > L/2 \end{cases}$$

Continuity of E_ϕ and H_ρ at $z = L/2$ gives:

$$E_\phi: -2jA \sin\beta L/2 = B e^{-\alpha L/2}$$

$$H_\rho: \frac{2A}{Z_d} \cos\beta L/2 = \frac{B}{Z_a} e^{-\alpha L/2}$$

dividing gives:

$$-jZ_d \tan\beta L/2 = Z_a$$

$$\frac{-j}{\beta} \tan\beta L/2 = j/a$$

$$\tan\beta L/2 + \beta/a = 0 \quad \checkmark$$

6.20

assume $a > b$

Because of the magnetic wall boundary conditions on the sidewalls, a rectangular dielectric waveguide along the z -axis would support TE modes with an H_z field of the form,

$$H_z = H_0 \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b},$$

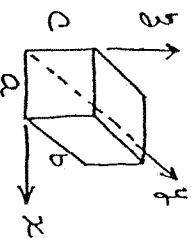
so the lowest order TE mode would be the TE_{11} mode. But $H_z \equiv 0$ for TM modes, so the lowest order TM mode would have,

$$H_x = H_0 \sin \frac{\pi x}{a}, \quad H_y = 0 \quad (\text{if } a > b)$$

So the dominant mode of this resonator must be the TM_{10S} mode. Thus we can write,

$$E_y = E_0 \sin \frac{\pi x}{a} e^{\pm j\beta z}$$

$$H_x = \pm \frac{E_0}{Z_{TM}} \sin \frac{\pi x}{a} e^{\pm j\beta z},$$



where, $\beta = \sqrt{(\epsilon_r k_0^2 - (\pi/a)^2)}$ for $|z| < c/2$

$$j\beta = \alpha = \sqrt{(\pi/a)^2 - \epsilon_r k_0^2} \quad \text{for } |z| > c/2,$$

and $Z_{TM} = Z_L = \beta \eta_0 / k = \beta \eta_0 / \epsilon_r k_0$ for $|z| < c/2$,

$$Z_{TM} = Z_a = j \alpha \eta_0 / k_0 \quad \text{for } |z| > c/2$$

Then the standing wave fields can be written as,

$$E_y = \begin{cases} A \sin \frac{\pi x}{a} [e^{j\beta z} + e^{-j\beta z}] = 2A \sin \frac{\pi x}{a} \cos \beta z & \text{for } |z| < c/2 \\ B \sin \frac{\pi x}{a} e^{-\alpha z} & \text{for } |z| > c/2 \end{cases}$$

$$H_x = \begin{cases} \frac{A}{Z_L} \sin \frac{\pi x}{a} [-e^{j\beta z} + e^{-j\beta z}] = \frac{2jA}{Z_L} \sin \frac{\pi x}{a} \sin \beta z & \text{for } |z| < c/2 \\ -\frac{B}{Z_a} \sin \frac{\pi x}{a} e^{-\alpha z} & \text{for } |z| > c/2 \end{cases}$$

Continuity of E_y, H_x at $z = c/2$:

$$2A \cos \beta c/2 = B e^{-\alpha c/2}$$

$$\frac{2jA}{Z_L} \sin \beta c/2 = -\frac{B}{Z_a} e^{-\alpha c/2}$$

divide to get:

$$\alpha \epsilon_r \tan \beta c/2 + \beta = 0$$

(6.21)

From (6.72) we have that $f_0 = \frac{c}{2d}$, or

$$f = \frac{2df_0}{c} = \frac{2(1.04)(94 \times 10^9)}{3 \times 10^8} = 25.$$

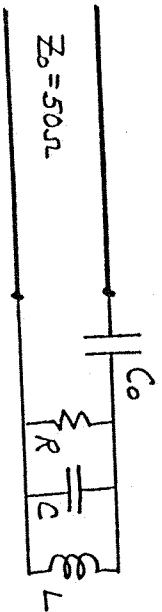
The surface resistivity is,

$$R_s = \sqrt{\frac{\omega \mu}{2\sigma}} = \sqrt{\frac{(2\pi)(94 \times 10^9)(4\pi \times 10^{-7})}{2(5.8 \times 10^7)}} = 0.08 \Omega$$

From (6.75) the Q is (due to conductor loss)

$$Q_c = \frac{\pi l Y_0}{4 R_s} = \frac{\pi(25)(377)}{4(0.08)} = \underline{92,500}.$$

(6.22)



$$R = 1000 \Omega$$

$$L = 1.26 \mu\text{H}$$

$$C = 0.804 \text{ pF}$$

The simplest way to solve this problem is graphically, with a Smith chart. The admittance of the resonator at frequencies near resonance is,

$$Y_R = \frac{1}{R} + j \frac{2Q\Delta\omega}{R\omega_0},$$

where $\omega_0 = \frac{1}{\sqrt{LC}} = 3.142 \times 10^8 \text{ RPS}$; $f_0 = \frac{\omega_0}{2\pi} = 5.00 \text{ GHz}$

$$Q = \frac{R}{\omega_0 L} = 25.3$$

Normalized to Z_0 , we have $Y_R = Z_0 Y_c = 0.05 + j2.53 \frac{\Delta\omega}{\omega_0}$. We can plot Y_R on a Smith chart, versus $\Delta\omega/\omega_0$. For $\Delta\omega = 0$, $Y_R = 0.05$. For $\Delta\omega = \pm 0.1\omega_0$, $Y_R = 0.05 \pm j0.253$.

Next, convert this locus to Z_R , an impedance locus. Then we see that a series capacitive reactance of $-jX_{C_0} = j4.2$ will yield an input impedance $Z_{in} = 1$. This corresponds to a resonator admittance

$Y_R = 0.05 - j0.22$. So the resonant frequency will be,

$$\Delta\omega = \frac{-0.22\omega_0}{2.53} = -0.0869\omega_0$$

$$\omega_r = \omega_0 + \Delta\omega = (1 - 0.0869)\omega_0 = 0.913\omega_0$$

at, $f_n = \frac{\omega T}{2\pi} = 4.566 \text{ GHz}$ (note lowering from f_0)

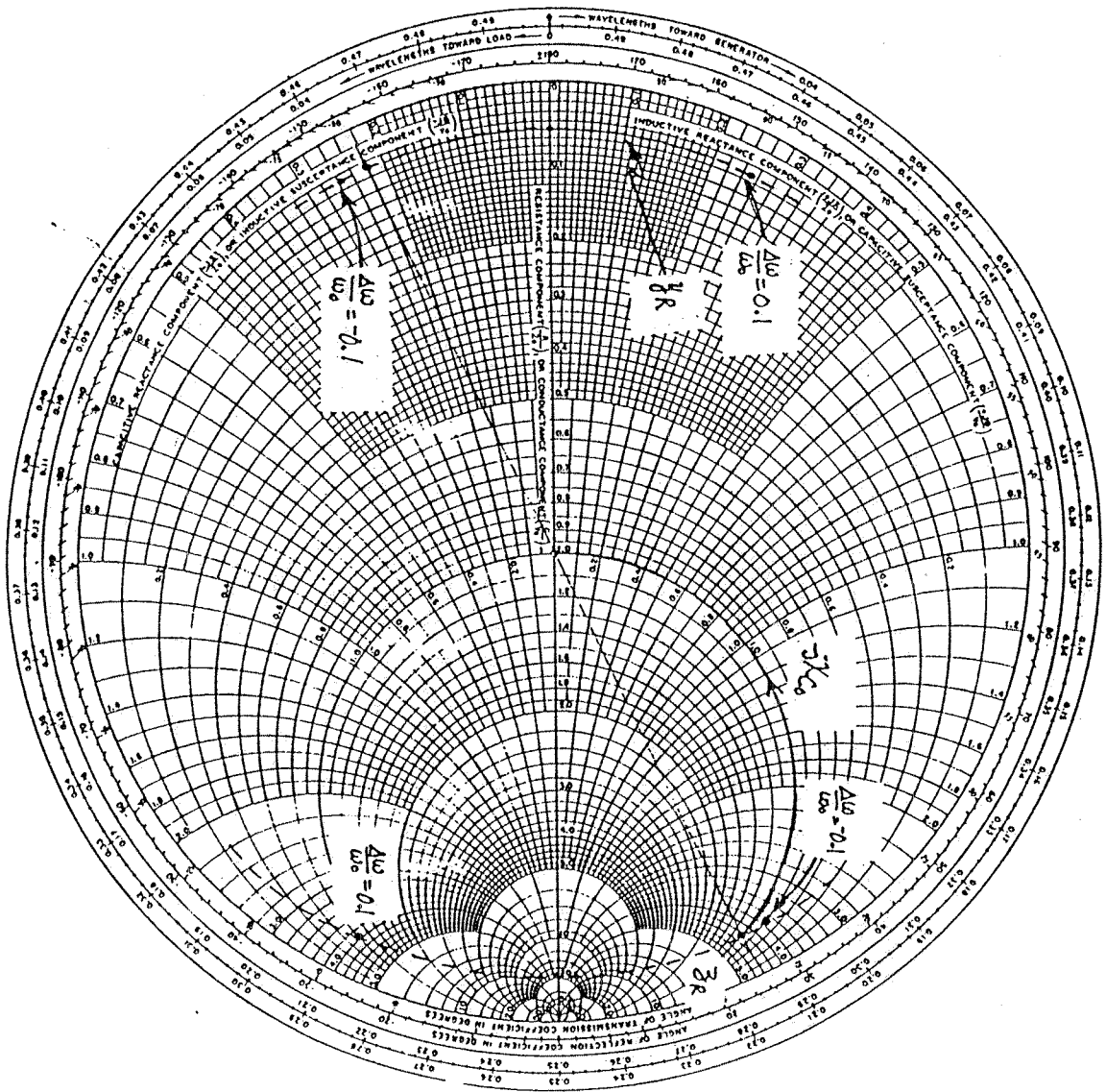
The coupling capacitor value is,

$$C_0 = \frac{1}{4.2Z_0\omega_n} = 0.166 \text{ pF.}$$

CHECK: at 4.566 GHz, $Y_R = (1-j4.39) \times 10^{-3} \text{ S}$

$$Z_R = 49.2 + j216.5 \Omega \approx 50 + jX_{C_0}$$

$$\frac{1}{j\omega C_0} = -j210.$$



6.23

Assume TE₁₀₁ mode, as in Section 6.7.

At 9 GHz, $k_0 = 188. \text{ m}^{-1}$; $\beta_0 = 140.5 \text{ m}^{-1}$; $L = \frac{\lambda_g}{2} = \frac{\pi}{\beta_0} = 2.24 \text{ cm}$.

$\frac{\omega}{2\pi} = f_0 = 9 \text{ GHz}$ is the resonant frequency of the closed cavity, and does not include the effect of the coupling aperture. For a high-Q cavity, the actual resonant frequency, ω_1 , will be close to ω_0 . So we can approximately compute χ_L using ω_0 . From (6.96),

$$\chi_L = \sqrt{\frac{\pi k_0 \omega_1}{20 \beta^2 c}} = 0.016 = \frac{\omega_L}{\omega_0} \Rightarrow \frac{L}{\lambda_0} = 2.83 \times 10^{-13}$$

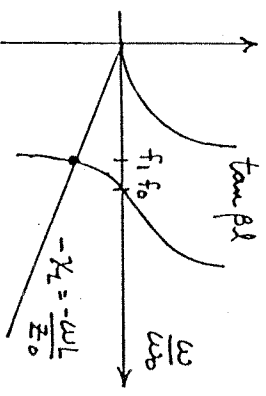
Then solve (6.92) for ω :

$$\tan \beta L + \chi_L = 0$$

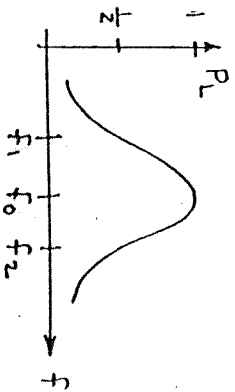
Numerical trial-and-error:

f	β	χ_L	$\tan \beta L + \chi_L$
9	140.	.0160	.01
8.9	137.7	.0158	-.04
8.97	139.65	.0159	.0025

Thus, $f_1 = 8.97 \text{ GHz}$



6.24



Assuming $Q \gg 1$, $f_0 = \frac{f_1 + f_2}{2} = 8.2325 \text{ GHz}$

$$BW = \frac{f_2 - f_1}{f_0} = 0.39\%$$

$$Q_L = \frac{1}{BW} = 329 \gg 1 \quad (\text{loaded } Q)$$

At resonance,

$$\Gamma = \frac{Z_L - 1}{Z_L + 1} = \frac{r - 1}{r + 1} = 0.33 \Rightarrow r = \frac{1 + \Gamma}{1 - \Gamma} = 1.985$$

From (6.83),

$$Q = g Q_e = \frac{1}{Q_L} \frac{g}{1 - \Gamma}$$

Solve for Q :

$$\frac{Q}{Q_L} - 1 = g$$

$$Q = Q_L (g + 1)$$

If we have a series resonator, $g = \frac{Z_0}{R} = \frac{1}{r} = 0.504$ (unloaded)

$$Q = (1 + g) Q_L = 495.$$

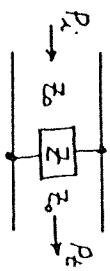
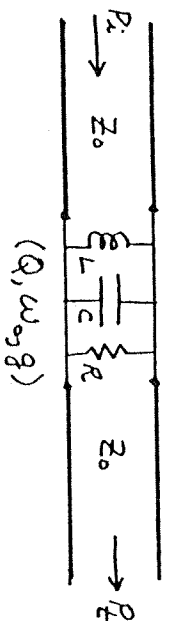
$$Q_e = \frac{Q}{g} = 982.$$

If we have a parallel resonator, $g = \frac{R}{Z_0} = r = 1.985$ (unloaded)

$$Q = (1 + g) Q_L = 982.$$

$$Q_e = \frac{Q}{g} = 495.$$

6.25



$$\frac{P_t}{P_i} = |S_{21}|^2$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$

$$S_{21} = \frac{2}{A + B/Z_0 + C Z_0 + D} = \frac{2}{1 + Z_0/Z + 1} = \frac{2}{2 + 2_0/Z}$$

So,
$$\frac{P_t}{P_i} = \frac{4}{|2 + Z_0/Z|^2}$$

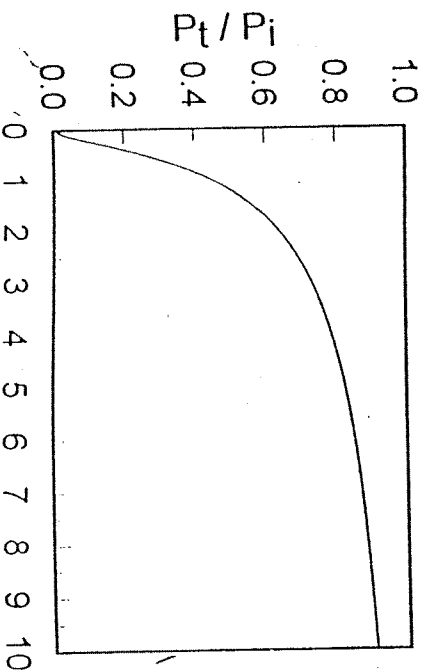
From Table 6.1, $Z_0 \approx \frac{Z_0}{R} + j \frac{2Q\Delta\omega Z_0}{R\omega_0}$

For a parallel RLC resonator, $g = R/Z_0$, so

$$\frac{P_t}{P_i} = \frac{4}{\left(2 + \frac{1}{g}\right)^2 + \left(\frac{2Q\Delta\omega}{g\omega_0}\right)^2} = \frac{4g^2}{(1 + 2g)^2 + \left(\frac{2Q\Delta\omega}{\omega_0}\right)^2}$$

at resonance $\Delta\omega = 0$, so this reduces to,

$$\frac{P_t}{P_i} = \frac{4g^2}{(1 + 2g)^2} = \left(\frac{2g}{1 + 2g}\right)^2$$



g

6.26

The unperturbed TE₁₀₁ cavity fields are,

$$E_y = A \sin \frac{\pi x}{a} \sin \frac{\pi z}{d}$$

$$H_x = \frac{-jA}{2} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \quad ; \quad z = kn/\beta$$

$$H_z = \frac{j\pi A}{k\eta a} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d}$$

Then the numerator in (6.102) is,

$$\int_{\mathcal{V}} (\Delta \epsilon |E_0|^2 + \Delta \mu |H_0|^2) dV = (\mu_r - 1) \mu_0 \int_0^a \int_0^b \int_0^t (|H_x|^2 + |H_z|^2) dz dy dx$$

$$= \mu_0 (\mu_r - 1) \frac{ab}{2} A^2 \int_0^t \left(\frac{1}{2z} \cos^2 \frac{\pi z}{d} + \frac{\pi^2}{k^2 \eta^2 a^2} \sin^2 \frac{\pi z}{d} \right) dz$$

$$= \mu_0 (\mu_r - 1) \frac{ab}{2} A^2 \left[\frac{1}{2z} \left(\frac{3}{2} + \sin \frac{2\pi z}{d} \right) \right]_0^t + \frac{\pi^2}{k^2 \eta^2 a^2} \left(\frac{3}{2} - \frac{\sin \frac{2\pi z}{d}}{4\pi/d} \right) \Big|_0^t$$

$$= \mu_0 (\mu_r - 1) \frac{ab}{2} A^2 \left[\frac{t}{2\eta^2} + \frac{\beta^2 - \pi^2/a^2}{4\pi} \frac{d}{\eta^2} \sin \frac{2\pi t}{d} \right]$$

The denominator in (6.102) is $\frac{abd\epsilon_0 A^2}{2}$, so

$$\frac{\omega - \omega_0}{\omega_0} = \frac{-(\mu_r - 1)ab\eta^2 [\dots]}{abd} \\ = \frac{-(\mu_r - 1)}{d} \left(\frac{t}{2} + \frac{\beta^2 - \pi^2/a^2}{4\pi} \frac{d}{\eta^2} \sin \frac{2\pi t}{d} \right)$$

For $t \ll d$ this simplifies to,

$$\frac{\omega - \omega_0}{\omega_0} \simeq -(\mu_r - 1) \left(\frac{t}{d} \right) \left(\frac{\beta^2}{k^2} \right)$$

6.27

Following Example 6.8:

$$\text{at } x = a/2, z = 0: \quad E_y = 0$$

$$H_x = \frac{-jA}{z}$$

$$, \quad z = k_0 \eta_0 / \beta$$

$$H_y = 0$$

Then,

$$\int_{\Delta V} (\mu_0 |\bar{H}_0|^2 - \epsilon |\bar{E}_0|^2) dV = \mu_0 \frac{A^2}{z^2} \Delta V \quad ; \quad \Delta V = \pi a \eta_0^2$$

$$\int_{\Delta V} (\mu_0 |\bar{H}_0|^2 + \epsilon |\bar{E}_0|^2) dV = \frac{V_0 \epsilon_0 A^2}{2}$$

So (6.109) reduces to,

$$\frac{\omega - \omega_0}{\omega_0} = \frac{2\mu_0 \Delta V}{z^2 \epsilon_0 V_0} = \frac{2\eta_0^2 \Delta V \beta^2}{k_0^2 \eta_0^2 V_0} = \frac{2\beta^2}{k_0^2} \frac{\Delta V}{V_0}$$

(an increase in resonant frequency)

Chapter 7

7.1

This is a special case of a lossless reciprocal 3-port network; it was shown in general that such a network could not be matched at all ports (using the [S] matrix). Alternatively, we can argue as follows: If the input to each port is to be matched to its respective characteristic impedance, we must have,

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_3} \quad (\text{port 1})$$

$$\frac{1}{Z_2} = \frac{1}{Z_1} + \frac{1}{Z_3} \quad (\text{port 2})$$

$$\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2} \quad (\text{port 3})$$

It is not possible to satisfy these three equations simultaneously:

$$\det \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = -4 \neq 0$$

7.2

$$RL = -20 \log |T| = -20 \log |S_{11}| = -20 \log (0.05) = 26. \text{ dB} \checkmark$$

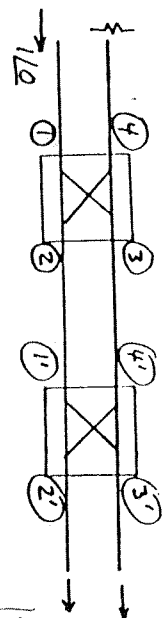
$$C = 10 \log \frac{P_i}{P_o} = -20 \log |S_{13}| = -20 \log (0.1) = 20. \text{ dB} \checkmark$$

$$D = 10 \log \frac{P_3}{P_2} = 20 \log \left| \frac{S_{13}}{S_{14}} \right| = 20 \log \left(\frac{0.1}{0.05} \right) = 6.0 \text{ dB} \checkmark$$

$$\text{I} = 10 \log \left(\frac{P}{P_4} \right) = -20 \log |S_{14}| = -20 \log (0.05) = 26. \text{ dB} \checkmark$$

isolation

7.3



$S_{13} = \beta e^{j\theta}$
 $S_{24} = \beta e^{-j\theta}$

$|S_{13}| = |S_{24}|$
 $|S_{12}| = |S_{34}| = \alpha$

$C = 8.34 \text{ dB} \Rightarrow \beta = |S_{13}| = 0.383$
 $\rightarrow \log |S_{13}|$
 $\alpha = \sqrt{1 - \beta^2} = 0.924$

$\theta \neq \delta = \pi + 2n\pi$

If $V_1^+ = 1 \angle 0^\circ$, then from (7.17),

$V_3^- = j\beta V_1^+ = 0.383 \angle 90^\circ$
 $V_2^- = \alpha V_1^+ = 0.924 \angle 0^\circ$
 $\vec{V} = \cos 90^\circ + j \sin 90^\circ$

Then the outputs of the second coupler are,

$V_3^- = j\beta V_1^+ + \alpha V_4^+ = j\beta V_2^- + \alpha V_3^-$
 $= (0.383)(0.924) \angle 90^\circ + (0.924)(0.383) \angle 90^\circ = 0.707 \angle 90^\circ \checkmark$
 $V_2^- = \alpha V_1^+ + j\beta V_4^+ = \alpha V_2^- + j\beta V_3^-$
 $= (0.924)(0.924) \angle 0^\circ - (0.383)(0.383) \angle 0^\circ = 0.707 \angle 0^\circ \checkmark$

Thus the outputs are identical to those for a single 3dB hybrid.

7.4

$P_1 = 4 \text{ W}, C = 20 \text{ dB}, D = 35 \text{ dB}, IL = 0.5 \text{ dB}$

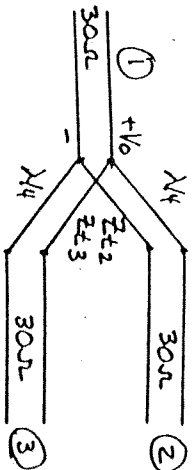
$P_1 = 4 \text{ W} = 10 \log \frac{4 \text{ W}}{0.001 \text{ W}} = 36.0 \text{ dBm}$

$IL = 0.5 \text{ dB} = -10 \log \frac{P_2}{P_1} \Rightarrow P_2 = P_1 - IL = 35.5 \text{ dBm} = 3.55 \text{ W} \checkmark$

$C = 20 \text{ dB} = -10 \log \frac{P_3}{P_1} \Rightarrow P_3 = P_1 - C = 16.0 \text{ dBm} = 0.04 \text{ W} \checkmark$

$D = 35 \text{ dB} = 10 \log \frac{P_3}{P_4} \Rightarrow P_4 = P_3 - D = -19.0 \text{ dBm} = 0.0126 \text{ mW} \checkmark$

7.5



$$\frac{P_2}{P_3} = 3$$

$$P_1 = \frac{1}{2} \frac{V_0^2}{Z_0}$$

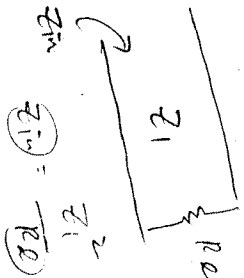
$$P_2 = \frac{1}{2} \frac{V_0^2}{Z_2} = \left(\frac{3}{4} P\right) = \frac{1}{2} V_0^2 \left(\frac{3}{4 Z_0}\right)$$

$$P_3 = \frac{1}{2} \frac{V_0^2}{Z_3} = \left(\frac{1}{4} P\right) = \frac{1}{2} V_0^2 \left(\frac{1}{4 Z_0}\right)$$

So,

$$Z_2 = 4 Z_0 / 3 = 40 \Omega \quad \checkmark$$

$$Z_3 = 4 Z_0 = 120 \Omega \quad \checkmark$$



$$Z_{in} = \frac{Z_1 Z_2}{R_0}$$

The N/4 matching transformers have impedances,

$$Z_{T1} = \sqrt{\frac{R_0 \cdot Z_{in}}{30}} = 34.6 \Omega \quad \checkmark$$

$$Z_{T3} = \sqrt{30(120)} = 60.0 \Omega \quad \checkmark$$

Then the S-parameters are, (phase ref. at 30Ω ports)

$$S_{11} = \frac{30 - 30}{30 + 30} = 0$$

$$S_{22} = \frac{30 \parallel 120 - 40}{30 \parallel 120 + 40} = \frac{24 - 40}{24 + 40} = -0.25 \quad \checkmark$$

$$S_{33} = \frac{30 \parallel 40 - 120}{30 \parallel 40 + 120} = \frac{17.1 - 120}{17.1 + 120} = -0.75 \quad \checkmark$$

$$S_{21} = S_{12} = \sqrt{\frac{P_2}{P_1}} e^{j\theta} = \sqrt{\frac{3}{4}} \angle -90^\circ = 0.866 \angle -90^\circ \quad \checkmark$$

$$S_{31} = S_{13} = \sqrt{\frac{P_3}{P_1}} e^{j\theta} = \sqrt{\frac{1}{4}} \angle -90^\circ = 0.50 \angle -90^\circ \quad \checkmark$$

Since the network is lossless, we should have,

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1$$

$$\text{As } |S_{23}|^2 + |S_{32}|^2 = \sqrt{1 - (0.25)^2 - (0.866)^2} e^{j\theta} = 0.433 \angle -180^\circ \quad \checkmark$$

7.6

T-NETWORK:

From Table 4.1 the ABCD parameters are

$$A = 1 + R_1/R_2$$

$$B = 2R_1 + R_1^2/R_2$$

$$C = 1/R_2$$

$$D = 1 + R_1/R_2$$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - C Z_0 - D}{A + B/Z_0 + C Z_0 + D} = 0 \Rightarrow 1 + \frac{R_1}{R_2} + \frac{2R_1}{Z_0} + \frac{R_1^2}{2R_2} - \frac{Z_0}{R_2} - \frac{Z_0}{R_2} - \frac{R_1}{R_2} = 0$$

$$R_1^2 + 2R_1R_2 - Z_0^2 = 0$$

$$R_2 = \frac{Z_0^2 - R_1^2}{2R_1}$$

$$S_{12} = \frac{2}{A + B/Z_0 + C Z_0 + D} = \alpha \Rightarrow 2 + \frac{2R_1}{R_2} + \frac{2R_1}{Z_0} + \frac{R_1^2}{2R_2} + \frac{Z_0}{R_2} = \frac{2}{\alpha}$$

$$2Z_0R_2 + 2Z_0R_1 + \underbrace{2R_1R_2 + R_1^2 + Z_0^2}_{=Z_0^2} = \frac{2}{\alpha} Z_0R_2$$

$$R_2 + R_1 + Z_0 = R_2/\alpha$$

$$(\frac{1}{\alpha} - 1)(Z_0 - R_1) = 2R_1$$

$$Z_0(\frac{1}{\alpha} - 1) = R_1(1 + \frac{1}{\alpha})$$

$$R_1 = Z_0 \frac{1 - \alpha}{1 + \alpha} \quad \checkmark$$

$$R_2 = \frac{2\alpha}{1 - \alpha^2} Z_0 \quad \checkmark$$

For $Z_0 = 50 \Omega$:

α (dB)	α	R_1 (Ω)	R_2 (Ω)
3	.708	8.6	141.9
10	.316	26.0	35.1
20	.100	40.9	10.1

TT-NETWORK: From Table 4.1 the ABCD parameters are,

$$A = 1 + R_2/R_1$$

$$B = R_2$$

$$C = \frac{R_2}{R_1} + \frac{R_2}{R_1^2}$$

$$D = 1 + R_2/R_1$$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A+B/Z_0 - C Z_0 - D}{A+B/Z_0 + C Z_0 + D} = 0 \Rightarrow \frac{R_2}{Z_0} - \frac{R_2}{R_1} - \frac{R_2 Z_0}{R_1^2} = 0$$

$$R_2 R_1^2 - 2 Z_0^2 R_1 - R_2 Z_0^2 = 0$$

$$R_2 = \frac{2 Z_0^2 R_1}{R_1^2 - Z_0^2}$$

$$S_{12} = \frac{2}{A+B/Z_0 + C Z_0 + D} = \alpha \Rightarrow 2 + \frac{2 R_2}{R_1} + \frac{R_2}{Z_0} + \frac{2 Z_0}{R_1} + \frac{Z_0 R_2}{R_1^2} = \frac{2}{\alpha}$$

$$\underbrace{\frac{2 Z_0}{R_1} + \frac{Z_0 R_2}{R_1^2}}_{= R_2/Z_0}$$

$$1 + \frac{R_2}{R_1} + \frac{R_2}{Z_0} = \frac{1}{\alpha}$$

$$R_1 Z_0 + R_2 (Z_0 + R_1) = \frac{1}{\alpha} Z_0 R_1$$

$$2 Z_0 R_1 = R_2 Z_0 (\frac{1}{\alpha} - 1) (R_1 - Z_0)$$

$$\frac{2 Z_0 \alpha}{1 - \alpha} = R_1 - Z_0$$

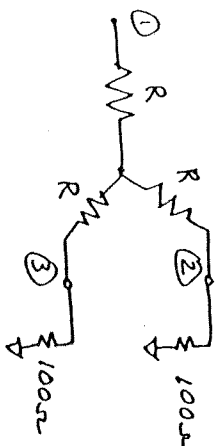
$$R_1 = Z_0 \left(1 + \frac{2 \alpha}{1 - \alpha} \right) = \frac{1 + \alpha}{1 - \alpha} Z_0 \quad \checkmark$$

$$R_2 = \frac{1 - \alpha^2}{2 \alpha} Z_0 \quad \checkmark$$

For $Z_0 = 50 \Omega$:

α (dB)	α	R_1 (Ω)	R_2 (Ω)
3	.708	292.5	17.6
10	.316	96.2	71.2
20	.100	611.1	247.5

7.7



DESIGN:

$$Z_0 = 100 \Omega$$

$$R = 33.3 \Omega \quad \checkmark$$

$$[S] = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

CASE a) ports 2 & 3 matched to 100Ω ($V_2^+ = V_3^+ = 0$);

$$\text{If } V_1^+ = 1,$$

$$V_3^- = \frac{1}{2} [V_1^+ + V_2^+] = \frac{1}{2}$$

$$V_3 = V_3^+ + V_3^- = \frac{1}{2}$$

$$P_3 = V_3^2 / Z_0 = 0.25 / Z_0$$

CASE b) port 3 matched, $\Gamma = 0.3$ at port 2 ($V_3^+ = 0$):

$$\text{If } V_1^+ = 1,$$

$$V_2^- = \frac{1}{2} [V_1^+ + V_3^+] = \frac{1}{2}$$

$$V_2^+ = \Gamma V_2^- = (0.3)(\frac{1}{2}) = 0.15$$

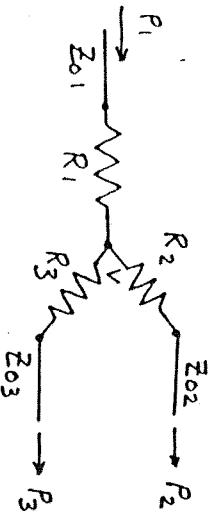
$$V_3^- = \frac{1}{2} [V_1^+ + V_2^+] = \frac{1}{2} (1.15) = 0.575 \text{ V}$$

$$V_3 = V_3^+ + V_3^- = 0.575 \text{ V}$$

$$P_3 = V_3^2 / Z_0 = 0.331 / Z_0$$

$$\frac{P_3 (\text{PORT 2 MISMATCHED})}{P_3 (\text{PORT 2 MATCHED})} (\text{dB}) = 10 \log \left(\frac{0.331}{0.25} \right) = \underline{1.2 \text{ dB}}$$

7.8



$$\alpha = \frac{P_2}{P_3}$$

$$\begin{aligned} \text{Let } Z_1 &= R_1 + Z_{01} \\ Z_2 &= R_2 + Z_{02} \\ Z_3 &= R_3 + Z_{03} \end{aligned}$$

For $\alpha = \frac{P_2}{P_3}$ we must have,

$$\frac{Z_{02} Z_3^2}{Z_2^2 Z_3} = \alpha$$

... (1)

For all three ports to be matched we must have,

$$Z_{01} = R_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

... (2)

$$Z_{02} = R_2 + \frac{Z_1 Z_3}{Z_1 + Z_3}$$

... (3)

$$Z_{03} = R_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

... (4)

We assume that Z_{01} and α are given; then we have 5 unknowns ($R_1, R_2, R_3, Z_{02}, Z_{03}$), and 4 equations.

So we need one more equation; we will choose the condition that,

$$\boxed{Z_{02} Z_{03} = Z_{01}^2}$$

... (5)

which will ensure that when N/4 transformers are used to match Z_{02} and Z_{03} to a final output impedance of Z_{01} , phase tracking will be maintained for ports 2 and 3. (See Problem 7.10). Now use (1) and (5) to eliminate Z_3 and Z_{03} from (2), (3), (4). Then (2) reduces to,

$$R_1 = \frac{Z_{01} [Z_{02} + \sqrt{\alpha} (Z_2 - Z_2)]}{Z_{02} + \sqrt{\alpha} Z_{01}}$$

$$Z_1 = \frac{Z_{01} [2Z_{02} + \sqrt{\alpha} (2Z_{01} - Z_2)]}{Z_{02} + \sqrt{\alpha} Z_{01}}$$

and (3) and (4) then reduce to,

$$Z_2 [(Z_{02} + \sqrt{\alpha} Z_{01})^2 - \alpha Z_{01} Z_{02}] = 2 Z_{02}^2 (Z_{02} + \sqrt{\alpha} Z_{01})$$

and,

$$Z_2 [(Z_{02} + \sqrt{\alpha} Z_{01})^2 - Z_{01} Z_{02}] = 2 Z_{01}^2 (Z_{02} + \sqrt{\alpha} Z_{01})$$

Then we obtain a quartic equation for Z_{02} :

$$Z_{02}^4 + Z_{01} (2\sqrt{\alpha} - 1) Z_{02}^3 + Z_{01}^2 (\alpha - 1) Z_{02}^2 + Z_{01}^3 (\alpha - 2\sqrt{\alpha}) Z_{02} - \alpha Z_{01}^4 = 0$$

After finding Z_{02} (either numerically or using the formula for a quartic equation), the above equations can be used to find Z_{03} , R_2 , R_3 , and R_1 .

EXAMPLE: Let $Z_{01} = 1$, $\alpha = 2$;

$$\text{Then, } Z_{02}^4 + 1.828 Z_{02}^3 + Z_{02}^2 - 0.828 Z_{02} - 2 = 0$$

The solution for Z_{02} was computed numerically using an HP-15C calculator to be,

$$Z_{02} = 0.8935$$

Then,

$$Z_2 = 1.041 ; R_2 = 0.1478$$

$$Z_3 = 1.6477 ; R_3 = 0.5285 ; Z_{03} = 1.1192$$

$$R_1 = 0.3621$$

$$\text{CHECK: } Z_{in} = R_1 + Z_2 Z_3 = 1.000 \checkmark$$

NOTE: There are other choices for the relation between Z_{02} and Z_{03} , or other pairs of variables. One possibility that may give a simpler solution would be to let $Z_1 = \alpha^{1/3} Z_2$.

