To complete this derivation, we use Equation 4.20 to find k_z in terms of ω and $k_{\perp,TM} = \omega_{co}/c = f_{co}/(2\pi c)$:

$$P_{TM} = \frac{ab}{8Z_0}\left[\left(\frac{n}{a}\right)^2 + \left(\frac{p}{b}\right)^2\right] \frac{1}{\left[1-\left(\frac{f_{co}}{f}\right)^2\right]^{1/2}} \min\left[\left(\frac{a}{n}\right)^2, \left(\frac{b}{p}\right)^2\right] E^2_{wall,max} \quad (4.44)$$

We see that as the waveguide dimensions increase, the power-handling capability increases for a constant $E_{wall,max}$. Also, the wall field for constant P decreases as $f \to f_{co}$.

In Table 4.4, we collect the expressions for the relationships between P and $E_{wall,max}$ for TM and TE modes in rectangular and circular waveguides. Separate expressions are required for TE modes in rectangular waveguides, depending on whether one of the field indices n or p vanishes, although both cannot vanish simultaneously. The factor of two difference stems from the fact that one of the cosine terms in the expression for B_z has a vanishing argument. One of the most important modes in a rectangular waveguide is the lowest-frequency $TE_{1,0}$ mode, for which Equation 4.42 reduces to

$$P_{TE}\left(TE_{1,0}\right) = \frac{ab}{4Z_0}\left[1-\left(\frac{c}{2af}\right)^2\right]^{1/2} E^2_{wall,max} \quad (4.45)$$

(see Problem 4).

In the case of circular waveguides, $E_{wall,max}$ is the maximum value of $E_r(r_0)$. The situation for TE modes is somewhat different, since E_r vanishes identically for p = 0. Therefore, the relation in the table holds only for p > 0. Two scaling relationships stand out in all of these expressions. First, the scaling with frequency is quite different for TM and TE modes. As $f \to f_{co}$ for a fixed value of the wall field, the power in the waveguide increases in the case of TM modes, while it must decrease for TE modes. Second, in the case of both TM and TE modes, for a fixed value of the wall field, the power increases roughly as the area of the cross section (see Problem 5).

The relationship between the electric field component perpendicular to the wall and the power flowing through a waveguide is important in the case of high-field breakdown on short timescales. When the length of the microwave pulse is long enough, or when a high power signal is pulsed rapidly enough at a high duty factor (defined as the product of the pulse repetition rate and the length of an individual pulse), the walls come into thermal equilibrium, and wall heating becomes an important operational factor for waveguides and cavities. To understand wall heating, we return to the analysis of the normal-mode fields. In our previous derivations, we assumed that the waveguide walls were perfectly conducting. In such ideal guides, a

TABLE 4.4
Relationship between the Power in a Given Mode, P, and the Maximum Value of the Electric Field at the Wall, $E_{wall,max}$

Mode	
Rectangular	
TM	$P_{TM} = \frac{ab}{8Z_0}\left[\left(\frac{n}{a}\right)^2 + \left(\frac{p}{b}\right)^2\right] \frac{1}{\left[1-\left(\frac{f_{co}}{f}\right)^2\right]^{1/2}} \min\left[\left(\frac{a}{n}\right)^2, \left(\frac{b}{p}\right)^2\right] E^2_{wall,max}$
TE n, p > 0	$P_{TE} = \frac{ab}{8Z_0}\left[\left(\frac{n}{a}\right)^2 + \left(\frac{p}{b}\right)^2\right]\left[1-\left(\frac{f_{co}}{f}\right)^2\right]^{1/2} \min\left[\left(\frac{a}{n}\right)^2, \left(\frac{b}{p}\right)^2\right] E^2_{wall,max}$
TE n or p = 0	$P_{TE} = \frac{ab}{4Z_0}\left[\left(\frac{n}{a}\right)^2 + \left(\frac{p}{b}\right)^2\right]\left[1-\left(\frac{f_{co}}{f}\right)^2\right]^{1/2} \min\left[\left(\frac{a}{n}\right)^2, \left(\frac{b}{p}\right)^2\right] E^2_{wall,max}$
Circular	
TM	$P_{TM} = \frac{\pi r_0^2}{4Z_0}(1+\delta_{p,0})\frac{E^2_{wall,max}}{\left[1-\left(\frac{f_{co}}{f}\right)^2\right]^{1/2}}$
TE p > 0	$P_{TE} = \frac{\pi r_0^2}{2Z_0}\left[1-\left(\frac{f_{co}}{f}\right)^2\right]^{1/2}\left[1+\left(\frac{\nu^2_{pn}}{p}\right)^2\right] E^2_{wall,max}$

Note: Min(x, y) is the smaller of x and y, Z_0 = 377 Ω, and $\delta_{p,0}$ is a Kronecker delta.

boundary current flows in an infinitely thin layer at the conductor surface to prevent the tangential component of **B** from penetrating the wall. When the wall conductivity σ is finite, the situation changes. In general, one would have to re-solve the problem, taking account of the penetration of the fields and surface currents into the walls. Fortunately, waveguide materials of interest have high-enough conductivity that we can treat the effect of finite σ as a perturbation to the infinite σ case, using the fields that we have already determined for that circumstance.

In considering wall heating, we confine our attention to circular waveguides, since this geometry is applicable to circular cross-section cavities, where this issue is perhaps most important. Further, as we did when we determined the relationship between wall fields and power, we treat the case of TM modes in the most detail, presenting the results for TE modes without derivation. First, consider the effect of finite σ at the wall. For