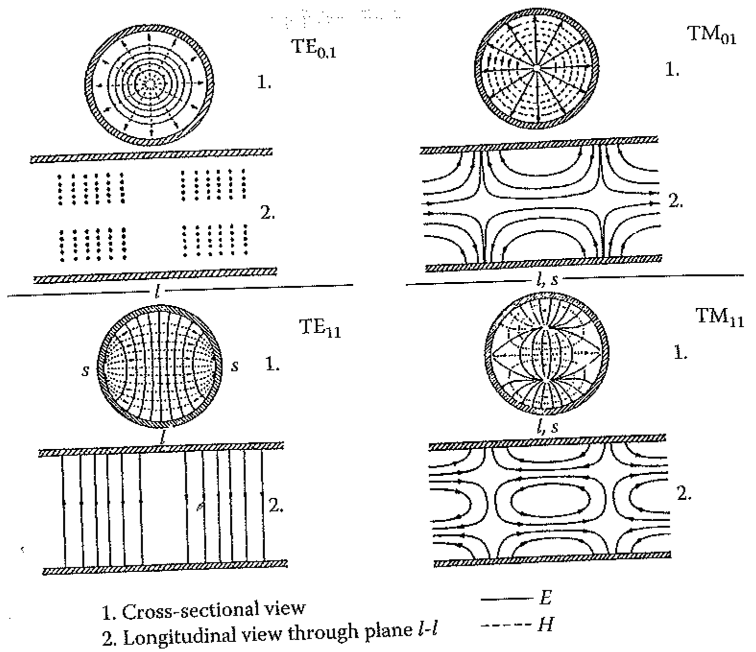

FIGURE 4.7
Field patterns for four normal modes of a circular waveguide. (From Saad, T.S. et al., *Microwave Engineers Handbook*, Vol. 1, Artech House, Norwood, MA, 1971. With permission.)

Figure 4.7 shows the field patterns for four of the circular waveguide modes. The $TE_{1,1}$ mode is commonly used because of the nearly plane-wave-like structure of the fields near the center of the waveguide. The $TM_{0,1}$ mode is also commonly seen, although it is less desirable in a number of applications because its intensity pattern has a minimum on the axis.

4.3.3 Power Handling in Waveguides and Cavities

The power-handling capability of a waveguide or cavity depends on two factors. At shorter pulses, roughly less than about 1 μsec, a key issue is breakdown, while at longer pulses, or at high duty factors so that the average power becomes significant, wall heating predominates. High-field breakdown of metallic waveguides and cavities is a complex subject that is reviewed elsewhere in greater detail.[1] Cleaning and conditioning of surfaces, maintaining stringent vacuum requirements, and applying surface coatings are elements necessary to maximize the power level at which breakdown occurs. The magnitude of the electric field at the wall is a key parameter determining when breakdown will occur. In a recent review, metal surfaces coated with a thin layer of titanium nitride (TiN) were found to break down when these fields exceeded one to several hundred MV/m. We therefore first consider the relationship between the power in a waveguide and the electric field at the wall. We will discuss how to determine this relationship for TM modes in a rectangular waveguide, and then provide the relationships for the TE mode in a rectangular waveguide, and the TM and TE modes in circular waveguides.

The *Poynting vector* $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ measures the direction and magnitude of the power flux carried by an electromagnetic wave. The integral of this vector over a surface Σ is interpreted as the power flowing through that surface. For oscillating fields of the form in Equation 4.11, the average power through that surface is given by

$$P = \int_\Sigma \langle \mathbf{S} \rangle \bullet d\mathbf{A} = \frac{1}{2}\int_\Sigma \mathbf{E}\times\mathbf{H}^* \bullet d\mathbf{A} = \frac{1}{2\mu_0}\int_\Sigma \mathbf{E}\times\mathbf{B}^* \bullet d\mathbf{A} \tag{4.39}$$

In a rectangular waveguide, this expression becomes

$$P = \frac{1}{2\mu_0}\int_0^b\int_0^a \left(E_x B_y^* - E_y B_x^*\right) dxdy \tag{4.40}$$

For TM waves, taking the form of the fields given in Table 4.1, integrating by parts, and using Equation 4.23 and the form for E_z in the table, we find

$$P_{TM} = \frac{1}{2\mu_0}\left(\frac{\omega k_z}{\omega_{co}^2}\right)\int_0^b\int_0^a |E_z|^2\, dxdy = \frac{ab}{8\mu_0}\left(\frac{\omega k_z}{\omega_{co}^2}\right)|D|^2 \tag{4.41}$$

Here, D is the field magnitude. We express this in terms of the wall field using the expression for the maximum value of the perpendicular field component at the wall, which is the maximum value of either E_x at the surfaces $x = 0$ and $x = a$ or E_y at the surfaces $y = 0$ and $y = b$. Defining $E_{wall,max}$ to be the larger of the two magnitudes, we use the expressions for E_x and E_y in Table 4.1 to eliminate $|D|^2$, finding

$$P_{TM} = \frac{ab}{8\pi^2 Z_0} k_{\perp TM}^2 \left(\frac{\omega}{k_z}\right)\min\left[\left(\frac{a}{n}\right)^2, \left(\frac{b}{p}\right)^2\right] E_{wall,max}^2 \tag{4.42}$$

where min(x, y) is the smaller of x or y, since the larger wall field, E_x or E_y, corresponds to the smaller of a/n or b/p (i.e., if $a/n < b/p$, $E_{wall,max}$ is the maximum value of E_x at the wall), and

$$Z_0 = \mu_0 c = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} = 377\ \Omega \tag{4.43}$$