7.9

From (7.37), $K^2 = P_3/P_2 = 1/3 \Rightarrow K = 0.577$

$$Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^2}} = 131.7 \Omega \checkmark$$

$$Z_{02} = K^2 Z_{03} = 43.9 \Omega \checkmark$$

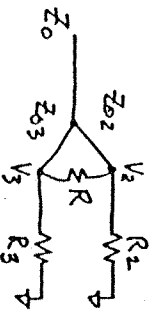
$$R = Z_0 (K+1/K) = 115.5 \Omega \checkmark$$

The output impedances are,

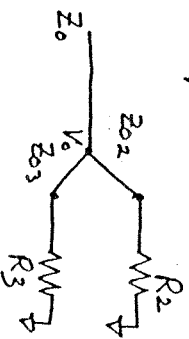
$$R_2 = Z_0 K = 28.9 \Omega \checkmark$$

$$R_3 = Z_0/K = 86.7 \Omega \checkmark$$

7.10



Assuming the output ports are matched, no power should be dissipated in resistor R (for lossless power division). Therefore $V_2 = V_3$, and the resistor R can be removed;



Input matching requires that,

$$\frac{1}{Z_0} = \frac{R_2}{Z_{02}^2} + \frac{R_3}{Z_{03}^2}$$

Power division requires that $P_3 = K^2 P_2$:

$$P_2 = \frac{1}{2} |V_0|^2 R_2 / Z_{02}^2$$

$$P_3 = \frac{1}{2} |V_0|^2 R_3 / Z_{03}^2 = K^2 P_2$$

Thus,

$$K^2 \frac{R_2}{Z_{02}^2} = \frac{R_3}{Z_{03}^2}$$

and,

$$Z_{02}^2 = Z_0 R_2 (1+K^2)$$

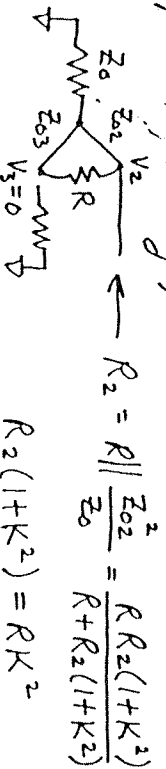
$$Z_{03}^2 = Z_0 R_3 (1+1/K^2)$$

... (1)

... (2)

For output matching,

PORT 2:

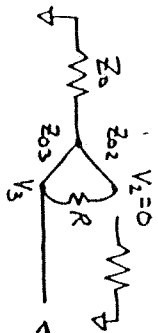


$$R_2 = R \parallel \frac{Z_{02}^2}{Z_0} = \frac{R R_2 (1+K^2)}{R + R_2 (1+K^2)}$$

$$R_2 (1+K^2) = R K^2$$

... (3)

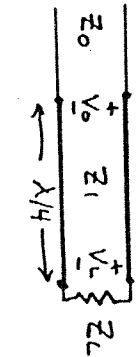
PORT 3:



$$R = R \parallel \frac{Z_{03}^2}{Z_0} = \frac{R R_3 (1 + 1/K^2)}{R + R_3 (1 + 1/K^2)}$$

$$R_3 (1 + K^2) = R \quad \dots (4)$$

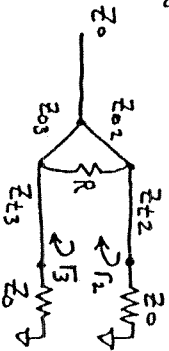
From (3) and (4) we have that $R_2 = K^2 R_3$. We need one more condition to obtain the design equations. This condition is that the output matching transformer R_2, R_3 be chosen so that output matching transformers have the same transfer phase at both ports:



It is easy to show that,

$$\frac{V_L}{V_0} = -j \frac{1 + \Gamma}{1 - \Gamma} ; \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

So the phase of V_L/V_0 does not change when Γ is replaced by $-\Gamma$, or when Z_L and Z_0 are interchanged (Z_L, Z_0 real).



$$\Gamma_2 = \frac{Z_0/Z_{T2} - 1}{Z_0/Z_{T2} + 1}$$

$$\Gamma_3 = \frac{Z_0/Z_{T3} - 1}{Z_0/Z_{T3} + 1}$$

So we choose $Z_{T2} Z_{T3} = Z_0^2 = \sqrt{Z_0 R_2} \sqrt{Z_0 R_3} = K Z_0 R_3$, which leads to the design equations,

$$R_2 = K Z_0 \quad \checkmark \quad Z_{02} = Z_0 \sqrt{K(1+K^2)} \quad \checkmark$$

$$R_3 = Z_0/K \quad \checkmark \quad Z_{03} = Z_0 \sqrt{\frac{1+K^2}{K^2}} \quad \checkmark$$

$$R = Z_0 (K + 1/K) \quad \checkmark$$

7.11

Setting $A_{10}^- = 0$ from (7.40b):

$$\left(\epsilon_0 \kappa_e + \frac{4\mu_0 \kappa_m}{Z_0^2} \right) \sin^2 \frac{\pi S}{a} - \frac{4\mu_0 \pi^2 \kappa_m}{\beta^2 a^2 Z_0^2} \cos^2 \frac{\pi S}{a} = 0$$

For a round aperture, $\kappa_e = 2\epsilon_0^3/3$; $\kappa_m = 4\epsilon_0^3/3$;

$$\left(\epsilon_0 + \frac{2\mu_0}{Z_0^2} \right) \sin^2 \frac{\pi S}{a} - \frac{2\mu_0 \pi^2}{\beta^2 a^2 Z_0^2} \cos^2 \frac{\pi S}{a} = 0$$

Since $Z_0 = k_0 \eta_0 / \beta$, this simplifies as follows:

$$(k_0^2 + 2\beta^2) \sin^2 \frac{\pi S}{a} - \frac{2\pi^2}{a^2} \cos^2 \frac{\pi S}{a} = 0$$

$$(3k_0^2 - \frac{2\pi^2}{a^2}) \sin^2 \frac{\pi S}{a} - \frac{2\pi^2}{a^2} (1 - \sin^2 \frac{\pi S}{a}) = 0$$

$$3k_0^2 \sin^2 \frac{\pi S}{a} = \frac{2\pi^2}{a^2}$$

$$\frac{6a^2}{\lambda^2} \sin^2 \frac{\pi S}{a} = 1, \text{ or } \sin^2 \frac{\pi S}{a} = \frac{\lambda_0}{16a} < 1 \text{ for } a > \lambda_0/2$$

7.12

at $f = 11.64 \text{ GHz}$, $k_0 = 230.4 \text{ m}^{-1}$; $\beta = 116.4 \text{ m}^{-1}$; $Z_0 = k_0 \eta_0 / \beta = 746.2 \Omega$
 $P_{10} = ab/Z_0 = 1.67 \times 10^{-7} \text{ W}$

From (7.41) the position of the coupling aperture is,

$$\sin^2 \frac{\pi S}{a} = \pi \sqrt{\frac{2}{4\pi^2 - k_0^2 a^2}} = 0.867 \Rightarrow S = 0.334a = \underline{0.528 \text{ cm}}$$

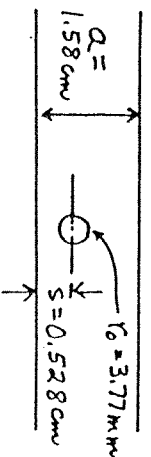
Then, $\sin^2 \frac{\pi S}{a} = 0.752$, $\cos^2 \frac{\pi S}{a} = 0.248$ For $C = 20 \text{ dB}$, $\left| \frac{A_{10}^-}{A} \right| = 0.1$. From (7.40b) we have,

$$0.1 = \frac{60}{P_{10}} \left[\frac{2}{3} \epsilon_0 \sin^2 \frac{\pi S}{a} + \frac{4\mu_0}{3Z_0^2} \left(\sin^2 \frac{\pi S}{a} - \frac{\pi^2}{\beta^2 a^2} \cos^2 \frac{\pi S}{a} \right) \right] \eta_0^3$$

$$0.1 P_{10} = \left[\frac{2}{3} \frac{\eta_0}{\epsilon_0} \sin^2 \frac{\pi S}{a} + \frac{4\eta_0 \mu_0}{3Z_0^2} \left(\sin^2 \frac{\pi S}{a} - \frac{\pi^2}{\beta^2 a^2} \cos^2 \frac{\pi S}{a} \right) \right] \eta_0^3$$

$$1.67 \times 10^{-8} = 3.12 \times 10^{-1} \eta_0^3$$

$$\eta_0 = \underline{3.77 \text{ mm}}$$



7.13

at $f = 17 \text{ GHz}$, $k_0 = 356. \text{m}^{-1}$, $\beta = 295.3 \text{m}^{-1}$,
 $Z_{10} = k_0 \eta_0 / \beta = 454 \Omega$, $P_{10} = ab/Z_{10} = 2.75 \times 10^{-7} \text{ W}$

From (7.44) the necessary skew angle is, for $S = a/2$,
 $\cos \theta = k_0^2 / 2\beta^2 = 0.728 \Rightarrow \theta = 43^\circ \checkmark$

From (7.45) the aperture radius is,

$$\left| \frac{A_{10}}{A} \right| = 0.0316 = \frac{4k_0^2 r_0^3}{3ab\beta} = 4.58 \times 10^6 r_0^3 \Rightarrow r_0 = \underline{1.90 \text{ mm}} \checkmark$$

7.14

$N = 4$ (5-holes)

$k_0 = 366.5 \text{m}^{-1}$, $\beta = 307.9 \text{m}^{-1}$, $Z_{10} = k_0 \eta_0 / \beta = 448.8 \Omega$

$P_{10} = ab/Z_{10} = 2.78 \times 10^{-7} \text{ W}$

From (7.40a), with $S = a/2$,

$$|K_{\pm}| = \frac{2k_0}{3\eta_0 P_{10}} \left| 1 - 2\beta^2/k_0^2 \right| = 9.59 \times 10^5$$

From (7.55),

$$C = 20 \text{ dB} = -20 \log |K_{\pm}| - 20 \log k - 20 \log \sum_{n=0}^N C_n^N$$

For $N = 4$, $\sum_{n=0}^N C_n^N = 1 + 4 + 6 + 4 + 1 = 16$, so

$$20 = -19.6 - 20 \log k - 24.1$$

$$k = 6.53 \times 10^{-9}$$

From (7.54) the aperture radii are,

$$r_0 = k^3 = 1.87 \text{ mm} = r_2$$

$$r_1 = (4k)^3 = 2.97 \text{ mm} = r_3$$

$$r_2 = (6k)^3 = 3.97 \text{ mm}$$

The spacing between the apertures is $\lambda g/4 = 5.1 \text{ mm}$.

(Smaller apertures would result if $S = a/4$ were used.)

7.15

$N=4$ (5 holes)

$$k_0 = 366.5 \text{ m}^{-1}, \beta = 307.9 \text{ m}^{-1}, Z_0 = 448.8 \Omega$$

$$P_{i0} = 2.78 \times 10^{-7} \text{ W}$$

From (7.40a,b) with $S=a/2$,

$$|K_f| = \frac{2k_0}{3\eta_0 P_{i0}} \left| 1 - \frac{Z\beta^2}{k_0^2} \right| = 9.59 \times 10^5$$

From (7.59),

$$30 \text{ dB} = D_{\text{min}} = 20 \log T_N(\text{acc } \theta_m)$$

$$T_4(\text{acc } \theta_m) = \cosh [4 \cosh^{-1}(\text{acc } \theta_m)] = 31.6$$

$$\text{acc } \theta_m = 1.587$$

From (7.57),

$$c = 20 \text{ dB} = -20 \log |K_f| - 20 \log k - 30$$

$$k = 3.30 \times 10^{-9}$$

From (7.56) and (6.60d):

$$2 [r_0^3 \cos 4\theta + r_1^3 \cos 2\theta + r_2^3] = k [\text{acc } \theta_m (\cos 4\theta + 4 \cos 2\theta + 3) - 4 \text{acc}^2 \theta_m (\cos 2\theta + 1) + 1]$$

Then,

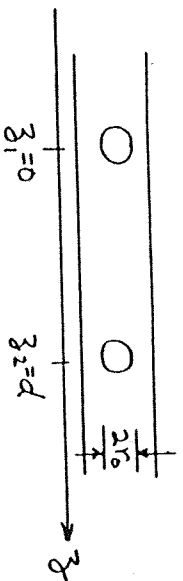
$$2r_0^3 = k \text{acc}^4 \theta_m \Rightarrow r_0 = r_4 = 2.19 \text{ mm}$$

$$2r_1^3 = k [4 \text{acc}^4 \theta_m - 4 \text{acc}^2 \theta_m] \Rightarrow r_1 = r_3 = 2.93 \text{ mm}$$

$$2r_2^3 = k [3 \text{acc}^4 \theta_m - 4 \text{acc}^2 \theta_m + 1] \Rightarrow r_2 = 2.54 \text{ mm}$$

The spacing between the apertures is $\beta/4 = 5.1 \text{ mm}$.

7.16



The incident TE₁₀ fields are, for $x=0$, $y=b/2$, $z=z_n$:

$$E_y = A \sin \frac{\pi x}{a} e^{j\beta z} = 0$$

$$H_x = \frac{-A}{z_0} \sin \frac{\pi x}{a} e^{j\beta z} = 0$$

$$H_z = \frac{j\pi A}{\beta a z_0} \cos \frac{\pi x}{a} e^{j\beta z} = \frac{j\pi A}{\beta a z_0} e^{j\beta z_n}$$

$$P_{10} = ab/z_0 \int z_0 = -k_0 \eta_0 / \beta$$

From (4.124) - (4.125) the equivalent polarization currents are: ($\hat{n} = \hat{x}$)

$$\bar{P}_e = 0$$

$$\bar{P}_m = -\alpha_m H_z S(x) S(y-b/2) S(z-z_n) e^{j\beta z_n}$$

Then the amplitudes of the forward and reverse coupled waves from a single aperture are,

$$A_{10}^+ = \frac{1}{P_{10}} \int \bar{H}_{10}^- \cdot j\omega \mu_0 \bar{P}_m dV = \frac{-j\omega \mu_0 \alpha_m}{P_{10}} \left(\frac{j\pi A}{\beta a z_0} \right) e^{j\beta z_n} H_z^- = \frac{j\pi^2 \alpha_m A}{P_{10} a^2 k_0 \eta_0}$$

$$A_{10}^- = \frac{1}{P_{10}} \int \bar{H}_{10}^+ \cdot j\omega \mu_0 \bar{P}_m dV = \frac{j\pi^2 \alpha_m A}{P_{10} a^2 k_0 \eta_0} e^{-j\beta z_n}$$

Then the total forward and backward wave amplitudes from two apertures at $z_1=0$ and $z_2=d$ are,

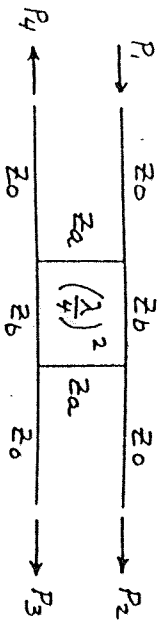
$$A_{10}^+ = \frac{j\pi^2 \alpha_m A (2)}{P_{10} a^2 k_0 \eta_0} \quad A_{10}^- = \frac{j\pi^2 \alpha_m A}{P_{10} a^2 k_0 \eta_0} (1 + e^{-2j\beta d})$$

For $A_{10}^- = 0$, we must have $(1 + e^{-2j\beta d}) = 0$, or $d = \lambda g/4$.

Then the coupling factor is,

$$C = 20 \log \left| \frac{A}{A_{10}^+} \right| = 20 \log \left| \frac{P_{10} a^2 k_0 \eta_0}{2\pi^2 \alpha_m} \right| \\ = 20 \log \left| \frac{30 a^3 b B}{8\pi^2 \eta_0^3} \right| \quad dB$$

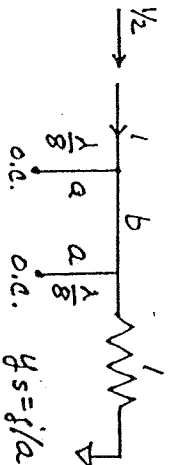
7.17



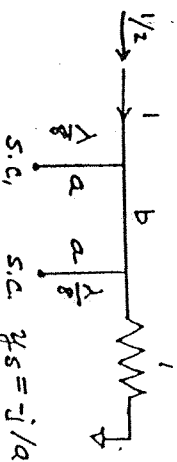
$$\alpha = P_2/P_3$$

$$\text{let } a = \frac{Z_a}{Z_0}, b = \frac{Z_b}{Z_0}$$

Following the analysis of Section 7.5, the even and odd circuits are: (in normalized form)



EVEN MODE



ODD MODE

The ABCD matrices are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} \begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} = \begin{bmatrix} -b/a & jb \\ j/b - j/b/a^2 & -b/a \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} \begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} = \begin{bmatrix} b/a & jb \\ j/b - j/b/a^2 & b/a \end{bmatrix}$$

$$\Gamma_e = \frac{S_{11}^o}{S_{11}^e} = \frac{A+B-C-D}{A+B+C+D} = \frac{j(b-1/b+b/a^2)}{-2b/a+j(b+1/b-b/a^2)}$$

$$\Gamma_o = \frac{S_{11}^e}{S_{11}^o} = \frac{A+B-C-D}{A+B+C+D} = \frac{j(b-1/b+b/a^2)}{2b/a+j(b+1/b-b/a^2)}$$

The reflection at port 1 is then,

$$B_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{j}{2}(b - \frac{1}{b} + \frac{b}{a^2}) \frac{-2j(b + \frac{1}{b} - \frac{b}{a^2})}{(\frac{2b}{a})^2 + (b + \frac{1}{b} - \frac{b}{a^2})^2} = 0$$

Thus, $b - \frac{1}{b} + \frac{b}{a^2} = 0$ (can't have $b + \frac{1}{b} - \frac{b}{a^2} = 0$, or else $B_2 = B_3 = 0$)

$$\text{So, } a = \frac{b}{\sqrt{1-b^2}} \quad \text{Then } \frac{b}{a^2} = \frac{1}{b} - b$$

$$\text{Then, } \Gamma_e = \frac{S_{11}^e}{S_{11}^o} = \frac{2}{A+B+C+D} = \frac{2}{-2\frac{b}{a} + j(b + \frac{1}{b} - \frac{b}{a^2})} = \frac{1}{-\frac{b}{a} + jb}$$

$$T_0 = \frac{2}{A+B+C+D} = \frac{2}{a^2 + j(b + \frac{b}{a^2})} = \frac{1}{\frac{b}{a} + jb}$$

So the output wave amplitudes at ports 2 and 3 are,

$$B_2 = \frac{1}{2} (T_e + T_0) = \frac{1}{2} \left[\frac{1}{-b/a + jb} + \frac{1}{b/a + jb} \right] = \frac{-j}{b(1 + 1/a^2)}$$

$$B_3 = \frac{1}{2} (T_e - T_0) = \frac{1}{2} \left[\frac{1}{-b/a + jb} - \frac{1}{b/a + jb} \right] = \frac{-1/a}{b(1 + 1/a^2)}$$

This shows a 90° phase shift between ports 2 and 3.

For $P_2/P_3 = \alpha$,

$$\frac{P_2}{P_3} = \alpha \Rightarrow \frac{|B_2|^2}{|B_3|^2} = \alpha \Rightarrow |B_3|^2 = \frac{1}{\alpha} |B_2|^2$$

$$1 = \frac{\alpha}{a^2} \Rightarrow a = \sqrt{\alpha} \quad \text{or} \quad Z_a = \sqrt{\alpha} Z_0 \quad \checkmark$$

Then, $b = \frac{a}{\sqrt{1+a^2}} = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} = \sqrt{\frac{\alpha}{1+\alpha}}$, or $Z_b = \sqrt{\frac{\alpha}{1+\alpha}} Z_0 \quad \checkmark$

CHECK: When $\alpha = 1$, $Z_a = Z_0 \quad \checkmark$, $Z_b = Z_0/\sqrt{2} \quad \checkmark$
at the isolated port,

$$B_4 = \frac{1}{2} (T_e - T_0) = \frac{1}{2} \left(b - \frac{b}{a^2} + \frac{b}{a^2} \right) (1) = 0$$

So there is isolation.

EXAMPLE: $\alpha = 3$ (6dB), $Z_0 = 50 \Omega$

Then $Z_a = 87 \Omega \quad \checkmark$

$Z_b = 43.5 \Omega \quad \checkmark$

7.18

$b = 0.32 \text{ cm}$, $\epsilon_r = 2.2$, $Z_{0e} = 70 \Omega$, $Z_{0o} = 40 \Omega$
 Thus, $\sqrt{\epsilon_r} Z_{0e} = 104 \Omega$; $\sqrt{\epsilon_r} Z_{0o} = 59 \Omega$.

From Figure (7.29),

$$S/b = 0.075 \implies S = 0.24 \text{ mm} \checkmark$$

$$W/b = 0.67 \implies W = 2.1 \text{ mm} \checkmark$$

7.19

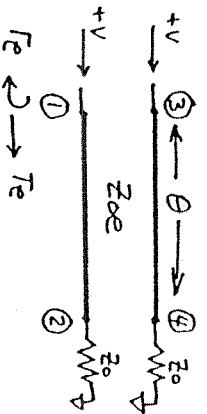
$\epsilon_r = 10$; $d = 0.16 \text{ cm}$; $W = 0.16 \text{ cm}$; $S = 0.064 \text{ cm}$
 $W/d = 1$
 $S/d = 0.4$

From Figure 7.30,

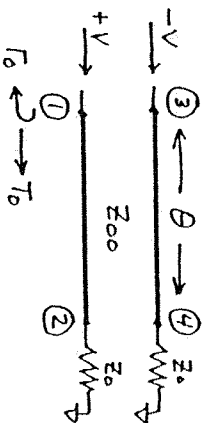
$$Z_{0e} = 61 \Omega$$

$$Z_{0o} = 35 \Omega$$

7.20



EVEN MODE



ODD MODE

For $\theta = \pi/2$,

$$\Gamma_e = \frac{Z_{0e} Z_0 / Z_0 - Z_0}{Z_{0e} / Z_0 + Z_0} = \frac{Z_{0e}^2 - Z_0^2}{Z_{0e}^2 + Z_0^2}$$

$$\Gamma_o = \frac{Z_{0o} / Z_0 - Z_0}{Z_{0o} / Z_0 + Z_0} = \frac{Z_{0o}^2 - Z_0^2}{Z_{0o}^2 + Z_0^2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos \theta & j Z_{0e} \sin \theta \\ \frac{j}{Z_{0e}} \sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos \theta & j Z_{0o} \sin \theta \\ \frac{j}{Z_{0o}} \sin \theta & \cos \theta \end{bmatrix}$$

$$\Gamma_e = S_{21} = \frac{2}{2 \cos \theta + j \left(\frac{Z_{0e}}{Z_0} + \frac{Z_0}{Z_{0e}} \right) \sin \theta}$$

$$\Gamma_o = S_{21} = \frac{2}{2 \cos \theta + j \left(\frac{Z_{0o}}{Z_0} + \frac{Z_0}{Z_{0o}} \right) \sin \theta}$$

For a unit amplitude wave incident at port 1, the output wave amplitudes are,

$$B_1 = \frac{1}{2} (\Gamma_e + \Gamma_o)$$

$$B_2 = \frac{1}{2} (\Gamma_e + \Gamma_o)$$

$$B_3 = \frac{1}{2} (\Gamma_e - \Gamma_o)$$

$$B_4 = \frac{1}{2} (\Gamma_e - \Gamma_o)$$

So the reflection at port 1 is,

$$B_1 = \frac{1}{2} \left[\frac{Z_{0e}^2 - Z_0^2}{Z_{0e}^2 + Z_0^2} + \frac{Z_{0o}^2 - Z_0^2}{Z_{0o}^2 + Z_0^2} \right] = \frac{Z_{0o}^2 Z_{0e}^2 - Z_0^4}{(Z_0^2 + Z_{0e}^2)(Z_0^2 + Z_{0o}^2)} = 0$$

Thus, $Z_0 = \sqrt{Z_{0e} Z_{0o}}$ ✓ (As all ports are matched ✓)

Then, $\left(\frac{Z_{0e}}{Z_0} + \frac{Z_0}{Z_{0e}} \right) = \left(\frac{Z_{0o}}{Z_0} + \frac{Z_0}{Z_{0o}} \right)$

So, $T_e = T_o$, and $B_4 \equiv 0$ ✓

The output waves at ports 2 and 3 are,

$$B_2 = \frac{2}{2} \frac{2 \cos \theta + j \left(\frac{Z_{0e}}{Z_0} + \frac{Z_0}{Z_{0e}} \right) \sin \theta}{2 \cos \theta + j \frac{Z_{0e} + Z_{0o}}{2 Z_0} \sin \theta} = \frac{1}{\cos \theta + j \frac{Z_{0e} + Z_{0o}}{2 Z_0} \sin \theta}$$

$$B_3 = \frac{1}{2} \left[\frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} - \frac{Z_{0o} - Z_{0e}}{Z_{0o} + Z_{0e}} \right] = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

$$\text{Let } C = B_3 = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$$

Then $\sqrt{1-C^2} = \frac{2 Z_0}{Z_{0e} + Z_{0o}}$, and so,

$$B_2 = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2} \cos \theta + j \sin \theta}$$

For $\theta = \pi/2$, the midband responses are,

$$B_3 = C \quad \checkmark$$

$$B_2 = -j \sqrt{1-C^2} \quad \checkmark$$

which agree with (7.85) - (7.86).

7.21

$$C = 10^{-19.1/20} = 0.1109 ; f = 8 \text{ GHz} ; Z_0 = 60 \Omega$$

From (7.87),

$$Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}} = 67.1 \Omega \quad Z_{0o} = Z_0 \sqrt{\frac{1-C}{1+C}} = 53.7 \Omega$$

For a stripline with $\epsilon_r = 2.2$, $b = 0.32 \text{ cm}$,

$$\sqrt{\epsilon_r} Z_{0e} = 99.5 \Omega , \quad \sqrt{\epsilon_r} Z_{0o} = 79.7 \Omega$$

From Figure 7.29,

$$s/b = 0.36 \implies S = 1.15 \text{ mm}$$

$$w/b = 0.60 \implies W = 1.92 \text{ mm}$$

The line lengths are,

$$l = \frac{\lambda g}{4} = \frac{c}{4\sqrt{\epsilon_r} f} = 6.32 \text{ mm}$$

7.22

$$C = 5 \text{ dB} = 10^{-5/20} = 0.562 ; f_0 = 8 \text{ GHz} ; Z_0 = 60 \Omega$$

From (7.87),

$$Z_{0e} = Z_0 \sqrt{\frac{1+C}{1-C}} = 113.3 \Omega \quad Z_{\infty} = Z_0 \sqrt{\frac{1-C}{1+C}} = 31.8 \Omega$$

Then, $\sqrt{\epsilon_r} Z_{0e} = 168.1 \Omega$ $\sqrt{\epsilon_r} Z_{0o} = 47.2 \Omega$ ($\epsilon_r = 2.2$)

From Figure 7.29,

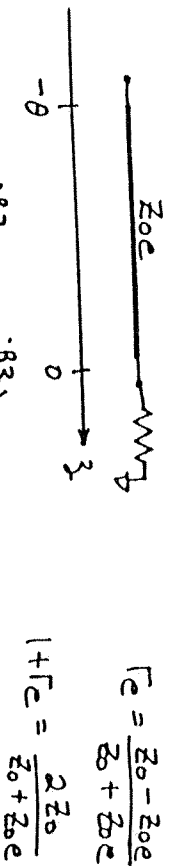
$$s/b \cong 0.009 \implies S = 0.029 \text{ mm (1)}$$

$$w/b \cong 0.34 \implies W = 1.09 \text{ mm}$$

This design is probably not practical due to the extremely close spacing of the lines.

7.23

For V_{1e} or V_{1o} at port 1, we first find V_{2e} or V_{2o} :



$$V(z) = V + (e^{-j\beta z} + \Gamma_e e^{j\beta z})$$

$$V_{1e} = V(-\theta) = V + (e^{j\theta} + \Gamma_e e^{-j\theta})$$

$$V_{2e} = V(l) = V + (1 + \Gamma_e)$$

$$\begin{aligned} \text{So, } V_{2e} &= \frac{V_{1e} (1 + \Gamma_e)}{e^{j\theta} + \Gamma_e e^{-j\theta}} = \frac{2Z_0 V_{1e}}{(Z_0 + Z_0 e)e^{j\theta} + (Z_0 - Z_0 e)e^{-j\theta}} \\ &= \frac{Z_0 V_{1e}}{Z_0 \cos \theta + j Z_0 e \sin \theta} = \frac{Z_0 V_{1e} \sec \theta}{Z_0 + j Z_0 e \tan \theta} \end{aligned}$$

$$\text{Similarly, } V_{2o} = \frac{Z_0 V_{1o} \sec \theta}{Z_0 + j Z_0 e \tan \theta}$$

Then using (7.74) and the results following (7.79) gives,

$$V_4 = V_{2e} - V_{2o} = \frac{Z_0 V_{1e} \sec \theta}{Z_0 + j Z_0 e \tan \theta} \left[\frac{Z_0 + j Z_0 e \tan \theta}{Z_0 + j Z_0 e \tan \theta} - \frac{Z_0 + j Z_0 e \tan \theta}{Z_0 + j Z_0 e \tan \theta} \right]$$

$$V_2 = V_{2e} + V_{2o} = \frac{2Z_0 V_{1e} \sec \theta}{2Z_0 + j Z_0 e \tan \theta} = \frac{2Z_0}{Z_0 e + Z_0} V \frac{Z_0}{Z_0 e + Z_0} \cos \theta + j \sin \theta$$

$$= \frac{V \sqrt{1 - c^2}}{\sqrt{1 - c^2} \cos \theta + j \sin \theta} \quad \text{since } \sqrt{1 - c^2} = \frac{2Z_0}{Z_0 e + Z_0}$$

7.24

$$V_3 = 2 \int V_1 \sin \theta e^{-jN\theta} \left[C_1 \cos(N-1)\theta - C_2 \cos(N-2)\theta + \dots + \frac{1}{2} C_M \right]$$

$$N=3, \alpha=25\text{dB}, Z_0=50\Omega, \text{BINOMIAL}$$

From (7.90) and Example 7.8:

$$M = \frac{N+1}{2}$$

$$C_0 = \left| \frac{V_3}{V_1} \right| \theta = \frac{25}{2}$$

$$C = \left| \frac{V_3}{V_1} \right| = 2 \sin \theta \left[C_1 \cos 2\theta + \frac{1}{2} C_2 \right] \quad ; \quad C_3 = C_1$$

$$= C_1 \sin 3\theta + (C_2 - C_1) \sin \theta \quad ; \quad \text{for } \theta = \pi/2, \alpha = C_0 = C_2 - 2C_1$$

$$\frac{dC}{d\theta} = [3C_1 \cos 3\theta + (C_2 - C_1) \cos \theta] \Big|_{\pi/2} \equiv 0$$

$$\frac{d^2C}{d\theta^2} = [-9C_1 \sin 3\theta - (C_2 - C_1) \sin \theta] \Big|_{\pi/2} = 10C_1 - C_2 = 0$$

At midband ($\theta = \pi/2$), $C_0 = 10^{-25/20} = 0.0562 = C_2 - 2C_1$

Solving for C_1, C_2 :

$$0.0562 = 10C_1 - 2C_1 = 8C_1$$

$$C_1 = C_3 = 0.00703$$

$$C_2 = 10C_1 = 0.0703$$

Using (7.87) gives the even and odd mode characteristic impedances at,

$$Z_{0e}^1 = Z_{0e}^3 = Z_0 \sqrt{\frac{1+C_1}{1-C_1}} = 50.35\Omega \quad \checkmark$$

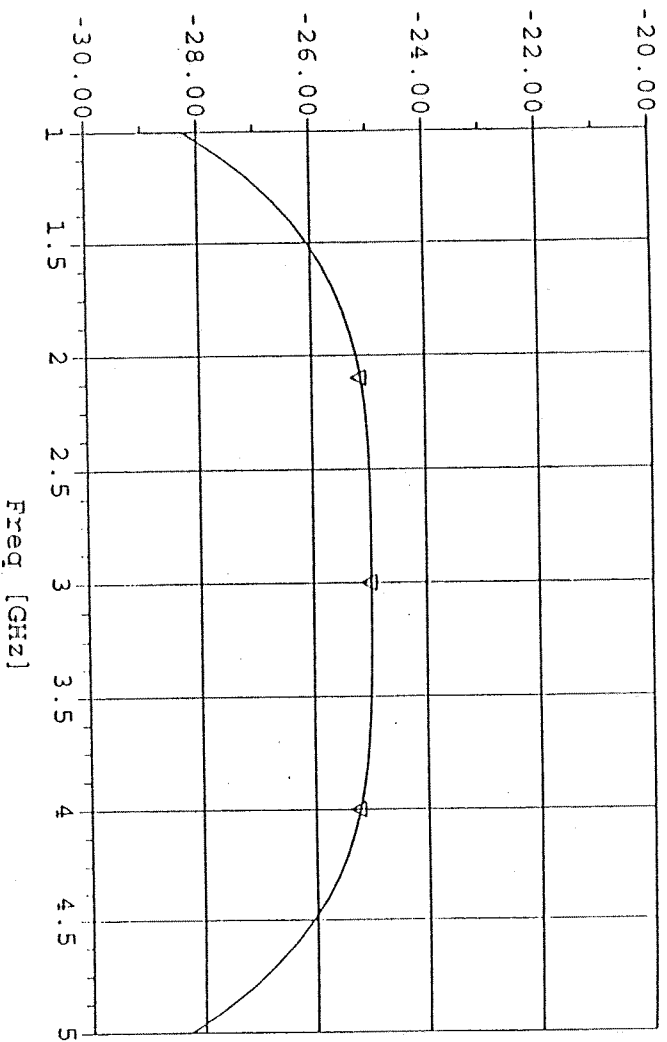
$$Z_{0o}^1 = Z_{0o}^3 = Z_0 \sqrt{\frac{1-C_1}{1+C_1}} = 49.65\Omega \quad \checkmark$$

$$Z_{0e}^2 = Z_0 \sqrt{\frac{1+C_2}{1-C_2}} = 53.65\Omega \quad \checkmark$$

$$Z_{0o}^2 = Z_0 \sqrt{\frac{1-C_2}{1+C_2}} = 46.60\Omega \quad \checkmark$$

The coupling vs. frequency is plotted on the following page.

▽ MS21 [dB] A



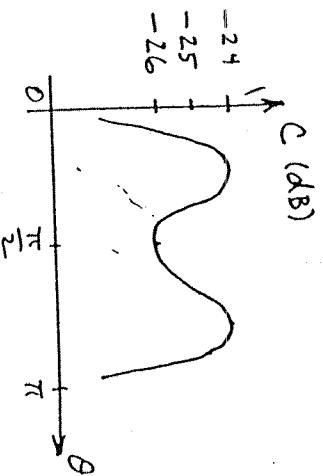
7.25

$N=3$, $C=25\text{dB}$, $Z_0=50\Omega$, 1dB EQUAL RIPPLE

From (7.90) and Example 7.8,

$$C = \left| \frac{V_3}{V_1} \right| = 2 \sin \theta [C_1 \cos 2\theta + \frac{1}{2} C_2] \\ = C_1 \sin 3\theta + (C_2 - C_1) \sin \theta$$

Ordinarily, we would try to equate this to a Chebyshev polynomial, but this will not work here because C is a polynomial in $\sin \theta$. The desired response, based on the above form of C , is as shown:



So at $\theta = \pi/2$, $C_0 = -26\text{dB} = 0.0501$
 $= C_2 - 2C_1$

Then there is only one parameter to determine, from the ripple level. We can easily do this by trial-and-error. The maximum value of C is $C_{MAX} = -24\text{dB} = 0.0631$. So we compute C vs. θ , for various values of C_1 , with $C_2 = 0.0501 + 2C_1$, using a simple computer program, and pick out C_{MAX} :

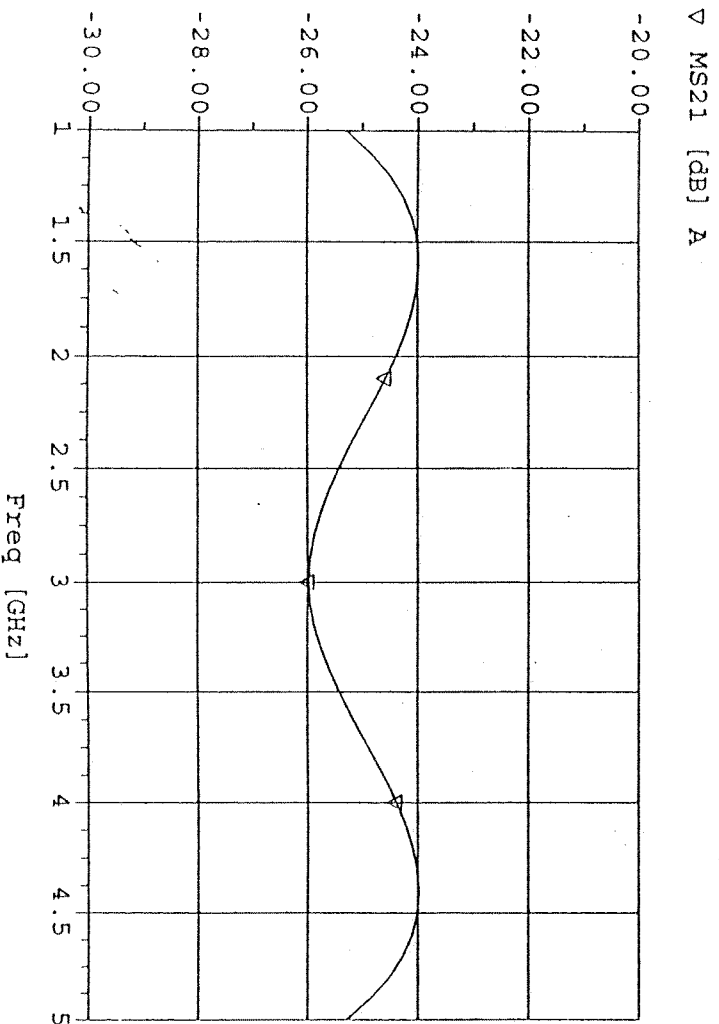
C_1	C_{MAX}
0.01	0.0520
0.02	0.0638
0.019	0.0625
0.0195	0.0632

← close to desired ripple

Thus, $C_1 = 0.0195$; $C_2 = 0.0891$

Then, $Z_{0e} = 50.98\Omega$; $Z_{1o} = 49.03\Omega$
 $Z_{0e}^2 = 54.67\Omega$; $Z_{1o}^2 = 45.73\Omega$
 $Z_{0e}^3 = 50.98\Omega$; $Z_{1o}^3 = 49.03\Omega$

The coupling vs frequency is plotted below. Note that the bandwidth is much greater than that of Problem 7.24.



7.26

From (7.98) and (7.99) we can show that,

$$Z_{e4} = z_0 \sqrt{\frac{1+c}{1-c}}$$

Equating this to (7.97a) gives:

$$z_0 \sqrt{\frac{1+c}{1-c}} = \frac{z_{00} + z_{0e}}{3z_{00} + z_{0e}} = \frac{1 + \frac{z_{0e}}{z_{00}}}{3 + \frac{z_{0e}}{z_{00}}} z_{0e}$$

Now solve (7.99) for z_{0e} in terms of z_{00} :

$$3c(z_{0e}^2 + z_{00}^2) + 2c z_{0e} z_{00} = 3(z_{0e}^2 - z_{00}^2)$$

$$3(c-1)z_{0e}^2 + 2c z_{0e} z_{00} + 3(c+1)z_{00}^2 = 0$$

$$z_{0e} = \frac{-2c z_{00} \pm \sqrt{4c^2 z_{00}^2 - 36(c^2-1)z_{00}^2}}{6(c-1)} = \frac{-c - \sqrt{9-8c^2}}{3(c-1)} z_{00}$$

(Choose negative root since z_{0e} and z_{00} are positive, and $c < 1$)
Substituting for z_{0e}/z_{00} in the above expression gives,

$$z_0 \sqrt{\frac{1+c}{1-c}} = \frac{2c-3-\sqrt{9-8c^2}}{8c-9-\sqrt{9-8c^2}} z_{0e}$$

or

$$z_{0e} = z_0 \sqrt{\frac{1+c}{1-c}} \frac{[8c-9-\sqrt{9-8c^2}][2c-3+\sqrt{9-8c^2}]}{(2c-3)^2 - (9-8c^2)}$$

$$= z_0 \sqrt{\frac{1+c}{1-c}} \frac{(24c^2 - 42c + 18) + 6(c-1)\sqrt{9-8c^2}}{12c(c-1)}$$

$$= z_0 \sqrt{\frac{1+c}{1-c}} \frac{4c-3+\sqrt{9-8c^2}}{2c} \quad \checkmark$$

To find z_{00} , simply replace c by $-c$ (because of symmetry of (7.98) and (7.99)):

$$z_{00} = z_0 \sqrt{\frac{1-c}{1+c}} \frac{4c+3-\sqrt{9-8c^2}}{2c} \quad \checkmark$$

7.27

$$f = 5 \text{ GHz}, \quad \epsilon_r = 10, \quad d = 1 \text{ mm}$$

$$C = 10^{-3/20} = 0.708$$

Assume $Z_0 = 50 \Omega$ (not stated in problem!)

From (7.100) we have,

$$Z_{0E} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C \sqrt{\frac{1-C}{1+C}}} \quad Z_0 = 176.4 \Omega$$

$$Z_{0O} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C \sqrt{\frac{1+C}{1-C}}} \quad Z_0 = 52.5 \Omega$$

(These results are very approximate; SuperCompact gives $Z_{0E} = 121 \Omega$ and $Z_{0O} = 21 \Omega$ for this design)

From Figure 7.30,

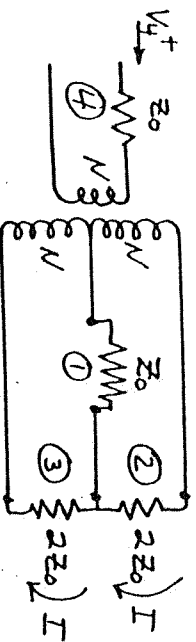
$$S/d \cong 0.075 \Rightarrow S = 0.075 \text{ mm}$$

$$W/d \cong 0.07 \Rightarrow W = 0.07 \text{ mm}$$

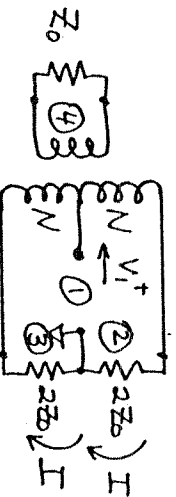
(Super Compact gives 0.071 mm and 0.075 mm for S and W , respectively, starting with the values calculated above for Z_{0E} and Z_{0O} . These are reasonably close to our values for S and W , even though Z_{0E} , Z_{0O} are not close to the Super Compact values.)

