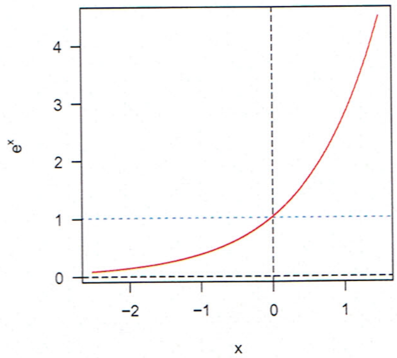
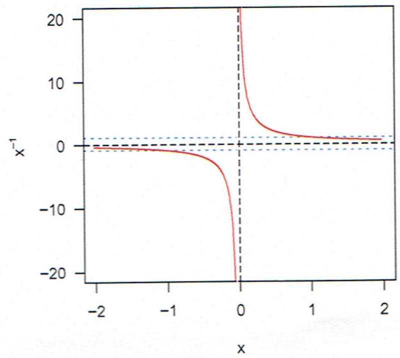
B

2.1. ZÁKLADNÍ VLASTNOSTI FUNKCE e^x

2.2.

B





1. DEFINIČNÍ OBOR: $D(f) = (-\infty; +\infty)$
2. OBOR HODNOT: $H(f) = (0; \infty)$
3. SPOJITOST FCE: SPOJITÁ
4. OHRANIČENOST: ZDOLA OHRANIČENÁ
5. PERIODICITA FCE: NEMÁ PERIODU
6. PARITA FCE: NENÍ SUDÁ ANI LICHÁ
7. MONOTÓNNOST: ROSTOUCÍ FCE
8. $+$: $\lim\limits_{x \to -\infty} f(x) = 0$

1. DEFINIČNÍ OBOR: $D(f) = (-\infty; 0) \cup (0; \infty)$
2. OBOR HODNOT: $H(f) = (-\infty; 0) \cup (0; \infty)$
3. SPOJITOST FCE: NESPOJITÁ
4. OHRANIČENOST: NENÍ CELKOVĚ OHRANIČENÁ
5. PERIODICITA: NEMÁ PERIODU
6. PARITA FCE: LICHÁ FCE
7. MONOTÓNNOST: KLESAJÍCÍ
8. $+$: $\lim\limits_{x \to \infty} f(x) = 0$

$\lim\limits_{x \to -\infty} f(x) = 0$

$\lim\limits_{x \to 0^+} f(x) = \infty$

$\lim\limits_{x \to 0^-} f(x) = -\infty$

2.3. VÝPOČTY LIMIT

1. $x^2 - 5x + 4$ $\quad \pm 1 \quad \pm 2 \quad \pm 4$

$$\begin{array}{c|ccc} & 1 & -5 & 4 \\ \hline 1 & 1 & -4 & 0 \\ & (x-4) & & \end{array}$$

$x = 1 \Rightarrow (x-1)$

$x^2 - 5x + 4 = (x-4)(x-1)$

1. $x^3 - 7x - 6$ $\quad \pm 1 \quad \pm 2 \quad \pm 3 \quad \pm 6$

	1	0	-7	6	
1	1	1	-6	-12	
-1	1	-1	-6	0	$x = -1 \Rightarrow (x+1)$
~~-1~~	~~1~~	~~-2~~	~~-4~~	×	
~~2~~	~~1~~	~~1~~	~~-4~~	×	
-2	1	-3	0		$x = -2 \Rightarrow (x+2)$

$(x-3)$

$$x^3 - 7x - 6 = (x+1)(x-3)(x+2)$$

2.4. LIMITY FCÍ VE VLASTNÍM BODĚ

1. $$\lim_{x\to -2} \frac{x^3-3x^2-4x+12}{x^5-2} = \lim_{x\to -2} \frac{(-2)^3-3(-2)^2-4(-2)+12}{(-2)^5-2} = \lim_{x\to -2} \frac{-8-12+8+12}{-32-2} =$$

$$\lim_{x\to -2} \frac{0}{-34} = 0$$

2. $$\lim_{x\to 0} \frac{2^x - 6^x - 3^x}{2^x+4^x} = \lim_{x\to 0} \frac{1-1-1}{1+1} = \lim_{x\to 0} -\frac{1}{2}$$

3. $$\lim_{x\to 4} x^2 - 3x - 4 = \lim_{x\to 4} 4^2 - 3\cdot 4 - 4 = 16-12-4 = 0$$

4. $$\lim_{x\to -2} \frac{x^3-x^2-4x+4}{x^2-3x-10} = \frac{(-2)^3-(-2)^2-4\cdot(-2)+4}{(-2)^2-3\cdot(-2)-10} = \lim_{x\to -2} \frac{-8-4+8+4}{4+6-10} = \frac{0}{0} \quad !$$

rozložit na závorky

$$\lim_{x\to -2} \frac{x^2(x-1)+4(x-1)}{(x-5)(x+2)} = \lim_{x\to -2} \frac{(x-1)(x-2)\cancel{(x+2)}}{(x-5)\cancel{(x+2)}} = \lim_{x\to -2} \frac{(-2-1)(-2-2)}{(-2-5)} = \frac{-3\cdot(-4)}{-7}$$

$$= \lim_{x\to -2} -\frac{+12}{7}$$

2.5.

