Cvičení - stáří Země

zirkon $ZrSiO_4$ - sledování izotopů U, Pb

při krystalizaci jen U

$^{238}U \xrightarrow{\tau_1} {}^{206}Pb$

$^{235}U \xrightarrow{\tau_2} {}^{207}Pb$

T .... doba rozpadu

$\tau_1 = 4{,}5 \cdot 10^9$ let

$\tau_2 = 0{,}71 \cdot 10^9$ let

$$N(^{238}U) = N_1 e^{-\frac{T}{\tau_1}}$$

$$N(^{206}Pb) = N_1 \left(1 - e^{-\frac{T}{\tau_1}}\right)$$

$$N(^{235}U) = N_2 e^{-\frac{T}{\tau_2}}$$

$$N(^{207}Pb) = N_2 \left(1 - e^{-\frac{T}{\tau_2}}\right)$$

$$\frac{^{238}U}{^{235}U} = \frac{137}{1}$$

$$\frac{^{206}Pb}{^{207}Pb} = \frac{11.7}{7.1}$$

$$\frac{^{206}Pb}{^{207}Pb}\,\frac{^{235}U}{^{238}U} = \frac{N_1 \left(1 - e^{-\frac{T}{\tau_1}}\right)}{N_2 \left(1 - e^{-\frac{T}{\tau_2}}\right)}\,\frac{N_2 e^{-\frac{T}{\tau_2}}}{N_1 e^{-\frac{T}{\tau_1}}}$$

$$r = \frac{^{206}Pb}{^{207}Pb}\,\frac{^{235}U}{^{238}U} = \frac{N_1\left(1 - e^{-\frac{T}{\tau_1}}\right)}{N_2\left(1 - e^{-\frac{T}{\tau_2}}\right)}\,\frac{N_2\, e^{-\frac{T}{\tau_2}}}{N_1\, e^{-\frac{T}{\tau_1}}}$$

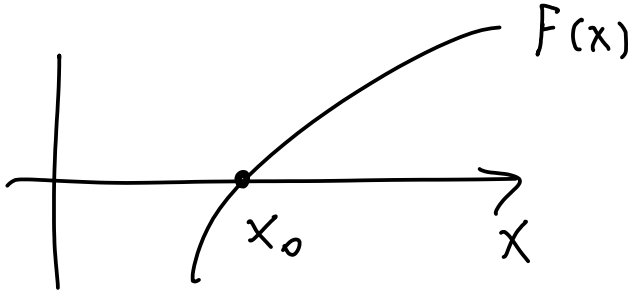
$$r = \frac{e^{\frac{T}{\tau_1}} - 1}{e^{\frac{T}{\tau_2}} - 1} \qquad F(x) = 0$$

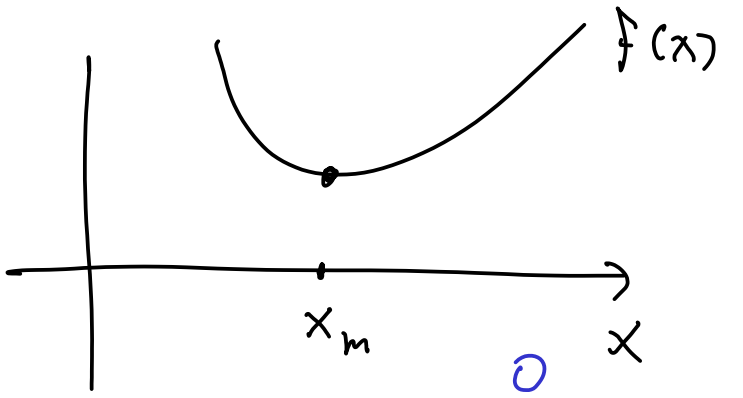
$$\frac{e^{\frac{T}{\tau_1}} - 1}{e^{\frac{T}{\tau_2}} - 1} - r = 0$$

Minima fce 1 proměnné

$f(x)$ minimum v x_m

1) přesnost





$f(x_0) = 0$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \mathcal{O}(\delta^2)$$

$f(x_0)$: 0; $f'(x_0)$: konst.

Δ_f chyba f

Δ_x chyba x_0

15-16 míst

$$\Delta_x = \frac{\Delta_f}{|f'(x_0)|}$$

15-16 míst

$$f(x) = f(x_m) + \underbrace{f'(x_m)}_{0}(x - x_m) + \frac{1}{2} f''(x_m)(x - x_m)^2 + \mathcal{O}(\delta^3)$$