7.28

First consider an incident wave at port 4, with the other ports matched:



There is no voltage drop across the resistor at port 1, as $S_{41} = S_{44} = 0$. The load impedance across the secondary is $4Z_0$, as the input impedance at port 4 is $Z_{in} = (N/2N)^2 (4Z_0) = Z_0$, as $S_{44} = 0$. The voltages at ports 2 and 3 have the same magnitude, but opposite signs (relative to the center terminal). Power conservation then gives $S_{24} = S_{42} = 1/\sqrt{2}$; $S_{34} = S_{43} = -1/\sqrt{2}$. Now consider an incident wave at port 1, with matched loads at the other ports:



This excites the transformer in an "odd mode", as $V_4 = 0$. (Consistent with $S_{41} = 0$). Ports 2 and 3 are now equally excited, as $S_{21} = S_{12} = S_{31} = S_{13} = 1/\sqrt{2}$. The input impedance at port 1 is Z_0 , as $S_{11} = 0$.

Finally, the unitary properties of the S-matrix for a lossless network lead to $S_{22} = S_{33} = 0$. Thus, the S-matrix is similar in form to (7.101).

7.29

From (7.101) the $[S]$ matrix of a 180° (3dB) hybrid is,

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

For V_1 at port 1 and V_4 at port 4, the output voltages are (note: hybrid is matched, so $V_1 = V_1^+$, $V_4 = V_4^+$)

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_1^+ \\ 0 \\ V_4^+ \end{bmatrix}$$

$$= \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & \checkmark & \checkmark & \checkmark \\ V_1 - V_4 & \checkmark & \checkmark & \checkmark \\ V_1 + V_4 & \checkmark & \checkmark & \checkmark \\ 0 & \checkmark & \checkmark & \checkmark \end{bmatrix} \begin{matrix} \checkmark \\ \text{(difference)} \\ \text{(sum)} \\ \checkmark \end{matrix}$$

7.30

$\alpha = \beta = \sqrt{2}/2$ for $\alpha = 3dB$

From (7.115a), $\beta = \frac{2\sqrt{k}}{k+1} \implies k = 0.1716 \checkmark$

From (7.115b), $\alpha = \frac{1-k}{1+k} \implies k = 0.1716 \checkmark$

Then,

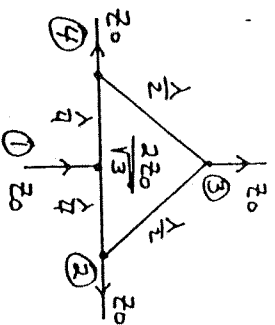
$$Z_{oe}(0) = Z_{oo}(0) = Z_0 = 50 \Omega$$

$$Z_{oe}(L) = Z_0/k = 29.1 \Omega$$

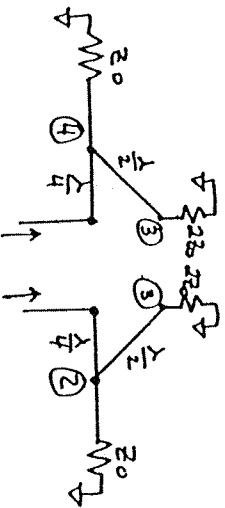
$$Z_{oo}(L) = kZ_0 = 8.6 \Omega$$

a Klopfenstein taper can be used for these taper variations.

7.31



First, let $V_1^+ = 1V$ at port 1, with matched loads at other ports. Then we can direct the network as follows:

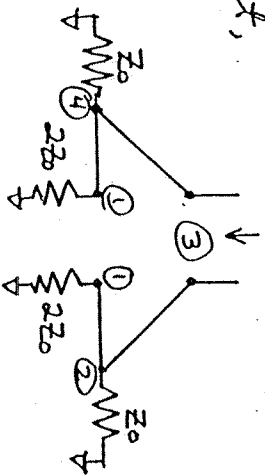


The input impedance of one of these halves is, $(\frac{2Z_0}{\sqrt{3}})^2 \frac{(Z_0 + 2Z_0)}{2Z_0} = 2Z_0$, so $Z_{in} = Z_0$, and $S_{11} = 0$. ✓

Because of the $1/2$ line, the voltage magnitude at port 3 is equal to the voltage magnitude at port 4.

By power conservation, $P_2 = P_4 = P_3 = P_{in}$. Thus, $S_{41} = S_{21} = 1/\sqrt{3} \angle -90^\circ$, $S_{31} = 1/\sqrt{3} \angle -270^\circ$. ($|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$)

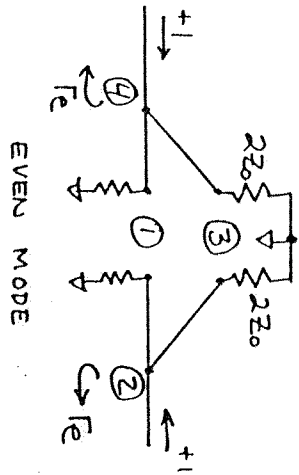
Next, let $V_3^+ = 1V$ at port 3, with other ports matched, and direct,



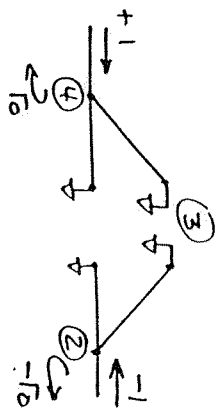
The input impedance of one of these halves is $Z_0 \parallel (\frac{2Z_0}{\sqrt{3}})^2 \frac{1}{2Z_0} = Z_0 \parallel \frac{2Z_0}{3} = \frac{Z_0}{5}$, so $Z_{in} = 2Z_0/5$, and $S_{33} = -2/3$.

So the power delivered to each half is $\frac{1}{2} P_{in} (1 - |S_{33}|^2) = 5/8 P_{in}$. Of this, $2/5$ goes to port 4 and $3/5$ goes to port 1. So, $S_{43} = S_{23} = 1/3 \angle -180^\circ$. The total power to port 1 is then $1/3 P_{in}$, so $S_{13} = 1/\sqrt{3} \angle -270^\circ$. (Then $|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{43}|^2 = \frac{1}{3} + \frac{1}{3} + \frac{4}{9} + \frac{4}{9} = 1$)

Now drive ports 2 and 4 with even and odd excitations:



EVEN MODE



ODD MODE

$$Z_{in}^e = 2Z_0 \parallel \frac{2Z_0}{3} = Z_0/2$$

$$Z_{in}^o = 0$$

$$\Gamma_e = \frac{Z_e - 1}{Z_e + 1} = -1/3$$

$$\Gamma_o = -1$$

Then, $S_{22} = S_{44} = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{1}{2}(-1/3 - 1) = -2/3$

$$S_{24} = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1}{2}(-1/3 + 1) = 1/3$$

So the complete S-matrix is,

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{3} \angle -90^\circ & 1/\sqrt{3} \angle -270^\circ & 1/\sqrt{3} \angle -90^\circ \\ 1/\sqrt{3} \angle -90^\circ & -2/3 & -1/3 & 1/3 \\ 1/\sqrt{3} \angle -270^\circ & -1/3 & -2/3 & -1/3 \\ 1/\sqrt{3} \angle -90^\circ & 1/3 & -1/3 & -2/3 \end{bmatrix}$$

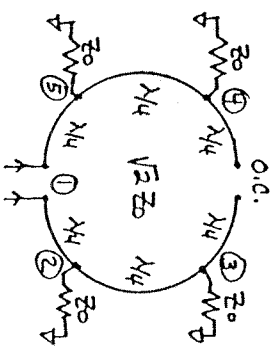
This checks with an analysis using SuperCompact.

7.32

$$\text{Let } V_1^T = 1 \angle 0$$

Breaking the network places an effective short circuit at ports 3 and 4, due to the $N/4$ o.c. stubs. Thus $S_{41} = S_{31} = 0$. Then there is an effective open circuit in parallel with the Z_0 loads at ports 2 and 5. So the input impedance of one of the halves is $(\sqrt{2}Z_0)^2/Z_0 = 2Z_0$.

The total input impedance at port 1 is then Z_0 , so $S_{11} = 0$, and the input power divided evenly to ports 2 and 5. Thus, $V_2^T = 0$; $V_2^T = V_5^T = 0.707 \angle -90^\circ$; $V_3^T = V_4^T = 0$.

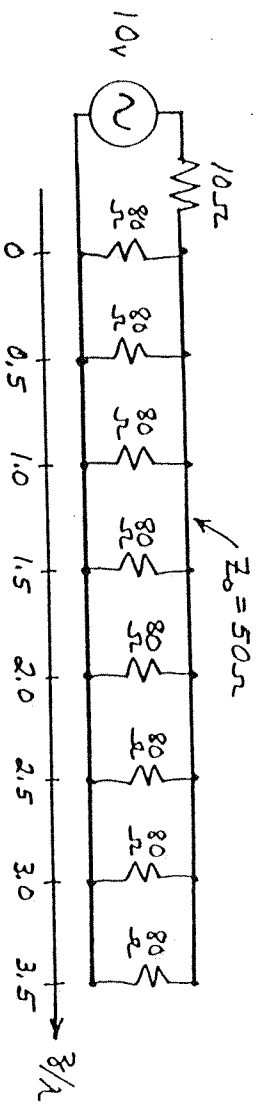


CHECK:

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1 \quad \checkmark$$

Chapter 8

8.1



The input impedance is $80\Omega/8 = 10\Omega$, so $V(0) = 5V$. In between the resistors, we must use the transmission line equations. For $0 < z < \lambda/2$, we have,

$$V(x) = V^+ (e^{j\beta x} + \Gamma e^{-j\beta x}), \text{ where } x = z - \lambda/2$$

$$V(z=0) = V(x=-\lambda/2) = V^+ (-1 - \Gamma) = 5V$$

Thus,
$$V(x) = \frac{-5}{1+\Gamma} (e^{j\beta x} + \Gamma e^{-j\beta x}), \quad \Gamma = \frac{80j7 - 50}{80j7 + 50} = -0.628$$

$$|V(z)| = \frac{5}{1+\Gamma} |1 + \Gamma e^{-2j\beta z}| \quad \text{for } 0 < z < \lambda/2$$

So $|V(\lambda/2)| = +5V$ ($V(\lambda/2) = -5V$). The peak occurs for $z = \lambda/4$:

$$|V(z = \lambda/4)| = \frac{5}{1+\Gamma} (1 - \Gamma) = 5 \frac{Z_0}{Z_L} = \frac{5(50)}{80j7} = 21.8V$$

Intermediate values can also be calculated in the same manner. For $\lambda/2 < z < \lambda$ we repeat the above procedure, but with $Z_L = 80j\lambda$. Thus we have,

$$|V(z = 0.75\lambda)| = 18.8V \quad \checkmark$$

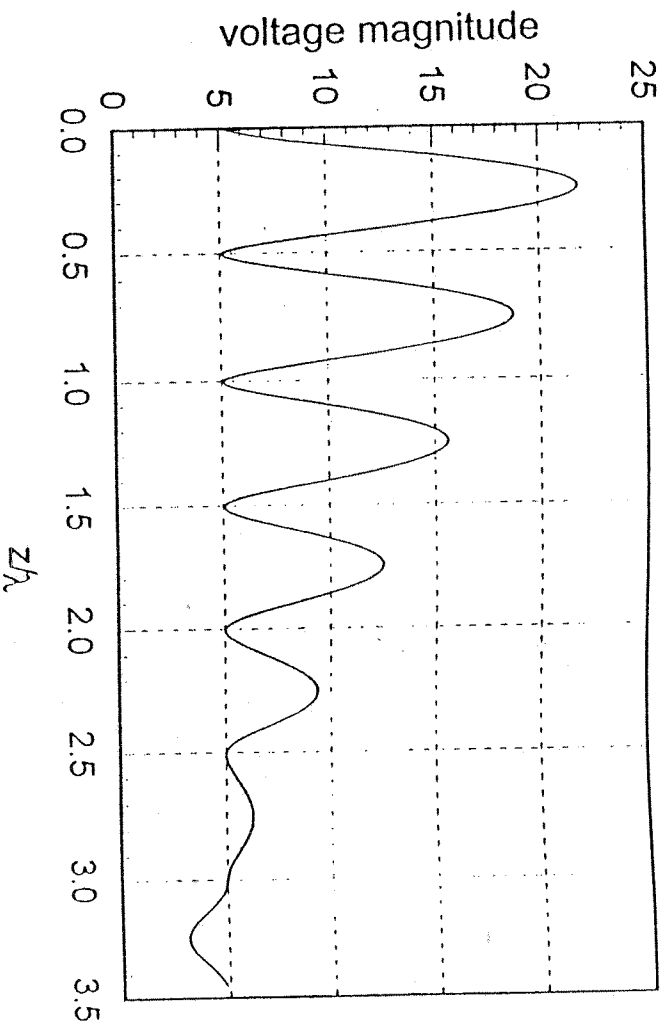
$$|V(z = 1.75\lambda)| = 12.5V \quad \checkmark$$

$$|V(z = 2.25\lambda)| = 9.37V \quad \checkmark$$

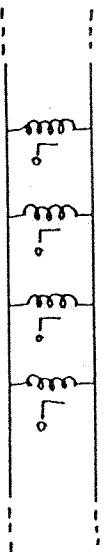
$$|V(z = 2.75\lambda)| = 6.25V \quad \checkmark$$

$$|V(z = 3.25\lambda)| = 3.12V \quad \checkmark$$

$|V(z)|$ is plotted on the following page.



8.2



$Z_0 = 100\Omega$; $d = 1.00m$
 $k = k_0$; $L_0 = 3nH$

Let $\theta = k_0 d$, $b = \frac{-Z_0}{\omega L_0} = \frac{-Z_0}{ck_0 L_0} = \frac{-111}{k_0}$

From (8.9) a passband occurs when,

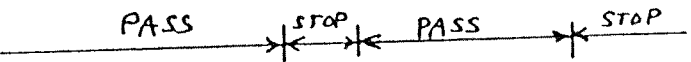
$$|\cos \beta d| = |\cos \theta - \frac{b}{2} \sin \theta| \leq 1,$$

and a stopband occurs when,

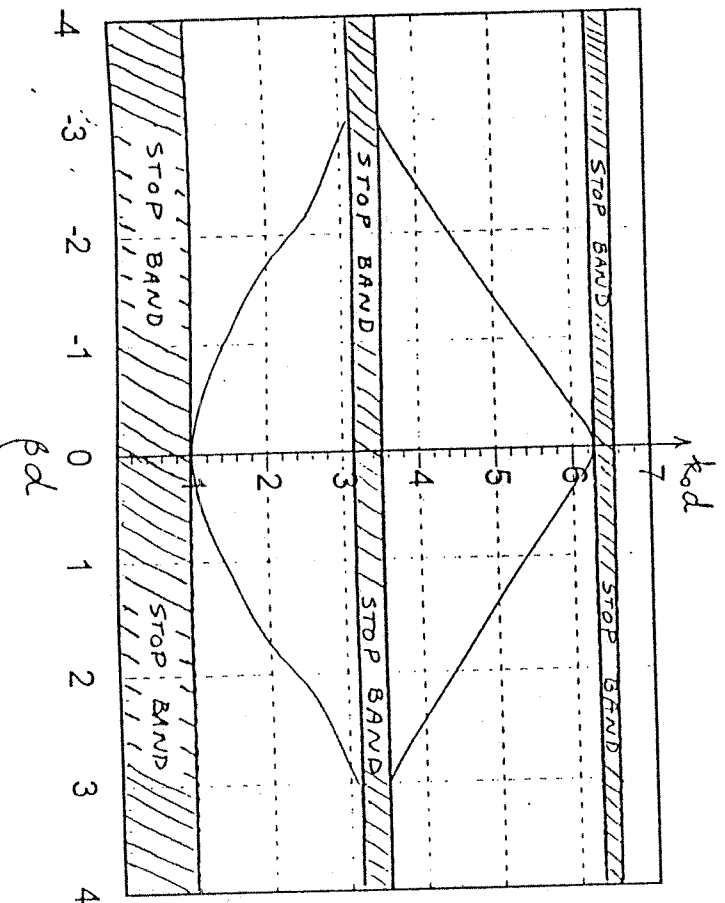
$$\cosh \alpha d = |\cos \theta - \frac{b}{2} \sin \theta| \geq 1 \quad \checkmark$$

So we can compute the following data table:

k_0	θ	$\cos \theta - 1/2 \sin \theta$	βd (rad)
10	5.7°	1.55	—
30	17.2°	1.50	—
100	57.3°	1.007	—
110	63.0°	0.903	0.444
150	85.9°	0.441	1.11
200	114.6°	-0.164	1.74
250	143.2°	-0.535	2.14
300	171.9°	-0.964	2.87
310	177.6°	-0.992	3.02
320	183.3°	-1.02	—
340	194.5°	-1.009	—
350	200.5°	-0.992	3.02
360	206.3°	-0.965	2.87
400	229.2°	-0.758	2.43
450	257.8°	-0.332	1.91
500	286.5°	0.177	1.39
550	315.1°	0.637	0.88
600	343.8°	0.934	0.36
625	358.1°	0.996	0.08



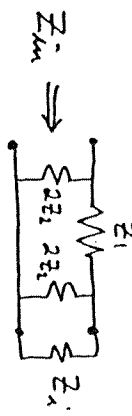
These passbands and stopbands are plotted below:



8.3

$Z_{i\pi}$ can easily be derived from $Z_i = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{B}{C}}$

Verify as follows:



$$Z_i = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1 + Z_1/4 Z_2}} = \frac{Z_2 \sqrt{Z_1}}{\sqrt{Z_2 + Z_1/4}}$$

$$\text{Let } Z = 2Z_2 \parallel Z_1 = \frac{\frac{2Z_2^2 \sqrt{Z_1}}{\sqrt{Z_2 + Z_1/4}}}{2Z_2 + \frac{Z_2 \sqrt{Z_1}}{\sqrt{Z_2 + Z_1/4}}} = \frac{2Z_2 \sqrt{Z_1}}{2\sqrt{Z_2 + Z_1/4} + \sqrt{Z_1}}$$

$$Z_{in} = 2Z_2 \parallel (Z_1 + Z) = \frac{4Z_1 Z_2 \sqrt{Z_2 + Z_1/4} + 2Z_2 \sqrt{Z_1} (2Z_2 + Z_1)}{2\sqrt{Z_2 + Z_1/4} + \sqrt{Z_1}}$$

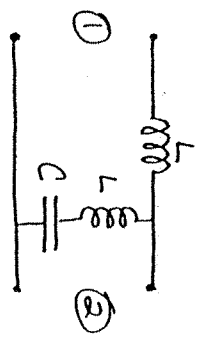
$$= \frac{2Z_1 \sqrt{Z_2 + Z_1/4} + \sqrt{Z_1} (2Z_2 + Z_1)}{2\sqrt{Z_2 + Z_1/4} + \sqrt{Z_1}}$$

$$= \frac{4Z_1 Z_2 \sqrt{Z_2 + Z_1/4} + 2Z_2 \sqrt{Z_1} (2Z_2 + Z_1)(2)}{4Z_2 \sqrt{Z_2 + Z_1/4} + 2Z_2 \sqrt{Z_1} + 2Z_1 \sqrt{Z_2 + Z_1/4} + \sqrt{Z_1} (2Z_2 + Z_1)}$$

$$= \frac{Z_2 [\sqrt{Z_1} (2Z_2 + Z_1) + 2Z_1 \sqrt{Z_2 + Z_1/4}]}{2\sqrt{Z_1} (Z_2 + Z_1/4) + Z_1 \sqrt{Z_2 + Z_1/4} + 2Z_2 \sqrt{Z_2 + Z_1/4}}$$

$$= \frac{Z_2 \sqrt{Z_1}}{\sqrt{Z_2 + Z_1/4}} = Z_i \quad \checkmark$$

8.4



$$\frac{1}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega C}{1 - \omega^2 LC}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\omega C}{1 - \omega^2 LC} & 1 \end{bmatrix} = \begin{bmatrix} 1 - 2\omega^2 LC & j\omega L \\ \frac{j\omega C}{1 - \omega^2 LC} & 1 \end{bmatrix} \quad \checkmark$$

From (8.27),

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{j\omega L (1 - 2\omega^2 LC)}{j\omega C}} = \sqrt{\frac{L}{C} (1 - 2\omega^2 LC)} \quad \checkmark$$

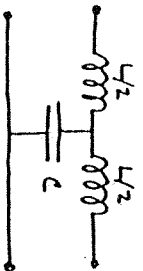
$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \frac{1}{1 - \omega^2 LC} \sqrt{\frac{j\omega L}{j\omega C (1 - 2\omega^2 LC)}} = \frac{1}{1 - \omega^2 LC} \sqrt{\frac{L}{C (1 - 2\omega^2 LC)}}$$

From (8.31); $\cosh \gamma = \sqrt{AD} = \sqrt{\frac{1-2\omega^2 LC}{1-\omega^2 LC}}$

8.5 $R_0 = 50 \Omega$, $f_c = 50 \text{ MHz}$, $f_\infty = 52 \text{ MHz}$, LOW-PASS

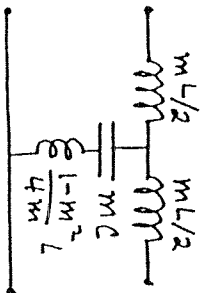
Design equations from Table 8.2:

CONSTANT - k SECTION:



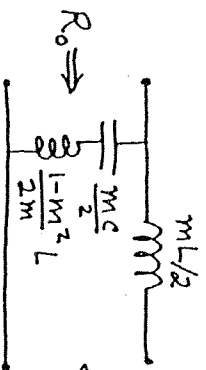
$L = 2R_0/\omega_c = 3.18 \times 10^{-7} \text{ H}$; $L/2 = 159 \text{ nH}$ ✓
 $C = 2/\omega_c R_0 = 127. \text{ pF}$ ✓

M-DERIVED SECTION:



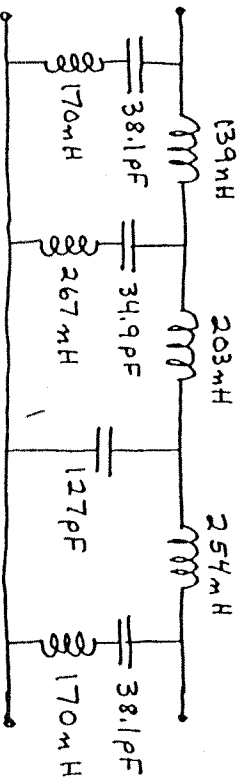
$m = \sqrt{1 - (f_c/f_\infty)^2} = 0.275$ ✓
 $\frac{mL}{2} = 43.7 \text{ nH}$ ✓
 $mC = 34.9 \text{ pF}$ ✓
 $\frac{1-m^2}{4m} L = 267. \text{ nH}$ ✓

MATCHING SECTIONS: ($m = 0.6$)



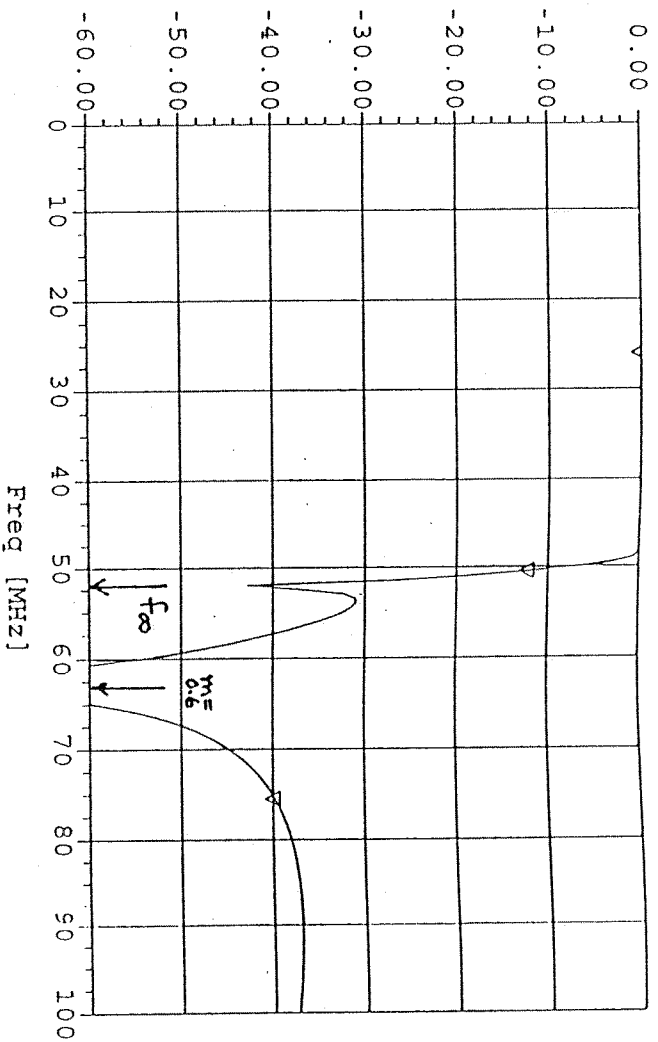
$\frac{mL}{2} = 95.4 \text{ nH}$ ✓
 $\frac{mC}{2} = 38.1 \text{ pF}$ ✓
 $\frac{1-m^2}{2m} L = 170. \text{ nH}$ ✓

COMPLETE FILTER:



The calculated response is shown on the following page.

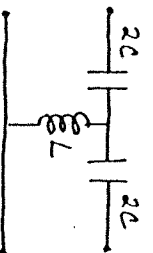
▽ MS12 [dB] FILTER



8.6

$R_0 = 75 \Omega$, $f_c = 50 \text{ MHz}$, $f_{\infty} = 48 \text{ MHz}$, HIGH-PASS
 Design equations are given in Table 8.2

CONSTRUCT - R SECTION:

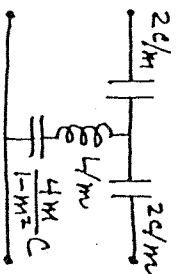


$$L = R_0 / 2\omega_c = 119, \text{ nH}$$

$$C = 1 / 2R_0\omega_c = 21.2 \text{ pF}$$

$$2C = 42.4 \text{ pF}$$

M - DERIVED SECTION:



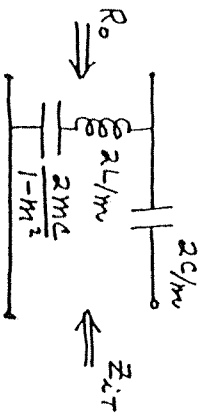
$$M = \sqrt{1 - (f_{\infty}/f_c)^2} = 0.280$$

$$\frac{2C}{m} = 151, \text{ pF}$$

$$L/m = 425, \text{ nH}$$

$$\frac{4M^2 C}{1 - M^2} = 25.8 \text{ pF}$$

MATCHING SECTION : ($\Gamma_M = 0.6$)

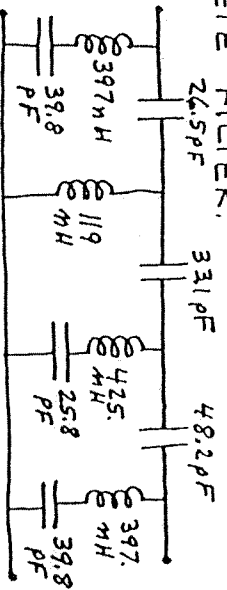


$$\frac{Z_0}{\Gamma_M} = 70.7 \text{ PF} \quad \checkmark$$

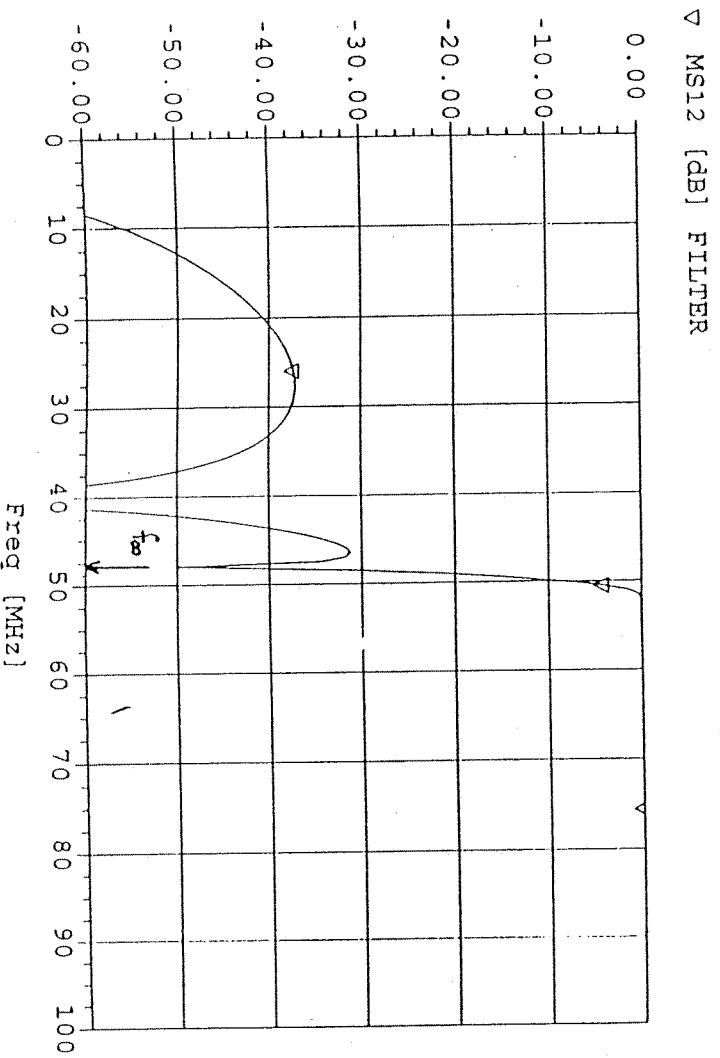
$$\frac{Z_0}{\Gamma_M} = 397. \text{ mH.} \quad \checkmark$$

$$\frac{2M\Omega}{1-\Gamma_M^2} = 39.8 \text{ PF} \quad \checkmark$$

COMPLETE FILTER:



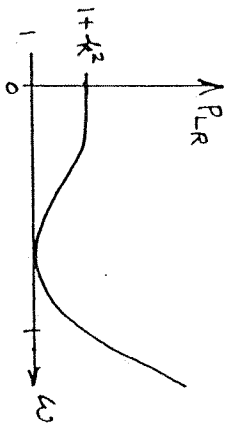
The calculated filter response is shown below.



8.7

From (8.61),

$$P_{LR} = 1 + k^2 T_N^2(\omega) = 1 + k^2 (2\omega^2 - 1)^2 \quad \text{for } N=2$$



$$P_{LR} = 1 = 0 \text{ dB}$$

$$P_{LR} = 1 + k^2 = 1 \text{ dB} = 1.259$$

$$\text{At } k = \pm 0.509$$

(We must choose $k = -0.509$, otherwise L, C are not real.)

Then from (8.63),

$$R = 1 + 2k^2 - 2k\sqrt{1+k^2} = 2.66 \quad \checkmark$$

We also have that,

$$4k^2 = \frac{1}{4R} L^2 C^2 R^2 \quad \Rightarrow \quad L = \frac{-4k}{C\sqrt{R}}$$

$$-4k^2 = \frac{1}{4R} (R^2 C^2 + L^2 - 2LCR^2)$$

$$-16k^2 R = R^2 C^2 + \frac{16k^2}{C^2 R} + 8kR\sqrt{R}$$

$$R^2 C^4 + (16k^2 R + 8kR\sqrt{R}) C^2 + \frac{16k^2}{R} = 0$$

$$7.08 C^4 - 6.64 C^2 + 1.56 = 0$$

Thus,

$$C = 0.685 \quad \checkmark$$

$$L = 1.822 \quad \checkmark$$

(R, L, C check with results given in Mathai, Young, and Tzeng.)

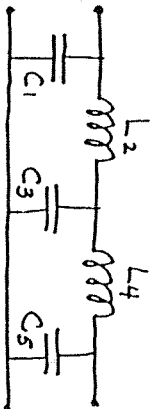
8, 8

$f_0 = 3 \text{ GHz}$, LOW-PASS, M.F., $Z_0 = 75 \Omega$, $\alpha = 20 \text{ dB}$ at 5 GHz
Following Example 8.4:

$$\left| \frac{w}{w_c} \right|^{-1} = \frac{\alpha}{5} - 1 = 0.667,$$

As from Figure 8.26 we see that $N=5$ will give $\alpha > 20 \text{ dB}$.
Then from Table 8.3, the LP prototype values are,

$$\begin{aligned} g_1 &= 0.618 \\ g_2 &= 1.618 \\ g_3 &= 2.000 \\ g_4 &= 1.618 \\ g_5 &= 0.618 \end{aligned}$$

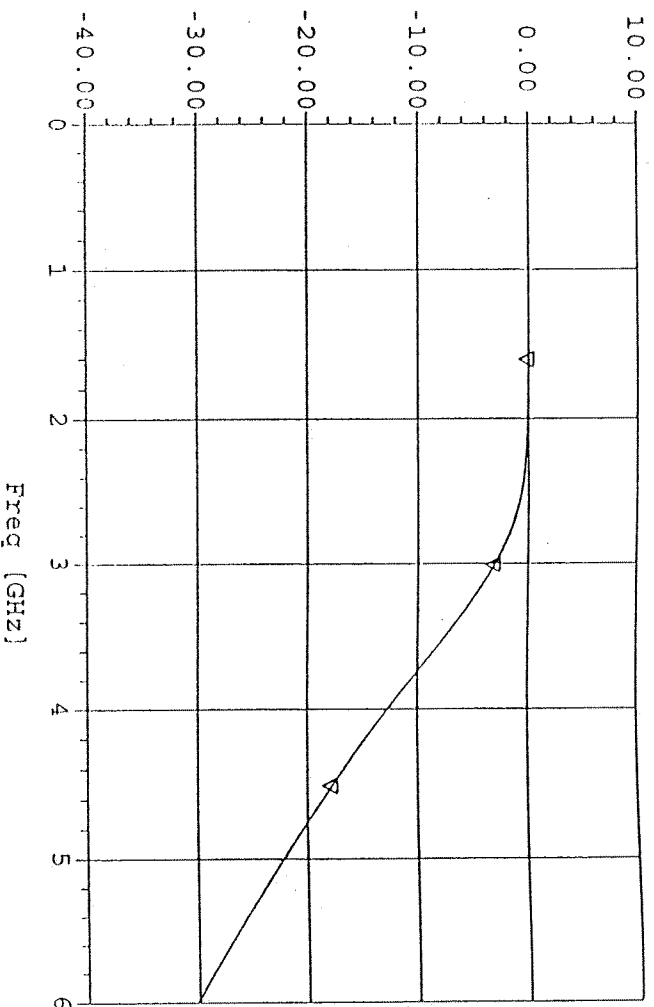


Use (8.67) to scale cutoff frequency and impedance:

$$\begin{aligned} C_1 &= \frac{g_1}{R_0 \omega_c} = 0.437 \text{ pF} \checkmark \\ L_2 &= \frac{R_0 g_2}{\omega_c} = 6.44 \text{ nH} \checkmark \\ C_3 &= \frac{g_3}{R_0 \omega_c} = 1.41 \text{ pF} \checkmark \\ L_4 &= \frac{R_0 g_4}{\omega_c} = 6.44 \text{ nH} \checkmark \\ C_5 &= \frac{g_5}{R_0 \omega_c} = 0.437 \text{ pF} \checkmark \end{aligned}$$

The calculated filter response is shown on the following page. Note that the insertion loss at 5 GHz is more than 20 dB .

▽ MS12 [dB] FILTER



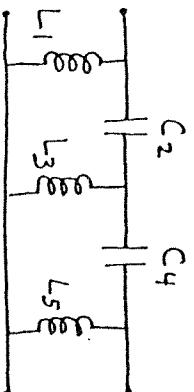
8.9

$f_0 = 1 \text{ GHz}$, HIGH PASS, 3dB E.R., $N=5$, $Z_0 = 50 \Omega$

at $f = 0.6 \text{ GHz}$, $\left| \frac{W}{W_0} \right|^{-1} = \frac{1}{0.6} - 1 = 0.667$, α from Figure 8.27b,

the attenuation for $N=5$ should be about 41 dB. From Table 8.4 the prototype values are,

- $g_1 = 3.4817$
- $g_2 = 0.7618$
- $g_3 = 4.5381$
- $g_4 = 0.7618$
- $g_5 = 3.4817$



Impedance and frequency scaling using (8.70):

$$L_1 = \frac{Z_0}{\omega_c g_1} = 2.28 \text{ nH} \quad \checkmark$$

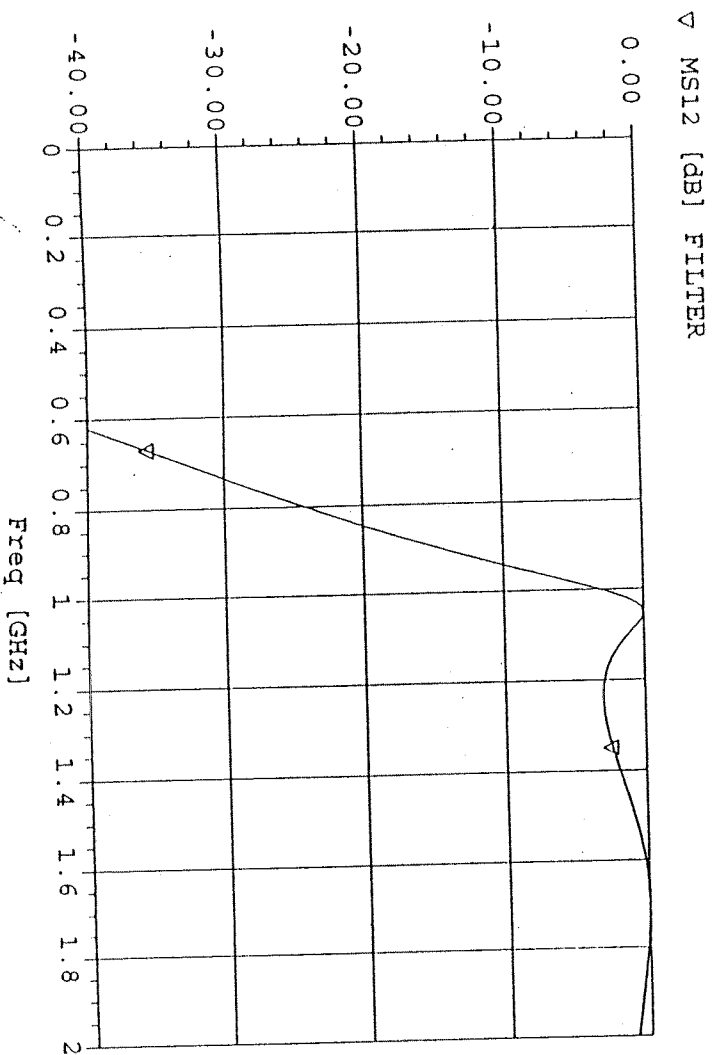
$$C_2 = \frac{1}{Z_0 \omega_c g_2} = 4.18 \text{ pF} \quad \checkmark$$

$$L_3 = \frac{Z_0}{\omega_c g_3} = 1.75 \text{ nH} \quad \checkmark$$

$$C_4 = \frac{1}{Z_0 \omega_c g_4} = 4.18 \text{ pF} \quad \checkmark$$

$$L_5 = \frac{Z_0}{\omega_c g_5} = 2.28 \text{ nH} \quad \checkmark$$

The calculated filter response is shown below. Note that the insertion loss at $f = 0.6 \text{ GHz}$ is just a bit more than 40dB.