1. $$\lim_{x \to \infty} \frac{2}{x^2+x} \cdot x = \frac{2}{\infty} \cdot \infty = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{2x}{x^2+x} = \lim_{x \to \infty} \frac{x^2\left(\frac{2}{x}\right)}{x^2\left(1+x\right)} = \frac{0}{1} = 0$$

2. $$\lim_{x \to \infty} \frac{3^x+5^x}{8^x} = \frac{\infty}{\infty} = \lim_{x \to \infty} \left(\frac{3}{8}\right)^x + \left(\frac{5}{8}\right)^x = 0+0 = 0$$

3. $$\lim_{x \to -\infty} \frac{6x^5-2x^3-8x+2}{3+x^2+4x^4} = \frac{\pm\infty}{\pm\infty}$$

$$\lim_{x \to -\infty} \frac{\cancel{x^5}\left(\overbrace{6-\frac{2}{x^2}-\frac{8}{x^4}+\frac{2}{x^5}}^{-\infty}\right)}{\cancel{x^5}\left(\underbrace{\frac{3}{x^5}+\frac{1}{x^3}+\frac{4}{x}}_{-\infty}\right)} = \lim_{x \to \infty} -\infty$$

4. $$\lim_{x \to \infty} \frac{\overset{\infty}{\frac{1}{4^x}}+1}{\underset{\infty}{\frac{1}{3^x}}-3} = -\frac{1}{3}$$

5. $$\lim_{x \to -\infty} \frac{2x^6-x^5+3x^4-5x}{3x^4+4x^8-3} = \lim_{x \to -\infty} \frac{2x^6}{4x^8} = \lim_{x \to -\infty} \frac{1}{2x^2} = 0$$

6. $$\lim_{x \to -\infty} \frac{2^x+4^x}{5^x} = \lim_{x \to -\infty} \left(\frac{2}{5}\right)^{-\infty} + \left(\frac{4}{5}\right)^{-\infty} = \lim_{x \to -\infty} \left(\frac{5}{2}\right)^{\infty} + \left(\frac{5}{4}\right)^{\infty} = \infty+\infty = \infty$$

7. $$\lim_{x \to \infty} \frac{3^x-2^x}{3^x} = \lim_{x \to \infty} \frac{3^x}{3^x} - \frac{2^x}{3^x} = \lim_{x \to \infty} 1-\left(\frac{2}{3}\right)^x = 1-0 = 1$$

8. $$\lim_{x \to \infty} \frac{-4x^4-x-3}{x^3-x+9x^4-5} = -\frac{4x^4}{9x^4} = -\frac{4}{9}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\, g'(x)}{g^2(x)}$$

2.6. DERIVACE 1. ŘÁDU FCE

1. $(x^5 - 4x^4 - 5x^2 - x - 3)' = 5x^4 - 12x^3 - 10x - 1$

2. $(\cos^4(x) + \tan(3x))' = 4\cos^3(x)\cdot(-\sin(x)) + \frac{3}{\tan 3x} =$

$= -4\cos^3(x)\cdot \sin(x) + \frac{3}{\tan(3x)}$

3. $\left(\frac{4-\cos x}{e^x}\right)' = \frac{\sin(x)\cdot e^x - (4-\cos x)\cdot e^x}{(e^x)^2} = \frac{\cancel{e^x}(\sin(x)+\cos(x)) - 4}{\cancel{(e^x)^2}\, e^x} =$

$= \frac{\sin(x)+\cos(x)-4}{e^x}$

4. $(e^x\cdot \sin(x) - 4\ln(x)\cdot\cos(x))' = e^x\cdot\sin + e^x\cdot\cos(x) - \frac{4}{x}\cdot\cos(x) + 4\ln(x)\cdot$

$(\sin(x)) = \sin(x)\,(e^x + 4\ln(x)) + \cos(x)\,(e^x - \frac{4}{x})$

5. $\left(\frac{x^2+x-6}{x+3}\right)' = \frac{(2x+1)(x+3) - [(x^2+x-6)(1)]}{(x+3)} =$

$= \frac{(2x+1)(x+3) - (x+3)(x-2)}{(x+3)^2} = \frac{\cancel{(x+3)}\,(2x+1-x+2)}{(x+3)^{\cancel{2}}} =$

