EXERCISES IN CATEGORY THEORY 1

1. Exercises

1.1. **Isomorphisms.**

(1) Suppose that $f : A \to B$ is an isomorphism. Show that it has a unique inverse $B \to A$.

1.2. **Epis and monos.**

(1) Show that in Set the monos are exactly the injective functions.
(2) Show that in the categories of monoids and groups the monos are the injective homomorphisms.
(3) Show that the monos in the category of topological spaces are the injective continuous functions.
(4) Show that the epis in Set are the surjective functions.
(5) Can you find an example of a homomorphism of monoids/rings or a continuous map of metric spaces which is epi but not a surjective function?

1.3. **Initial and terminal objects.**

(1) Let $\mathcal{K}$ be the following category. Objects of $\mathcal{K}$ are triples (X, a, s) where X is a set, $a \in X$ is an element of X, and $f : X \to X$ is a function. A morphism $f : (X, a, s) \to (Y, b, t)$ of $\mathcal{K}$ is a function $f : X \to Y$ such that $f(a) = b$ and $t \circ f = f \circ s$.
Can you describe the objects and morphisms of this category using diagrams in the category Set? For instance, an element $x \in X$ of a set can be viewed using diagrams as an arrow
$$1 \to X$$
where $1 = \{\star\}$ is the 1-element set.
What is the initial object of $\mathcal{K}$?

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