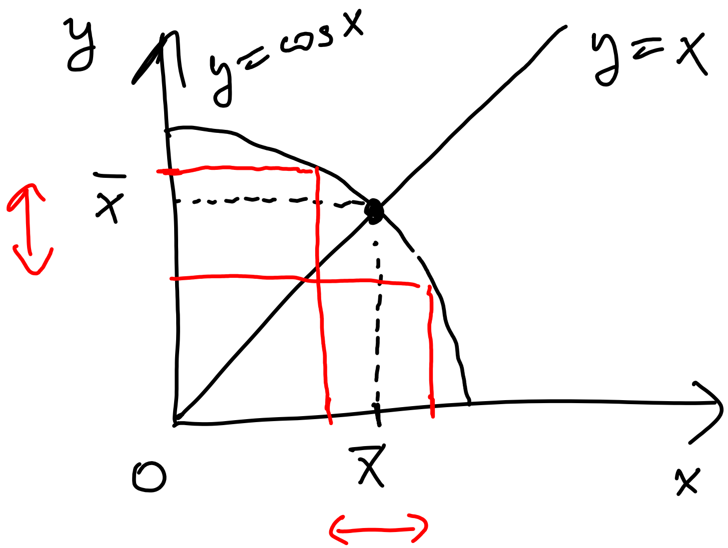
Řešení transcendentních rovnic s jednou neznámou

$f(x) = 0$ metody — iterační $x_0, x_1, x_2, \dots$ — intervalové $[a_0, b_0], [a_1, b_1], \dots$

A1) "dosazovací" metoda

$x - \cos x = 0 \rightarrow x = \cos x$ $\quad x_{n+1} = \cos x_n \quad \bar{x} = 0.739\dots$

$\arccos x = x \quad x_{n+1} = \arccos x_n$ nefunguje



$|g'(\bar{x})| < 1$ po zobrazení $x_{n+1} = g(x_n)$ užší interval

$g(x) = \cos x \qquad g'(x) = -\sin x \qquad |g'| < 1$

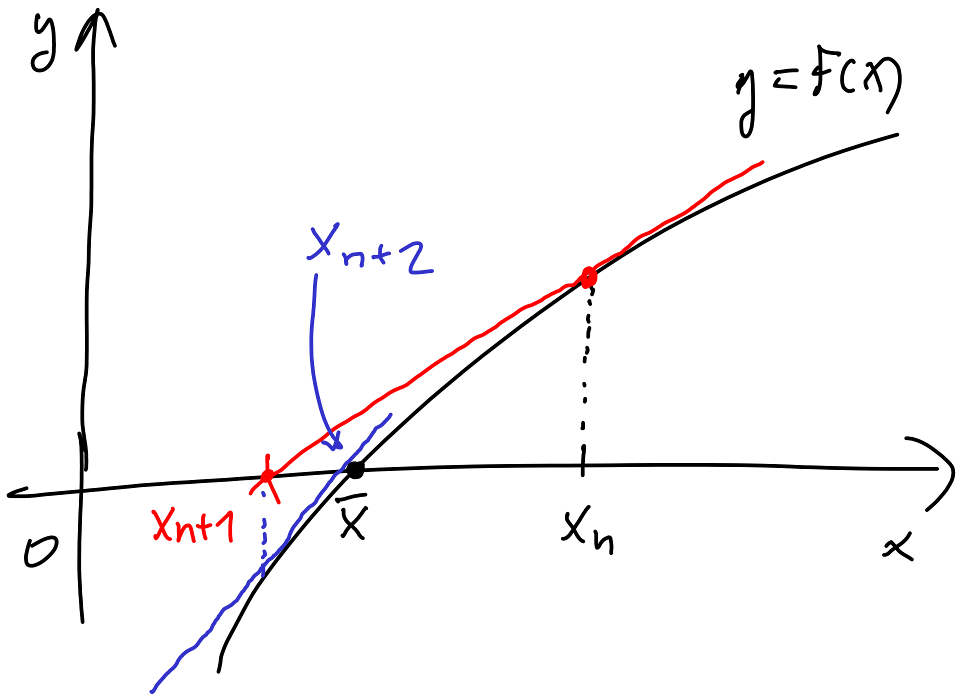
$g(x) = \arccos x \qquad g'(x) = \dfrac{-1}{\sqrt{1-x^2}} \qquad |g'| > 1$

$$x_{n+1} = g(x_n)$$

$$x_{n+1} = (1-a)\, g(x_n) + a\, x_n \qquad 0 \leq a < 1$$

derivace $\quad (1-a)\, g' + a \quad \rightarrow$ urychlení / zpomalení konvergence