$= \frac{x+3}{x+3} = \underline{\underline{1}}$ $\qquad x_{1,2} = \frac{-1 \pm \sqrt{25}}{2} \left\langle \begin{matrix} 2 \\ -3 \end{matrix}\right.$

6. $(x^7 - x^{-7} - x^0 - \ln(x) + \tan(x))' = 7x^6 + 7x^{-8} - \frac{1}{x} + \frac{1}{\cos^2 x}$

7. $\left((2-x^2)\cdot \sin(x) - x^3\cdot\cos(x)\right)' = -2x\cdot\sin(x) + (2-x^2)\cdot\cos(x) -$

$\left[3x^2\cdot\cos(x) + x^3\cdot(-\sin(x))\right] = x\cdot\sin(x)\cdot(x^2-2) + \cos(x)(2-x^2-3x^2) =$

$= x\cdot\sin(x)\cdot(x^2-2) + 2\cos(x)(1-2x^2)$

? 8. $\left(\dfrac{x\cdot e^{4x}-2}{2x}\right)' = \dfrac{4e^{4x}\cdot 2x\,(x\cdot e^{4x}-2)\cdot 2}{4x^2} = \dfrac{6(x\cdot e^{4x})+4}{4x^2} =$

$= \dfrac{3e^{4x}}{2x} + \dfrac{1}{x^2} = \dfrac{3x e^{4x}-2}{2x^2}$

2.7. DERIVACE 2. ŘÁDU FCE

1. $(\cos(x)\ln(x))'' = \left(-\sin(x)\cdot\ln(x) + \cos(x)\cdot\frac{1}{x}\right)' =$

? $= -\cos(x)\cdot\ln(x) - \sin(x)\frac{1}{x} + (-\sin(x))\cdot\frac{1}{x} + \cos(x)\cdot(-x^{-2}) =$

$= -\frac{2}{x}\cdot\sin(x) - \cos(x)\cdot\left(\ln(x) + \frac{1}{x^2}\right)$

$= -2x\cdot\sin(x) = -2x\cdot\sin(x) - \cos(x)\cdot(\ln(x)\cdot x^2+1)$

2. $(x^5 - x^4 - 5x^2 + x - 3)'' = (5x^4 - 4x^3 - 10x + 1)' =$

$= 20x^3 - 12x^2 - 10 = 2(10x^3 - 6x^2 - 5)$

3. $\left(\dfrac{\ln(x^2)}{x}\right)'' = \frac{1}{x^2}\cdot 2x\cdot x - \ln(x^2)\cdot 1 = \left(\dfrac{2-\ln(x^2)}{x^2}\right)' =$

$= \dfrac{-\frac{1}{x^2}\cdot 2x\cdot x^2 - (2-\ln(x^2))\cdot 2x}{x^4} = \dfrac{-2x - (2-\ln(x^2))\cdot 2x}{x^4} =$

$= \dfrac{2x(\ln(x^2)-3)}{x^4} = \dfrac{2(\ln(x^2)-3)}{x^3}$

4. $(x\cdot\cos(x))'' = (1\cdot\cos(x) + x\cdot(-\sin(x)))' = -\sin(x) + 1\cdot(-\sin(x)) + x\cdot(-\cos(x)) =$

$= -2\cdot\sin(x) - x\cdot\cos(x)$

2.8. l'Hospitalovo pravidlo

1. $$\lim_{x \to -2} \frac{x^3 - x^2 - 4x + 4}{x^2 - 3x - 10} = \frac{-8 - 4 + 8 + 4}{4 + 6 - 10} = \left/\frac{0}{0}\right/$$

$$= \lim_{x \to -2} \frac{3x^2 - 2x - 4}{2x - 3} = \frac{12 + 4 - 4}{-4 - 3} = -\frac{12}{7}$$

2. $$\lim_{x \to -2} \frac{x^3 - x^2 + 4x + 4}{x^2 - 3x - 10} = \frac{-8 - 4 - 8 + 4}{4 + 6 - 10} = \frac{-16}{0}$$

nelze řešit l'Hospitalovým pravidlem (pouze pro $\left/\frac{0}{0}\right/$ a $\left/\frac{\infty}{\infty}\right/$)

3. $$\lim_{x \to 1} \frac{2x^3 - 3x^2 - 2x + 3}{5x^2 - 8x + 3} = \frac{2 - 3 - 2 + 3}{5 - 8 + 3} = \left/\frac{0}{0}\right/$$

$$= \lim_{x \to 1} \frac{6x^2 - 6x - 2}{10x - 8} = \frac{6 - 6 - 2}{10 - 8} = -\frac{2}{2} = 1$$