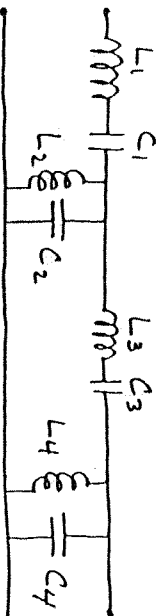


8.10

$f_0 = 2 \text{ GHz}$, B.P., M.F.G.D., $\Delta = 0.05$, $N = 4$, $Z_0 = 50 \Omega$

From Table 8.5 the prototype element values are,

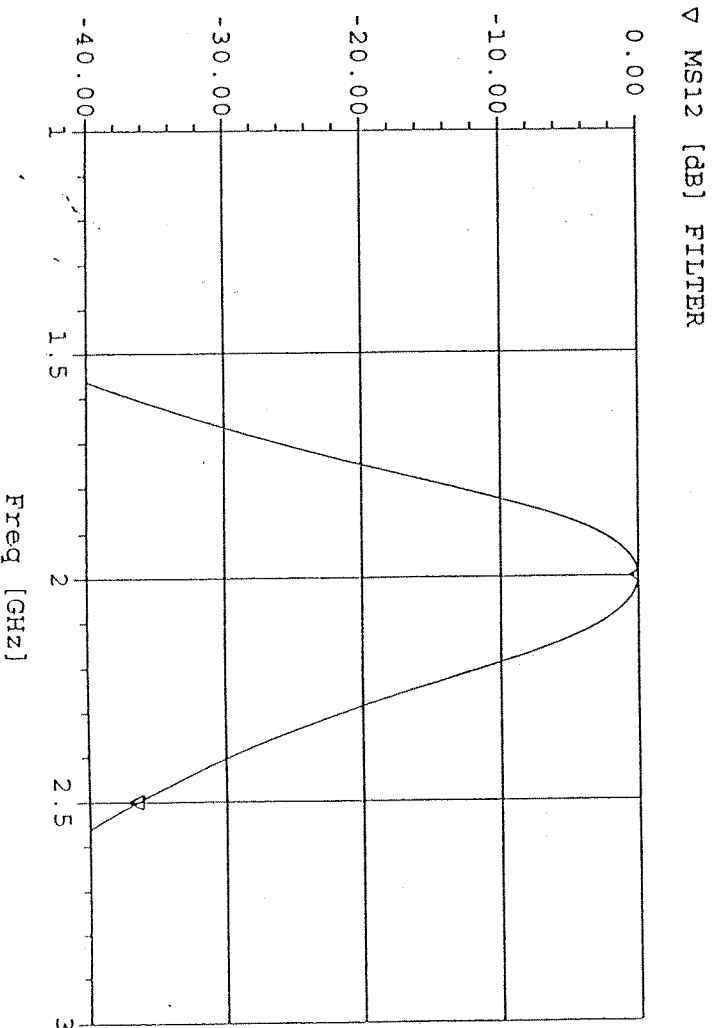
$$\begin{aligned} g_1 &= 1.0598 \\ g_2 &= 0.5116 \\ g_3 &= 0.3181 \\ g_4 &= 0.1104 \end{aligned}$$



From Table 8.6 and (8.64) the scaled element values are,

$$\begin{aligned} L_1 &= \frac{g_1 Z_0}{\omega_0 \Delta} = 84.3 \text{ nH} \quad \checkmark & C_1 &= \frac{\Delta}{\omega_0 g_1 Z_0} = 0.075 \text{ pF} \quad \checkmark \\ L_2 &= \frac{\Delta Z_0}{\omega_0 g_2} = 0.388 \text{ nH} \quad \checkmark & C_2 &= \frac{g_2}{\omega_0 \Delta Z_0} = 16.3 \text{ pF} \quad \checkmark \\ L_3 &= \frac{g_3 Z_0}{\omega_0 \Delta} = 25.3 \text{ nH} \quad \checkmark & C_3 &= \frac{\Delta}{\omega_0 g_3 Z_0} = 0.25 \text{ pF} \quad \checkmark \\ L_4 &= \frac{\Delta Z_0}{\omega_0 g_4} = 1.80 \text{ nH} \quad \checkmark & C_4 &= \frac{g_4}{\omega_0 \Delta Z_0} = 3.51 \text{ pF} \quad \checkmark \end{aligned}$$

The calculated filter response is shown below.



8.11

$$f_0 = 3 \text{ GHz}, \quad Z_0 = 75 \Omega, \quad N = 3, \quad B_5, \quad 0.5 \text{ dB E.R.}$$

First use (8.75) to transform 3.1 GHz to a L.P. prototype response frequency:

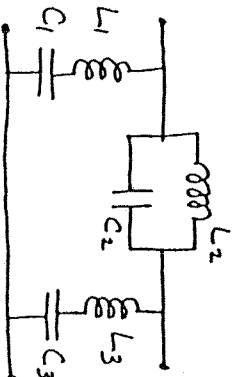
$$\omega \leftarrow \Delta \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1} = 0.1 \left(\frac{3.1}{3} - \frac{3}{3.1} \right)^{-1} = 1.52$$

So, $\left| \frac{\omega}{\omega_c} \right| - 1 = 0.52$, and Figure 8.27a gives an attenuation of 11 dB for $N=3$. From Table 8.4, the prototype values are,

$$g_1 = 1.5963$$

$$g_2 = 1.0967$$

$$g_3 = 1.5963$$



From Table 8.6 and (8.64) the scaled element values are,

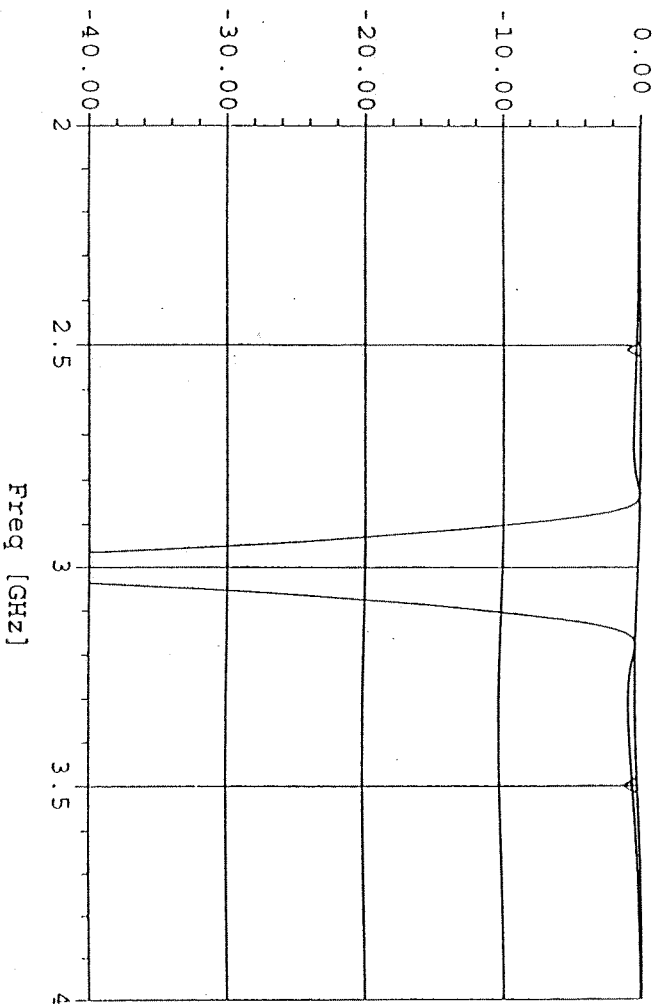
$$L_1 = \frac{Z_0}{\omega_0 g_1 \Delta} = 24.9 \text{ nH} \quad \checkmark \quad C_1 = \frac{g_1 \Delta}{\omega_0 Z_0} = 0.113 \text{ pF} \quad \checkmark$$

$$L_2 = \frac{g_2 \Delta Z_0}{\omega_0} = 0.436 \text{ nH} \quad \checkmark \quad C_2 = \frac{1}{Z_0 \omega_0 g_2 \Delta} = 6.45 \text{ pF} \quad \checkmark$$

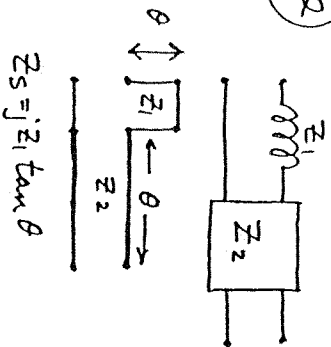
$$L_3 = \frac{Z_0}{\omega_0 g_3 \Delta} = 24.9 \text{ nH} \quad \checkmark \quad C_3 = \frac{g_3 \Delta}{Z_0 \omega_0} = 0.113 \text{ pF} \quad \checkmark$$

The calculated response for this filter is shown on the following page. Note that the insertion loss at 3.1 GHz is about 10 dB.

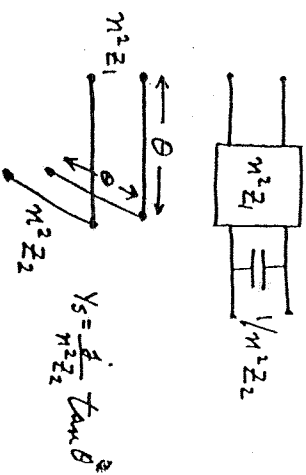
▽ MS12 [dB] FILTER



8.12



$$\begin{aligned}
 \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} 1 & jZ_1 \tan \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & jZ_2 \sin \theta \\ \frac{j}{Z_2} \sin \theta & \cos \theta \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta - \frac{Z_1}{Z_2} \frac{\sin^2 \theta}{\cos \theta} & j(Z_1 + Z_2) \sin \theta \\ \frac{j}{Z_2} \sin \theta & \cos \theta \end{bmatrix}
 \end{aligned}$$



$$\begin{aligned}
 \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} \cos \theta & j n^2 Z_2 \sin \theta \\ \frac{j}{n^2 Z_2} \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j \tan \theta}{n^2 Z_2} & 1 \end{bmatrix} \\
 &= \begin{bmatrix} \cos \theta - \frac{Z_1}{Z_2} \frac{\sin^2 \theta}{\cos \theta} & j n^2 Z_2 \sin \theta \\ \frac{j}{n^2} \left(\frac{1}{Z_2} + \frac{1}{Z_1} \right) \sin \theta & \cos \theta \end{bmatrix}
 \end{aligned}$$

So these two matrices are equal if,

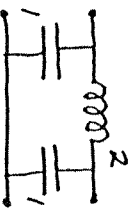
$$Z_1 + Z_2 = n^2 Z_1$$

$$n^2 = 1 + Z_2/Z_1 \quad \checkmark$$

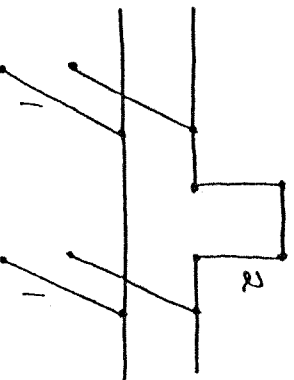
8.13

$f_0 = 6 \text{ GHz}$, $N=3$, $M.F.$, $Z_0 = 50 \Omega$

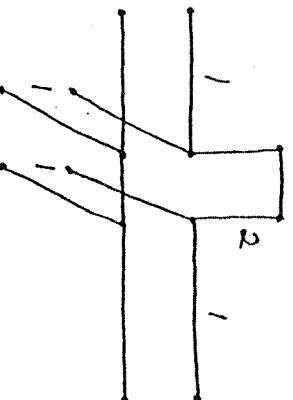
From Table 8.3 the L.P. prototype is,



(Choosing a π -circuit simplifies the problem)
 Richards' transform:



Add unit elements at ends:



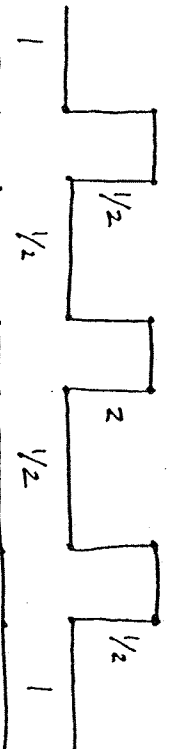
Apply first Kennelly identity (twice):

$$Z_1 = 1$$

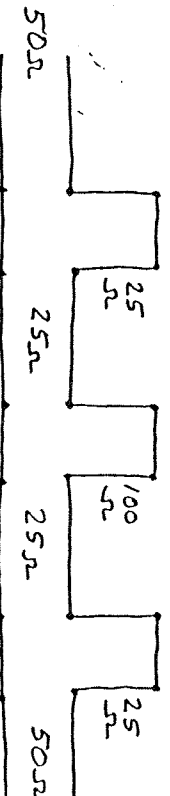
$$Z_2 = 1$$

$$n^2 = 1 + \frac{Z_1}{Z_2}$$

$$= 2$$

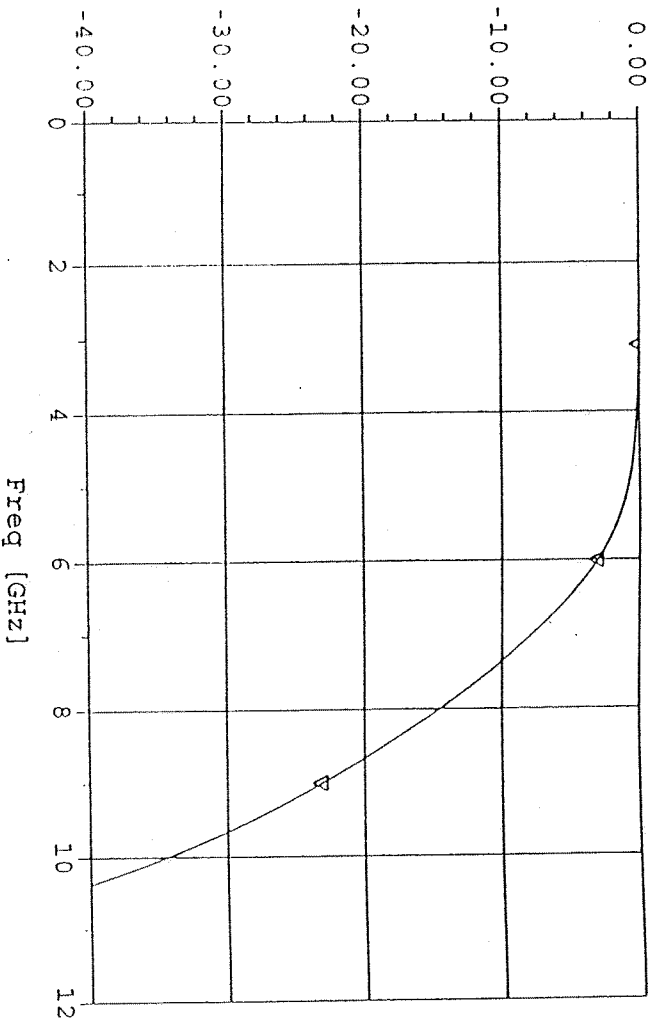


Scale to 50 Ω :



All line lengths and stub lengths are $\lambda/8$ long at 6 GHz. The calculated filter response is shown below.

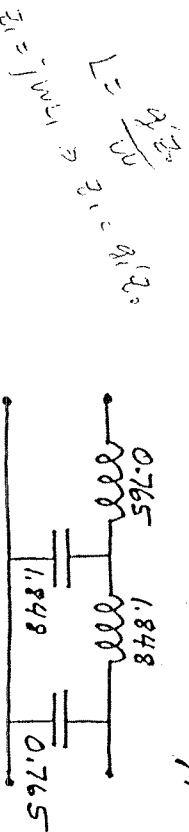
▽ MS12 [dB] FILTER



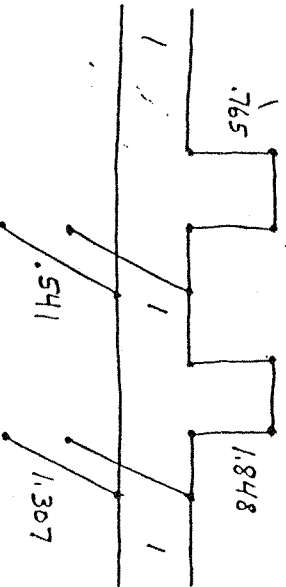
8.14

$f_0 = 8 \text{ GHz}$, $N=4$, L.P., M.F., $Z_0 = 50 \Omega$

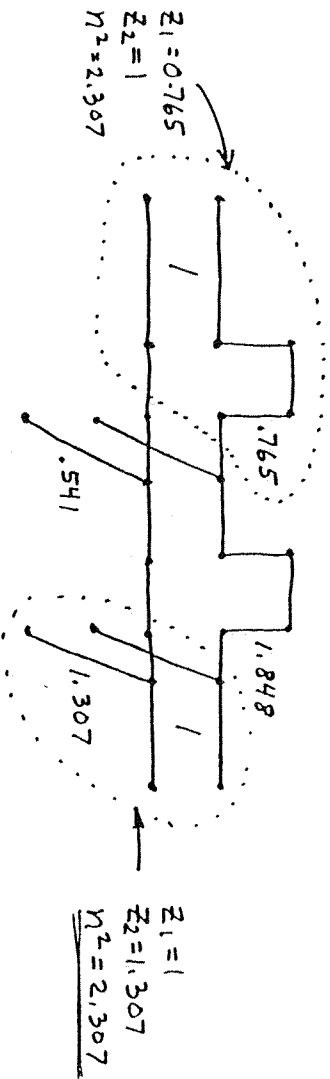
From Table 8.3 the L.P. prototype is,



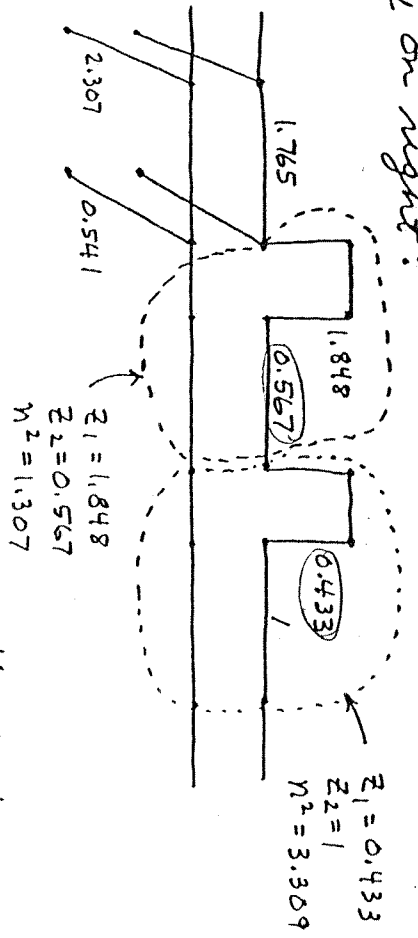
Applying Richards transform:



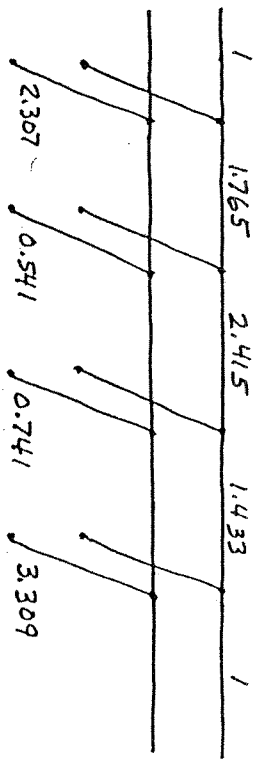
add unit elements:



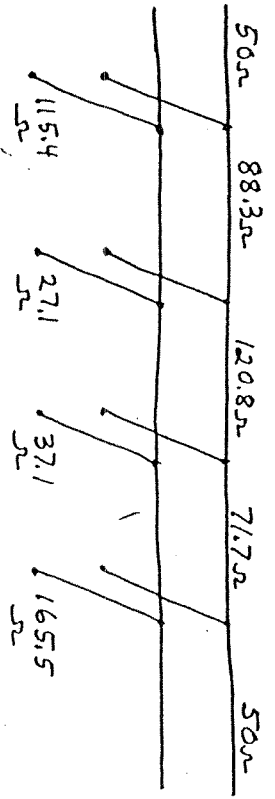
use the second Kuroda identity on left; first Kuroda identity on right:



Now use the second Kuroda identity twice:

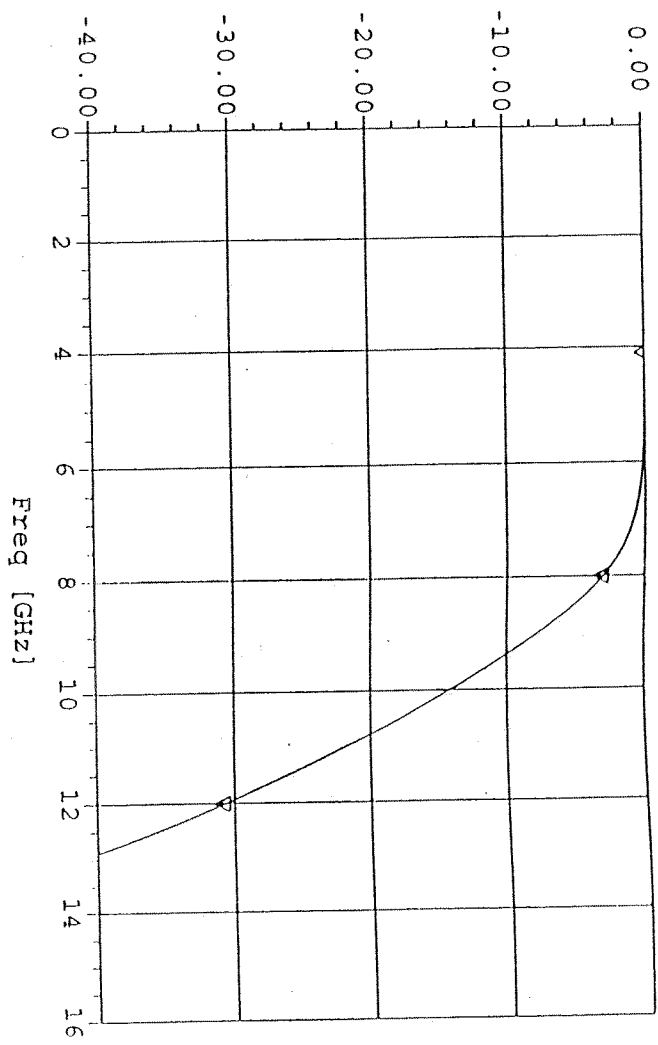


Scale to 50Ω:

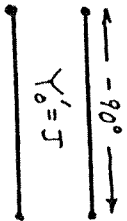


all lines are $\lambda/8$ long at 8543. The calculated filter response is shown on the following page.

▽ MS12 [dB] FILTER



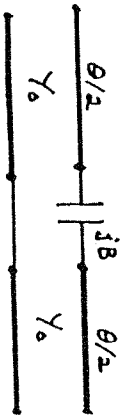
8.15

Quarter-wave line ($-\lambda/4$ long, since $\theta < 0$)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix} \quad j^{-2} = 1$$

ADMITTANCE INVERTER:

$$j \sqrt{\frac{B}{A}}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \theta/2 & jY_0 \sin \theta/2 \\ jY_0 \sin \theta/2 & \cos \theta/2 \end{bmatrix} \begin{bmatrix} 1 & j/B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta/2 & jY_0 \sin \theta/2 \\ jY_0 \sin \theta/2 & \cos \theta/2 \end{bmatrix}$$

$$= \begin{bmatrix} (\cos \theta + \frac{Y_0}{2B} \sin \theta) & j(\frac{1}{Y_0} \sin \theta - \frac{1}{B} \cos^2 \frac{\theta}{2}) \\ jY_0 (\sin \theta + \frac{Y_0}{B} \sin^2 \frac{\theta}{2}) & (\cos \theta + \frac{Y_0}{2B} \sin \theta) \end{bmatrix} \quad \checkmark$$

Equivalence requires the following conditions:

$$A, D: \cos \theta + \frac{Y_0}{2B} \sin \theta = 0 \Rightarrow \theta = -\tan^{-1} \left(\frac{2B}{Y_0} \right) < 0 \quad \checkmark$$

$$B: \frac{Y_0}{J} = -\sin \theta + \frac{Y_0}{B} \cos^2 \theta/2$$

$$C: \frac{J}{Y_0} = -\sin \theta - \frac{Y_0}{B} \sin^2 \theta/2$$

$$\ominus \frac{Y_0}{J} - \frac{J}{Y_0} = \frac{Y_0}{B}$$

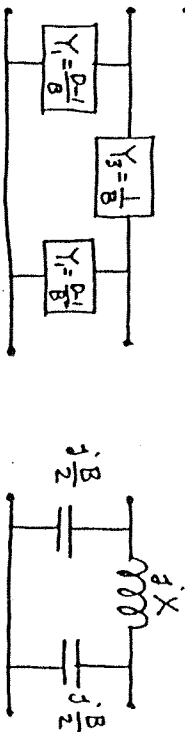
$$\text{or, } B = \frac{Y_0}{\frac{Y_0}{J} - \frac{J}{Y_0}} = \frac{J}{1 - (J/Y_0)^2} \quad \checkmark$$

$$\text{also, } \tan |\theta| = \frac{2B}{Y_0} = \frac{2(J/Y_0)}{1 - (J/Y_0)^2} = \frac{2 \tan |\frac{\theta}{2}|}{1 - \tan^2 |\frac{\theta}{2}|}$$

$$\text{or, } \tan |\frac{\theta}{2}| = J/Y_0 \quad \checkmark$$

8.16

The easiest way to do this problem is to use the π -network of Table 4.1, with the shunt and series element values given in terms of the ABCD parameters:



Then using the ABCD parameters for a transmission line gives the equivalent circuit elements as,

$$jX = B = jZ_0 \sin \beta l$$

$$j\frac{B}{2} = \frac{D-1}{B} = \frac{\cos \beta l - 1}{jZ_0 \sin \beta l} = \frac{2}{Z_0} \tan \beta l / 2$$

For $\beta l < \pi/4$ and large Z_0 , these results reduce to:

$$X \approx Z_0 \beta l \quad \checkmark$$

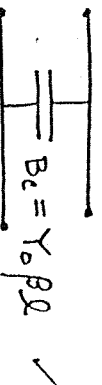
$$\frac{B}{2} \approx 0 \quad \checkmark$$



For $\beta l < \pi/4$ and small Z_0 , these results reduce to:

$$X \approx 0$$

$$\frac{B}{2} \approx \frac{\beta l}{2Z_0}$$



(NOTE: This problem can also be done using g -parameters, but a sign change for g_2 will be required because of the reference direction for I_2 for the ABCD parameters.)

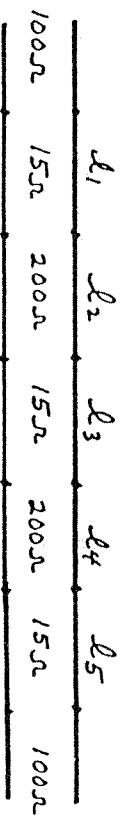
8.17

$f_0 = 4 \text{ GHz}$, $N = 5$, L.P., 0.5 dB E.R., $Z_0 \approx 100 \Omega$

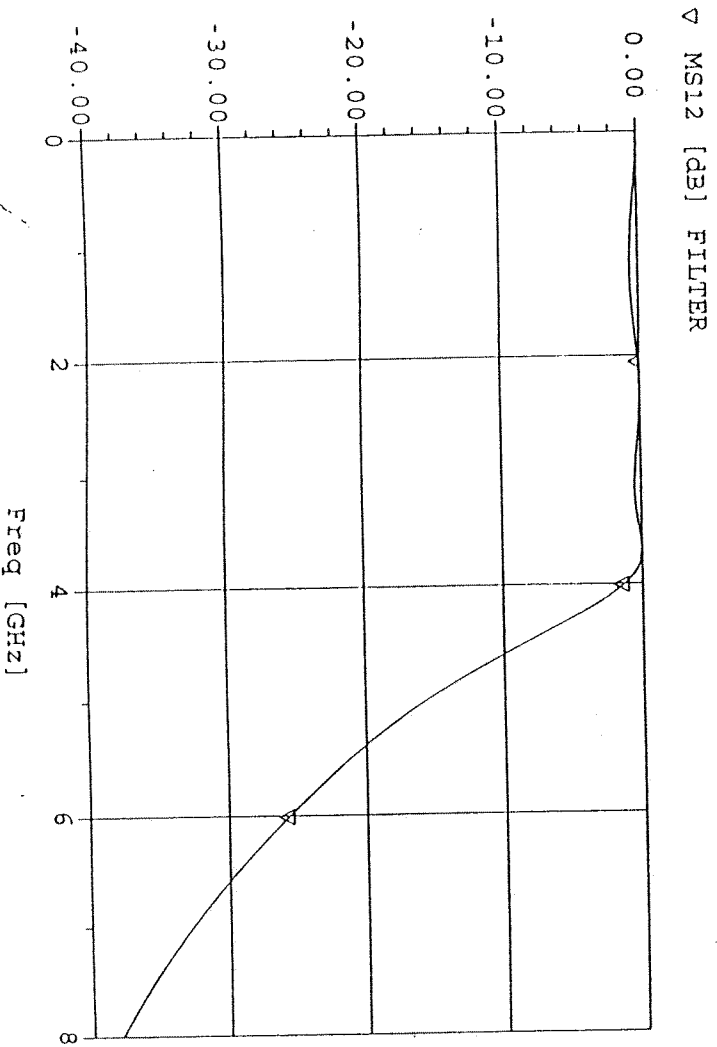
From Table 8.4 and (8.86), with $Z_L = 15 \Omega$ and $Z_H = 200 \Omega$,

$g_1 = 1.7058 = C_1$	\Rightarrow	$\beta_{L1} = g_1 Z_L / Z_0 = 14.7^\circ$	\checkmark
$g_2 = 1.2296 = L_2$	\Rightarrow	$\beta_{L2} = g_2 Z_0 / Z_H = 35.2^\circ$	\checkmark
$g_3 = 2.5408 = C_3$	\Rightarrow	$\beta_{L3} = g_3 Z_L / Z_0 = 21.8^\circ$	\checkmark
$g_4 = 1.2296 = L_4$	\Rightarrow	$\beta_{L4} = g_4 Z_0 / Z_H = 35.2^\circ$	\checkmark
$g_5 = 1.7058 = C_5$	\Rightarrow	$\beta_{L5} = g_5 Z_L / Z_0 = 14.7^\circ$	\checkmark

NOTE: $\beta_L < 45^\circ$ for all cases (lengths at $f = 4 \text{ GHz}$)



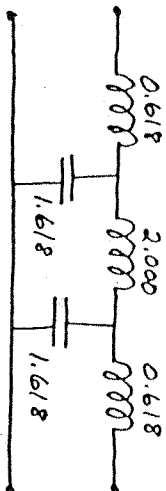
The calculated filter response is shown below:



8.18

$f_0 = 2 \text{ GHz}$, L.P., M.F., $Z_0 = 50 \Omega$

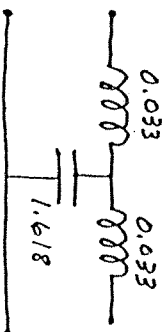
From Table 8.3 the LP prototype is,



The shunt capacitors can be implemented with a length of Z_1 line. Using (8.83b) gives,

$$B = \frac{1.618}{Z_0} = \frac{\sin \beta l}{Z_1} \Rightarrow \beta l = 18.9^\circ$$

$$\frac{1}{1.618} \Rightarrow \frac{18.9^\circ}{Z_1 = 10 \Omega} \Rightarrow$$



The model for this line (see Figure 8.39) shows inductors on either side, with values given by (8.83a):

$$\frac{X}{Z_0} = \frac{Z_0}{Z_1} \tan \beta l / 2 = 0.033$$

Then the end inductor of value 0.618 can be implemented as length of Z_1 line. Using (8.83a) gives,

$$0.618 - 0.033 = 0.585$$

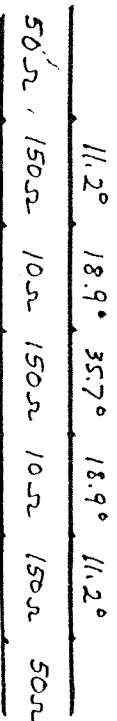
$$\frac{11.2^\circ}{Z_1 = 150 \Omega}$$

and the middle inductor of value 2.000 can be implemented

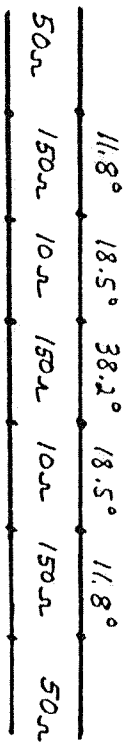
$$\text{as, } 2 - 2(0.033) = 1.934$$

$$\frac{35.7^\circ}{Z_1 = 150 \Omega}$$

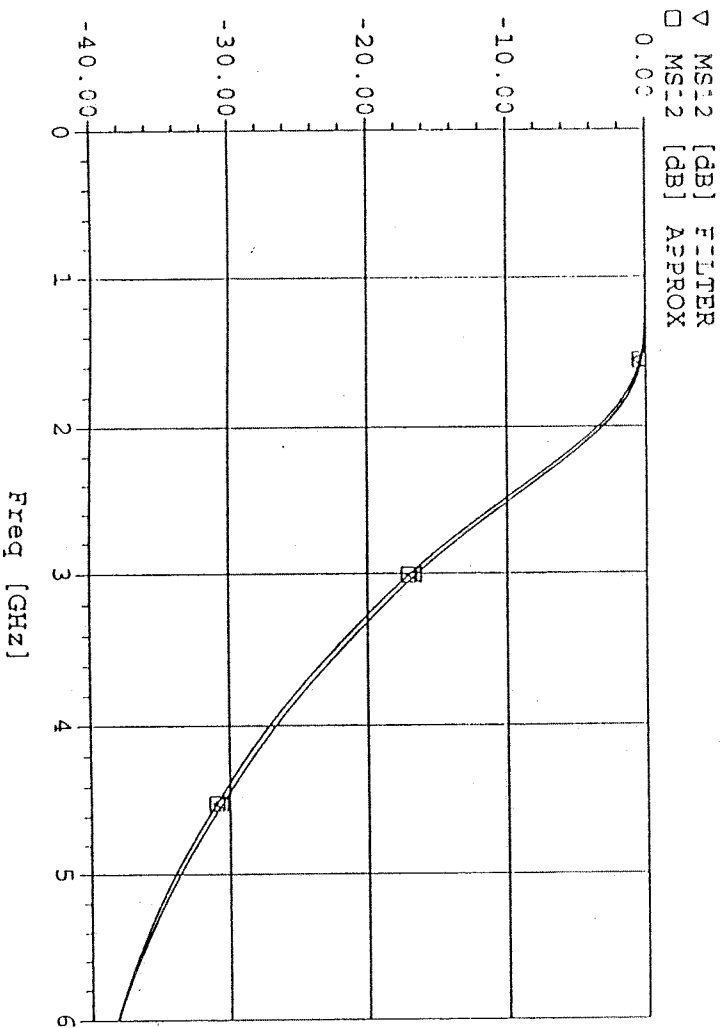
The final filter:



For comparison, the design using the approximations of (8.84) and (8.85) is,



Note that only the middle section differs very much in these two designs. The calculated filter response is shown below for both designs. Note that there is very little difference.



8.19

3.00-3.50 GHz, B.P., M.F., N=3, Z₀ = 50Ω

$$f_0 = 3.25 \text{ GHz}; \quad \Delta = \frac{3.5 - 3.0}{3.25} = 0.154$$

Use (8.71) to transform 2.9 GHz to normalized LP form:

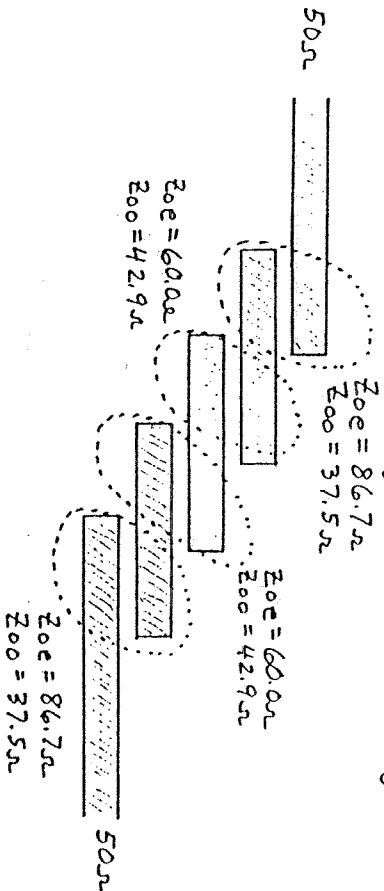
$$\omega \leftarrow \frac{1}{\Delta} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{0.154} \left(\frac{2.9}{3.25} - \frac{3.25}{2.9} \right) = -1.48$$

Then, $\left| \frac{\omega}{\omega_0} \right| - 1 = 0.48$, and Figure 8.26 gives $\alpha = 10.5 \text{ dB}$ ✓

The L.P. prototype values are given in Table 8.3, and Z_{0In} can be found from (8.121). Then Z_{0e} and Z_{0o} follow from (8.108).

