(5) Zeitabhängige Schrödinger gleichung

(13)

$$\hat{\mathcal{H}}\Psi = -\frac{\hbar}{i}\frac{\partial}{\partial t}|\Psi\rangle$$

Každý spin. $|\Psi\rangle = c_\alpha|\alpha\rangle + c_\beta|\beta\rangle$ $\qquad c_\alpha^* c_\alpha + c_\beta^* c_\beta = 1$

stav lze psát jako $\cancel{c_\alpha|\alpha\rangle}$ kombinaci vl. stavů

$$\curvearrowright \underbrace{c_\alpha E_\alpha|\alpha\rangle + c_\beta E_\beta \cdot |\beta\rangle}_{\hat{H}|\Psi\rangle} = -\frac{\hbar}{i}|\alpha\rangle\frac{\partial}{\partial t}c_\alpha - \frac{\hbar}{i}|\beta\rangle\frac{\partial c_\beta}{\partial t}$$

Multipliziere mit $\langle\alpha|$ oder $|\beta\rangle$ und integriere

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$$\curvearrowright \quad c_\alpha \cdot E_\alpha + 0 = -\frac{\hbar}{i}\frac{\partial}{\partial t}c_\alpha - 0$$

$$\curvearrowright \quad 0 + c_\beta E_\beta = -0 - \frac{\hbar}{i}\frac{\partial}{\partial t}c_\beta$$

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$$\curvearrowright \quad \frac{\partial c_\alpha}{c_\alpha} = \frac{-i}{\hbar}E_\alpha\, dt \qquad \frac{\partial c_\beta}{c_\beta} = -\frac{i}{\hbar}E_\beta\, dt$$

Integration über Zeit:

$$c_\alpha(t) = c_\alpha(0)\,e^{-\frac{i}{\hbar}E_\alpha \cdot t} = c_\alpha(0)\,e^{-\frac{i}{2}\omega t}$$

$$c_\beta(t) = c_\beta(0)\,e^{-\frac{i}{\hbar}E_\beta \cdot t} = c_\beta(0)\,e^{+\frac{i}{2}\omega t}$$

$$|\Psi\rangle(t) = c_\alpha(0)\,e^{-\frac{i}{2}\omega t}|\alpha\rangle + c_\beta(0)\,e^{+\frac{i}{2}\omega t}|\beta\rangle$$

Betrachte $\langle\hat{S}^2\rangle$, $\langle\hat{S}_z\rangle$, $\langle\hat{S}_x\rangle$, $\langle\hat{S}_y\rangle$

$$\langle\hat{S}^2\rangle = \langle\Psi^*|\hat{S}^2|\Psi\rangle = \left\{c_\alpha^*\langle\alpha|\,e^{+\frac{1}{2}\omega t} + c_\beta^*\,e^{-\frac{1}{2}\omega t}\langle\beta|\right\}\hat{S}^2 \cdot$$

$$\left\{c_\alpha(0)\,e^{-\frac{1}{2}\omega t}|\alpha\rangle + c_\beta(0)\,e^{+\frac{1}{2}\omega t}|\beta\rangle\right\}$$

$$= \frac{3}{4}\Big\{\underbrace{c_\alpha^*(0)\,c_\alpha(0) + c_\beta^*\,c_\beta(0)}_{1}\Big\} = \frac{3}{4} = \text{konst.}$$

$$\langle \hat{S}_z \rangle = \ldots = \frac{1}{2}\left\{ c_\alpha^*(0)\, c_\alpha(0) - c_\beta^*(0)\, c_\beta(0) \right\} = \text{konst.}$$

$$\langle \hat{S}_x \rangle = \langle \psi^* | S^x | \psi \rangle = \ldots\ldots =$$

$$= \frac{1}{2}\left\{ c_\alpha^*(0)\, e^{+\frac{i}{2}\omega t} \langle \alpha | + c_\beta^*(0)\, e^{-\frac{i}{2}\omega t} \langle \beta | \right\}$$

$$\left\{ c_\alpha(0)\, e^{-1/2\, \omega t} | \beta \rangle + c_\beta(0)\, e^{+1/2\, \omega t} | \alpha \rangle \right\} =$$

$$= \frac{1}{2}\left\{ c_\alpha^*(0)\, c_\beta(0)\, e^{+i\omega t} + c_\beta^*(0)\, c_\alpha(0)\, e^{-i\omega t} \right\}$$

$$e^{i\omega t} = \cos \omega t + i \sin \omega t$$

$$\langle \hat{S}_x \rangle = \frac{1}{2}\left\{ c_\alpha^*(0)\, c_\beta(0) + c_\beta^*(0)\, c_\alpha(0) \right\} \cos(\omega t) +$$

$$+ \frac{i}{2}\left\{ c_\alpha^*(0)\, c_\beta(0) - c_\beta^*(0)\, c_\alpha(0) \right\} \sin(\omega t)$$

Analog:

$$\langle S_y \rangle = \frac{1}{2}\left\{ c_\alpha^*(0)\, c_\beta(0) + c_\beta^*(0)\, c_\alpha(0) \right\} \sin \omega t$$

$$- \frac{i}{2}\left\{ c_\alpha^*(0)\, c_\beta(0) - c_\beta^*(0)\, c_\alpha(0) \right\} \cos \omega t$$

$t = 0$:

$$\langle S_x(0) \rangle = \frac{1}{2}\left\{ c_\alpha^*(0)\, c_\beta(0) + c_\beta^*(0)\, c_\alpha(0) \right\}$$

$$\langle S_y(0) \rangle = -\frac{i}{2}\left\{ c_\alpha^*(0)\, c_\beta(0) - c_\beta^*(0)\, c_\alpha(0) \right\}$$

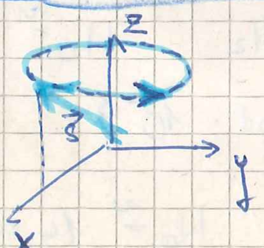
$$\langle \hat{S}_x \rangle = \langle \hat{S}_x(0) \rangle \cos(\omega t) - \langle \hat{S}_y(0) \rangle \sin(\omega t)$$

$$\cancel{\langle S_y \rangle =}$$

$$\langle \hat{S}_y \rangle = \langle \hat{S}_x(0) \rangle \sin \omega t + \langle \hat{S}_y(0) \rangle \cos(\omega t)$$

$$\langle \hat{S}_z \rangle = \langle \hat{S}_z(0) \rangle$$

Lamor prezession um Magnetfeld axe



Rechts~~kurve~~schraube für $\gamma < 0$ $\gamma_S = -\frac{g_e \mu_B}{\hbar}$

Für $\gamma > 0$ (Proton)

gilt Linksschraube

$$\omega = \frac{g_e \cdot \mu_B \cdot H_0}{\hbar} = \text{ESR-Frequenz}$$