A2) Newtonova – Raphsonova metoda (metoda tečen)



$$F(x) = F(x_n) + F'(x_n)(x - x_n) + \frac{1}{2} F''(x_n)(x - x_n)^2 + \mathcal{O}(\delta^3)$$

$$F(x_n) + F'(x_n)(\underset{n+1}{x} - x_n) = 0$$

$$\hookrightarrow \boxed{x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}}$$

$$f(x) = f(x_n) + f'(x_n)(x - x_n) + \frac{1}{2} f''(x_n)(x - x_n)^2 + \cancel{O(o^3)}$$

$$f(x_n) + f'(x_n)(x_{n+1} - x_n) = 0$$

odhad chyby v průběhu jednoho kroku

$$0 = f(x_n) + f'(x_n)(x - x_n) + \frac{1}{2} f''(x_n) \underbrace{(x - x_n)^2}_{-h_n}$$

→ lepší x k porovnání s x_{n+1}

chyba v kroku n $\quad x_n - x = h_n$

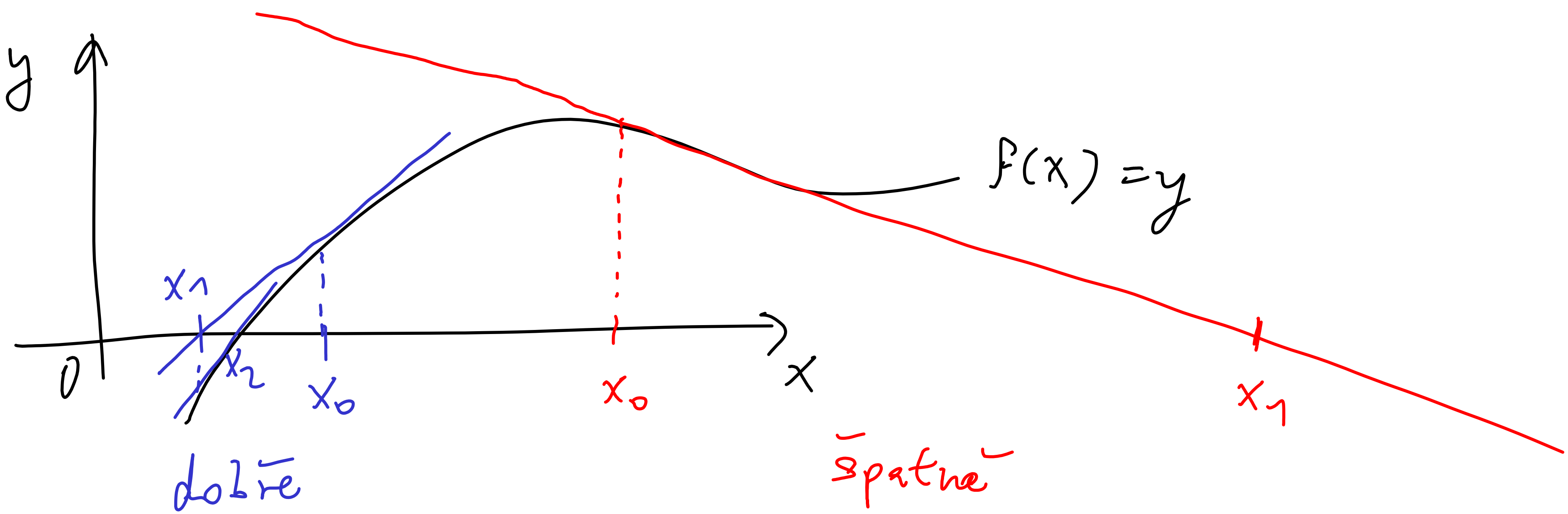
$n+1$ $\quad x_{n+1} - x = h_{n+1}$

$$0 = \underbrace{f(x_n) + f'(x_n)(x_{n+1} - x_n)}_{0} + f'(x_n)\underbrace{(x - x_{n+1})}_{-h_{n+1}} + \frac{1}{2} f''(x_n) h_n^2$$