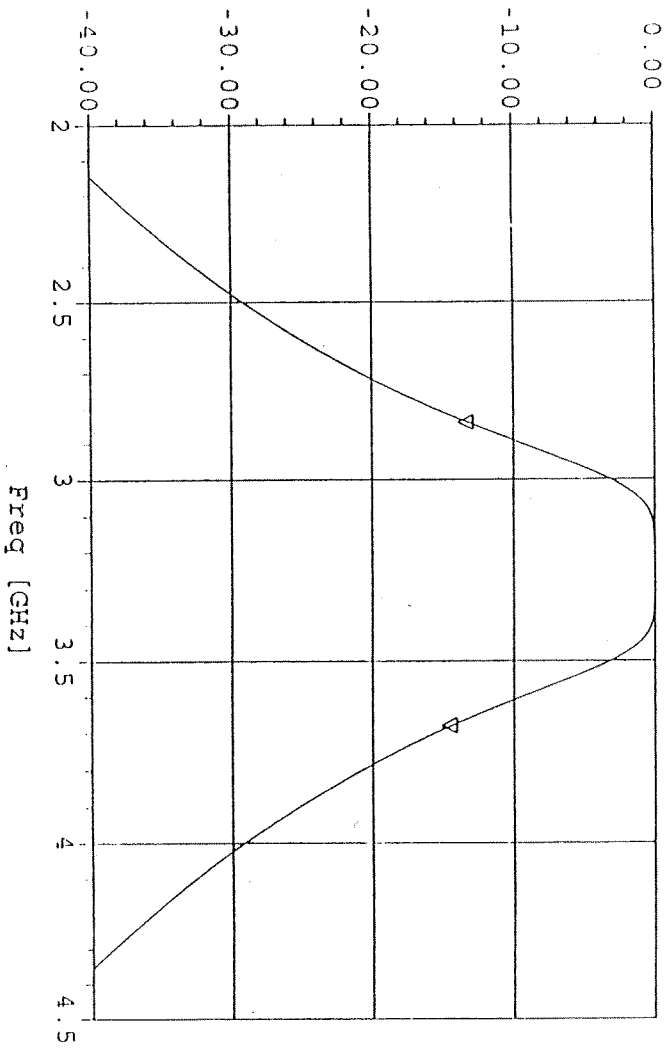
N	g _N	Z _{0In}	Z _{0e} (Ω)	Z _{0o} (Ω)
1	1.000	0.492	86.7	37.5
2	2.000	0.171	60.0	42.9
3	1.000	0.171	60.0	42.9
4	1.000	0.492	86.7	37.5

all lines are λ/4 long at 3.25 GHz.



The calculated filter response is shown on the following page. The insertion loss at 2.9 GHz is a bit less than 10 dB.

▽ MS12 [dB] FILTER



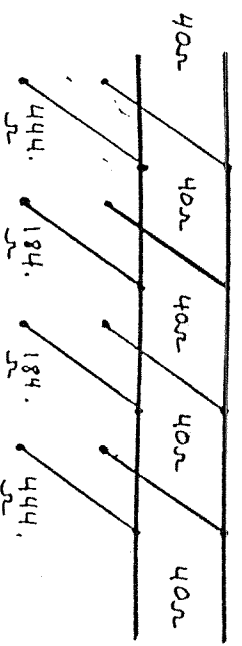
8.20

$f_0 = 3 \text{ GHz}$, B.S., M.F., $N=4$, $\Delta=0.15$, $Z_0 = 40 \Omega$.

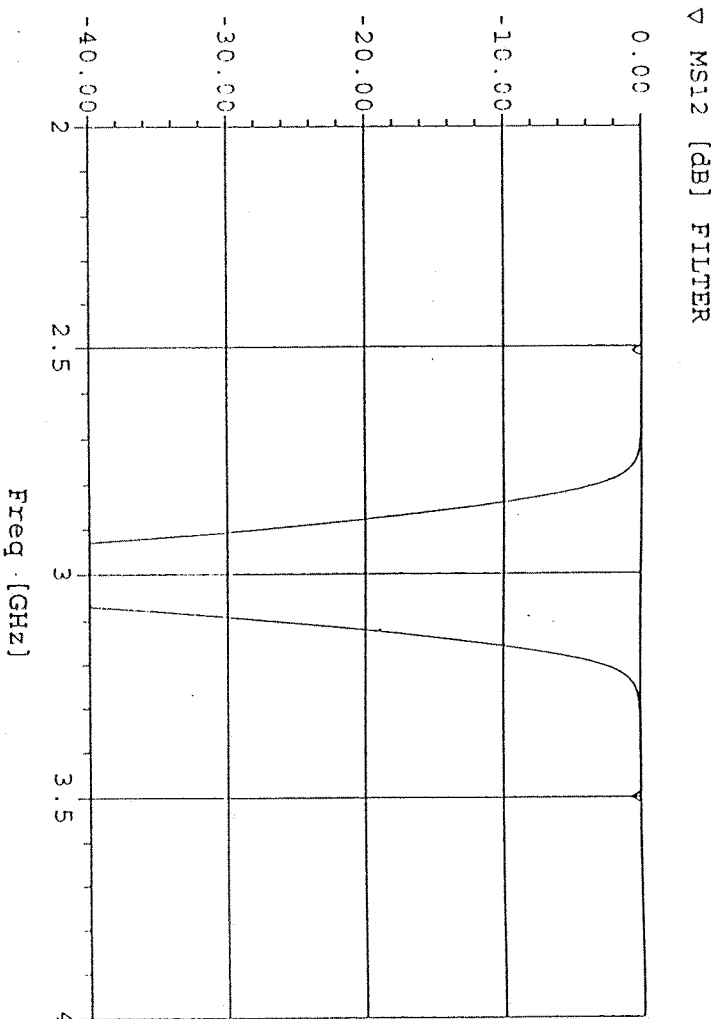
We find the g_n values from Table 8.3. Then the $\lambda/4$ O.C. stub characteristics' impedances can be found from (8.130):

n	g_n	$Z_{0n}(\Omega)$
1	0.765	444.
2	1.848	184.
3	1.848	184.
4	0.765	444.

(Z_{01} and Z_{04} may be too high to be practical)
 The final filter is shown below:



All lines and stubs are $\lambda/4$ long at 3 GHz. The calculated filter response is shown below:



8.21

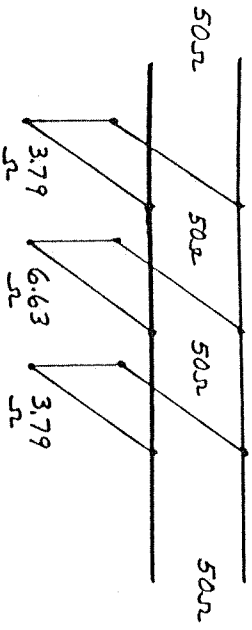
3.00 - 3.50 GHz, $N=3$, B.P., 0.5 dB E.R., $Z_0 = 50 \Omega$
 $f_0 = 3.25 \text{ GHz}$; $\Delta = 0.154$

We find g_n from Table 8.4. Then the $\lambda/4$ short-circuit stub characteristic impedances can be found from

(8.131):

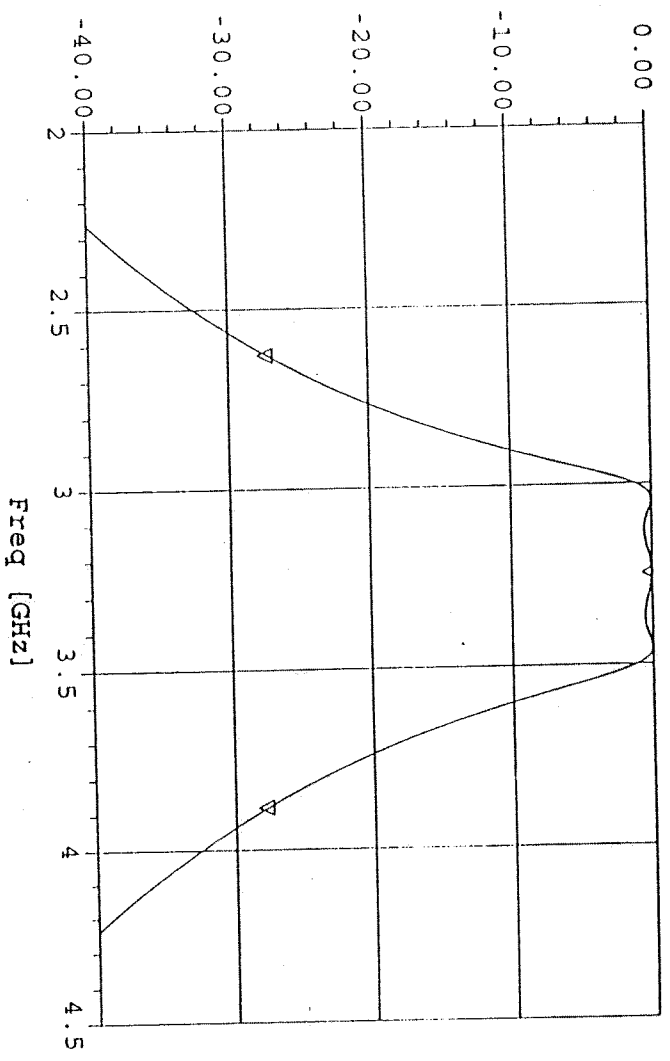
n	g_n	$Z_{0n} (\Omega)$	
1	1.5963	3.79	$Z_{0n} = \frac{\pi Z_0 \Delta}{4 g_n}$
2	1.0967	6.63	
3	1.5963	3.79	

(These impedances may be too low to be practical.)
 The complete filter is shown below:



The calculated filter response is shown below.

▽ MS21 [dB] FILTER



8.22 The bandpass $\lambda/4$ resonator filter uses short-circuited stubs, which have an input admittance of,

$$Y = -jY_{0n} \cot \theta, \text{ where } \theta = \pi/2 \text{ for } \omega = \omega_0.$$

Let $\omega = \omega_0 + \Delta\omega$. Then $\theta = \pi/2 (1 + \Delta\omega/\omega_0)$, and

$$Y = jY_{0n} \tan \frac{\pi \Delta\omega}{2\omega_0} \approx \frac{j\pi Y_{0n} (\omega - \omega_0)}{2\omega_0}.$$

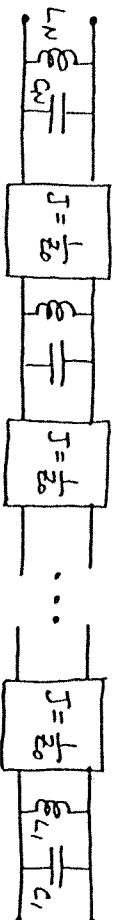
Now the admittance of a parallel LC resonator is, from Table 6.1,

$$Y \approx j\omega C_n (\omega - \omega_0).$$

So the characteristic impedance of the stub is,

$$Z_{0n} = \frac{1}{Y_{0n}} = \frac{\pi}{4\omega_0 C_n}$$

The filter circuit can be redrawn as follows:



This is the same as the circuit in part (e) of Figure 8.45, for coupled line bandpass filters (with $Z_0 \gamma_n = 1$). Correspondance with the lumped-element bandpass filter requires that,

$$\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C'_1}{L'_1}}$$

$$Z_0^2 \sqrt{\frac{C_2}{L_2}} = \sqrt{\frac{L'_2}{C'_2}}$$

where C'_n and L'_n are the lumped element filter values, and $L_n C_n = L'_n C'_n = 1/\omega_0^2$. Solving for C_n gives,

$$C_1 = C'_1$$

$$C_2 = \frac{1}{\omega_0^2 Z_0^2 C'_2}$$

Using Table 8.6 to transform back to LP prototype values gives,

$$C_1 = C'_1 = \frac{g'_1}{\Delta \omega_0 Z_0}$$

$$C_2 = \frac{1}{\omega_0^2 Z_0^2} \left(\frac{\omega_0 g'_2 Z_0}{\Delta} \right) = \frac{g'_2}{\Delta \omega_0 Z_0}$$

So the characteristic impedances are,

$$Z_{0n} = \frac{\pi}{4\omega_0} \left(\frac{\Delta \omega_0 Z_0}{g'_n} \right) = \frac{\pi \Delta Z_0}{4 g'_n} \quad \checkmark$$

which agrees with (8.131).

8.23

$f_0 = 4 \text{ GHz}$, $\Delta = 0.12$, B.P., M.F., $Z_0 = 50 \Omega$

First transform 3.6 GHz to L.P. prototype form:

$$\omega \leftarrow \frac{1}{\Delta} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{0.12} \left(\frac{3.6}{4} - \frac{4}{3.6} \right) = -1.76$$

Then,

$$\left| \frac{\omega}{\omega_0} \right| - 1 = 0.76$$

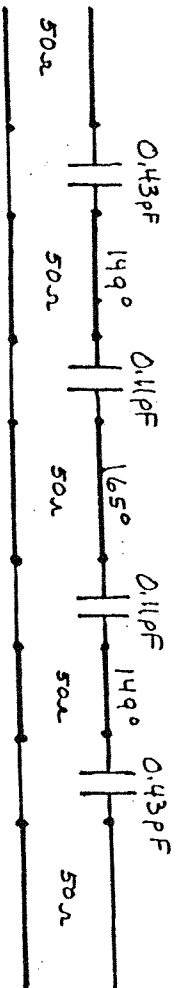
So from Figure 8.26 we see that $N=3$ is required to achieve $\alpha > 12 \text{ dB}$ at 3.6 GHz . (or, analytically, $P_{LR} = 1 + (\omega/\omega_0)^{2N} = 14.9 \text{ dB}$ for $N=3$)

The prototype values are given in Table 8.3. Then $Z_0 J_n$ can be found from (8.121). Then, $Z_0 B_n = \frac{J_n Z_0}{1 - (J_n Z_0)^2}$ and $\theta_n = \pi - \frac{1}{2} [\tan^{-1}(2Z_0 B_n) + \tan^{-1}(2Z_0 B_{n+1})]$. Also,

$$C_n = B_n / \omega_0.$$

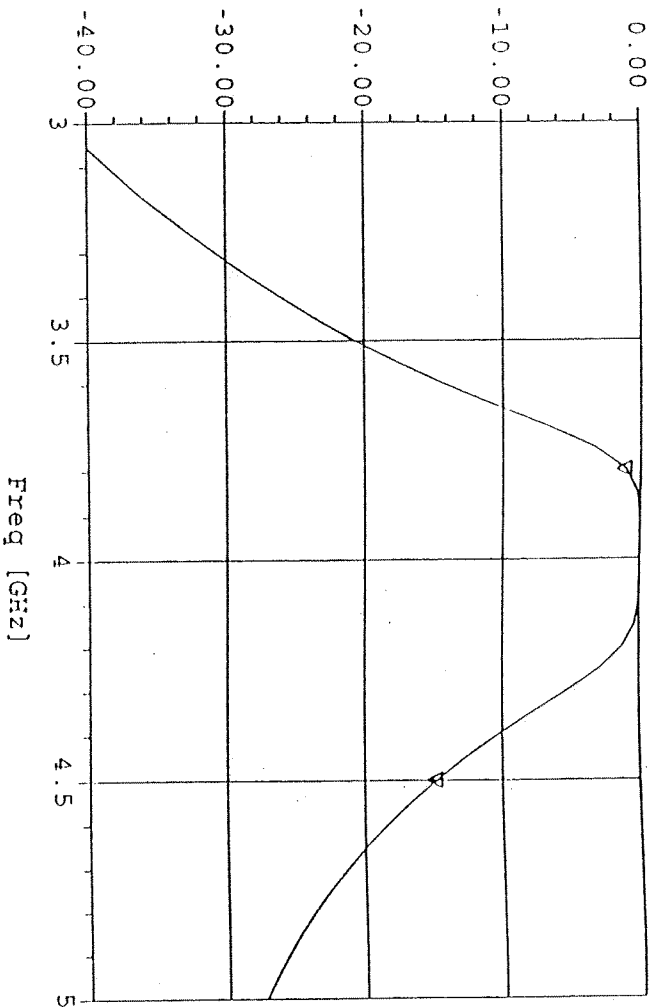
n	g_n	$Z_0 J_n$	$Z_0 B_n$	$C_n \text{ (PF)}$	$\theta_n @ 4 \text{ GHz}$
1	1.000	0.434	0.535	0.443	149°
2	2.000	0.133	0.135	0.11	165°
3	1.000	0.133	0.135	0.11	149°
4	1.000	0.434	0.535	0.443	—

The filter circuit is shown below:



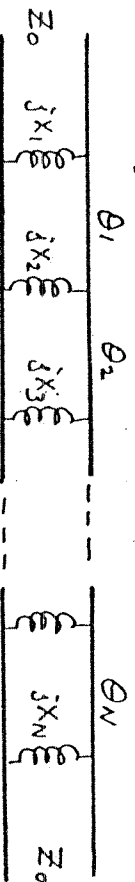
The calculated filter response is shown on the following page.

▽ MS21 [dB] FILTER

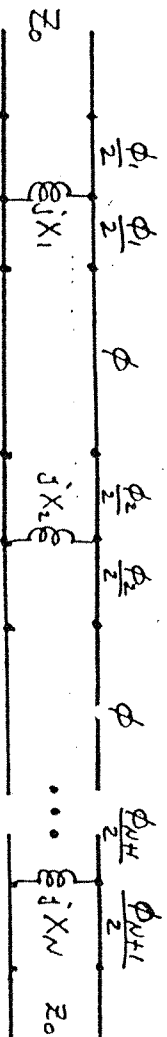


8.24

The equivalent circuit is,

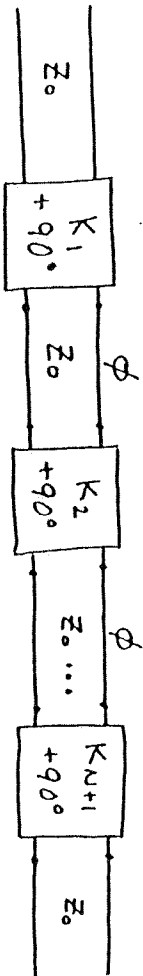


which can be redrawn as,



where $\theta_i = \pi + \frac{1}{2}\phi_i + \frac{1}{2}\phi_{i+1}$ for $i=1, 2, \dots, N$

From Figure 8.38, the circuit can be redrawn using impedance inverters:



From the properties of the impedance inverter, we have,
 $\phi_i = -\tan^{-1} \left(\frac{2X_i}{Z_0} \right)$ ✓

$$X_i = \frac{K_i}{1 - (K_i/Z_0)^2} \quad \checkmark$$

Then,
 $\theta_i = \pi - \frac{1}{2} \left[\tan^{-1} (2X_i/Z_0) + \tan^{-1} (2X_{i+1}/Z_0) \right]$ ✓
 The inverter constants are taken from (8.121):
 $(J = K/Z_0^2)$

$$\frac{K_1}{Z_0} = \sqrt{\frac{\pi \Delta}{2g_1}}$$

$$\frac{K_n}{Z_0} = \frac{\pi \Delta}{2 \sqrt{g_n g_{n-1}}}$$

$$\frac{K_{N+1}}{Z_0} = \sqrt{\frac{\pi \Delta}{2g_N g_{N+1}}}$$

Note: Δ should be the fractional bandwidth in terms of guide wavelength:

$$\Delta = \frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g0}} \approx \left(\frac{\lambda_{g0}}{\lambda_0} \right)^2 \left(\frac{\Delta \omega}{\omega_0} \right)$$

Chapter 9

9.1

$F = 10 \text{ GHz}$, $4\pi M_s = 1780 \text{ G}$
 From (9.24) - (9.25),

$$[u] = \begin{bmatrix} u & jX & 0 \\ -jX & u & 0 \\ 0 & 0 & u_0 \end{bmatrix}$$

where,

$$u = u_0 \left(1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \right), \quad X = u_0 \frac{\omega \omega_m}{\omega_0^2 - \omega^2}$$

1) $H_0 = M_s = 0 \implies \omega_b = \omega_m = 0 \implies u = u_0, X = 0$

As,

$$[u] = \begin{bmatrix} u_0 & 0 & 0 \\ 0 & u_0 & 0 \\ 0 & 0 & u_0 \end{bmatrix} \checkmark$$

2) $H_0 = 1000 \text{ Oe}$

Then, $f_0 = 2.8 \frac{\text{MHz}}{\text{Oe}} (1000 \text{ Oe}) = 2.8 \text{ GHz}$

$$F_m = 2.8 \frac{\text{MHz}}{\text{Oe}} (1780 \text{ G}) \left(\frac{10 \text{ Oe}}{1 \text{ G}} \right) = 4.98 \text{ GHz}$$

As,

$$u = u_0 \left(1 + \frac{f_0 f_m}{f_0^2 - f^2} \right) = 0.849 u_0$$

$$X = u_0 \frac{f f_m}{f_0^2 - f^2} = -0.540 u_0$$

$$[u] = \begin{bmatrix} 0.849 & -j0.540 & 0 \\ j0.540 & 0.849 & 0 \\ 0 & 0 & 1 \end{bmatrix} u_0 \checkmark$$

9.2

$$\begin{aligned} B_x &= \mu H_x + j\chi H_y \\ B_y &= -j\chi H_x + \mu H_y \\ B_z &= \mu_0 H_z \end{aligned}$$

Then,

$$\begin{aligned} B^+ &= \frac{1}{2}(B_x + jB_y) = \frac{1}{2}[\mu H_x + j\chi H_y + \chi H_x + j\mu H_y] \\ &= \frac{1}{2}(\mu + \chi)(H_x + jH_y) = (\mu + \chi)H^+ \\ B^- &= \frac{1}{2}(B_x - jB_y) = \frac{1}{2}[\mu H_x + j\chi H_y - \chi H_x - j\mu H_y] \\ &= \frac{1}{2}(\mu - \chi)(H_x - jH_y) = (\mu - \chi)H^- \end{aligned}$$

Thus,

$$\begin{bmatrix} B^+ \\ B^- \\ B_z \end{bmatrix} = \begin{bmatrix} (\mu + \chi) & 0 & 0 \\ 0 & (\mu - \chi) & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \begin{bmatrix} H^+ \\ H^- \\ H_z \end{bmatrix} \quad \checkmark$$

9.3

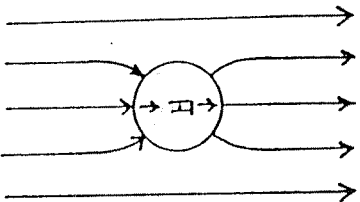
$$4\pi M_s = 1780 \text{ G}, \quad H_e = 1200 \text{ Oe}$$

From (9.41) the internal field is,

$$\bar{H} = \bar{H}_e - N\bar{M}$$

From Table 9.1 the demagnetization factors for a sphere are $N_x = N_y = N_z = 1/3$, so

$$\begin{aligned} H_z &= H_e - N_z M_z = 1200 - \frac{1}{3}(1780) \\ &= 607 \text{ Oe.} \end{aligned}$$



9.4

$$4\pi M_s = 600 \text{ G}$$

From Table 9.1 the demagnetization factors for a thin rod are $N_x = N_y = \frac{1}{2}$, $N_z = 0$. Then (9.46) gives the gyromagnetic resonance frequency as,

$$\begin{aligned} \omega_r &= \mu_0 \gamma \sqrt{(H_a + \frac{1}{2} M_s)(H_a + \frac{1}{2} M_s)} \\ &= \mu_0 \gamma (H_a + \frac{1}{2} M_s) \\ &= \mu_0 \gamma (1300 \text{ Oe}) \left(\frac{1 \text{ A/m}}{4\pi \times 10^{-3} \text{ Oe}} \right) \\ f_r &= \frac{\omega_r}{2\pi} = \frac{\mu_0 \gamma}{2\pi} \left(\frac{1 \text{ A/m}}{4\pi \times 10^{-3} \text{ Oe}} \right) (1300 \text{ Oe}) \\ &= 2.8 \frac{\text{MHz}}{\text{Oe}} (1300 \text{ Oe}) = 3.64 \text{ GHz} \end{aligned}$$

9.5

$$4\pi M_s = 1200 \text{ G}, \epsilon_r = 10, H_0 = 500 \text{ Oe}, f = 8 \text{ GHz}$$

(FARADAY ROTATION)

$$\begin{aligned} f_0 &= (2.8 \frac{\text{MHz}}{\text{Oe}}) (500 \text{ Oe}) = 1.4 \text{ GHz} \\ f_m &= (2.8 \frac{\text{MHz}}{\text{Oe}}) (1200 \text{ G}) = 3.36 \text{ GHz} \\ k_0 &= 167.6 \text{ m}^{-1} \end{aligned}$$

Then from (9.25),

$$\begin{aligned} \mu &= \mu_0 \left(1 + \frac{f_0 f_m}{f_0^2 - f^2} \right) = 0.924 \mu_0 \checkmark \\ K &= \mu_0 \frac{f f_m}{f_0^2 - f^2} = -0.433 \mu_0 \checkmark \end{aligned}$$

From (9.52) the propagation constants of the CP waves are,

$$\begin{aligned} \text{RHCP: } \beta_+ &= \omega \sqrt{\epsilon(\mu+K)} = k_0 \sqrt{\epsilon_r} \sqrt{0.924 - 0.433} = 371.4 \text{ m}^{-1} \checkmark \\ \text{LHCP: } \beta_- &= \omega \sqrt{\epsilon(\mu-K)} = k_0 \sqrt{\epsilon_r} \sqrt{0.924 + 0.433} = 617.4 \text{ m}^{-1} \checkmark \end{aligned}$$

$$\text{Then, } \Delta\beta = \beta_+ - \beta_- = -246.0 \text{ m}^{-1}$$

From (9.57) the polarization rotation of an LP wave is,

$$\begin{aligned} \phi &= -(\beta_+ - \beta_-) z / 2 \\ \text{As, } z &= \frac{2\phi}{\beta_- - \beta_+} = \frac{2(\pi/2)}{246 \text{ rad/m}} = 12.8 \text{ mm} \checkmark \end{aligned}$$

9.6

$4\pi M_s = 1200 \text{ G}$, $\epsilon_r = 10$, $H_0 = 2000 \text{ Oe}$, $f = 4 \text{ GHz}$
(BIREFRINGENCE)

$$f_0 = (2.8 \text{ MHz/Oe})(2000 \text{ Oe}) = 5.6 \text{ GHz}$$

$$f_m = (2.8 \text{ MHz/Oe})(1200 \text{ Oe}) = 3.36 \text{ GHz}$$

$$k_0 = 83.8 \text{ m}^{-1}$$

Then from (9.25),

$$u = u_0 \left(1 + \frac{f_0 f_m}{f_0^2 - f^2} \right) = 2.23 u_0$$

$$x = u_0 \frac{f f_m}{f_0^2 - f^2} = 0.875 u_0$$

The \hat{x} polarized wave has $\bar{H} = \hat{y} H_y$, and is the extraordinary wave. From (9.64) - (9.65),

$$u_e = \frac{u^2 - x^2}{u} = 1.89 u_0$$

$$\beta_e = \omega \sqrt{\epsilon} u_e = k_0 \sqrt{\epsilon_r} u_e / u_0 = 364.3 \text{ m}^{-1}$$

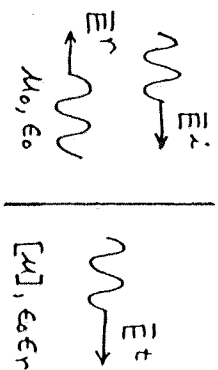
The \hat{y} polarized wave has $\bar{H} = \hat{x} H_x$, and is the ordinary wave. Thus,

$$\beta_0 = \sqrt{\epsilon_r} k_0 = 265.0 \text{ m}^{-1}$$

So the distance required for a differential phase shift of 270° is,

$$z = \frac{3\pi/2}{\beta_e - \beta_0} = 47.5 \text{ mm.}$$

9.7



The incident, reflected, and transmitted fields for a RHP are can be written as,

$$\begin{aligned} \vec{E}^i &= E_0 (\hat{x} - j\hat{y}) e^{j\beta_0 z} & \vec{H}^i &= \frac{E_0}{\eta_0} (\hat{y} + j\hat{z}) e^{j\beta_0 z} \\ \vec{E}^r &= \Gamma^+ E_0 (\hat{x} - j\hat{y}) e^{j\beta_0 z} & \vec{H}^r &= \frac{-E_0}{\eta_0} \Gamma^+ (\hat{y} + j\hat{z}) e^{j\beta_0 z} \\ \vec{E}^t &= T^+ E_0 (\hat{x} - j\hat{y}) e^{j\beta_0 z} & \vec{H}^t &= \frac{E_0}{\eta_0} T^+ (\hat{y} + j\hat{z}) e^{j\beta_0 z} \end{aligned}$$

where, $\beta_0 = \omega \sqrt{\mu_0 \epsilon_0}$, $\beta_+ = \omega \sqrt{\epsilon(\mu + K)}$, $\eta_+ = \frac{1}{\sqrt{\epsilon}} = \sqrt{\frac{\mu + K}{\epsilon}}$

Matching fields at $z=0$ gives (for both \hat{x} and \hat{y} components)

$$\begin{aligned} 1 + \Gamma^+ &= T^+ \\ \eta_+ (1 - \Gamma^+) &= \eta_0 T^+ \end{aligned}$$

Solving gives,

$$\Gamma^+ = \frac{\eta_+ - \eta_0}{\eta_+ + \eta_0}, \quad T^+ = \frac{2\eta_+}{\eta_+ + \eta_0} \quad \checkmark$$

Similarly, for a LHP we are obtain,

$$\Gamma^- = \frac{\eta_- - \eta_0}{\eta_- + \eta_0}, \quad T^- = \frac{2\eta_-}{\eta_- + \eta_0} \quad \checkmark$$

where,

$$\eta_- = \frac{1}{\sqrt{\epsilon}} = \sqrt{\frac{\mu - K}{\epsilon}}.$$

9.8

$$4\pi M_S = 1200 \text{ G}, \quad \bar{H}_0 = \hat{x} H_0, \quad f = 4 \text{ GHz}, \quad \mathbf{E} = \hat{x} E_0$$

$$f_M = (2.8 \text{ MHz/Oe})(1200 \text{ G}) = 336 \text{ GHz}$$

This is a case of birefringence. From (9.64)-(9.65),

$$\beta_e = \omega \sqrt{\mu \epsilon}$$

$$\mu_e = \frac{\mu^2 - K^2}{\mu}$$

The wave will be cutoff when $\mu_e \leq 0$:

$$\frac{\mu^2 - K^2}{\mu} < 0$$

$$\frac{\left(1 + \frac{f_0 f_M}{f_0^2 - f^2}\right)^2 - \left(\frac{f f_M}{f_0^2 - f^2}\right)^2}{1 + \frac{f_0 f_M}{f_0^2 - f^2}} < 0$$

$$\frac{(f_0^2 - f^2)^2 + 2f_0 f_M (f_0^2 - f^2) + f_M^2 (f_0^2 - f^2)}{(f_0^2 - f^2) [(f_0^2 - f^2) + f_0 f_M]} < 0$$

$$\frac{(f_0^2 - f^2) + 2f_0 f_M + f_M^2}{f_0^2 - f^2 + f_0 f_M} < 0$$

If $f_0^2 - f^2 + f_0 f_M > 0$, then $(f_0 + f_M)^2 - f^2 < 0$
 $f_0 + f_M < f$

$$f_0 < f - f_M = 4.0 - 3.36 = 0.64 \text{ GHz} \Rightarrow H_0 = 2290 \text{ Oe} \quad \checkmark$$

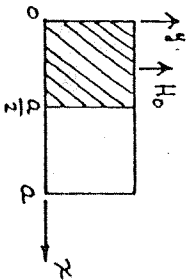
If $f_0^2 - f^2 + f_0 f_M < 0$, then $(f_0 + f_M)^2 - f^2 > 0$

$$f_0 = \frac{-f_M \pm \sqrt{f_M^2 + 4f^2}}{2} = -1.68 \pm 4.34 = 2.66 \text{ GHz}$$

$$H_0 = 950 \text{ Oe} \quad \checkmark$$

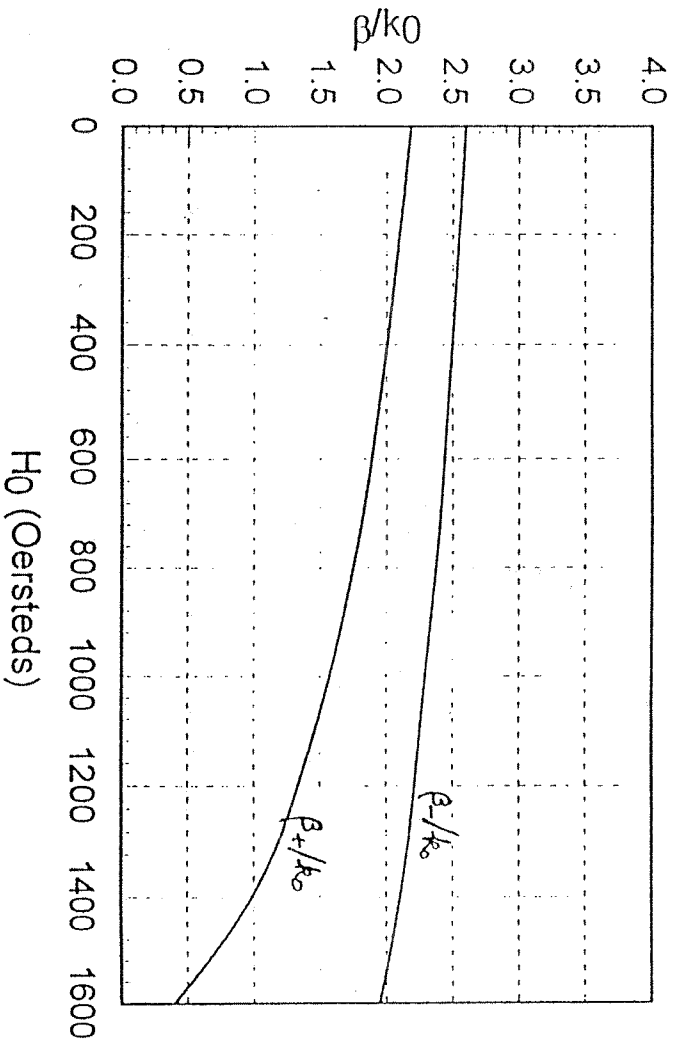
So the cutoff range is between 2290e and 9500e.

9.9



$a = 1.0 \text{ cm}$
 $F = 10 \text{ GHz}$
 $4\pi M_S = 1700 \text{ G}$
 $G_r = 13$

The propagation constants are found by solving (9.79) numerically. This was done using the computer program listed below, with the following results:



Computer Program Listing for the solution of Problem 9.9 :

```

c plot cp propagation coefficients for ferrite
c loaded waveguide with one slab
c Ho in Oersted's
c 4PIms in Gauss
c delH in Oersted's
c a is width of guide
c c is left-hand side of slab
c t is slab thickness

real ms
complex w0,cu,ck,cue,epsc,betp
common a,c,t,cu,cue,ck,xk0,epsc
external func
fmc=10000.
eps=13.
tand=.001
ms=1700.
delh=.001
a=.01
c=1.e-8
t=.5*a
pi=3.1415926
c calculate constants
xk0=2.*pi*fmc/300.
w=2.*pi*fmc*1.e6
epsc=eps*cplx(1.,-tand)
do 10 i=1,9
  h0=(i-1)*200.
  write(6,*) 'Ho=',h0
  do 10 k=1,2
    w0=2.8e6*(h0+(0.,1.)*delh/2.)*(2.*pi)
    wm=2.8e6*ms*(2.*pi)
    cu=1.+w0*wm/(w0*w0-w*w)
    ck=w*wm/(w0*w0-w*w)
    cue=(cu*cu-ck*ck)/cu
    if(x.eq.2) ck=-ck
    call root(func,xk0,(3,-.00001)*xk0,(3.,0.)*xk0,betp)
    write(6,20) c/a,real(betp)/xk0,-aimag(betp)*8.7/100.
    format(' c/a=',f7.3,' bet/xk0=',f10.5,' alp(dB/cm)=' ,e12.5)
  continue
10
20
10
stop
end
complex function func(bet)
common a,c,t,cu,cue,ck,xk0,epsc
complex cu,cue,ck,epsc,xka,xkf,ctc,ctd,ctt,bet
xka=csqrt((1.,0.)*xk0*xk0-bet*bet)
xkf=csqrt((1.,0.)*xk0*xk0*cue*epsc-bet*bet)
if(aimag(xka).lt.0.) xka=-xka
if(aimag(xkf).lt.0.) xkf=-xkf
d=a-c-t
ctc=ccos(xka*c)/csin(xka*c)
ctd=ccos(xka*d)/csin(xka*d)
ctt=ccos(xkf*t)/csin(xkf*t)
func=(xkf/cue)**2+(ck*bet/cu/cue)**2
func=func-xka*ctc*(xkf*ctt/cue-ck*bet/cu/cue)
func=func-xka*xka*ctc*(ck*bet/cu/cue-xkf*ctt/cue)
return
end

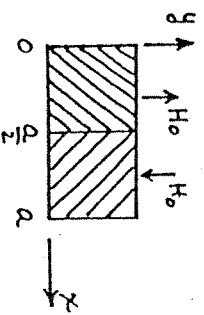
```

```

SUBROUTINE root(func, xk0, BET1, BET2, broot)
  COMPLEX bet1, bet2, bc, bcn, bco, broot, f, fo, fn, fd, fp, func
  NDIV=600*real((BET2-BET1))/XK0
  DB=(BET2-BET1)/NDIV
  DO 30 I=1, NDIV+1
    BCN=1.00001*BET1+(I-1)*DB
    fn=func(bcn)
    IF(I.EQ.1) GOTO 20
    if(real(fo)*real(fn).gt.0. .and. aimag(fo)*aimag(fn).gt.0.) goto 20
  C A ZERO CROSSING EXISTS BETWEEN BCO AND BCN
  C USE INTERVAL HALVING AND INTERPOLATION TO FIND COMPLEX ROOT
  EPSILON=1.E-6
  DEL=.0001*XK0
  ITER=0
  C COMPUTE NUMERICAL DERIVATIVE OF DENOMINATOR
  BC=BCN
  F=FN
  fd=func(BC+DEL)
  FP=(FD-F)/DEL
  BC=BC-F/FP
  f=func(BC)
  IF(CABS(F/FP).LT.EPSILON*XK0) GOTO 70
  ITER=ITER+1
  IF(ITER.GT.100) WRITE(6,74)
  FORMAT(' ITERATIONS EXCEEDED-NO ROOT FOUND-CONTINUING SEARCH')
  IF(ITER.GT.100) GOTO 20
  GOTO 40
  C ROOT AT BC
  70 continue
  broot=bc
  return
  C RETURN TO DO LOOP
  20 FO=FN
  BCO=BCN
  30 CONTINUE
  c no root found
  return
end

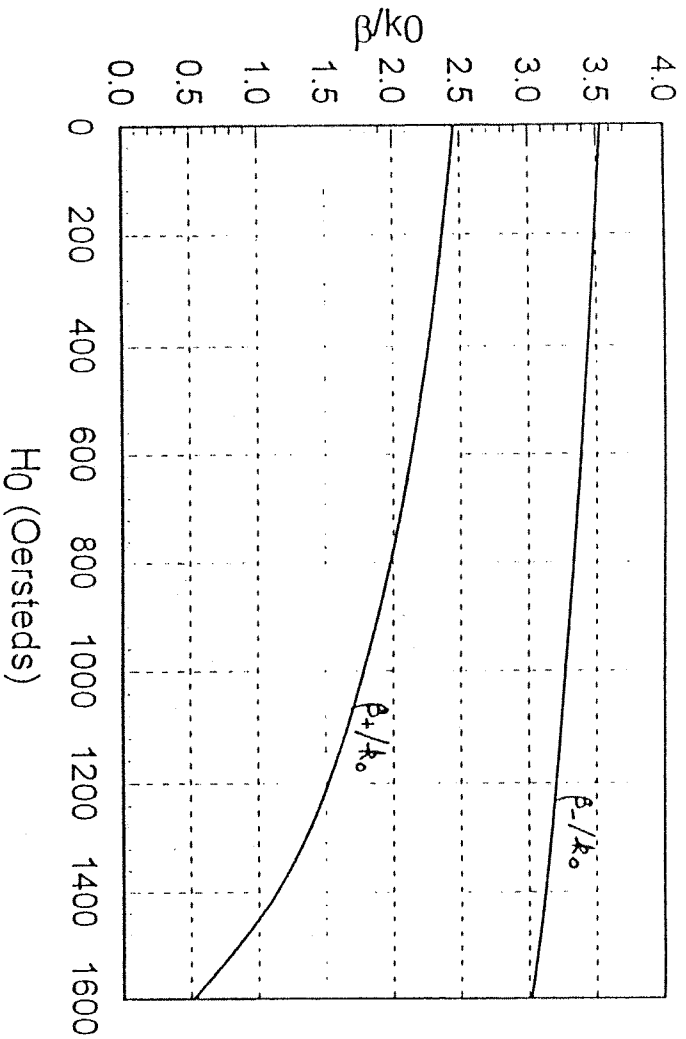
```

9.10

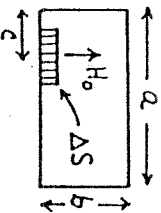


$a = 1.0 \text{ cm}$
 $f = 10 \text{ GHz}$
 $4\pi Ms = 1700 \text{ G}$
 $\epsilon_r = 13$

The propagation constants were found by solving (9.84) numerically using a computer program similar to the one listed above for problem 9.9. Results are shown on the following page.



