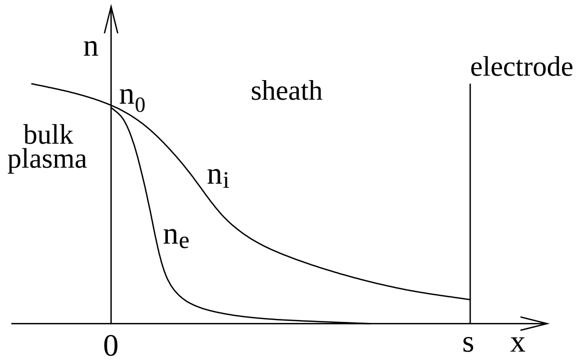
1 DC sheath bez srážek, Bohmova rychlost



$$\Phi(0) = 0 \tag{1}$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}x} \approx 0 \tag{2}$$

$$n_e = n_0\,\mathrm{e}^{\frac{q\Phi}{kT_e}} \tag{3}$$

$$n_i v_i = n_0 v_B \tag{4}$$

$$\frac{1}{2}m_i v_i^2 + q\Phi = \frac{1}{2}m_i v_B^2 \tag{5}$$

Řešíme Poissonovu rovnici $\triangle\Phi = -\frac{\rho}{\varepsilon_0}$:

$$\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} = -\frac{q}{\varepsilon_0}(n_i - n_e) = \frac{n_0 q}{\varepsilon_0}\left(\mathrm{e}^{q\Phi/kT_e} - \frac{1}{\sqrt{1-\frac{2q\Phi}{m_i v_B^2}}}\right) \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}x}\right)^2 = 2\,\frac{\mathrm{d}\Phi}{\mathrm{d}x}\,\frac{n_0 q}{\varepsilon_0}\left(\mathrm{e}^{q\Phi/kT_e} - \frac{1}{\sqrt{1-\frac{2q\Phi}{m_i v_B^2}}}\right)$$

$$E^2 - E_0^2 = \frac{2n_0 q}{\varepsilon_0}\int_0^{\Phi}\left(\mathrm{e}^{q\Phi'/kT_e} - \frac{1}{\sqrt{1-\frac{2q\Phi'}{m_i v_B^2}}}\right)\mathrm{d}\Phi' \tag{7}$$

Protože levá strana rovnice (7) je nezáporná a $\mathrm{d}\Phi' < 0$, musí platit

$$\mathrm{e}^{q\Phi/kT_e} - \frac{1}{\sqrt{1-\frac{2q\Phi}{m_i v_B^2}}} \leq 0,$$

z čehož vyplývá

$$v_B^2 \geq \frac{1}{m_i}\,\frac{2q\Phi}{1-\mathrm{e}^{-2q\Phi/kTe}}.$$

Tato nerovnost musí platit i pro malé hodnoty potenciálu Φ, takže

$$v_B^2 \geq \frac{kT_e}{m_i}. \tag{8}$$

Rychlost iontů $\sqrt{kT_e/m_i}$ se nazývá Bohmova rychlost.

2 Plovoucí potenciál

$$\frac{1}{4}\,n_0\sqrt{\frac{8kT_e}{\pi m_e}}\,\mathrm{e}^{\frac{q\Phi_{fl}}{kT_e}} = n_0\sqrt{\frac{kT_e}{m_i}}$$

$$q\Phi_{fl} = -\frac{kT_e}{2}\,\ln\frac{m_i}{2\pi m_e} \tag{9}$$

3 Child-Langmuirův zákon pro bezsrážkový sheath

Rovnici (7) upravíme pro předpoklady $n_e \approx 0$, $\frac{1}{2} m_i v_B^2 \ll q\Phi$ a $E_0 \ll E$ a za využití $j = qn_0 v_B$ na

$$\begin{aligned}
E^2 &\approx -\frac{2n_e q}{\varepsilon_0} \int_0^{\Phi} \frac{\mathrm{d}\Phi'}{\sqrt{-\frac{2q\Phi'}{m_i v_B^2}}} = -\frac{\sqrt{2qm_i}}{\varepsilon_0} n_0 v_B \int_0^{\Phi} \frac{\mathrm{d}\Phi'}{\sqrt{-\Phi'}} = \frac{j}{\varepsilon_0}\sqrt{\frac{2m_i}{q}} 2\sqrt{-\Phi} \\
\frac{\mathrm{d}\Phi}{\mathrm{d}x} &= -\sqrt{\frac{j}{\varepsilon_0} 2 \sqrt{\frac{2m_i}{q}}} (-\Phi)^{\frac{1}{4}} \\
(-\Phi)^{\frac{3}{4}} &= \frac{3}{2}\sqrt{\frac{j}{\varepsilon_0}\sqrt{\frac{m_i}{2q}}} \\
j &= \frac{4\varepsilon_0}{9}\sqrt{\frac{2q}{m_i}} \frac{U_{sh}^{\frac{3}{2}}}{s^2} \qquad (10)
\end{aligned}$$

4 Srážkový sheath

$$j = q\, n_i\, v_i = q\, n_i\, \mu_i\, E \qquad (11)$$

Pro jednoduchost předkpokládejme konstantní pohyblivost iontů μ_i:

$$\begin{aligned}
\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} &= -\frac{qn_i}{\varepsilon_0} = \frac{j}{\varepsilon_0\, \mu_i \frac{\mathrm{d}\Phi}{\mathrm{d}x}} \\
2\frac{\mathrm{d}\Phi}{\mathrm{d}x}\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} &= \frac{2j}{\varepsilon_0\, \mu_i} \\
\frac{\mathrm{d}\Phi}{\mathrm{d}x} &= -\sqrt{\frac{2j}{\varepsilon_0\, \mu_i} x} \\
-\Phi &= \frac{2}{3}\sqrt{\frac{2j}{\varepsilon_0\, \mu_i}}\, x^{\frac{3}{2}} \\
j &= \frac{9\varepsilon_0\, \mu_i}{8}\frac{U_{sh}^2}{s^3} \qquad (12)
\end{aligned}$$

5 Sheath s konstantní koncentrací iontů

$$E = \frac{qn_0\, x}{\varepsilon_0} \qquad (13)$$

$$U_{sh} = \frac{qn_0\, s^2}{2\varepsilon_0} \qquad (14)$$