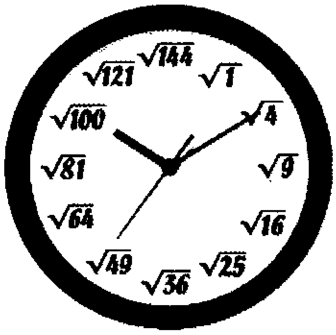
How to calculate a square root

http://www.youtube.com/watch?v=3i94NWF39nU



Pre-listening

1) **How can you find out what a square root of a number is without a calculator?**
2) **Which numbers are "perfect squares"?**

Listening.

Listen to and watch the video and decide whether the statements are true or false.

1) Approximation technique will produce a number which is not accurate.
2) The method proposed by the professor is exact decimal by decimal.
3) He learned it in 1968 when he was at a university.
4) There were no calculators at that time, so he had to ask the teacher about the square roots.
5) We start with number 1, which is a perfect square.
6) 2 is not possible because it is not a perfect square.
7) We put a random number in the blank space.
8) 7 is the smallest digit we can use.
9) The important step is to double the underlined digit.
10) When the last digit is 0, we must subtract another place.
11) He can´ t present the explanation why it works because it is extremely difficult.
12) Square roots of integers that are not perfect squares are called irrational.
13) They have two important features: decimals go on forever and there is a certain pattern of repetition.
14) They go on forever because you never get a zero remainder.
15) Cube roots cannot be solved in a similar way.

1. **Look and read:**

Exact calculations and approximations

Some square roots may be calculated exactly

e.g. $\sqrt{4} = 2$

$\sqrt{6{\cdot}25} = 2{\cdot}5$

$\sqrt{14{\cdot}44} = 3{\cdot}8$

Other square roots may be calculated only approximately

e.g. $\sqrt{2} = 1{\cdot}414213......$

$\sqrt{3} = 1{\cdot}7320508......$

$\sqrt{5} = 2{\cdot}236068......$

These approximate square roots are called irrational numbers i.e. we can continue the numbers after the decimal point as long as we wish.

Look at the following and say whether they can be calculated *exactly* or *only approximately*:

a) $\sqrt{9}$ c) $\sqrt{12{\cdot}25}$ e) The area of a circle g) $\sqrt{110{\cdot}25}$
b) $\sqrt{13}$ d) π f) Any irrational number h) $\sqrt{23{\cdot}5}$

2. **Read this:**

Approximations to square roots

To find $\sqrt{7}$:
First, we guess a value for $\sqrt{7}$, say $2\frac{1}{2}$.
$7/2\frac{1}{2} = 2{\cdot}8$. Thus $2\frac{1}{2}$ is too small.
So we try a value half-way between $2\frac{1}{2}$ and $2{\cdot}8$ i.e. $2{\cdot}65$.
$7/2{\cdot}65 = 2{\cdot}64$. Thus $2{\cdot}65$ is slightly too large.
So we try $(2{\cdot}65+2{\cdot}64)/2 = 2{\cdot}645$.
$7/2{\cdot}645 = 2{\cdot}646$.
$\sqrt{7}$ may be calculated to an *arbitrary* degree of accuracy i.e. we can calculate it to *any required* degree of accuracy, but $2{\cdot}645$ is a reasonably good approximation.

Now write similar paragraphs using the following examples:

a) $\sqrt{11}$; first guess $3\frac{1}{2}$ b) $\sqrt{34}$: first guess $5\frac{1}{2}$

3. **Read this:**

- 4 exceeds $\sqrt{7}$ by a considerable amount.
- $2{\cdot}65$ exceeds $\sqrt{7}$ by a very small amount.

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4. **Look and read:**

$0 < x < 1$	No integral value of x satisfies this inequality.
$x = \sqrt{-1}$	No real number satisfies this equation.
x is a prime number and x is even	Only one number satisfies these requirements.

Now write similar sentences about the following:

x is the square of an integer and the last digit of x is 3	a) ...... value ......
$x^2+5=0$	b) ...... real number ......
$0<x<2$	c) ...... integral value ......
$x^2+2x-35=0$	d) ...... positive value ......
$x=\sqrt{2}$	e) ...... rational number ......
x is divisible by both 7 and 9 and $x<100$	f) ...... value ......

5. **Look and read:**

We are given the length of one side of a regular hexagon. This is sufficient for the area to be calculated.

We are given the length of one side of a triangle. This is insufficient for the area to be calculated.

Now write sentences about the following in the same way:

Given	Required
a) one side of a square	area
b) one side of a rectangle	area
c) the altitude of a cone	volume
d) the area of one face of a regular dodecahedron	surface area
e) the length of the non-parallel sides of a trapezium	area
f) the surface area of a sphere	volume
g) the area of the lateral faces of a prism	volume
h) a chord of a circle	area

6. Look and read:

Congruence of triangles

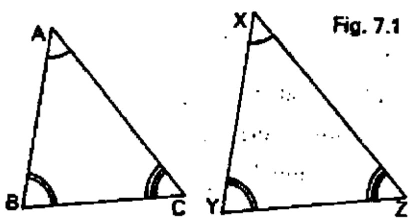


Fig. 7.1

- In Figure 7.1 $\hat{A} = \hat{X}$, $\hat{B} = \hat{Y}$, $\hat{C} = \hat{Z}$ (i.e. the angles are equal). This is a *necessary* condition for the two triangles to be congruent, but it is not a *sufficient* condition, i.e. the two triangles may be congruent, but we have insufficient information.