9.11



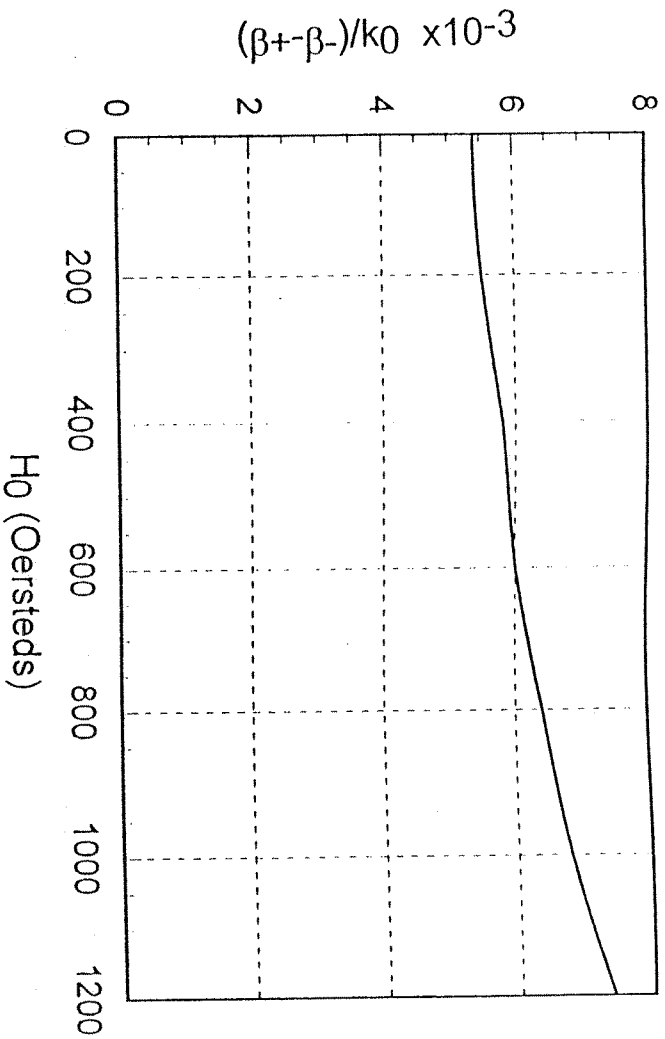
$f = 10 \text{ GHz}$, $a = 2.286 \text{ cm}$, $b = 1.016 \text{ cm}$
 $4\pi M_s = 1700 \text{ G}$, $c = a/4$, $\Delta S = 2\pi \text{ mrad}$
 $k_0 = 209.4 \text{ m}^{-1}$, $S = 232.3 \text{ mrad}^2$, $f_M = 4.76 \text{ GHz}$ ✓

From (9.80) the differential phase shift is,

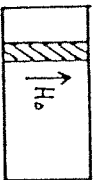
$$(\beta_+ - \beta_-)/k_0 = -2 \frac{k_c}{k_0} \frac{X}{S} \Delta S \sin 2k_c c = -0.0113 X/S$$

H_0 (Oe)	f_0 (GHz)	X/μ_0	S/μ_0	$(\beta_+ - \beta_-)/k_0$
0	0	-0.476	1.000	0.0054 ✓
200	0.56	-0.477	0.973	0.0055
400	1.12	-0.482	0.946	0.0058
600	1.68	-0.489	0.918	0.0060
800	2.24	-0.501	0.891	0.0064
1000	2.80	-0.516	0.855	0.0068
1200	3.36	-0.537	0.820	0.0074 ✓

This data is plotted on the following page



9.12



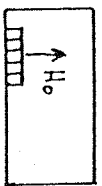
$$f = 8 \text{ GHz}, \quad 4\pi M_S = 1500 \text{ G}, \quad f_M = 4.2 \text{ GHz}$$

Magnon resonance for this geometry is given approximately by (9.87): $(N_x=1, N_y=N_z=0)$

$$f = \sqrt{f_0(f_0 + f_M)}$$

$$\text{Solve for } f_0: \quad f_0^2 + 4.2f_0 - 64 = 0 \Rightarrow f_0 = -2.1 \pm 8.27 = 6.17 \text{ GHz}$$

$$H_0 = \frac{6170 \text{ MHz}}{2.8 \text{ MHz/Oe}} = 2204 \text{ Oe. } \checkmark$$

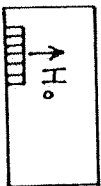


Magnon resonance for this geometry is given by the condition,

$$f_0 = f = 8 \text{ GHz} \quad (N_x = N_z = 0, N_y = 1)$$

$$H_0 = \frac{8000 \text{ MHz}}{2.8 \text{ MHz/Oe}} = 2857. \text{ Oe } \checkmark$$

9,13



$f_0 = 10 \text{ GHz}$, $4\pi M_S = 1700 \text{ G}$, $\Delta H = 2000 \text{ e}$

$$\frac{\Delta S}{S} = 0.01$$

Gyromagnetic resonance is given by the condition,

$$f_0 = f = 10 \text{ GHz} \quad (\text{since } N_x = N_z = 0, N_y = 1)$$

$$H_0 = \frac{10,000 \text{ MHz}^2}{2.8 \text{ MHz}/10 \text{ e}} = 3571.0 \text{ e}$$

The position of the ferrite slab is given by (9.86), since the RF magnetic fields, H_x and H_z , have demagnetization factors of zero. Thus,

$$\tan k_0 x = \pm \frac{k_0}{\beta_0}$$

$$k_0 = 209.4 \text{ m}^{-1} \quad \checkmark$$

$$k_0 = \pi/a = 137.4 \text{ m}^{-1}$$

$$\beta_0 = \sqrt{k_0^2 - k_z^2} = 158. \text{ m}^{-1}$$

$$s = x = \frac{1}{k_0} \tan^{-1} \frac{k_0}{\beta_0} = \underline{0.521 \text{ cm}}$$

The perturbation result of (9.81) must be used to find the attenuation constants, since this geometry cannot be analysed exactly:

$$\alpha_{\pm} = \frac{\Delta S}{S \beta_0} (\beta_0^2 \chi''_{xx} \sin^2 k_0 x + k_0^2 \chi''_{zz} \cos^2 k_0 x \mp \chi''_{xy} k_0 \beta_0 \sin 2k_0 x)$$

where the susceptibilities are given by (9.39):

$$f_0 = f = 10 \text{ GHz}$$

$$f_m = 1700 (2.18) = 4.76 \text{ GHz}$$

$$\alpha = \frac{\Delta H}{2\omega \mu_0 \chi} = \frac{(200)(2.8 \text{ MHz})}{2(19,000 \text{ MHz})} = 0.028$$

$$\chi''_{xx} = \frac{\alpha f f_m [f_0^2 + f^2 (1 + \alpha^2)]}{[f_0^2 - f^2 (1 + \alpha^2)]^2 + 4 f_0^2 f^2 \alpha^2} = 8.50$$

$$\chi''_{zz} = \chi''_{xx} = 8.50$$

$$\chi''_{xy} = \frac{2 f_0 f_m f^2 \alpha}{[f_0^2 - f^2 (1 + \alpha^2)]^2 + 4 f_0^2 f^2 \alpha^2} = 8.49$$

Then,

$$\alpha_{\pm} = 6.3 \times 10^{-5} (9.139 \times 10^4 + 9.136 \times 10^4 \mp 1.825 \times 10^5)$$

$$\alpha_+ = 0.0158 \text{ nepers/m} = 0.137 \text{ dB/m}$$

$$\alpha_- = 23.0 \text{ nepers/m} = 200 \text{ dB/m}$$

For 30 dB reverse attenuation, the required length is,

$$L = \frac{30 \text{ dB}}{200 \text{ dB/m}} = 0.15 \text{ m} = 15 \text{ cm}$$

Then the forward insertion loss is,

$$IL = (0.137)(0.15) = 0.02 \text{ dB}$$

Note: the calculation of α_{\pm} is numerically sensitive.

9.14

The magnetic fields for the TE_{10} waveguide mode can be written as,

$$H_x = \frac{2\beta A}{k_c} \sin k_c x e^{-j\beta z}$$

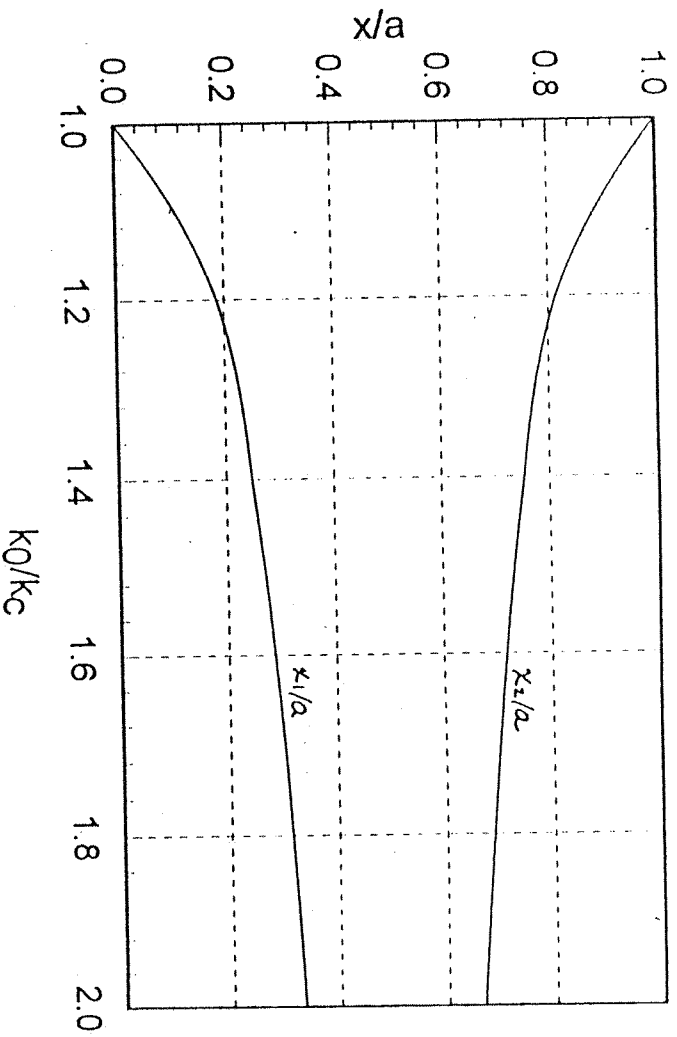
$$H_z = A \cos k_c x e^{-j\beta z}$$

with $k_c = \pi/a$, $\beta = \sqrt{k_0^2 - k_c^2}$. Circular polarization occurs when,

$$\frac{H_x}{H_z} = \pm j = \frac{2\beta}{k_c} \tan k_c x, \text{ or } \tan k_c x = \pm \frac{k_c}{\beta} \quad (9.86)$$

k_0/k_c	β/k_c	$x_{1/a}$	$x_{2/a}$
1.0	0	0	1.000
1.2	0.663	0.186	0.814
1.4	0.980	0.247	0.753
1.6	1.249	0.285	0.715
1.8	1.497	0.313	0.687
2.0	1.732	0.333	0.666

These two positions for circular polarization are plotted on the following page.



9.15

$$f = 6 \text{ GHz}, \quad \epsilon_r = 10, \quad 4\pi M_r = 1500 \text{ G}, \quad L = 2.78 \text{ cm}$$

$$\text{So } f_m = 4.2 \text{ GHz } \checkmark, \quad k_0 = 125.7 \text{ m}^{-1}$$

In the resonant state, $H_0 = f_0 = 0$. So,

$$\mu = \mu_0, \quad X = -\frac{f_m}{f} \mu_0 = -0.70 \mu_0 \checkmark$$

$$\mu_e = \frac{\mu^2 - X^2}{\mu} = 0.51 \mu_0 \checkmark$$

In state #1, $\bar{m} = M_r \hat{x}$ and $\bar{H} = \hat{y} H_y$, so this is an extraordinary wave:

$$\beta_e = k_0 \sqrt{\epsilon_r \mu_e / \mu_0} = 283.9 \text{ m}^{-1} \checkmark$$

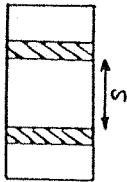
In state #2, $\bar{m} = M_r \hat{y}$ and $\bar{H} = \hat{x} H_x$, so this is an ordinary wave:

$$\beta_o = \sqrt{\epsilon_r} k_0 = 397.5 \text{ m}^{-1} \checkmark$$

Then the differential phase shift is,

$$\Delta\phi = (\beta_2 - \beta_1)L = (\beta_o - \beta_e)L = 181^\circ \checkmark$$

9.16



$$S = 2 \text{ mm}, \quad f = 10 \text{ GHz}, \quad 4\pi M_r = 1000 \text{ G}$$

From Figure 9.18, maximum differential phase shift for $S = 2 \text{ mm}$ occurs for $t/a \approx 0.112$, so $t = 0.112a = 2.6 \text{ mm}$.
 Then $(\beta_+ - \beta_-)/k_0 = 0.24$, for $4\pi M_r = 1786 \text{ G}$. If we assume $(\beta_+ - \beta_-)$ is proportional to X (and so M_r), then for $4\pi M_r = 1000 \text{ G}$ we have

$$\frac{(\beta_+ - \beta_-)}{k_0} = 0.24 \left(\frac{1000}{1786} \right) = 0.134,$$

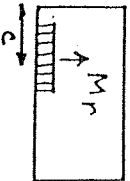
so, $(\beta_+ - \beta_-) = 0.134 k_0 = 16.1^\circ / \text{cm}$ ✓

Then the slab lengths for 180° and 90° sections are,

$$L = \frac{180^\circ}{16.1^\circ / \text{cm}} = 11.2 \text{ cm} \checkmark$$

$$L = \frac{90^\circ}{16.1^\circ / \text{cm}} = 5.6 \text{ cm} \checkmark$$

9.17



$$a = 2.286 \text{ cm}, \quad b = 1.016 \text{ cm}, \quad f = 9 \text{ GHz}$$

$$4\pi M_r = 1200 \text{ G}, \quad c = a/4, \quad \Delta S = 2 \text{ mm}^2$$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a} X \frac{\Delta S}{S} \sin \frac{2\pi c}{a}$$

Now for $H_0 = 0$,

$$\frac{X}{a} = \frac{-f m}{f} \approx \frac{-2.8(1200)}{9000} = -0.373$$

and,

$$S = ab = 232.3 \text{ mm}^2, \quad \text{so}$$

$$\beta_+ - \beta_- = 0.883 \text{ rad/m} = 0.506^\circ / \text{cm} \checkmark$$

So the required length is,

$$L = \frac{22.5^\circ}{0.506^\circ / \text{cm}} = 44.5 \text{ cm} \checkmark$$

(a bit long!)

9.18

$f = 9 \text{ GHz}$, $4\pi M S = 1700 \text{ G}$, $\Delta S = 6 \text{ mm}^2$
 $a = 2.286 \text{ cm}$, $b = 1.016 \text{ cm}$, $H_a = 4000 \text{ Oe}$

From (9.41) the internal bias field is $(N_3 - 1)$

$H_0 = H_a - N M S = 4000 - 1700 = \underline{2300 \text{ Oe}}$

From (9.80) the (approximate) differential phase shift is,

$\beta_+ - \beta_- = \frac{-2\pi}{a} \chi \frac{\Delta S}{5} \sin \frac{2\pi d}{a}$

Maximum phase shift will occur for $C = a/4 = \underline{0.572 \text{ cm}}$.

Then, $f_m = 2.8 (1700) = 4.76 \text{ GHz}$, $f_0 = 2.8 (2300) = 6.44 \text{ GHz}$.

So,

$\chi = \chi_0 \left(1 + \frac{f_0 f_m}{f_0^2 - f^2} \right) = 0.224 \chi_0$

$\chi = \chi_0 \frac{f f_m}{f_0^2 - f^2} = -1.08 \chi_0$

Then,

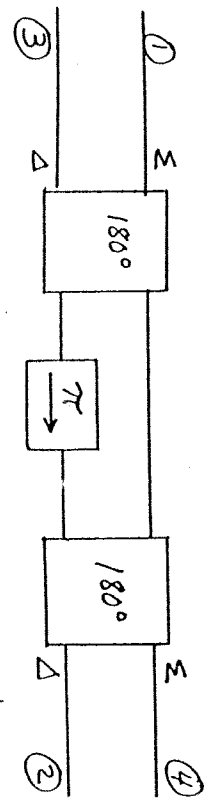
$\beta_+ - \beta_- = 0.342 \text{ rad/cm} = 19.6^\circ/\text{cm}$

So the required length for a 180° phase shift is,

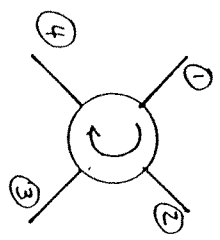
$L = \frac{180^\circ}{19.6^\circ/\text{cm}} = \underline{9.2 \text{ cm}}$

9.19

a four-port circulator can be made using a gyrator and two 180° hybrid couplers:



a three port circulator can be obtained, by shorting any one of the ports.



9.20

$R_L = 10\text{dB}, 20\text{dB}.$

From (9.92) the scattering matrix of a mismatched circulator is,

$$[S] = \begin{bmatrix} \Gamma & \beta & \alpha \\ \alpha & \Gamma & \beta \\ \beta & \alpha & \Gamma \end{bmatrix}$$

Then $|\beta| \approx |\Gamma|$

$$|\alpha| \approx |\Gamma|^2$$

For $R_L = 10\text{dB}$, $\Gamma = |\beta| = 10\text{dB}$

For $R_L = 20\text{dB}$, $\Gamma = |\beta| = 20\text{dB}$

Chapter 10

10.1

From (10.8) the ratio of power outputs is,

$$Y = \frac{T_1 + T_e}{T_2 + T_e} = \frac{300 + 250}{77 + 250} = 1.68 \checkmark$$

10.2

$$T_e = \frac{T_1 - Y T_2}{Y - 1}$$

$$T_e + \Delta T_e = \frac{T_1 - (Y + \Delta Y) T_2}{(Y + \Delta Y) - 1}$$

$$\Delta T_e = \frac{T_1 - (Y + \Delta Y) T_2}{(Y + \Delta Y) - 1} - \frac{T_1 - Y T_2}{Y - 1}$$

$$= \frac{T_1 - (Y + \Delta Y) T_2}{(Y - 1) \left(1 + \frac{\Delta Y}{Y - 1}\right)} - \frac{T_1 - Y T_2}{Y - 1}$$

$$\approx \frac{[T_1 - (Y + \Delta Y) T_2] T_2 \left[1 - \frac{\Delta Y}{Y - 1}\right] - (T_1 - Y T_2)}{Y - 1}$$

$$\approx \frac{-\frac{T_1}{Y - 1} + \frac{Y T_2}{Y - 1} - T_2}{Y - 1} \Delta Y = \frac{(T_2 - T_1)}{(Y - 1)^2} \Delta Y$$

$$\frac{\Delta T_e}{T_e} = \frac{(T_2 - T_1) Y}{(Y - 1)^2 T_e} \frac{\Delta Y}{Y} = \frac{(T_1 + T_e)(T_2 + T_e)}{T_e (T_2 - T_1)} \frac{\Delta Y}{Y}$$

Minimize with respect to T_e :

$$\frac{d}{dT_e} \left[\frac{\Delta T_e}{T_e} \right] = \frac{\left(\frac{T_1}{T_e} + 1\right) \left(\frac{T_2}{T_e} + 1\right) + T_e \left(\frac{-T_1}{T_e^2}\right) \left(\frac{T_2}{T_e} + 1\right) + T_e \left(\frac{T_1}{T_e} + 1\right) \left(\frac{-T_2}{T_e^2}\right)}{T_2 - T_1} = 0$$

Thus,

$$T_e = \sqrt{T_1 T_2} \checkmark$$

10.3

Find attenuation for each line.

X-BAND W.G.: From Example 3.1 the attenuation of copper X-band guide at 10 GHz is $\alpha_c = 0.11 \text{ dB/m}$. So the total loss is $L = 0.22 \text{ dB}$.

RG-8/U: From Appendix J, $\alpha_c = 35 \text{ dB/100 ft}$, or $\alpha_c = 1.15 \text{ dB/m}$. So the total loss is $L = 2.30 \text{ dB}$.

CIRCULAR W.G.: From Example 3.1, $R_S = 0.026 \Omega$. From (3.133) the attenuation for TE_{11} mode propagation is, ($k = 209. \text{ m}^{-1}$, $k_c = \pi/2 = 1.57 \text{ m}^{-1}$, $\beta = 99.8$)

$$\alpha_c = \frac{R_S}{2k\eta\beta} \left(k_c^2 + \frac{k^2}{\eta^2 - 1} \right) = 0.0172 \text{ nepers/m} = 0.15 \text{ dB/m}$$

So the total loss is $L = 0.30 \text{ dB}$. Thus, the rectangular waveguide is the best choice, since it has the lowest attenuation.

10.4

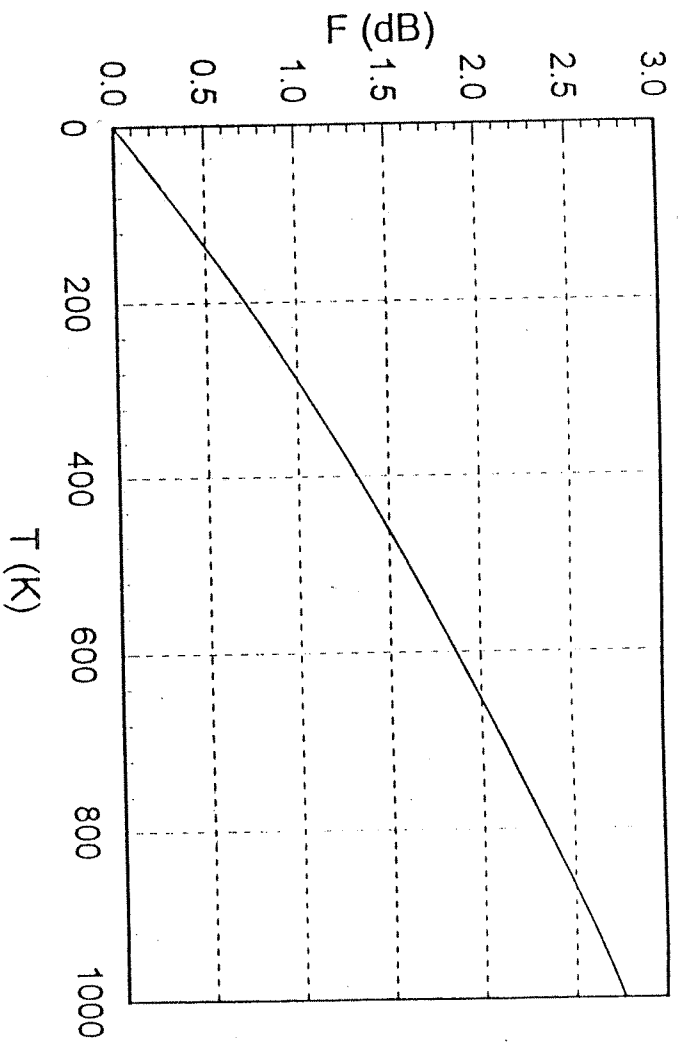
From (10.16), the noise figure of a lossy line is,

$$F = 1 + (L-1) \frac{T_0}{T}$$

When $T = T_0$, $F = L = 1 \text{ dB} = 1.259$. Thus, $F = 1 + 0.259 \left(\frac{T}{290 \text{ K}} \right)$

T (K)	F (dB)
0	0
250	0.88
500	1.60
750	2.23
1000	2.77

This data is plotted on the following page.



10.5

The equivalent noise power input is,

$$P_{min} = kTB = (1.38 \times 10^{-23})(250)(10^9) = 3.5 \times 10^{-12} \text{ W}$$

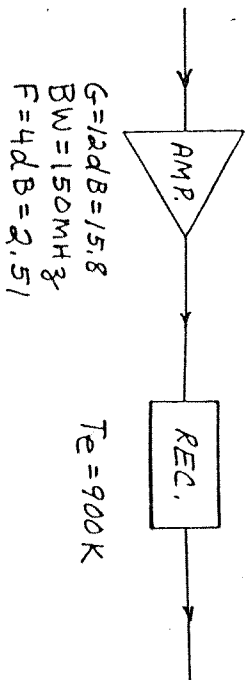
If we assume the upper limit is set by the 1 dB compression point, then

$$P_{max} = -10 \text{ dBm} = 0.1 \text{ mW} = 10^{-4} \text{ W}.$$

Thus the dynamic range is,

$$10 \log \frac{P_{max}}{P_{min}} = 10 \log \left(\frac{10^{-4}}{3.5 \times 10^{-12}} \right) = 75 \text{ dB}$$

10.6



The noise figure of the receiver is, from (10.11),

$$F_2 = 1 + \frac{T_e}{T_0} = 1 + \frac{900}{290} = 4.10$$

Then the noise figure of the cascade is, from (10.21),

$$F_{ca} = F_1 + \frac{1}{G_1} (F_2 - 1) = 2.51 + \frac{4.10 - 1}{15.8} = 2.77 = 4.3 \text{ dB} \checkmark$$

10.7

From (10.4) the equivalent noise temperature of the source is

$$T_e = \frac{P_s}{k B} = \frac{(10^{-85/10})(10^{-3})}{(1.38 \times 10^{-23})(10^9)} = 229 \text{ K}$$

From (10.11) the noise figure of the amplifier is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{180}{290} = 1.62 = 2.1 \text{ dB} \checkmark$$

From (10.16) the noise figure of the transmission line is,

$$F = 1 + (L-1) \frac{T_0}{T_1} = 1 + (1.41-1) \left(\frac{300}{290} \right) = 1.43$$

The noise figure of the transmission line and amplifier cascade is,

$$F_{ca} = F_1 + \frac{1}{G_1} (F_2 - 1) = 1.43 + (1.41)(1.62 - 1) = 2.30 = 3.6 \text{ dB} \checkmark$$

The total noise power output due to the line and amplifier is,

$$P_{ca} = k T_e G_{ca} B G_{ca} = k (F_{ca} - 1) T_0 B G_{ca} \\ = (1.38 \times 10^{-23})(2.30 - 1)(10^9)(10^{10.5/10})(290) = 5.84 \times 10^{-11} \text{ W} \checkmark$$

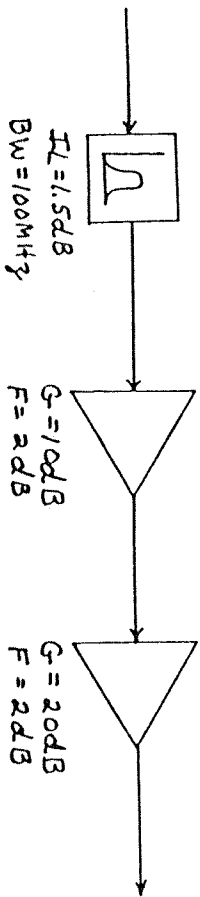
The noise power output due to the source is,

$$P_s = -85 \text{ dBm} - 1.5 \text{ dB} + 12 \text{ dB} = -74.5 \text{ dBm} = 3.55 \times 10^{-14} \text{ W} \checkmark$$

Thus the total noise power output is,

$$P_n = 9.39 \times 10^{-11} \text{ W} = -70.3 \text{ dBm}$$

10.8



From (10.23) the noise figure of the cascade is ($F_1 = I_L = 1.5 \text{ dB}$)

$$F_{\text{CAS}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.41 + (1.41)(1.58 - 1) + \frac{1.41}{10} (1.58 - 1)$$

$$= 2.31 = 3.64 \text{ dB}$$

If $P_{\text{in}} = -90 \text{ dBm}$, then $P_{\text{out}} = -90 \text{ dBm} - 1.5 \text{ dB} + 10 \text{ dB} + 20 \text{ dB} + 20 \text{ dB} = -61.5 \text{ dBm}$.

The noise power output is then,

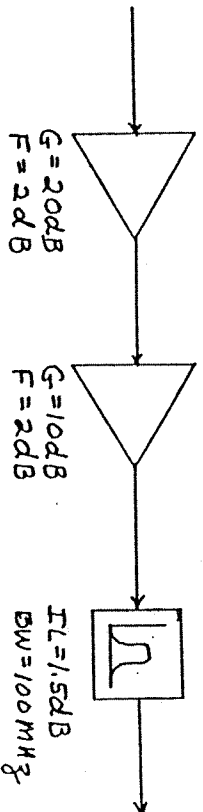
$$P_n = G_{\text{cas}} k T_{\text{cas}} B = k (F_{\text{cas}} - 1) T_0 B G_{\text{cas}}$$

$$= (1.38 \times 10^{-23}) (2.31 - 1) (290) (10^8) (10^{28.5/10}) = 3.71 \times 10^{-10} \text{ W}$$

$$= -64.3 \text{ dBm}$$

Thus, $\frac{S_o}{N_o} = -61.5 + 64.3 = 2.8 \text{ dB}$

The best noise figure would be achieved with the arrangement shown below:



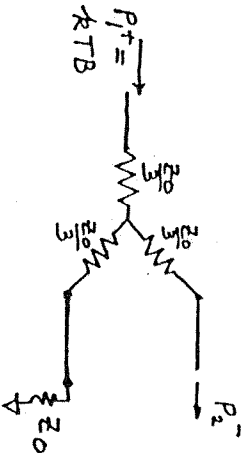
Then,

$$F_{\text{CAS}} = 1.58 + \frac{(1.58 - 1)}{100} + \frac{(1.41 - 1)}{1000} = 1.586 = 2.0 \text{ dB}$$

(In practice, however, the initial filter may serve to prevent overload of the amplifier, and may not be allowed to be moved.)

10.9

a) RESISTIVE DIVIDER



$$[S] = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & \\ 1 & 1 & 1 & \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

When the input noise power at port 1 is kTB , and the divider is at temperature T , the system is in thermodynamic equilibrium. Thus the output noise power at port 2 must be kTB . We can also express this as due to the attenuated input noise power and noise power added by the network (ref. at input). Thus,

$$P_2^- = kTB = \frac{kTB}{4} + \frac{N_{added}}{4}$$

$$\therefore N_{added} = 3kTB$$

The equivalent noise temperature is then,

$$T_e = \frac{N_{added}}{k_B} = 3T$$

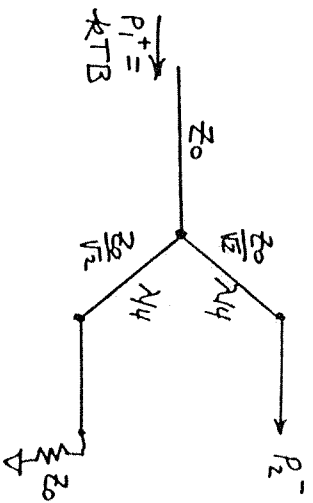
And the noise figure is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0}$$

At room temperature, $T = T_0$, so $F = 4 = 6.01\text{dB}$.

(this result checks with that obtained using the available gain method)

b) WILKINSON DIVIDER



$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$

In this case, if the input noise power is kTB , and the system is in thermodynamic equilibrium, the net output power at port 2 is $\frac{3}{4} kTB$, because of the mismatch of the output ports ($1/4$ of output power is reflected). Then we have,

$$P_2^- = \frac{3}{4} kTB = \frac{kTB}{2} + \frac{N_{added}}{2} \quad (N_{added} \text{ ref. at input})$$

$$\therefore N_{added} = \frac{1}{2} kTB$$

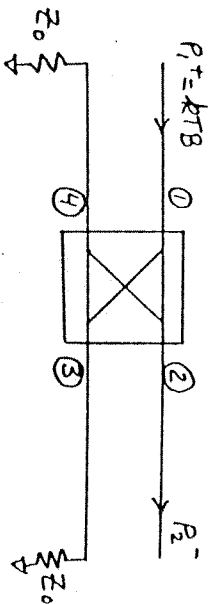
$$T_e = \frac{N_{added}}{kB} = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0}$$

$$\text{If } T = T_0, \quad F = \frac{3}{2} = 1.76 \text{ dB.}$$

(Result verified with HP-MDS, calculator's using available gain, and direct measurement)

c) QUADRATURE HYBRID



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 0 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Using the same thermodynamic arguments as above, the output noise power is kTB (outputs are matched).
Thus,

$$P_2^- = kTB = \frac{kTB}{2} + \frac{N_{added}}{2}$$

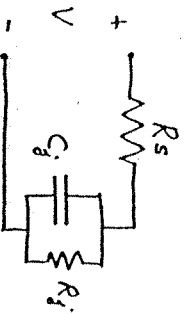
$$\therefore N_{added} = kTB$$

$$T_e = \frac{N_{added}}{k_B} = T$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{T_0}$$

If $T = T_0$, we have $F = 2 = 3dB$

(10.10) The AC model of the diode is shown below:



The voltage sensitivity is, from (10.32)-(10.33),

$$\beta_V = \beta_i R_j = \frac{\Delta I_{dc}}{P_{in}} R_j$$

where P_{in} is the RF input power and ΔI_{dc} is the change in DC current through the diode. If the RF voltage across the diode is V , then the RF input power is,

$$P_{\text{in}} = \frac{|V|^2}{2} \operatorname{Re}\{Y_d\} = \frac{|V|^2}{2} \operatorname{Re}\left\{ \frac{1}{R_s + \frac{R_i(j\omega C_j)}{R_i + j\omega C_j}} \right\}$$

$$= \frac{|V|^2}{2} \operatorname{Re}\left\{ \frac{R_i + j\omega C_j}{(1 + R_s/R_i) + j\omega C_j R_s} \right\} = \frac{|V|^2}{2} \frac{R_i(1 + R_s/R_i) + \omega^2 C_j^2 R_s}{(1 + R_s/R_i)^2 + (\omega C_j R_s)^2}$$

From (10.31) the change in DC current is,

$$\Delta I_{d0} = \frac{|V_0|^2}{4} G_d' = \frac{|V_0|^2}{4} \frac{\alpha}{R_j}$$

where v_0 is the peak RF junction voltage. This is the current when the junction is short-circuited. When the packaged diode is shorted, the effect of R_s must be included:

$$\Delta I_{d0} = \frac{|V_0|^2}{4} \frac{\alpha}{R_j} \frac{R_j}{R_j + R_s} = \frac{\alpha |V_0|^2}{4(R_j + R_s)}$$

The relation between v_0^2 and $|V|^2$ is,

$$v_0 = V \frac{\frac{R_i/j\omega C_j}{R_i + j\omega C_j}}{R_s + \frac{R_i/j\omega C_j}{R_j + j\omega C_j}} = V \frac{1}{(1 + R_s/R_j) + j\omega C_j R_s}$$

So,

$$|v_0|^2 = \frac{|V|^2}{(1 + R_s/R_j)^2 + (\omega C_j R_s)^2}$$

Finally,

$$\beta_v = \frac{\alpha R_j}{2(1 + R_s/R_j)[(1 + R_s/R_j) + (\omega C_j)^2 R_s R_j]} \quad \text{V/W} \quad \checkmark$$

at $f = 10 \text{ GHz}$, $\omega C_j = 2\pi(10^{10})(0.1 \times 10^{-12}) = 0.0063$; $\alpha = 1/25 \text{ mV}$

I_0 (μA)	R_j (Ω)	β_v (V/mW)
0	12.5×10^5	33.
20	1.24×10^3	14.
50	4.99×10^2	3.

