Listening

The Hilbert Hotel Paradox

http://www.youtube.com/watch?v=BRn_GNcglKo

Q: Can you think of some problems in mathematics which have not been solved yet?

Listen to and watch the video, decide whether the statements are true or false. Correct the false statements.

1) Hilbert was probably one of the last mathematicians who understood a wide range of areas of maths.
2) He wanted to illustrate some surprising aspects of infinity.
3) The Hilbert Hotel has infinitely many people who are employed there.
4) The customer is surprised because the clerk is unable to find the keys of his room.
5) There are infinitely many customers and rooms.
6) When all customers move to the next room, room 1 would be vacant.
7) The second evening: The customer does not know how many people he wants to accommodate.
8) Friends of a customer can occupy the even-numbered rooms.
9) The third evening: there is an infinite number of buses and customers.
10) The people from the first bus will get rooms numbered under the products of three.
11) The solution is possible because the powers of primes are always different.
12) Hilbert called the mentioned infinity aleph null.

David Hilbert From Wikipedia, the free encyclopedia

Pre-reading Match the terms from the text and their descriptions or definitions.

a) Transfinite numbers	It generalizes the notion of Euclidean space. It extends the methods of vector algebra and calculus from the two-dimensional Euclidean plane and three-dimensional space to spaces with any finite or infinite number of dimensions.
b) Invariant theory	The study of mathematics itself using mathematical methods
c) Hilbert spaces	Cardinal numbers or ordinal numbers that are larger than all finite numbers, yet not necessarily absolutely infinite.
d) Proof theory	A branch of abstract algebra that studies actions of groups on algebraic varieties from the point of view of their effect on functions.
e) Metamathematics	A branch of mathematical logic that represents proofs as formal mathematical objects, facilitating their analysis by mathematical techniques.

David Hilbert, January 23, 1862 – February 14, 1943) was a German mathematician, recognized as one of the most influential and universal mathematicians of the 19th and early 20th centuries. He discovered and developed a broad range of fundamental ideas in many areas, including invariant theory and the axiomatization of geometry. He also formulated the theory of Hilbert spaces, one of the foundations of functional analysis.

Hilbert adopted and warmly defended Georg Cantor's set theory and transfinite numbers. A famous example of his leadership in mathematics is his 1900 presentation of a collection of problems that set the course for much of the mathematical research of the 20th century.

Hilbert and his students contributed significantly to establishing rigor and some tools to the mathematics used in modern physics. He is also known as one of the founders of proof theory, mathematical logic and the distinction between mathematics and metamathematics.

Hilbert's problems from Wikipedia, the free encyclopedia

Read the text on Hilbert´s problems and answer the questions.

1) **How many problems were suggested by Hilbert?**
2) **How many of them were presented in Paris?**
3) **When were these problems published?**

Hilbert's problems are a list of twenty-three problems in mathematics published by German mathematician David Hilbert in 1900. The problems were all unsolved at the time, and several of them were very influential for 20th century mathematics. Hilbert presented ten of the problems (1, 2, 6, 7, 8, 13, 16, 19, 21 and 22) at the Paris conference of the International Congress of Mathematicians, speaking on 8 August in the Sorbonne; the complete list of 23 problems was published later.

Read the second part and decide whether the statements are true or false. Correct the false statements.

1) **Hilbert proposed various topics in his problems.**
2) **Only some of the problems allow a clear positive/negative answer.**
3) **The third problem was solved by a non-specialist.**
4) **Only a part of the fifth problem has been solved.**

5) **Some recent problems correspond to Hilbert' s statements of problems.**
6) **Real algebraic curves are not a developed field of maths today.**
7) **The fourth problem is not precise enough to enable solution.**
8) **Axiomatization of physics has been completed at the time of Hilbert.**
9) **Paul Cohen received a prize for his work on the tenth problem.**

Nature and influence of the problems

Hilbert's problems ranged greatly in topic and precision. Some of them are propounded precisely enough to enable a clear affirmative/negative answer, like the 3rd problem (probably the easiest for a nonspecialist to understand and also the first to be solved) or the notorious 8th problem (the Riemann hypothesis). There are other problems (notably the 5th) for which experts have traditionally agreed on a single interpretation and a solution to the accepted interpretation has been given, but for which there remain unsolved problems which are so closely related as to be, perhaps, part of what Hilbert intended. Sometimes Hilbert's statements were not precise enough to specify a particular problem but were suggestive enough so that certain problems of more contemporary origin seem to apply, e.g. most modern number theorists would probably see the 9th problem as referring to the (conjectural) Langland's correspondence on representations of the absolute Galois group of a number field. Still other problems (e.g. the 11th and the 16th) concern what are now flourishing mathematical subdisciplines, like the theories of quadratic forms and real algebraic curves.

There are two problems which are not only unresolved but may in fact be unresolvable by modern standards. The 6th problem concerns the axiomatization of physics, a goal that twentieth century developments of physics (including its recognition as a discipline independent from mathematics) seem to render both more remote and less important than in Hilbert's time. Also, the 4th problem concerns the foundations of geometry, in a manner which is now generally judged to be too vague to enable a definitive answer.

Remarkably, the other twenty-one problems have all received significant attention, and late into the twentieth century work on these problems was still considered to be of the greatest importance. Notably, Paul Cohen received the Fields Medal during 1966 for his work on the first problem, and the negative solution of the tenth problem during 1970 by Matiyasevich (completing work of Davis, Putnam and Robinson) generated similar acclaim. Aspects of these problems are still of great interest today.

Word study – sentence patterns too/enough

Some of them are propounded precisely enough to enable a clear affirmative/negative answer.

Connect the sentences using *too* or *enough*.

a) I can' t wear long skirts. I' m too short. ....
b) We can' t see the Alps yet. We' re not high enough. ....
c) I don' t need a break yet. I' m not tired enough. ....
d) There isn' t a library in the village. It' s too small. ....
e) He' s not going to accept an idea like that. He' s too traditional.
....

Fill in a suitable adjective using *too* or *enough*.
complicated expensive fast high light long reliable shallow similar small

a) The number was...........................for me to remember.
b) It was far ....for me to jump over.
c) Is the room ....for you to paint in?
d) The rules are ....for a young child to understand.
e) The two boys looked ....for everyone to believe they were brothers.
f) The children' s pool was ....for us adults to swim in.
g) This print is probably ....for most customers to read.
h) The car was driving ....for him to be able to read its number-plate.
i) I' m afraid he simply isn' t ....for us to be able to trust him with the task.
j) The restaurant' s ....for us to be able to afford to eat there regularly.

(Adapted from Sarah Peters, Tomáš Gráf: Time to Practise, pp. 322 and 314)