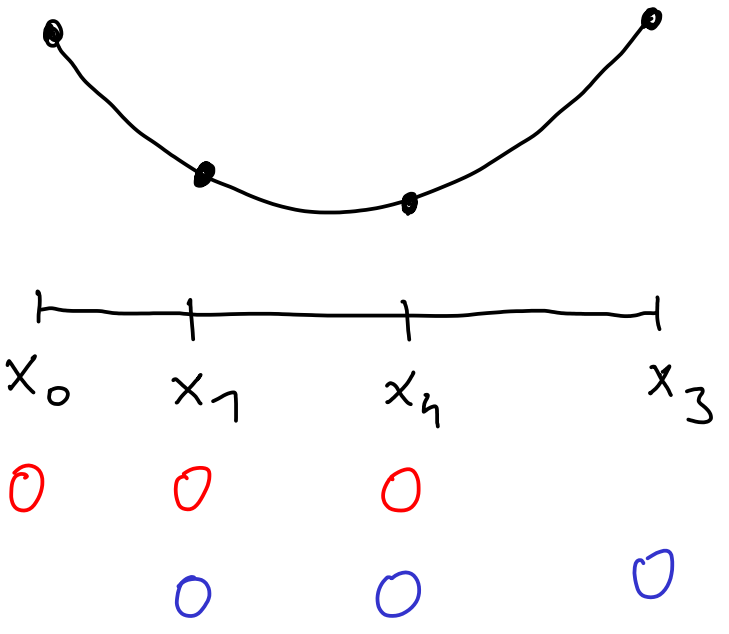
$$\Delta_f = \frac{1}{2} f''(x_m) \Delta_x^2$$

15-16 míst

7-8 míst

$$\Delta_x = \sqrt{\frac{\Delta_f}{\frac{1}{2} f''(x_m)}}$$

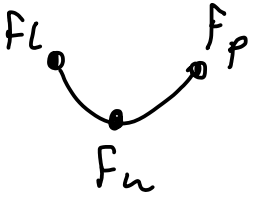
2) metoda zlatého řezu



přidáme x_4 do delšího podintervalu

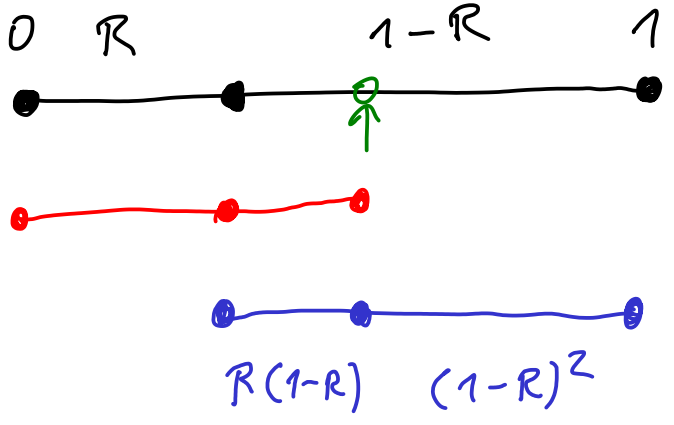
vybereme x_0, x_1, x_4 nebo x_1, x_4, x_3

kritériem



$f_4 < f_L, f_P$

opakujeme, až je interval dostatečně malý



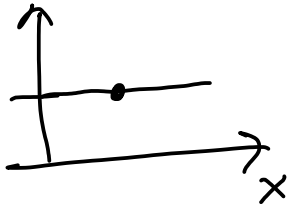
$$R + R(1-R) = 1-R \rightarrow R = \frac{3-\sqrt{5}}{2} \approx 0.38$$

zlatý řez $\frac{1-R}{R} = \frac{1+\sqrt{5}}{2} \approx 1.618$

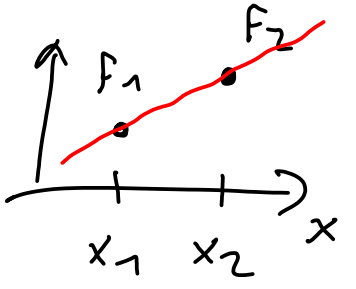
Interpolace polynomem + numerická derivace

$N+1$ bodů $\rightarrow$ polynom stupně N $\quad P_N(x) = \sum_{n=0}^{N} a_n x^n$

$N = 0$



$N = 1$



$$f(x) = f_1 + \frac{f_2 - f_1}{x_2 - x_1}(x - x_1)$$

obecné N

$(x_i, f_i) \quad i = 0, 1, \dots, N$

$P_N(x_i) = f_i \quad$ pro $\forall i$

$$a_0 + a_1 x_0 + a_2 x_0^2 + \dots = f_0$$

$$a_0 + a_1 x_1 + a_2 x_1^2 + \dots = f_1$$

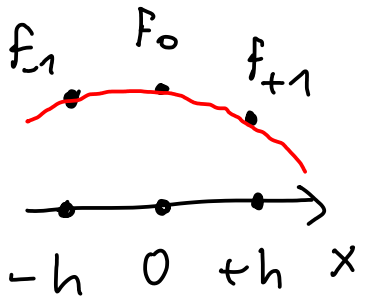
$$a_0 + a_1 x_2 + a_2 x_2^2 + \dots = f_2$$