$R_j' = \frac{1}{\alpha(I_0 + I_S)}$

10.11

The input signal can be written as,

$$v(t) = \cos \omega_1 t + \cos \omega_2 t$$

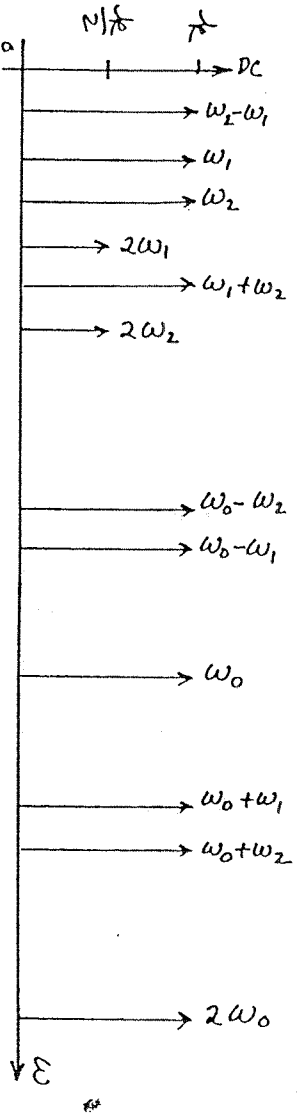
and the LO signal is,

$$v_{LO}(t) = \cos \omega_0 t$$

Then the v^2 mixer output will be,

$$\begin{aligned}
 i &= k (\cos \omega_1 t + \cos \omega_2 t + \cos \omega_0 t)^2 \\
 &= k [\cos^2 \omega_1 t + \cos^2 \omega_2 t + \cos^2 \omega_0 t] \\
 &\quad + 2k [\cos \omega_1 t \cos \omega_2 t + \cos \omega_1 t \cos \omega_0 t + \cos \omega_2 t \cos \omega_0 t] \\
 &= \frac{k}{2} [3 + \cos 2\omega_1 t + \cos 2\omega_2 t + \cos 2\omega_0 t] \\
 &\quad + k [\cos(\omega_1 - \omega_2)t + \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_0)t + \cos(\omega_1 + \omega_0)t \\
 &\quad + \cos(\omega_2 - \omega_0)t + \cos(\omega_2 + \omega_0)t]
 \end{aligned}$$

(All possible combinations of the sums and differences of ω_1, ω_2 , and ω_0)



(ω_1, ω_2 , and ω_0 are due to the linear term, not the v^2 term; they are shown here for reference only)

1212

$$v_1 = v_0 \cos \omega t \quad ; \quad v_2 = v_0 \cos(\omega t + \theta)$$

As in (10.42)-(10.45), the diode currents in a mixer using a quadrature hybrid will be,

$$i_1 = k v_0^2 [\cos(\omega t - \pi/2) + \cos(\omega t + \theta - \pi)]^2 \\ = k v_0^2 [\sin \omega t - \cos(\omega t + \theta)]^2$$

$$i_2 = -k v_0^2 [\cos(\omega t - \pi) + \cos(\omega t + \theta - \pi/2)]^2 \\ = -k v_0^2 [-\cos \omega t + \sin(\omega t + \theta)]^2$$

Low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \frac{1}{2} \sin \theta)$$

$$i_2 = -k v_0^2 (1 - \frac{1}{2} \sin \theta)$$

So the output is $i_1 + i_2 = k v_0^2 \sin \theta$ ✓
If a mixer with a 180° hybrid is used, the diode currents become,

$$i_1 = k v_0^2 [\cos \omega t + \cos(\omega t + \theta)]^2$$

$$i_2 = -k v_0^2 [\cos \omega t - \cos(\omega t + \theta)]^2$$

Then low-pass filtering leaves the following DC components:

$$i_1 = k v_0^2 (1 + \frac{1}{2} \cos \theta)$$

$$i_2 = -k v_0^2 (1 - \frac{1}{2} \cos \theta)$$

So the output is $i_1 + i_2 = k v_0^2 \cos \theta$ ✓

10.13

Retaining only the terms that give rise to the third order intermodulation products:

$$v_0 \sim k (V_1 \cos \omega_1 t + V_2 \cos \omega_2 t)^3$$

$$\sim k (V_1^3 \cos^3 \omega_1 t + 3 V_1^2 V_2 \cos^2 \omega_1 t \cos \omega_2 t + V_1 V_2^2 \cos \omega_1 t \cos^2 \omega_2 t)$$

$$\sim k (V_1^3 \cos 3\omega_1 t + 3 V_1^2 V_2 \cos \omega_1 t \cos \omega_2 t + V_1 V_2^2 \cos \omega_1 t \cos 3\omega_2 t)$$

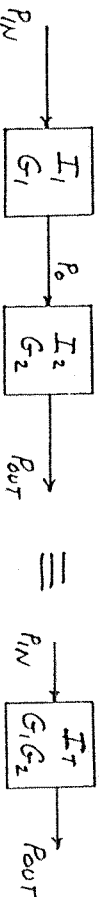
$$\sim \frac{k}{2} [V_1^3 V_2 \cos(2\omega_1 - \omega_2)t + V_1 V_2^3 \cos(2\omega_2 - \omega_1)t]$$

So the ratio of the powers in the two outputs is,

$$\left(\frac{V_1^3 V_2}{V_1 V_2^3} \right)^2 = \left(\frac{V_1}{V_2} \right)^2 = 6 \text{ dB}$$

Note that the individual output powers vary as P_{in}^3 .

10.14



LINEAR BEHAVIOR: $P_0 = G_1 P_{IN}$

THIRD-ORDER BEHAVIOR: $P_0 = C P_{IN}^3$ (C is a constant)

at the third order intercept point, $P_0 = I_1$, so

$$C = \frac{P_0}{P_{in}^3} = \frac{I_1}{P_{in}^3} = \frac{I_1 G_1^3}{I_1^3} = \frac{G_1^3}{I_1^2} \quad \checkmark$$

So the distortion power at the output of the first stage is, for input power P_{in} ,

$$P_{d1} = \left(\frac{G_1^3}{I_1^2} \right) P_{in}^3$$

Similarly, the distortion power at the output of the second stage (as caused by the second stage) is,

$$P_{d2} = \left(\frac{G_2^3}{I_2^2} \right) P_0^3 = \left(\frac{G_2^3}{I_2^2} \right) G_1^3 P_{in}^3$$

The distortion voltages at the output of the cascade are,

$$V_{d1} = \sqrt{Z_0 G_1 P_{d1}} = \sqrt{Z_0 P_{in}^3} \sqrt{\frac{G_1 G_1^3}{I_1^2}}$$

$$V_{d2} = \sqrt{Z_0 P_{d2}} = \sqrt{Z_0 P_{in}^3} \sqrt{\frac{G_1^3 G_2^3}{I_2^2}}$$

Assuming these voltages add in phase, the total distortion voltage is,

$$V_d = \sqrt{Z_0 P_{in}^3} \left(\sqrt{\frac{G_1^3 G_1^3}{I_1^2}} + \sqrt{\frac{G_1^3 G_2^3}{I_2^2}} \right) \\ = \sqrt{Z_0 P_{in}^3 G_1^3 G_2^3} \left(\frac{1}{G_2 I_1} + \frac{1}{I_2} \right)$$

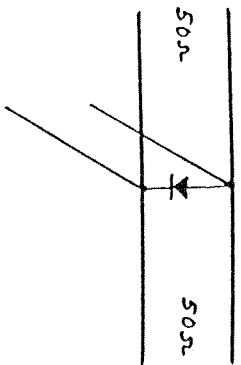
So the total distortion power output is,

$$P_d = \frac{V_d^2}{Z_0} = P_{in}^3 G_1^3 G_2^3 \left(\frac{1}{G_2 I_1} + \frac{1}{I_2} \right)^2 = \left(\frac{G_1^3 G_2^3}{I_1^2} \right) P_{in}^3$$

Thus,

$$I_T = \left(\frac{1}{G_2 I_1} + \frac{1}{I_2} \right)^{-1} \quad \checkmark$$

10.15

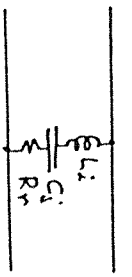


$$\omega L_i = 7.5 \Omega$$

$$1/\omega g = 79.6 \Omega$$

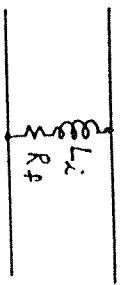
SWITCH ON: (DIPPER OFF)

SWITCH OFF: (DIPPER ON)



$$Z_d = 0.5 - j72 \Omega$$

$$Y_d = (0.096 + j13.9) \text{ mS}$$



$$Z_d = 0.3 + j7.5 \Omega$$

$$Y_d = (5.3 - j133.) \text{ mS}$$

To minimize the insertion loss for the ON state, let the stub susceptance be $Y_s = -j0.0139 = -j0.695/Z_0$. So the stub length should be $l = 0.403 \lambda$.

In the ON state, the shunt impedance is then,

$$Z = 1/0.096 \times 10^{-3} = 10,417 \Omega, \text{ so the insertion loss is, from}$$

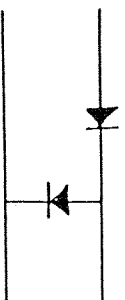
(10.596),

$$IL = -20 \log \left| \frac{2Z}{2Z + Z_0} \right| = 0.021 \text{ dB}$$

In the OFF state, the shunt impedance is $Z = 0.246 + j6.8 \Omega$, so the insertion loss is,

$$IL = -20 \log \left| \frac{2Z}{2Z + Z_0} \right| = 11.7 \text{ dB}$$

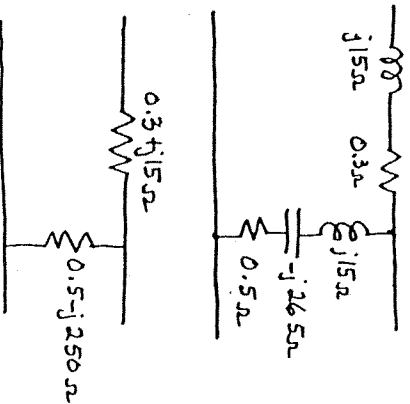
10.16



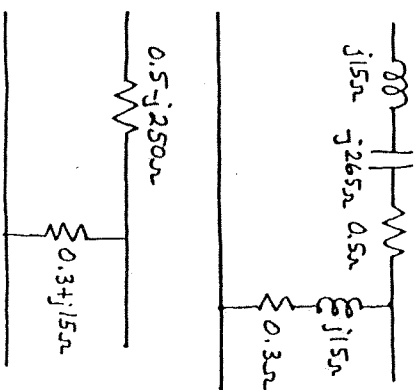
$$\omega L_i = 15\Omega$$

$$1/\omega C_j = 265\Omega$$

SWITCH ON:



SWITCH OFF:



ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} = \begin{bmatrix} 1+Z_1/Z_2 & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$



Convert to S_{21} :

$$S_{21} = \frac{2}{A+B/Z_0+CZ_0+D} = \frac{2}{1+Z_1/Z_2+Z_1/Z_0+Z_0/Z_2+1} = \frac{2}{2+\frac{Z_1}{Z_2}+\frac{Z_1+Z_0}{Z_2}}$$

ON STATE: $Z_1 = 0.3+j15\Omega$, $Z_2 = 0.5-j265\Omega$

$$S_{21} = 0.995 \angle -14^\circ$$

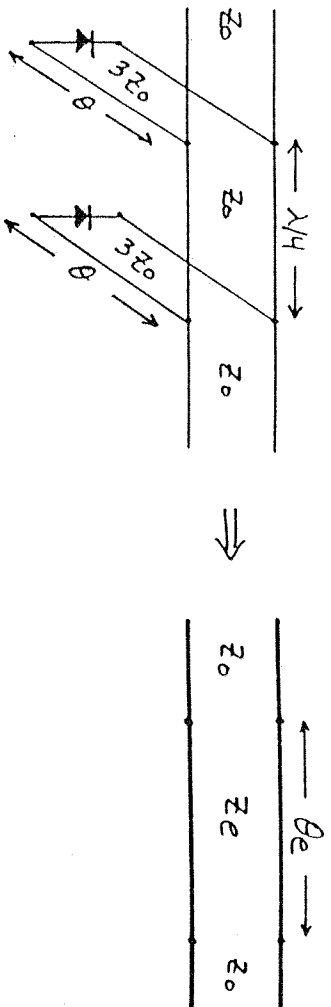
$$IL = 0.044 \text{ dB}$$

OFF STATE: $Z_1 = 0.5-j250\Omega$, $Z_2 = 0.3+j15\Omega$

$$S_{21} = 0.118 \angle 149^\circ$$

$$IL = 18.6 \text{ dB}$$

10.17



From (10.65),

$$\cos \theta_e = -b$$

$$Z_e = Z_0 / \sqrt{1-b^2}$$

where b is the normalized stub susceptance.

For diodes ON, $b = -\frac{1}{3} \cot \theta$

$$\cos \theta_e = \frac{1}{3} \cot \theta$$

For diodes OFF, $b = \frac{1}{3} \tan \theta$

$$\cos \theta_e = -\frac{1}{3} \tan \theta$$

$$\text{So } \Delta \phi = 45^\circ = \cos^{-1}(\frac{1}{3} \cot \theta) - \cos^{-1}(-\frac{1}{3} \tan \theta) \quad (\text{ON-OFF})$$

Solving this equation numerically:

θ	$\Delta \phi$
110°	73°
120°	46°
130°	40°
122°	44.3°
121°	45.2°

So we choose $\theta = 121^\circ$. (Using $\theta = 31^\circ$ gives $\Delta \phi = -45^\circ$)

Invertion loss for $\theta = 121^\circ$:

Using (10.64), $b = BZ_0$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-BZ_0 + j(1 - B^2Z_0^2) - BZ_0} = \frac{2}{-2B + j(2 - B^2)}$$

$$|S_{21}|^2 = \frac{4}{4b^2 + (a-b^2)^2}$$

DIODES ON: $b = \frac{1}{3} \text{ at } \theta = 0.20$

$$|S_{21}|^2 = 0.9996$$

$$IL = 0.0017 \text{ dB} \sim 0 \text{ dB} \checkmark$$

DIODES OFF: $b = \frac{1}{3} \text{ at } \theta = -0.555$

$$|S_{21}|^2 = 0.977$$

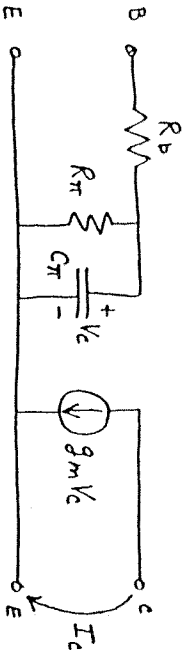
$$IL = 0.102 \text{ dB} \checkmark$$

(SuperCompact analysis gives $IL_{\text{ON}} = 0 \text{ dB}$, $\phi_{\text{ON}} = -101.5^\circ$,
 $IL_{\text{OFF}} = 0.10 \text{ dB}$, $\phi_{\text{OFF}} = -56.7^\circ$, thus $\Delta\phi = 44.8^\circ$)

Chapter 11

11.1

unilateral bipolar transistor model:



From (11.1) the short-circuit current gain is,

$$G_{\pi}^{SC} = \left. \frac{I_C}{I_B} \right|_{V_{ce}=0} = \frac{g_m V_e}{I_B} = \frac{g_m I_B}{I_B} \frac{\left| \frac{R_{\pi} \delta \omega C_{\pi}}{R_{\pi} + 1/j\omega C_{\pi}} \right|}{I_B}$$

$$= g_m \frac{R_{\pi}}{|1 + j\omega R_{\pi} C_{\pi}|} = \frac{g_m}{\left| \frac{1}{R_{\pi}} + j\omega C_{\pi} \right|} \approx \frac{g_m}{\omega C_{\pi}} \quad \text{since } R_{\pi} \gg 1/\omega C_{\pi}$$

(e.g., if $R_{\pi} = 110\Omega$, $C_{\pi} = 18\text{pF}$, $f = 1\text{GHz}$, then $1/\omega C_{\pi} = 9\Omega$)

11.2

$R_i = 7\Omega$, $C_{gs} = 0.12\text{pF}$, $R_{ds} = 400\Omega$, $C_{gs} = 0.3\text{pF}$, $C_{gd} = 0$, $g_m = 30\text{mS}$, $f = 5\text{GHz}$.

$$y_{11} = \frac{\delta \omega C_{gs}}{1 + j\omega R_i C_{gs}} = \frac{j0.00942}{1 + j0.06597} = 0.0094 \angle 86^{\circ} = 0.00062 + j0.0094 \checkmark$$

$$y_{21} = \frac{g_m}{1 + j\omega R_i C_{gs}} = \frac{0.03}{1 + j0.06597} = 0.03 \angle 4^{\circ} \quad \checkmark \quad y_{12} = 0 \checkmark$$

$$y_{22} = \frac{1}{R_{ds}} + j\omega C_{gs} = 0.0025 + j0.00377 = 0.00452 \angle 56.5^{\circ} \checkmark$$

$$\Delta y = (Y_{11} + Y_0)(Y_{22} + Y_0) - Y_{12} Y_{21} = (.0226 \angle 24.5^{\circ})(.0228 \angle 9.5^{\circ}) = 0.000515 \angle 34^{\circ}$$

$$S_{11} = \frac{(Y_0 - Y_{11})(Y_0 + Y_{22})}{Y_0 + Y_{11}} = \frac{Y_0 - Y_{11}}{Y_0 + Y_{11}} = \frac{.0215 \angle -25.9^{\circ}}{.0226 \angle 24.5^{\circ}} = 0.951 \angle -50^{\circ} \quad \checkmark$$

$$S_{12} = 0$$

$$S_{21} = \frac{-2Y_{21}Y_0}{\Delta y} = \frac{(-.04)(.03 \angle 4^{\circ})}{.000515 \angle 34^{\circ}} = 2.33 \angle 150^{\circ}$$

$$S_{22} = \frac{Y_0 - Y_{22}}{Y_0 + Y_{22}} = \frac{.0179 \angle -12^{\circ}}{.0228 \angle 9.5^{\circ}} = 0.785 \angle -22^{\circ} \quad \checkmark$$

If conjugately matched, the unilateral transducer gain is,

$$G_{TU} = \frac{1}{1-|S_{11}|^2} |S_{21}|^2 \frac{1}{1-|S_{22}|^2} = 148.8 = 21.7 \text{ dB}$$

The corresponding result from the circuit model, as given in (11.21), is

$$G_{TU} = \frac{g_m^2 R_d S}{4\omega^2 R_i C_g^2} = 21.6 \text{ dB}$$

(These results were verified via SuperCompact.)

(11.3) The $[S]$ matrix for a 3-dB matched attenuator is,

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

For $Z_L = 50 \Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = 0$, $\Gamma_{out} = 0$

Then from (11.15), (11.16), and (11.11) we have

$$G_A = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2 (1-|\Gamma_{out}|^2)} = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

$$G = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

For $Z_L = 25 \Omega$: $\Gamma_L = -1/3$, $\Gamma_S = \Gamma_{out} = 0$, $\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -1/6$

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = |S_{21}|^2 (1-|\Gamma_L|^2) = 0.444 \checkmark$$

$$G = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)} = 0.457 \checkmark$$

FOR $Z_S = 25 \Omega$, $Z_L = 50 \Omega$: $\Gamma_L = \Gamma_{in} = 0$, $\Gamma_S = -1/3$, $\Gamma_{out} = S_{22}' + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} = -1/6$

$$G_A = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)}{(1 - |\Gamma_{out}|^2)} = 0.457$$

$$G_T = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)}{1} = 0.444$$

$$G = |S_{21}|^2 = 0.5$$

11.4

From (11.28) - (11.29) the centers and radii of the stability circles are:

$$\Delta = S_{11} S_{22} - S_{12} S_{21} = 0.117 \angle -50^\circ$$

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 2.56 \angle 28^\circ \checkmark$$

$$R_L = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| = 1.37 \checkmark$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = 3.77 \angle 174^\circ \checkmark$$

$$R_S = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| = 2.53 \checkmark$$

From (11.131),

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 |S_{12} S_{21}|} = 1.35 > 1 \checkmark$$

Since $K > 1$ and $|\Delta| < 1$, the transistor is unconditionally stable.

11.5

$$S_{11} = 0.8 \angle -90^\circ, \quad S_{12} = 0.3 \angle 70^\circ, \quad S_{21} = 5.1 \angle 80^\circ, \quad S_{22} = 0.62 \angle -40^\circ$$

$$\Delta = 1.52 \angle -49^\circ$$

$$C_L = 0.66 \angle -70^\circ$$

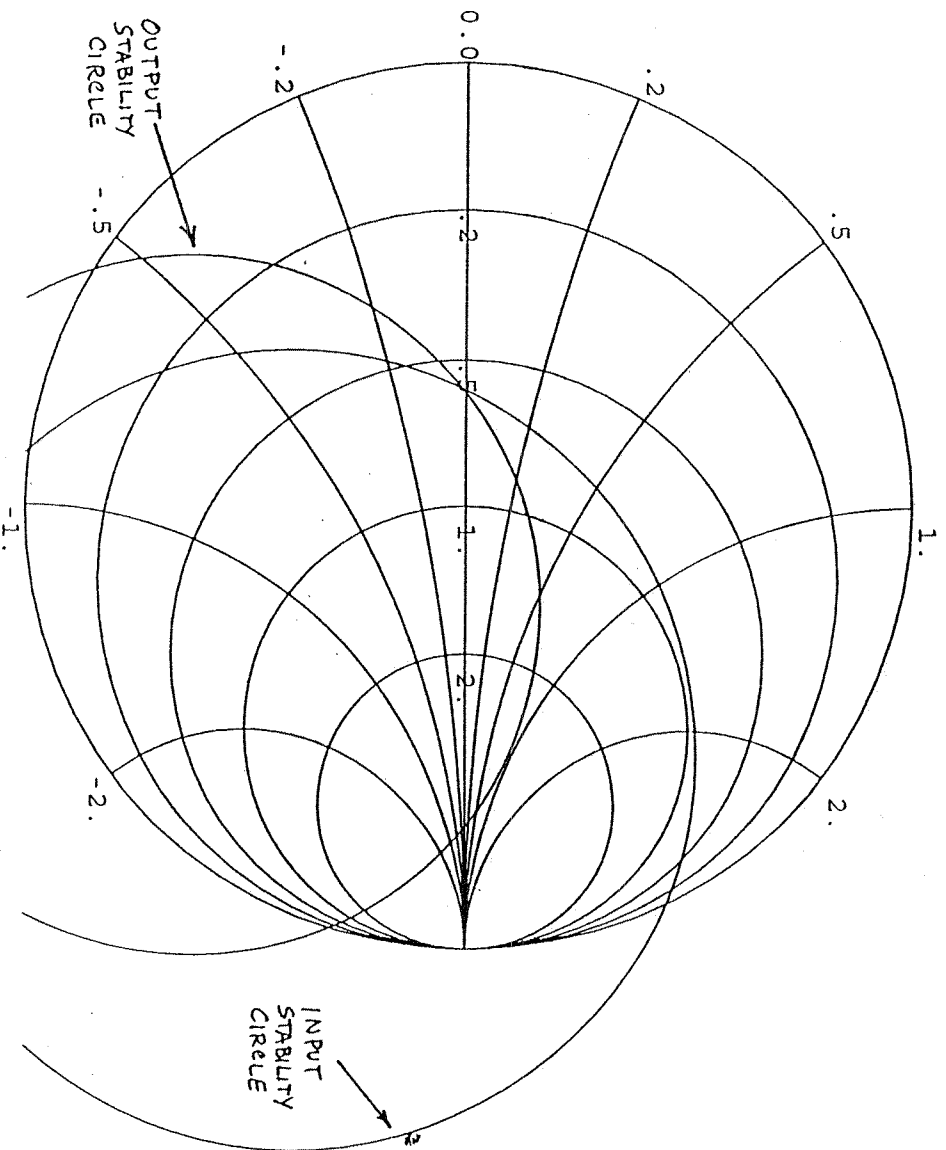
$$C_S = 0.68 \angle -35^\circ$$

$$R_L = 0.79$$

$$R_S = 0.91$$

$$K = 0.75 \quad \checkmark$$

Since $K < 1$ the transistor is potentially unstable. The stability circles are plotted below.



11.6 Using (11.38) to compute μ :

DEVICE	S_{11}	S_{12}	S_{21}	S_{22}	μ	
A	$0.34 \angle 170^\circ$	$0.06 \angle 70^\circ$	$4.3 \angle 80^\circ$	$0.45 \angle -25^\circ$	1.193	UNC. STABLE
B	$0.75 \angle -60^\circ$	$0.2 \angle 70^\circ$	$5.0 \angle 90^\circ$	$0.5 \angle 60^\circ$	0.283	POT. UNSTABLE
C	$0.65 \angle -140^\circ$	$0.04 \angle 60^\circ$	$2.4 \angle 50^\circ$	$0.7 \angle -65^\circ$	1.057	UNC. STABLE

Device A has the best stability.

11.7 From (11.38) the μ -parameter test is,

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta| + |S_{21} S_{12}|} > 1$$

If $S_{12} = 0$ (unilateral) then we have,

$$\Delta = S_{11} S_{22}$$

$$\text{So, } \mu = \frac{1 - |S_{11}|^2}{|S_{22} - |S_{11}|^2 S_{22}|} = \frac{1 - |S_{11}|^2}{|S_{22}| |1 - |S_{11}|^2|} > 1$$

Since the denominator is positive and μ is positive, the numerator must also be positive, thus $|S_{11}| < 1$. Then the above reduces to,

$$\mu = \frac{1}{|S_{22}|} > 1,$$

$$\text{So, } |S_{22}| < 1.$$

11.8

$$S_{11} = 0.65 \angle -140^\circ, S_{21} = 2.4 \angle 50^\circ, S_{12} = 0.04 \angle 60^\circ, S_{22} = 0.70 \angle -65^\circ$$

First we check stability:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.393 \angle 165^\circ$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.26$$

Since $|\Delta| < 1$ and $K > 1$ the transistor is unconditionally stable at 5 GHz. For maximum gain, the transistor should be conjugately matched: (using 11.43)

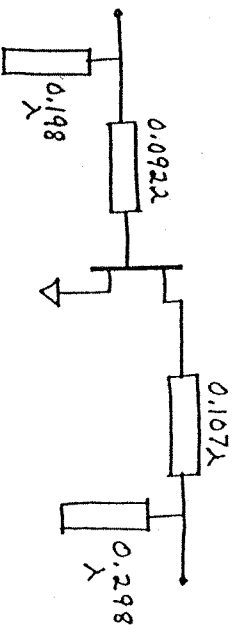
$$\Gamma_S = \Gamma_{in}^* = 0.826 \angle 147^\circ \checkmark$$

$$\Gamma_L = \Gamma_{out}^* = 0.850 \angle 71^\circ \checkmark$$

The gains can then be calculated as,

$$G_S = 3.14, G_0 = 5.76, G_L = 1.64$$

So the overall transducer gain is $G_T = 29.7 = 14.7 \text{ dB}$ ✓
Matching was done on a Smith chart. The final AC amplifier circuit is shown below:



SuperCompact analysis gives $|S_{11}| = 0.05, |S_{22}| = 0.04, G = 14.7 \text{ dB}$ ✓

11.9

$$S_{11} = 0.61 \angle -170^\circ, S_{21} = 2.24 \angle 32^\circ, S_{12} = 0, S_{22} = 0.72 \angle -83^\circ$$

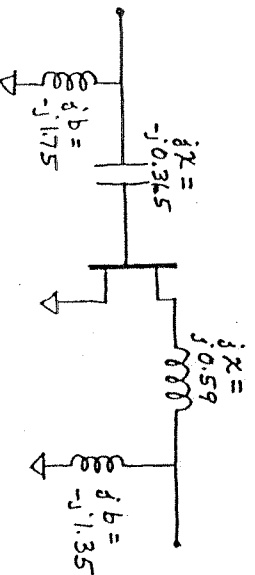
The transistor is unconditionally stable since $K = \infty$ and $|a_1| < 1$.
 Since the transistor is unilateral,

$$\Gamma_S = S_{11}^* = 0.61 \angle 170^\circ \quad \checkmark, \quad \Gamma_L = S_{22}^* = 0.72 \angle 83^\circ \quad \checkmark$$

and the maximum gain is, from (11.45),

$$G_{TU_{MAX}} = \frac{1}{1 - |S_{11}|^2} \frac{1}{1 - |S_{22}|^2} = 16.6 = 12.2 \text{ dB}$$

Matching was done with a Smith chart. The final circuit is:



The matching element values are, at 6 GHz,

$$C = \frac{-1}{\omega Z_0 Y_c} = 1.45 \text{ pF} \quad \checkmark$$

$$L = \frac{Z_0 Y_c}{\omega} = 0.78 \text{ nH} \quad \checkmark$$

$$L = \frac{-Z_0}{\omega b_L} = 0.76 \text{ nH} \quad \checkmark$$

$$L = \frac{-Z_0}{\omega b_L} = 0.98 \text{ nH} \quad \checkmark$$

SuperCompact analysis gives $|S_{11}| = 0.035$, $|S_{22}| = 0.008$,
 and $G = 12.2 \text{ dB} \quad \checkmark$

(11.10)

$$S_{11} = 0.61 \angle -170^\circ, \quad S_{21} = 2.24 \angle 32^\circ; \quad S_{12} = 0, \quad S_{22} = 0.72 \angle -83^\circ$$

$$G_S = 10 \text{ dB}, \quad G_S = 1 \text{ dB}, \quad G_L = 2 \text{ dB}$$

Since $K = \infty$ and $|A| < 1$, the transistor is unconditionally stable. From (11.48), we have

$$G_{S_{\text{MAX}}} = \frac{1}{1 - |S_{11}|^2} = 1.59 \checkmark, \quad G_{L_{\text{MAX}}} = \frac{1}{1 - |S_{22}|^2} = 2.08 \checkmark$$

So for $G_S = 1 \text{ dB} = 1.26$, and $G_L = 2 \text{ dB} = 1.58$, we have from (11.49),

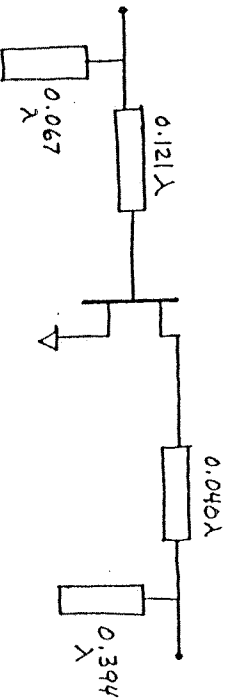
$$g_S = \frac{G_S}{G_{S_{\text{MAX}}}} = 0.792, \quad g_L = \frac{G_L}{G_{L_{\text{MAX}}}} = 0.760$$

Then the center and radii of the constant gain circles can be found from (11.52) - (11.53):

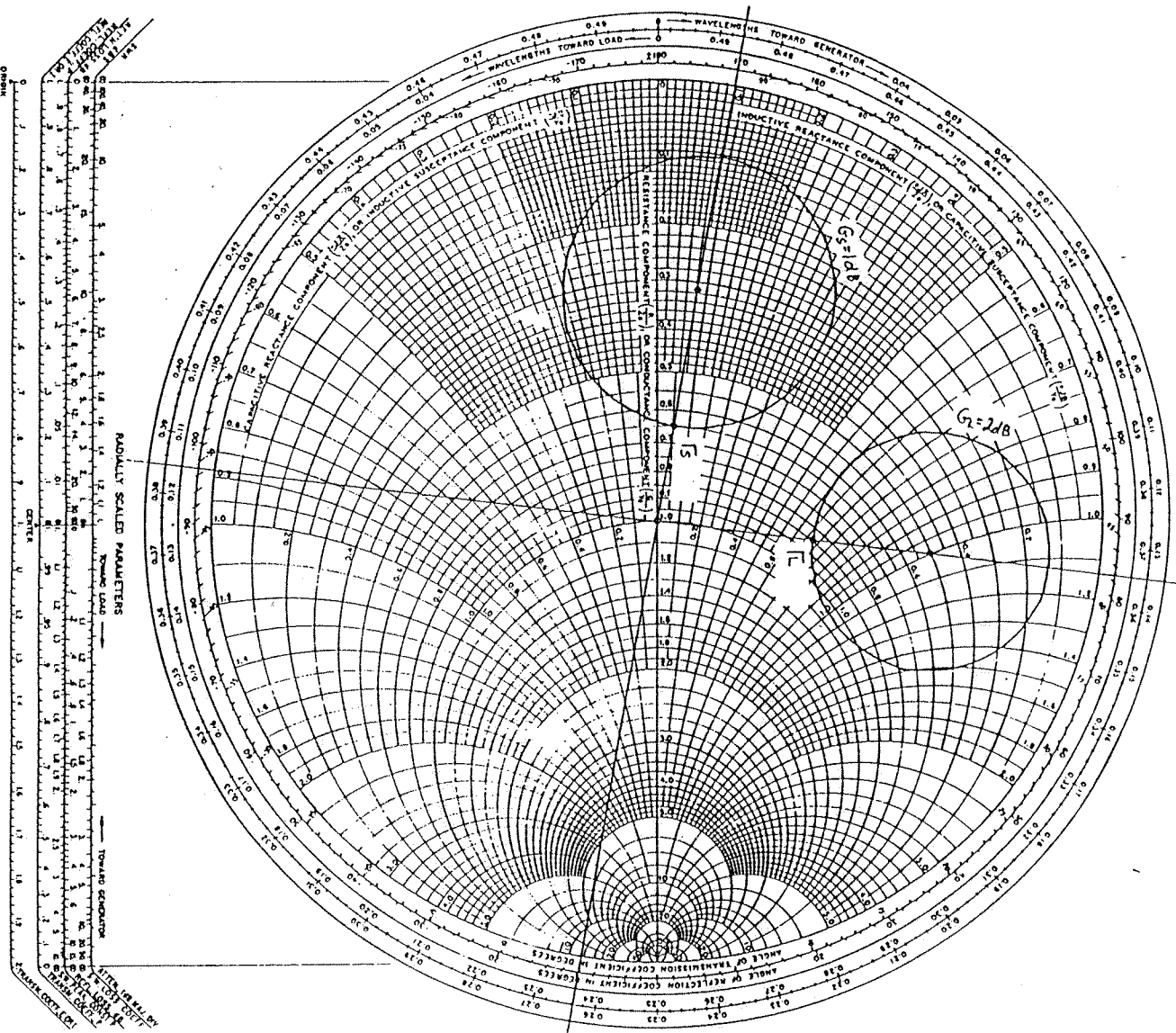
$$C_S = 0.524 \angle 120^\circ \checkmark, \quad C_L = 0.625 \angle 83^\circ \checkmark$$

$$R_S = 0.310 \checkmark, \quad R_L = 0.269 \checkmark$$

Since $G_0 = 10 \log |S_{21}|^2 = 7.0 \text{ dB}$, setting the $G_S = 1 \text{ dB}$ and the $G_L = 2 \text{ dB}$ gain circles will give an overall gain of 10 dB . We plot these circles on the Smith chart, and choose $\Gamma_S = 0.215 \angle 170^\circ \checkmark$ and $\Gamma_L = 0.361 \angle 83^\circ \checkmark$ to minimize the magnitude of these values. After matching, we have the following amplifier circuit:



SuperCompact analysis gives $|S_{11}| = 0.45$, $|S_{21}| = 0.48$, $G = 10.05 \text{ dB} \checkmark$ (reflections at input and output serve to reduce the gain to 10 dB). Smith chart shown on following page.



11.11

From (11.47) the unilateral figure of merit is,

$$U = \frac{|S_{22}| |S_{21}| |S_{11}| |S_{21}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} = \frac{(0.6)(4.3)(.34)(.45)}{[1 - (.34)^2][1 - (.45)^2]} = 0.056 \quad \checkmark$$

So from (11.46) this bounds the error in G_T/G_{TU} by,

$$0.897 = \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} = 1.122$$

$$-0.47 \text{ dB} < G_T(\text{dB}) - G_{TU}(\text{dB}) < 0.5 \text{ dB}$$

(11.12)

From (11.48a) and (11.49a), when $G_S = 1$ we have,

$$g_S = \frac{1}{G_{S_{\text{MAX}}}} = 1 - |S_{11}|^2, \quad 1 - g_S = |S_{11}|^2$$

So (11.52) reduces to,

$$C_S = \frac{(1 - |S_{11}|^2) S_{11}^*}{1 - |S_{11}|^4} = \frac{S_{11}^*}{1 + |S_{11}|^2}$$

$$R_S = \frac{|S_{11}|(1 - |S_{11}|^2)}{1 - |S_{11}|^4} = \frac{|S_{11}|}{1 + |S_{11}|^2}$$

So the equation of the constant gain circle becomes,

$$\left| \Gamma_S - \frac{S_{11}^*}{1 + |S_{11}|^2} \right| = \frac{|S_{11}|}{1 + |S_{11}|^2}$$

One solution to this equation occurs for $\Gamma_S = 0$, as the circle must pass through the center of the Smith chart.

(11.13)

$$S_{11} = 0.7 \angle 110^\circ, \quad S_{12} = 0.02 \angle 60^\circ, \quad S_{21} = 3.5 \angle 60^\circ, \quad S_{22} = 0.8 \angle -70^\circ$$
$$F_{\text{MIN}} = 2.5 \text{ dB}, \quad \Gamma_{\text{OPT}} = 0.7 \angle 120^\circ, \quad R_N = 15 \Omega$$

First check stability: $K = 1.07$, $|A| = 0.53$

Since $K > 1$ and $|A| < 1$ the device is unconditionally stable.

Minimum noise figure occurs for $\Gamma_S = \Gamma_{\text{OPT}} = 0.7 \angle 120^\circ$. Then we maximize gain by conjugate matching the output.

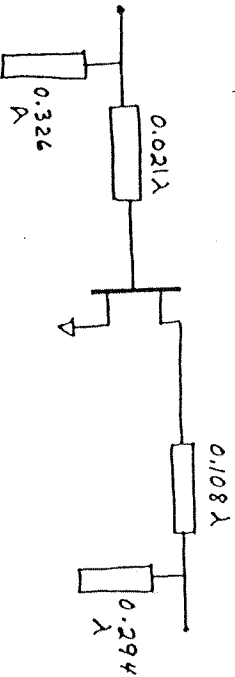
From (11.41b),

$$\Gamma_L = \left(S_{22} + \frac{S_{12} S_{21} \Gamma_S^*}{1 - S_{11} \Gamma_S} \right)^* = 0.873 \angle 74^\circ \quad \checkmark$$

So the noise figure will be $F = F_{\text{min}} = 2.5 \text{ dB}$, and the gain will be,

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - S_{11} \Gamma_S|^2} |S_{21}|^2 \frac{1 - |S_{22}|^2}{|1 - S_{22} \Gamma_L|^2}$$
$$= (1.85)(12.25)(3.81) = 86.3 = 19.4 \text{ dB}$$

Impedance matching is done with a Smith chart; the final amplifier circuit is shown below.



Supercompact analysis of this amplifier gives $|S_{11}| = 0.33$, $|S_{22}| = 0.13$, $G = 19.7 \text{ dB}$, and $F = 2.5 \text{ dB}$. The data file for this analysis is listed below. (The solution is a bit simpler if S_{12} is set to zero, resulting in $G = 18 \text{ dB}$.)

```

Compact Software - SUPER-COMPACT PC V6.52 27-JUN-97 17:07:09
File: C:\COMPACT\PRB11_13.CKT
NOI
OST 1 0 z=50 e=117. f=5.GHZ
TRL 1 2 z=50 e=8. f=5.GHZ
TWO 2 3 Q
TRL 3 4 z=50 e=39. f=5.GHZ
OST 4 0 z=50 e=106. f=5.GHZ
ZNF: 2POR 1 4
END
FREQ 5.GHZ
END
OUT
PRI AMP S
END
DATA
CI: S
SCHC .7 -110. 3.5 60. 0.02 60. .8 -70.
NOI RN
SCHC 2.5 .7 120. .3
END

```

$$(11.14)$$

$$S_{11} = 0.6 \angle -60^\circ, S_{21} = 2 \angle 81^\circ, S_{12} = 0, S_{22} = 0.7 \angle -60^\circ$$

$$F_{MIN} = 2.0 \text{ dB}, \Gamma_{OPT} = 0.62 \angle 100^\circ, R_N = 20 \Omega$$

Since $S_{12} = 0$ and $|S_{11}| |S_{22}| < 1$, the device is unconditionally stable. The overall gain is, $G_{TU} = G_S G_0 G_L$, where $G_0 = |S_{21}|^2 = 4 = 6 \text{ dB}$. ✓ So $G_S + G_L = 0 \text{ dB}$.