$$h_{n+1} = h_n^2 \left| \frac{\frac{1}{2} f''(x_n)}{f'(x_n)} \right|$$

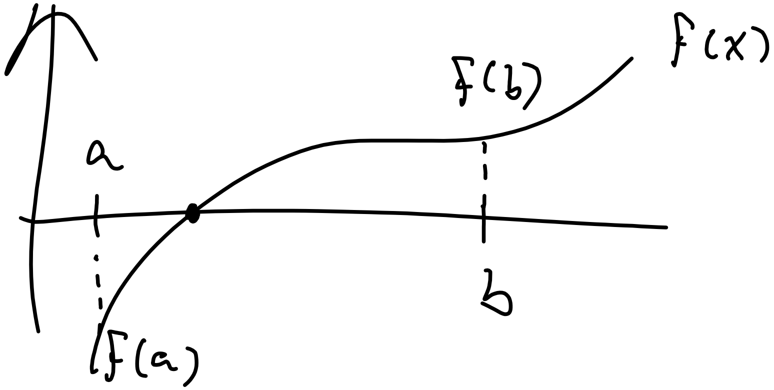
$h_n = 0.01$

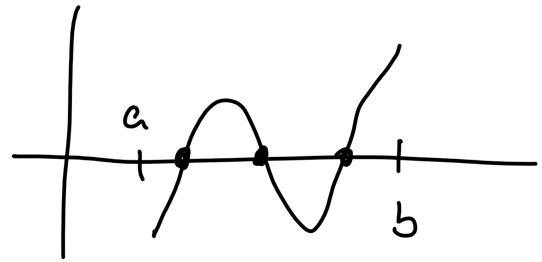
$h_{n+1} = 0.01^2 \, |\cdots|$

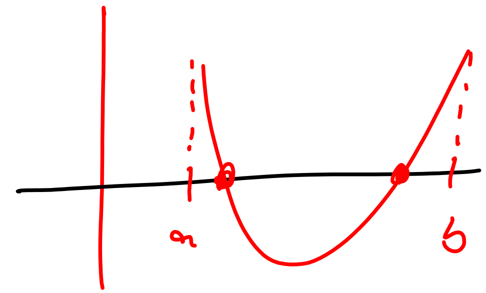

y
0
x
f(x) = y
x1
x2
x0
x0
x1
dobře
špatně

kritérium pro přítomnost kořene

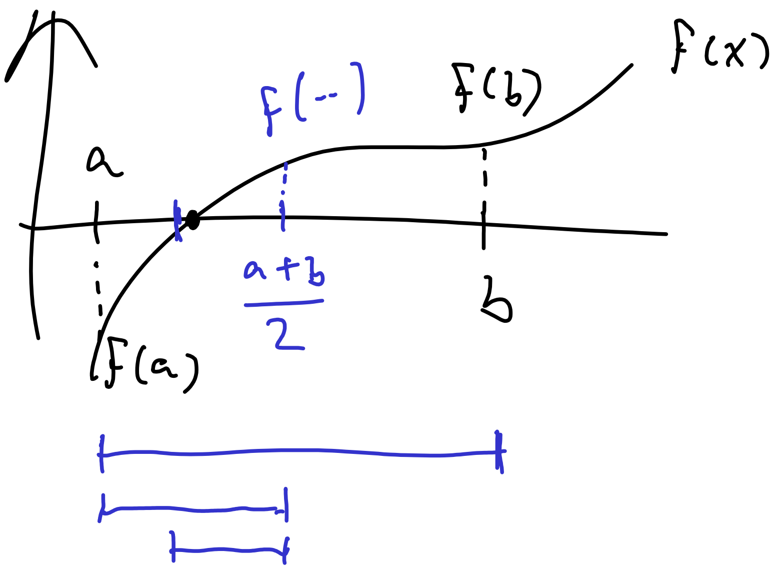
$f(a) \cdot f(b) < 0$







B1) bisekce (půlení intervalu)