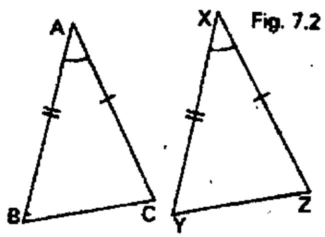


Fig. 7.2

- In Figure 7.2 $\hat{A} = \hat{X}$, AB = XY, AC = XZ (i.e. two sides and the included angle are equal). This is a sufficient condition for the triangles to be congruent, i.e. the triangles are congruent.

Now write about the following pairs of triangles in the same way:

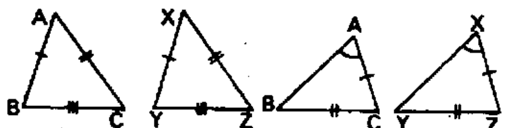


Fig. 7.3 Fig. 7.4

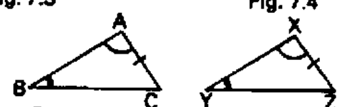


Fig. 7.5

Section 3 Reading

7. Read this:

The solution of triangles

A triangle has three sides and three angles. When three of these elements are known and at least one of the elements is a side, the other three elements can be calculated. Only one trigonometrical ratio, sine, is required in the calculation.

In a triangle ABC, we are given the lengths of AB and AC and the value of $A\hat{B}C$.

We use this formula:

$$\text{Sin } A\hat{C}B = \frac{AB \sin A\hat{B}C}{AC}$$

The fraction $\frac{AB \sin A\hat{B}C}{AC}$ may be of three kinds:

i) an improper fraction i.e. greater than one;
ii) a proper fraction i.e. less than one;
or iii) exactly equal to one.

In case (i), AC is smaller than AB sin $A\hat{B}C$. This requires sin $A\hat{C}B$ to be greater than one, which is impossible. Therefore no such triangle can exist.

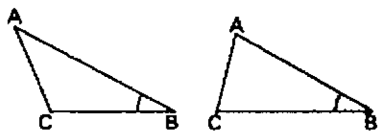


Fig. 7.6

In case (ii), AC is greater than AB sin $A\hat{B}C$. Two values of $A\hat{C}B$ may satisfy the equation (Figure 7.6). In this case, further information is required to solve the triangle exactly.

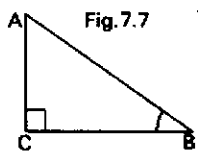


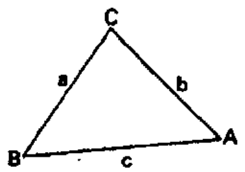
Fig. 7.7

In case (iii), AC is equal to AB sin $A\hat{B}C$. Only one solution satisfies the equation: $A\hat{C}B = 90°$ (Figure 7.7).

Say whether the following statements are true or false? Correct the false statements.

a) The sine ratio is sufficient for triangle ABC (given AB, AC and $A\hat{B}C$) to be solved.
b) Any three elements of a triangle are sufficient for it to be solved.
c) Case (i) would require $A\hat{C}B$ to be greater than two right angles.
d) Only one further element is required to solve the triangle in (ii).
e) In (ii) we are given the further information that $A\hat{C}B$ exceeds one right angle. This is sufficient for the triangle to be solved.

8. Look at these examples:



- $\hat{A} > 180°$. No such triangle can exist.
- $a = b = c = 3$ cm. Only one such triangle can exist.

Write similar sentences about the following cases:

a) $\frac{b \sin A}{a} > 1$
b) $\frac{b \sin A}{a} \geqslant 1$
c) $\frac{b \sin A}{a} = 1$
d) $\frac{b \sin A}{a} < 1$

Section 4 Listening

Approximate values

9. Listen to the passage and write down in figures each number you hear.

10. Listen to the passage again and say whether the following statements are true or false. Correct the false statements.

a) 3·1416 is an approximate value of π.
b) The difference between two approximate values is known as the absolute error.
c) The absolute error is the same as the true error.
d) The relative error is the true value divided by the absolute error.
e) The percentage error is found by multiplying the absolute value by 100.
f) The true value of 3·76, which is accurate to three significant figures, may be anywhere between 3·7 and 3·8.

11. Look at this example:

3·76 is an approximate value of 3·757 accurate to three significant figures.

Now make similar sentences about the following:

a) 3·1416; π
b) 0·108; 0·1077
c) 3 500; 3 498

12. Solve these problems:

Find a) the absolute error, b) the relative error and c) the percentage error in exercise 11 c).

13. PUZZLE:

How many different digits are needed to give the value of:

a) 1/3 b) $(1/3)^2$ c) $(1/3)^4$ to ten significant figures?

Unit 7
Approximate values
The value of π may be calculated to any required degree of accuracy. Correct to four decimal places, its value is 3·1416. This value is said to be correct to five significant figures.

If the population of a city is 346 268, then we may say that the population is approximately 350 000. This approximation is said to be correct to two significant figures.

In this last case the approximate value exceeds the true value by 3 732. This difference is known as the absolute error or the true error.

Another important value is *relative* error. We can use the formula $\frac{\text{absolute error}}{\text{true value}}$ to find the relative error. In this case, we have

$$\frac{3\,732}{346\,268} = 0{\cdot}0108.$$

Note that the calculation of the relative error, 0·0108, is accurate to three significant figures.

If we multiply one approximate value by another, the number of significant figures in the product is generally less than in the multiplier and multiplicand. For example, the product of 3·76, accurate to 3 significant figures and 2·012, accurate to four significant figures, is 7·56512, which is only accurate to two significant figures, 7·6, as the true answer may be anywhere between 7·553 and 7·577.