



"Populační ekologie živočichů"

Stano Pekár

## **Types of interactions**

#### DIRECT

Effect of species 1 on fitness of species 2IncreaseNeutralDecreaseIncrease+ +IncreaseNeutral0 +0 0Decrease+ -- 0

- ++ .. mutualism (plants and pollinators)
- 0 + .. commensalism (saprophytism, parasitism, phoresis)
- + .. predation (herbivory, parasitism), mimicry
- 0 .. amensalism (allelopathy)
  - .. competition
- **INDIRECT** Apparent competition Facilitation Exploitation competition

# Niche measures

## Niche breadth Levin's index (D):

- *p<sub>k</sub>* .. proportion of individuals in class *k*does not include resource availability
- $-1 < D < \infty$

#### Smith's index (FT):

-  $q_k$  ... proportion of available individuals in class k- 0 < FT < 1

## Niche overlap Pianka's index (a):

- does not account for resource availability
- 0 < *a* < 1
- Lloyd's index (L):
- $-0 < L < \infty$



 $L = \sum \frac{p_{1k} p_{2k}}{q_{1k}}$ 

$$FT = \sum_{k=1}^{n} \sqrt{p_k q_k}$$

 $D = \frac{1}{\sum_{k=1}^{n} p_{k}^{2}}$ 

## Model of competition

- based on the logistic differential model
- ▶ assumptions:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = Nr\left(1 - \frac{N}{K}\right)$$

- all parameters are constant
- individuals of the same species are identical
- environment is homogenous, differentiation of niches is not possible
- only exact compensation is present
- model of Lotka (1925) and Volterra (1926)

**species 1**:  $N_1$ ,  $K_1$ ,  $r_1$ **species 2**:  $N_2$ ,  $K_2$ ,  $r_2$ 

$$\frac{dN_1}{dt} = N_1 r_1 \left( 1 - \frac{N_1 + N_2}{K_1} \right)$$
$$\frac{dN_2}{dt} = N_2 r_2 \left( 1 - \frac{N_1 + N_2}{K_2} \right)$$

total competitive effect (intra + inter-specific)

 $(N_1 + \alpha N_2)$  where  $\alpha$ .. coefficient of competition  $\alpha = 0$ .. no interspecific competition

 $\alpha < 1$ .. species 2 has lower effect on species 1 than species 1 on itself  $\alpha = 0.5$ .. one individual of species 1 is equivalent to 0.5 individuals of species 2)

 $\alpha = 1$ ... both species has equal effect on the other one

 $\alpha > 1$ .. species 2 has greater effect on species 1 than species 1 on itself

species 1: 
$$\frac{dN_1}{dt} = N_1 r_1 \left( 1 - \frac{N_1 + \alpha_{12}N_2}{K_1} \right)$$
  
species 2: 
$$\frac{dN_2}{dt} = N_2 r_2 \left( 1 - \frac{\alpha_{21}N_1 + N_2}{K_2} \right)$$

• if competing species use the same resource then interspecific competition is equal to intraspecific

## Equilibrium analysis of the model

• examination of the model behaviour using null isoclines

• used to describe change in any two variables in coupled differential equations by projecting orthogonal vectors

▶ identification of isoclines: a set of abundances for which the change in populations is 0:



# ▶ species 1 $r_1N_1 (1 - [N_1 + \alpha_{12}N_2] / K_1) = 0$ $r_1N_1 ([K_1 - N_1 - \alpha_{12}N_2] / K_1) = 0$ trivial solution if r<sub>1</sub>, N<sub>1</sub>, K<sub>1</sub> = 0 and if K<sub>1</sub> - N<sub>1</sub> - α<sub>12</sub>N<sub>2</sub> = 0 then N<sub>1</sub> = K<sub>1</sub> - α<sub>12</sub>N<sub>2</sub>

if 
$$N_1 = 0$$
 then  $N_2 = K_1 / \alpha_{12}$   
if  $N_2 = 0$  then  $N_1 = K_1$ 