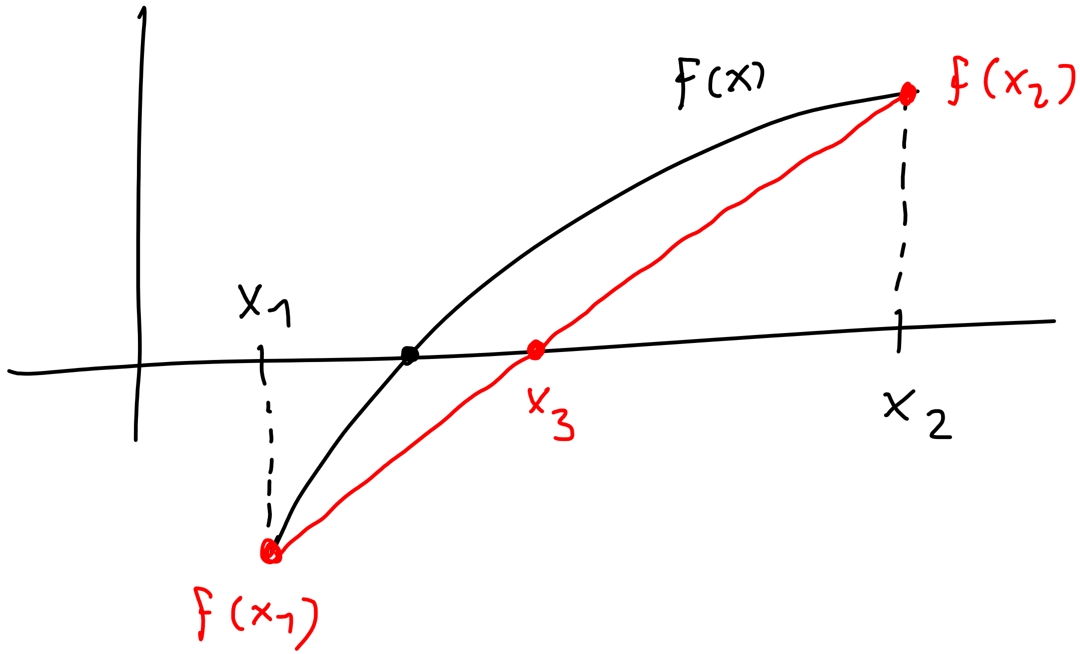


šířka po n krocích $\frac{b-a}{2^n}$ = chyba

$\log(\text{chyba}) \sim -n \underbrace{\log 2}_{\approx 0.3} \sim -0.3n$

počet des. míst $\approx |\log \text{chyba}| \approx 0.3n$

B2) metoda sečen, regula falsi



$$0 = y = f(x_1) + \frac{x_3 - x_1}{x_2 - x_1}\left[f(x_2) - f(x_1)\right]$$

$$x_3 = x_1 + (x_2 - x_1)\frac{-f(x_1)}{f(x_2) - f(x_1)}$$

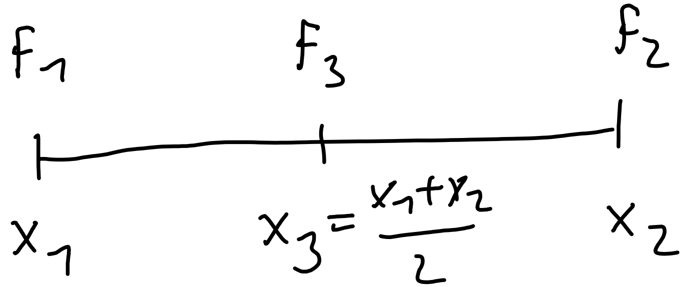
regula falsi – podinterval s kořenem

metoda sečen – $[x_2, x_3]$

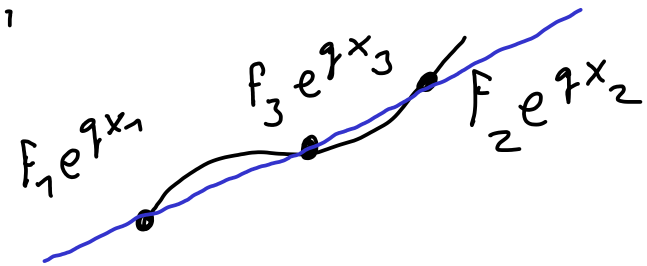
B3) Riddersova metoda

"napravíme" zakřivení $f(x)$ a použijeme regula falsi

$f(x) = 0 \qquad f(x)e^{qx} = 0$ .. "napravená" funkce



"napraveno!"



$$F_3 e^{qx_3} = \frac{1}{2}\left(F_1 e^{qx_1} + F_2 e^{qx_2}\right)$$

$$F_1 e^{qx_1} - 2F_3 e^{qx_3} + F_2 e^{qx_2} = 0 \quad / e^{qx_1}$$

$$e^{q(x_3-x_1)} = e^{Q}$$
$$e^{q(x_2-x_1)} = e^{2Q}$$

$$F_1 - 2F_3 e^{Q} + F_2 (e^{Q})^2 = 0 \quad \longrightarrow \quad e^{Q} = \frac{1}{F_2}\left(F_3 + \operatorname{sgn} F_2 \cdot \sqrt{F_3^2 - F_1 F_2}\right)$$

regula falsi

$$x_4 = x_1 + (x_3 - x_1)\frac{-F_1}{F_3 e^{Q} - F_1} = x_3 + (x_3 - x_1)\frac{\operatorname{sgn}(F_1 - F_2) F_3}{\sqrt{F_3^2 - F_1 F_2}}$$