Plot noise figure circles for $F = 2.0, 2.05, 2.2$, and 3.0 dB :

F (dB)	N	C _F	R _F
2.05	0.0134	$0.6 \angle 100^\circ$	0.09
2.20	0.055	$0.59 \angle 100^\circ$	0.18
3.00	0.30	$0.48 \angle 100^\circ$	0.40
2.00	0.	$0.62 \angle 100^\circ$	0

Now plot constant gain circles for $G_S = G_L = 0 \text{ dB}$:

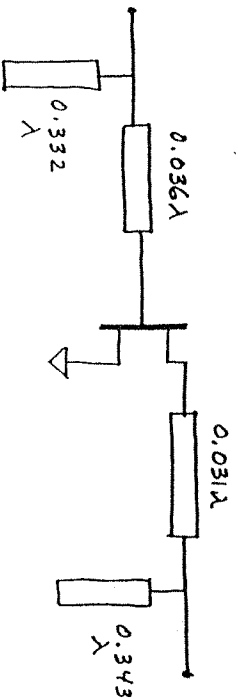
$$G_{S_{MAX}} = 1.56 \checkmark \quad G_{L_{MAX}} = 1.96 \checkmark$$

$$g_S = 0.641 \quad g_L = 0.510$$

$$c_S = 0.44 \angle 60^\circ \quad c_L = 0.47 \angle 60^\circ$$

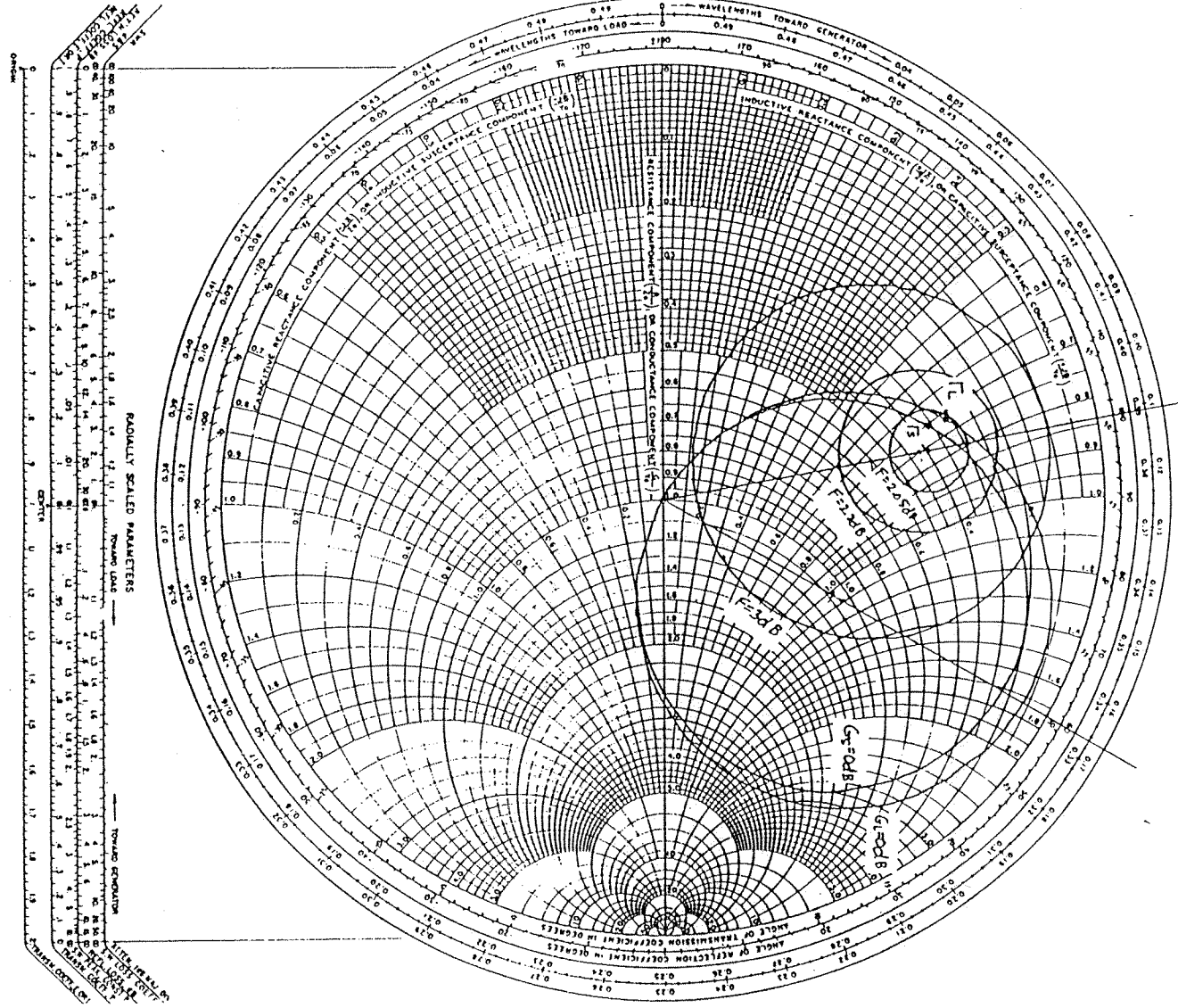
$$R_S = 0.44 \quad R_L = 0.47$$

These two circles are close together near the $F = 2 \text{ dB}$ point. We choose $\Gamma = 0.66 \angle 105^\circ$, $\Gamma_S = 0.62 \angle 105^\circ$. Then we should obtain $F \approx 2.04 \text{ dB}$. The final AC amplifier circuit is:



Super Compact analysis gives $|S_{11}| = 0.62$, $|S_{22}| = 0.67$, $G = 6.1 \text{ dB}$, and $F = 2.04 \text{ dB}$. ✓ The gain and noise circles are shown below.

240



11.15

S-parameters and noise parameters of Problem 11.14

Plot the $F=2.5$ dB constant noise figure circle:

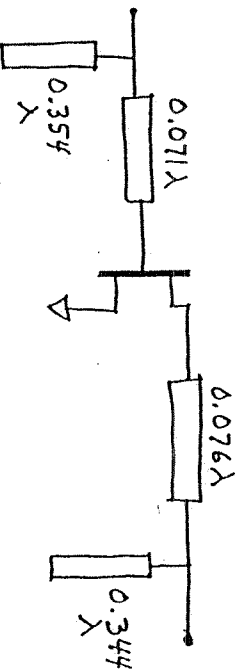
$$N=0.141, C_F=0.543 \angle 100^\circ, R_F=0.286$$

$$\text{Now, } G_{S_{MAX}}=1.56=1.93 \text{ dB}, G_{L_{MAX}}=1.96=2.92 \text{ dB}$$

But these points (S_{11}^* , S_{22}^*) do not lie on the $F=2.5$ dB circle. We can plot some gain circles to just give intersections with the $F=2.5$ dB noise circle:

$G_S=1.5$ dB	$g_S=0.905$	$C_S=0.56 \angle 60^\circ$	$R_S=0.204$
$G_L=2.5$ dB	$g_L=0.907$	$C_L=0.67 \angle 60^\circ$	$R_L=0.163$
$G_T=1.7$ dB	$g_S=0.948$	$C_S=0.58 \angle 60^\circ$	$R_S=0.149$
$G_S=1.8$ dB	$g_S=0.970$	$C_S=0.59 \angle 60^\circ$	$R_S=0.112$

The $G_S=1.8$ dB and $G_L=2.5$ dB circles are close to optimum (the $F=2.5$ dB noise circle). Thus we have $\Gamma_S=0.545 \angle 72^\circ$, $\Gamma_L=0.59 \angle 72^\circ$, which will yield a gain of $G_T=1.8+2.5+6=10.3$ dB. The final AC amplifier circuit is shown below:



SuperCompact analysis of this circuit gives $|S_{11}|=0.20$, $|S_{22}|=0.28$, $G=10.3$ dB, and $F=2.4$ dB. The noise and gain circles are shown on the following page.

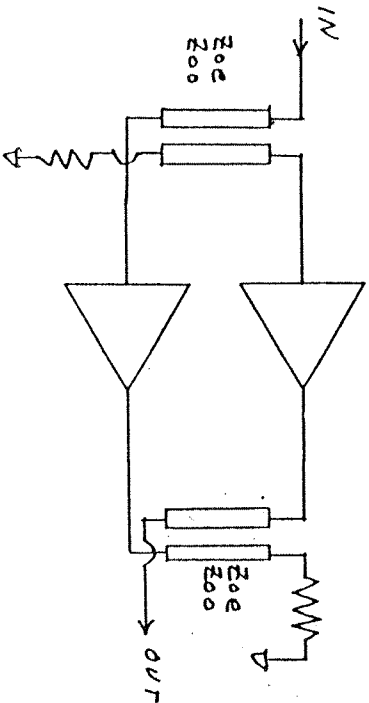
11.16

FOR THE COUPLED LINE COUPLERS:

$$C = 10^{-3} \gamma_{20} = 0.708$$

$$Z_{oe} = 50 \sqrt{\frac{1+C}{1-C}} = 121 \Omega$$

$$Z_{oo} = 50 \sqrt{\frac{1-C}{1+C}} = 21 \Omega$$

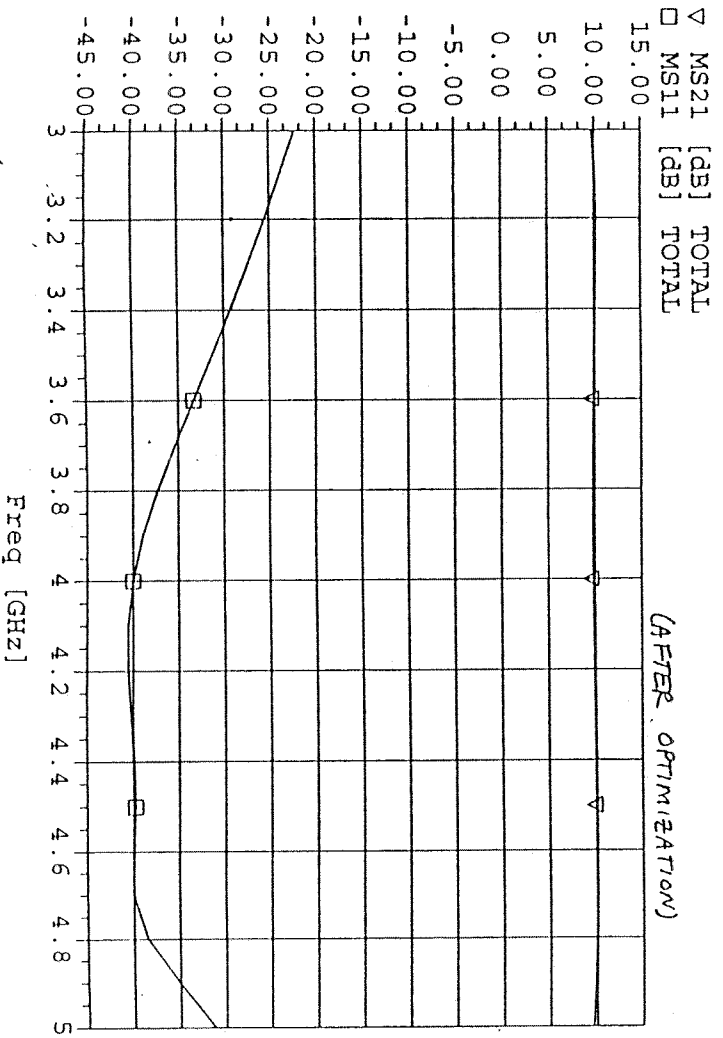
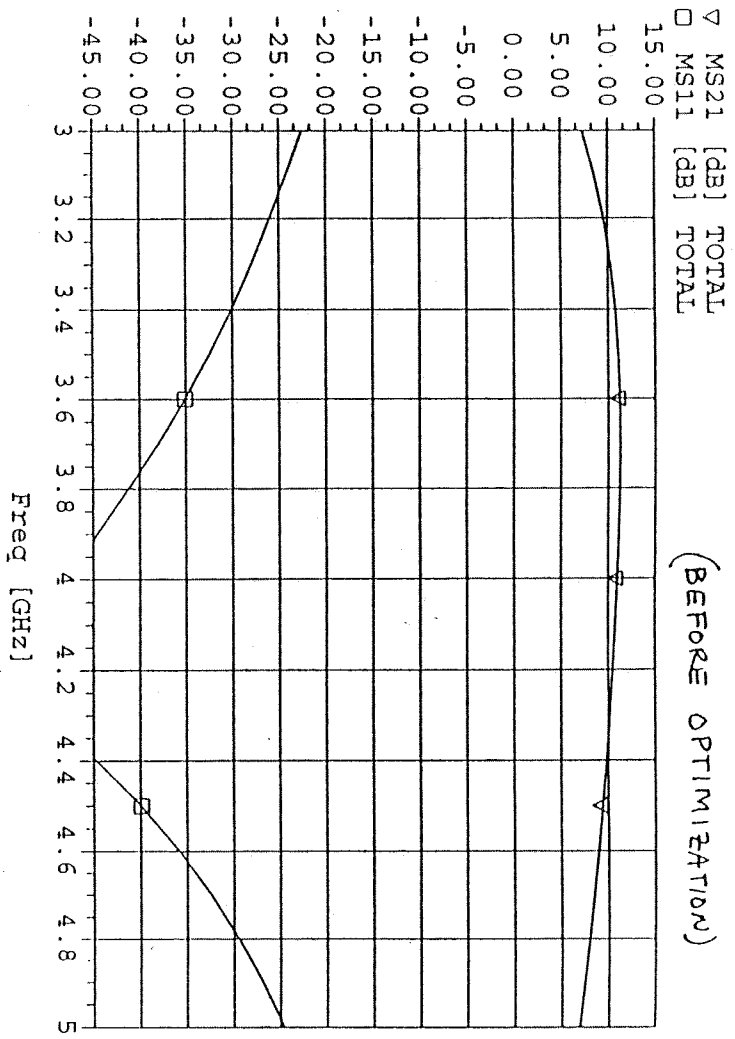


The amplifier circuit of Example 11.4 was used for both amplifiers here. As in Example 11.6, the amplifier matching networks were optimized using SuperCompact to give a flat 10dB gain response, with good input matching. Results of the optimization are given below, including the line and stub lengths before and after optimization, the SuperCompact data file, and the calculated gain and input return loss of the balanced amplifier before and after optimization. Results seem to be a bit better than those of Example 11.6.

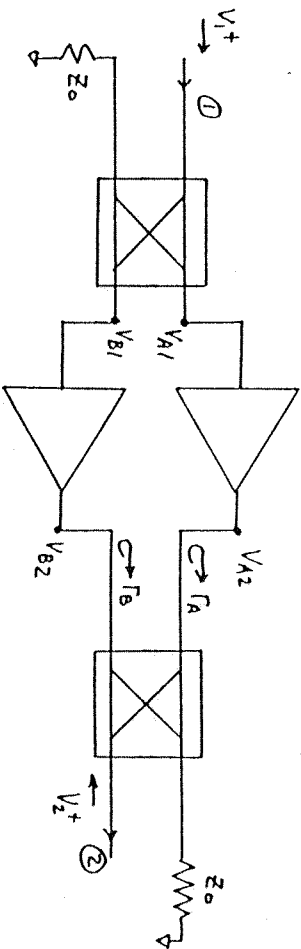
PARAMETER	BEFORE OPT.	AFTER OPT.
INPUT SECTION STUB LENGTH	0.100 λ	0.125 λ
INPUT SECTION LINE LENGTH	0.179 λ	0.119 λ
OUTPUT SECTION LINE LENGTH	0.045 λ	0.089 λ
OUTPUT SECTION LINE LENGTH	0.432 λ	0.458 λ

Compact Software - SUPER-COMPACT PC V6.52 16-MAY-97 16:25:15
File: c:\compact\prb1_16.ckt

```
LAD
OST 1 0 Z=50. E=?36.? F=4GHZ
TRL 1 2 Z=50. E=?64.? F=4GHZ
TWO 2 3 Q
TRL 3 4 Z=50. E=?16.? F=4GHZ
OST 4 0 Z=50. E=?156.? F=4GHZ
AMP: 2POR 1 4
END
BLK
cpl 1 2 3 4 ZE=121. ZO=21. E=90. F=4GHZ
cfc: 4POR 1 2 3 4
END
BLK
cfc 1 2 3 4
AMP 2 8
AMP 4 6
cfc 5 6 7 8
RES 3 0 R=50.
RES 7 0 R=50.
TOTAL: 2POR 1 3
END
FREQ
STEP 3GHZ 5GHZ 100MHZ
END
OPT
total MS21=10DB MS11
END
OUT
PRI total S
END
DATA
Q: S
3GHZ .80 -90. 2.80 100. .0 0. .66 -50.
4GHZ .75 -120. 2.50 80. .0 0. .60 -70.
5GHZ .71 -140. 2.30 60. .0 0. .58 -85.
END
```



(11.17)



The analysis for S_{22} is identical to that for S_{11} in eqn (11.62) - (11.66), but with input V_2^+ at port 2.

Thus, if the input at port 2 is V_2^+ , then the voltages incident at the amplifiers are,

$$V_{A2}^- = \frac{1}{\sqrt{2}} V_2^+$$

$$V_{B2}^- = \frac{-j}{\sqrt{2}} V_2^+$$

Then the reflected output voltage at port 2 is,

$$\begin{aligned} V_2^- &= \frac{1}{\sqrt{2}} V_{A2}^- + \frac{j}{\sqrt{2}} V_{B2}^- = \frac{1}{\sqrt{2}} \Gamma_A V_{A2}^- + \frac{-j}{\sqrt{2}} \Gamma_B V_{B2}^- \\ &= \frac{1}{2} V_2^+ (\Gamma_A - \Gamma_B) \end{aligned}$$

Thus,

$$S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+ = 0} = \frac{1}{2} (\Gamma_A - \Gamma_B) \quad \checkmark$$

(11.18)

$$\text{From (11.77), } G = \frac{g_m^2 Z_d Z_g}{4} \frac{(e^{-N\alpha_d} l_g - e^{-N\alpha_d} l_d)^2}{(e^{-\alpha_d} l_g - e^{-\alpha_d} l_d)^2}$$

Differentiating with respect to N and setting to zero gives,

$$\alpha_d l_g e^{-N\alpha_d} l_g - \alpha_d l_d e^{-N\alpha_d} l_d = 0$$

$$\ln \alpha_d l_g - N \alpha_d l_g = \ln \alpha_d l_d - N \alpha_d l_d$$

$$\ln \frac{\alpha_d l_g}{\alpha_d l_d} = N (\alpha_d l_g - \alpha_d l_d)$$

$$N = \frac{\ln(\alpha_d l_g / \alpha_d l_d)}{\alpha_d l_g - \alpha_d l_d} \quad \checkmark$$

11.19

$R_i = 5\Omega$, $R_{dS} = 200\Omega$, $C_{gs} = 0.35\text{ pF}$, $g_m = 40\text{ mS}$
Assume $Z_g = Z_d = 50\Omega$.

We use (11.69) and (11.72) to find $\alpha_g L_g$, $\alpha_d L_d$. Then use (11.77) to find G :

f	$G(N=4)$	$G(N=8)$	$G(N=16)$
2	9.9 dB	14.0 dB	16.5 dB
4	9.8	13.7	15.9
8	9.2	12.5	13.4
12	8.4	10.7	9.5
16	7.2	8.4	4.7
18	6.5	7.0	2.2
20	5.7	5.6	-0.2

Gain at $f = 18\text{ GHz}$ vs. N :

N	G (dB)
5	7.1
6	7.3
7	7.2
8	7.0
9	6.7

So the optimum value of N is seen to be $N_{opt} = 6$.
Using (11.78) to find N_{opt} directly:

$$\alpha_g L_g = 0.1958$$

$$\alpha_d L_d = 0.125$$

$$N_{opt} = \frac{\ln(\alpha_g L_g / \alpha_d L_d)}{\alpha_g L_g - \alpha_d L_d} = 6.33$$

(11.20)

Let $Z = R + jX$, $R > 0$

(POSITIVE RESISTANCE)

Then $Z = Z/Z_0 = r + jx$, $r > 0$

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{(r-1) + jx}{(r+1) + jx}$$

Now let $Z = -R + jX$, $R > 0$

(NEGATIVE RESISTANCE)

$Z = -r + jx$, $r > 0$

$$\Gamma_{\text{New}}, \quad \Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{-r-1 + jx}{(-r+1) + jx} = \frac{(r+1) - jx}{(r-1) - jx}$$

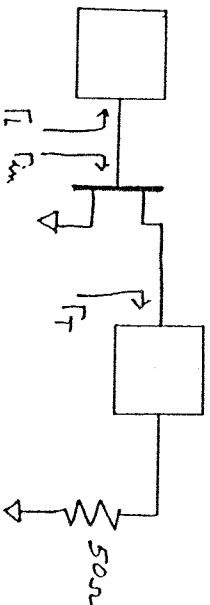
So,

$$\frac{1}{\Gamma^*} = \frac{(r-1) + jx}{(r+1) + jx}$$

which has the same form as Γ for positive resistance. So we can read the resistance circles as negative, and interpret the "reflector coefficient" read from the Smith chart as $1/\Gamma^*$.

(11.21)

$$S_{11} = 0.9 \angle -150^\circ, \quad S_{21} = 2.6 \angle 150^\circ, \quad S_{12} = 0.2 \angle -150^\circ, \quad S_{22} = 0.5 \angle -105^\circ$$



The output stability circle is,

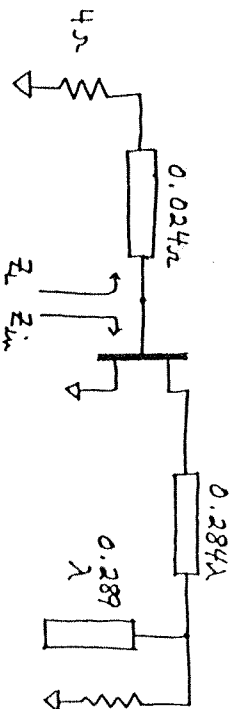
$$C_T = 8.09 \angle -15^\circ, \quad R_T = 8.28$$

From (11.6a),

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_T}{1 - S_{22} \Gamma_T}$$

Choose Γ_T so that $|\Gamma_T| < 1$ and Γ_{in} is large. By trial and error, we select $\Gamma_T = 0.9 \angle 130^\circ$. Then $\Gamma_{in} = 1.61 \angle -162^\circ$, σ_L
 $Z_{in} = -12.75 \Omega$. So $Z_L = -R_{in}/3 - jX_{in} = 4 + j7.5 \Omega = (0.08 + j0.15) Z_0$.

Matching networks were designed on the Smith chart. The final AC circuit is shown below.



(11.22)

From (11.79), we know that $Z_L + Z_{in} = 0$.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}, \quad Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

So,

$$\frac{1 + \Gamma_L}{1 - \Gamma_L} + \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 0$$

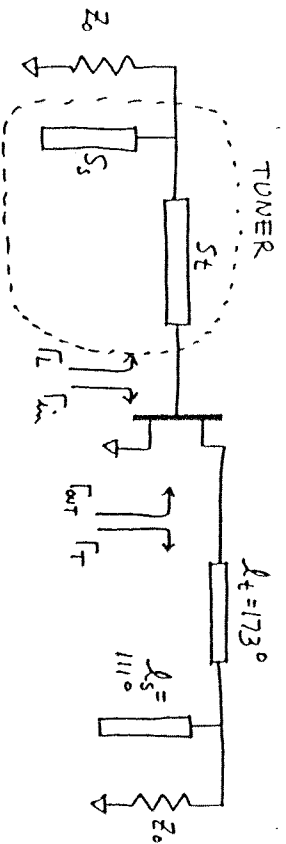
$$(1 + \Gamma_L)(1 - \Gamma_{in}) + (1 + \Gamma_{in})(1 - \Gamma_L) = 0$$

$$1 - \Gamma_{in} + \Gamma_L - \Gamma_L \Gamma_{in} + 1 - \Gamma_L + \Gamma_{in} - \Gamma_{in} \Gamma_L = 0$$

$$2 - 2\Gamma_{in}\Gamma_L = 0$$

$$\Gamma_{in}\Gamma_L = 1 \quad \checkmark$$

11.23

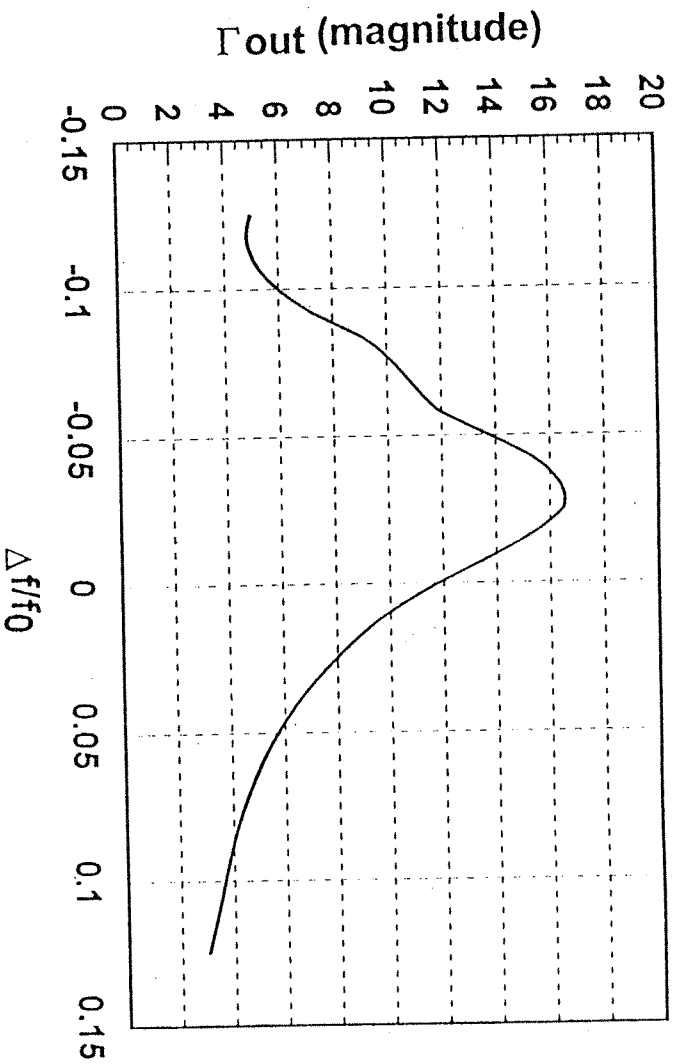


As in Example 11.10, choose $\Gamma = 0.6 \angle -130^\circ$. The Γ_{out} , Z_{out} , Z_T , L_T , and C_S are unchanged. Then we have the simple matching problem of using the stub tuner to match 50Ω to Γ . The stub susceptance is $jB_S = +j1.56$, or a stub length of $S_S = 0.158\lambda$. The line length is $S_T = 0.18 - 0.176 = 0.004\lambda$.

We then analyze the above circuit to compute Γ_{out} versus frequency:

f (GHz)	$\Delta f/f_0$	$ \Gamma_{out} $
2.10	-0.125	5.0
2.18	-0.092	7.2
2.20	-0.083	9.1
2.26	-0.058	11.9
2.30	-0.042	15.4
2.34	-0.025	16.5
2.38	-0.008	13.6
2.40	0	11.8
2.42	0.008	10.2
2.46	0.025	7.9
2.50	0.042	6.3
2.60	0.083	4.1
2.66	0.110	3.4
2.70	0.125	3.0

The maximum of $|\Gamma_{out}|$ does not occur at $\Delta f = 0$ because the tuner is not resonant at f_0 . The "Q" is much lower than in Example 11.10. This problem shows the advantage of using a high-Q resonator for the oscillator. $|\Gamma_{out}|$ vs f is plotted on the following page.



(11.24)

$$S_{11} = 1.2 \angle 150^\circ, \quad S_{12} = 0.2 \angle 120^\circ, \quad S_{21} = 3.7 \angle -72^\circ, \quad S_{22} = 1.3 \angle -67^\circ$$

As in Example 11.10, maximize $|\Gamma_{out}|$ by choosing $S_{11}\Gamma_L \approx 1$, \approx same

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Thus let $\Gamma_L = 0.8 \angle -150^\circ$. Then $|\Gamma_{out}| = 15.88 \angle -99.3^\circ$, and

$$Z_{out} = Z_0 \frac{1 + \Gamma_{out}}{1 - \Gamma_{out}} = -7.6 + j1.9 \Omega$$

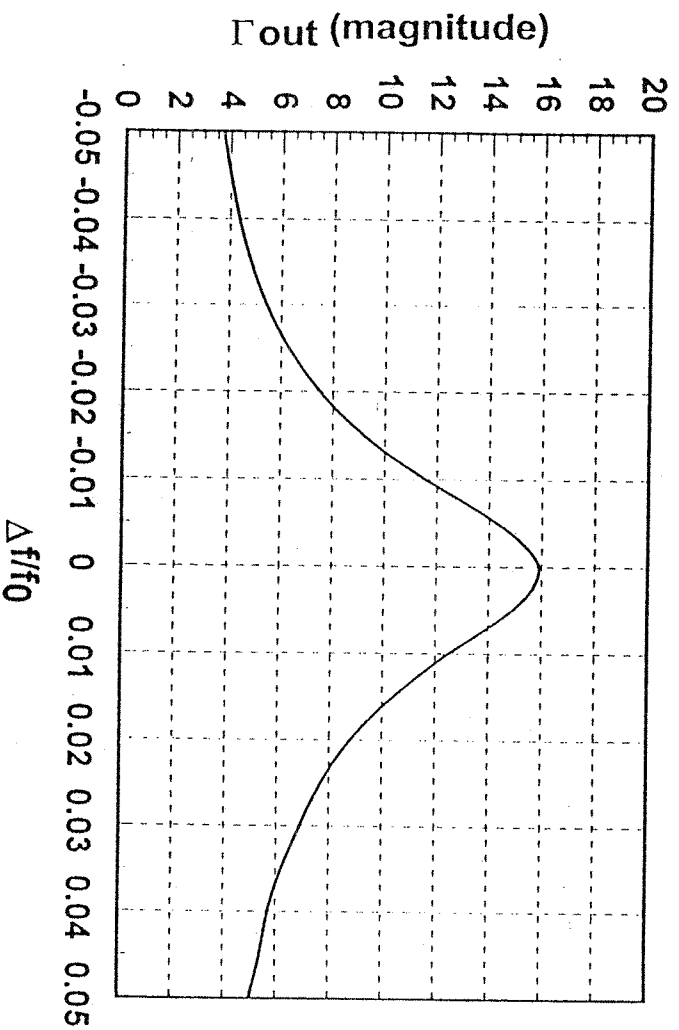
$$Z_T = \frac{-Z_{out}}{3} = -j1.9 \Omega \quad (Z_T = 0.0506 - j0.038)$$

Matching Z_T to the load impedance gives $L_T = 0.031 \lambda$ with a required short susceptance of $+j4$. Thus $L_S = 0.21 \lambda$.

At the diode's resonator, $\Gamma_L' = \Gamma_L e^{2j\beta L} = (0.8 \angle -150^\circ) e^{2j\beta L} = 0.8 \angle 180^\circ$. Thus $L_r = 0.4583 \lambda$.

$$Z'_L = Z_0 \frac{1 + \Gamma'_L}{1 - \Gamma'_L} = 5.55 Z_0 = N^2 R$$

$|\Gamma_{out}|$ vs f was calculated with Super Compact, and is plotted below:



Chapter 12

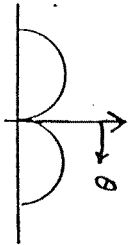
(12.1) From (12.10) and (12.11) the power density from an antenna with gain G is,

$$S = \frac{P_{in} G}{4\pi r^2} \quad \text{W/m}^2$$

So,

$$G = \frac{4\pi r^2 S}{P_{in}} = \frac{4\pi (5000)^2 (1.27 \times 10^{-6})}{100} = 4.0 = 6 \text{ dB}$$

(12.2) $E_{\theta} = \frac{a}{r} \sin \theta e^{j(kr - \omega t)}$ for $0 \leq \theta \leq 90^\circ$



From (12.2) and (12.1) the directivity is,

$$D = \frac{4\pi F_{MAX}}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin \theta d\theta d\phi}$$

$$F(\theta, \phi) = r^2 S(\theta, \phi) = \frac{r^2}{\eta_0} |E_{\theta}|^2 = A \sin^2 \theta \quad \text{for } 0 \leq \theta \leq \pi/2$$

Then

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin \theta d\theta d\phi = 2\pi A \int_{\theta=0}^{\pi/2} \sin^3 \theta d\theta = 2\pi A \left(\frac{2}{3}\right) = \frac{4\pi A}{3}$$

Thus, $D = \frac{4\pi A}{4\pi A/3} = 3 = 4.77 \text{ dB}$ ✓

(12.3) From (12.5), (12.6), and (12.11),

$$P_d = A_e S = \frac{G \lambda^2 S}{4\pi} = \frac{(15.8)(.0375)^2 (2 \times 10^{-6})}{4\pi}$$

($G=15.8$; $\lambda=0.0375 \text{ m}$)

$$= 3.54 \times 10^{-9} \text{ W}$$

12.4

From (12.12), $T_E = \eta T_b + (1-\eta) T_p$

T_E = equivalent noise temperature of antenna

T_b = brightness temperature of background

T_p = physical temperature of antenna

Solving for η :

$$\eta = \frac{T_p - T_E}{T_p - T_b} = \frac{295 - 345}{295 - 5} = 0.898$$

12.5

From the Friis power transmission formula of (12.15), the required transmitter power is, for

$$G_T^2 G_R = 10^{38/10} = 6310, \quad \lambda = 0.061 \text{ m}, \quad P_r = 10^{-6} \text{ mW}$$

$$P_T = \frac{(4\pi R)^2}{G_T G_R \lambda^2} P_r = \frac{(4\pi)^2 (27,000)^2}{(6310)^2 (0.061)^2} (10^{-6}) = 0.777 \text{ mW} = -1.1 \text{ dBm}$$

12.6

From (12.10) and (12.11),

$$S = \frac{P_{in} G}{4\pi r^2}$$

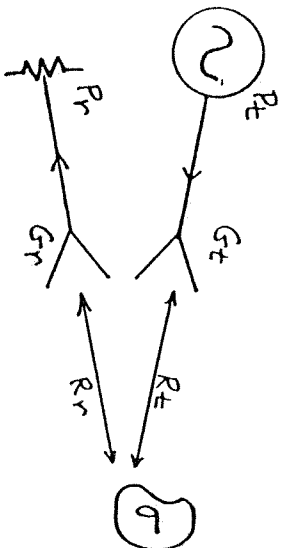
In the main beam of the antenna, $G = 10^{34/10} = 1000$,
As,

$$S = \frac{10 (1000)}{4\pi (30)^2} = 0.884 \text{ W/m}^2 = 0.088 \text{ mW/cm}^2$$

In the sidelobe region of the antenna, $G = 10^{15/10} = 31.6$,
As,

$$S = \frac{10 (31.6)}{4\pi (30)^2} = 0.028 \text{ W/m}^2 = 0.0028 \text{ mW/cm}^2$$

12.7



From (12.10)-(12.11) the power density incident on the target is,

$$S = \frac{P_T G_T}{4\pi R_T^2}$$

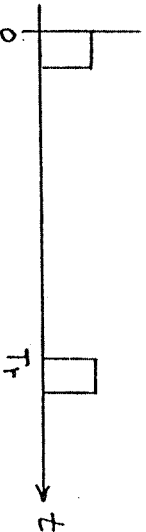
The scattered power density at the receiver is, from (12.26),

$$S_R = \frac{P_T G_T \sigma(\theta_E, \phi_E; \theta_R, \phi_R)}{(4\pi)^2 R_T^2 R_R^2},$$

where $\sigma(\theta_E, \phi_E; \theta_R, \phi_R)$ is the radar cross-section of the target seen at θ_R, ϕ_R with an incident wave at θ_E, ϕ_E . Then the received power can be found using (12.14):

$$P_R = P_T \frac{G_T G_R \lambda^2 \sigma(\theta_E, \phi_E; \theta_R, \phi_R)}{(4\pi)^3 R_T^2 R_R^2} \quad \checkmark$$

12.8



When a pulse is transmitted at $t=0$, the return pulse must come back before the next pulse is transmitted at $t=T_T$, to avoid an ambiguity in range. The round-trip time for a pulse return is,

$$T = 2R/c,$$

so the maximum unambiguous range is,

$$R_{MAX} = \frac{cT_T}{2} = \frac{c}{2f_T} \quad \checkmark$$

(12.9)

From (12.29) the doppler frequency are,

$$f_{d(MIN)} = \frac{2v_{MIN} f_0}{c} = \frac{2(\frac{1200}{3600}) (12 \text{ GHz})}{3 \times 10^8 \text{ m/sec}} = 80 \text{ Hz}$$

$$f_{d(MAX)} = \frac{2v_{MAX} f_0}{c} = \frac{2(20 \text{ M/sec}) (12 \text{ GHz})}{3 \times 10^8 \text{ m/sec}} = 1.6 \text{ kHz}$$

So the necessary passband is 80-1600 Hz.

(12.10)

From (12.27) the received power is,

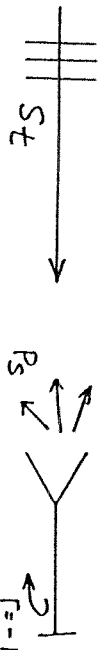
$$P_r = P_t \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} = 1000 \frac{(1000)^2 (0.15)^2 (20)}{(4\pi)^3 (10^4)^4} = 2.27 \times 10^{-11} \text{ W}$$

$$= -76 \text{ dBm}$$

The transmitter power is $10 \log(10^4)(10^3) = 70 \text{ dBm}$. So the isolation between receiver and transmitter must be,

$$I = 70 \text{ dBm} - (-76 \text{ dBm}) + 10 \text{ dB} = 156 \text{ dB}$$

(12.11)



Assume an incident plane wave with power density S_t . Then the received power of the antenna is, from (12.5) and (12.6):

$$P_R = S_t A_e = S_t \frac{\lambda^2 G}{4\pi}$$

Because of the short-circuit termination, all of this power is re-transmitted (assuming a lossless antenna), giving a radiated power in the main beam direction of, $P_S = G P_R$. Then the RCS can be found from (12.25):

$$\sigma = \frac{P_S}{S_t} = \frac{\lambda^2 G^2}{4\pi} \quad \checkmark$$

(12.12)

$$G_i = 31.6, \quad G_r = 1000.$$

From (12.41) we have the following relation for the self-screening parameter case:

$$\frac{J}{S} = \frac{P_i}{P_r} \frac{4\pi R^2 G_i}{\sigma G_r} \left(\frac{Br}{G_i} \right)$$

$$\text{As,} \quad I = \left(\frac{100}{10000} \right) \frac{4\pi R^2 (31.6)}{5 (1000)} \left(\frac{.5}{10} \right) = 3.97 \times 10^{-5} R^2$$

So the range for $J/S = 1$ is $R = 159 \text{ m}$. ($J/S > 1$ for $R > 159 \text{ m}$)

(12.13)

$$S = 1300 \text{ W/m}^2 = \frac{1}{2} |\mathbf{E}| |\mathbf{H}| = \frac{1}{2\eta_0} |\mathbf{E}|^2 = \frac{\eta_0}{2} |\mathbf{H}|^2$$

$$|\mathbf{E}| = \sqrt{2\eta_0 S} = \sqrt{2(377)(1300)} = 990 \text{ V/m}$$

$$|\mathbf{H}| = \sqrt{\frac{2S}{\eta_0}} = \sqrt{\frac{2(1300)}{377}} = 2.6 \text{ A/m}$$

CHECK:

$$\frac{1}{2} |\mathbf{E}| |\mathbf{H}| = \frac{1}{2} (990)(2.6) = 1300 \text{ W/m}^2 \quad \checkmark$$