• species 2  $r_2N_2 (1 - [N_2 + \alpha_{21}N_1] / K_2) = 0$   $N_2 = K_2 - \alpha_{21}N_1$ trivial solution if  $r_2, N_2, K_2 = 0$ if  $N_2 = 0$  then  $N_1 = K_2 / \alpha_{21}$ if  $N_1 = 0$  then  $N_2 = K_2$ 

## Isoclines



- above isocline  $i_1$  and below  $i_2$  competition is weak
- in-between  $i_1$  and  $i_2$  competition is strong

#### 1. Species 2 drives species 1 to extinction

K and α determine the model behaviour
disregarding initial densities species 2 (stronger competitor) will outcompete species 1 (weaker competitor)

• equilibrium  $(0, K_2)$ 



#### 2. Species 1 drives species 2 to extinction

▶ species 1 (stronger competitor) will outcompete species 2 (weaker competitor)

• equilibrium  $(K_1, 0)$ 

$$K_1 > \frac{K_2}{\alpha_{21}}$$
  $K_2 < \frac{K_1}{\alpha_{12}}$ 

$$r_1 = r_2$$
  $K_1 = K_2$   
 $N_{01} = N_{02}$   $\alpha_{12} < \alpha_{21}$ 



#### **3. Stable coexistence of species**

- disregarding initial densities both species will coexist at stable equilibrium (where isoclines cross)
- at at equilibrium population density of both species is reduced
- both species are weak competitors
- equilibrium  $(K_1^*, K_2^*)$





 $\begin{array}{ccc} r_1 < r_2 & K_1 = K_2 \\ N_{01} = N_{02} & \alpha_{12}, \ \alpha_{21} < 1 \end{array}$ 



### **Stability analysis**

#### Jacobian matrix of partial derivations for 2dimensional system

$$\mathbf{J} = \begin{pmatrix} \frac{\partial \, \mathrm{d}N_1/\mathrm{d}t}{\partial N_1} & \frac{\partial \, \mathrm{d}N_1/\mathrm{d}t}{\partial N_2} \\ \frac{\partial \, \mathrm{d}N_2/\mathrm{d}t}{\partial N_1} & \frac{\partial \, \mathrm{d}N_2/\mathrm{d}t}{\partial N_2} \end{pmatrix}$$

- evaluation of the derivations for densities close to equilibrium
  estimate eigenvalues of the matrix
  if all eigenvalues < 0 .. locally stable</li>
- Lotka-Volterra system is stable for  $\alpha_{12}\alpha_{21} < 1$

## Test of the model

• when *Rhizopertha* and *Oryzaephilus* were reared separately both species increased to 420-450 individuals (= K)

• when reared together *Rhizopertha* reached  $K_1 = 360$ , while *Oryzaephilus*  $K_2 = 150$  individuals

• combination resulted in more efficient conversion of grain ( $K_{12} = 510$  individuals)

 three combinations of densities converged to the same stable equilibrium

prediction of
 Lotka-Volterra model is correct



Crombie (1947)

# System for discrete generations

solution of the differential model – Ricker's model:

$$N_{1,t+1} = N_{1,t}e^{r_1\left(\frac{K_1 - N_{1,t} - \alpha_{12}N_{2,t}}{K_1}\right)} N_{2,t+1} = N_{2,t}e^{r_2\left(\frac{K_2 - N_{2,t} - \alpha_{21}N_{1,t}}{K_2}\right)}$$

♦ dynamic (multiple) regression is used to estimate parameters from a series of abundances
... a, b, c - regression  $ln \begin{pmatrix} N_{1,t+1} \end{pmatrix} = r N r_1 N r_1 \alpha_{12}$ 

parameters

$$\ln\left(\frac{N_{1,t+1}}{N_{1,t}}\right) = r_1 - N_{1,t} \frac{r_1}{K_1} - N_{2,t} \frac{r_1 \alpha_{12}}{K_1}$$

$$\ln\left(\frac{N_{2,t+1}}{N_{2,t}}\right) = r_2 - N_{2,t} \frac{r_2}{K_2} - N_{1,t} \frac{r_2 \alpha_{21}}{K_2}$$

$$r = a$$
  $\alpha = \frac{Kc}{r}$   $K = \frac{r}{b}$