$\dots$

maticový zápis pro N:

$$\begin{pmatrix} 1 & x_0 & x_0^2 & x_0^3 & \dots & x_0^N \\ 1 & x_1 & x_1^2 & x_1^3 & \dots & x_1^N \\ 1 & x_2 & x_2^2 & x_2^3 & \dots & x_2^N \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & x_N & x_N^2 & x_N^3 & & x_N^N \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_N \end{pmatrix}$$

Symetrický případ s $N = 2$



$$f(x) = a_0 + a_1 x + a_2 x^2$$

$$f'(x) = a_1 + 2a_2 x \qquad f'(0) = a_1$$

$$f''(x) = 2a_2$$

$$\begin{pmatrix} 1 & -h & h^2 \\ 1 & 0 & 0 \\ 1 & h & h^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} f_{-1} \\ f_0 \\ f_{+1} \end{pmatrix}$$

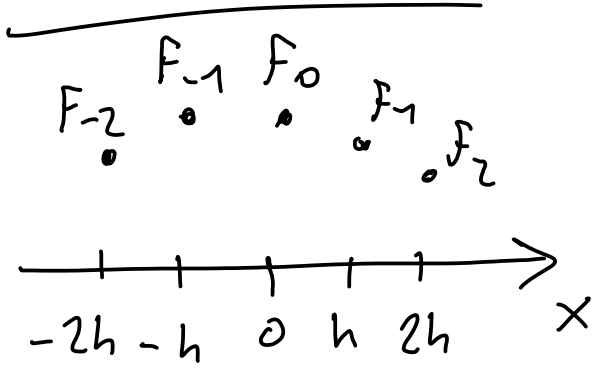
$$a_0 = f_0$$

$$2h a_1 = f_{+1} - f_{-1}$$

$$a_1 = \frac{f_{+1} - f_{-1}}{2h}$$

$$2h^2 a_2 = f_1 - 2f_0 + f_{-1}$$

$$2a_2 = \frac{f_1 - 2f_0 + f_{-1}}{h^2}$$



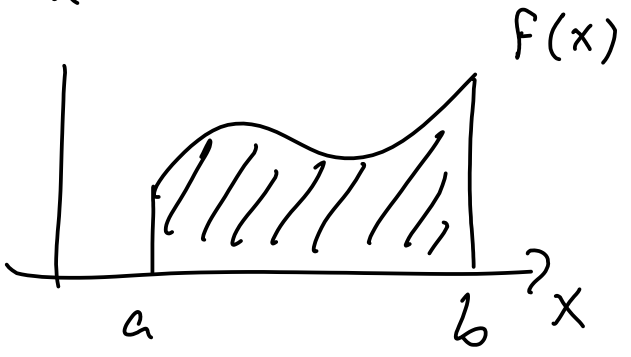
$$f'(0) \approx \frac{-f_2 + 8f_1 - 8f_{-1} + f_{-2}}{12h}$$

$$f''(0) \approx \frac{-f_2 + 16f_1 - 30f_0 + 16f_{-1} - f_{-2}}{12h^2}$$

Numerická kvadratura

integrační pravidlo

1) úvod



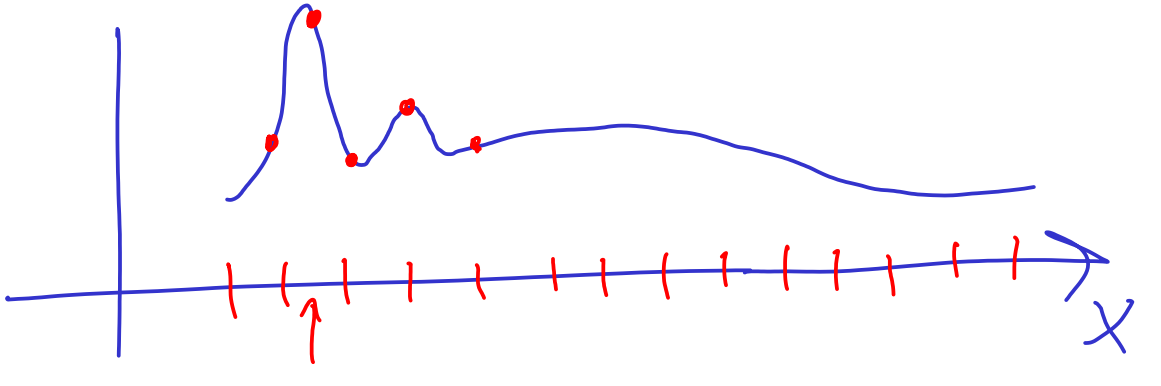
$$I = \int_a^b f(x)\,dx \approx \sum_{j=0}^{N} w_j\, f(x_j)$$

body (uzly) $x_0, x_1, \dots, x_N$

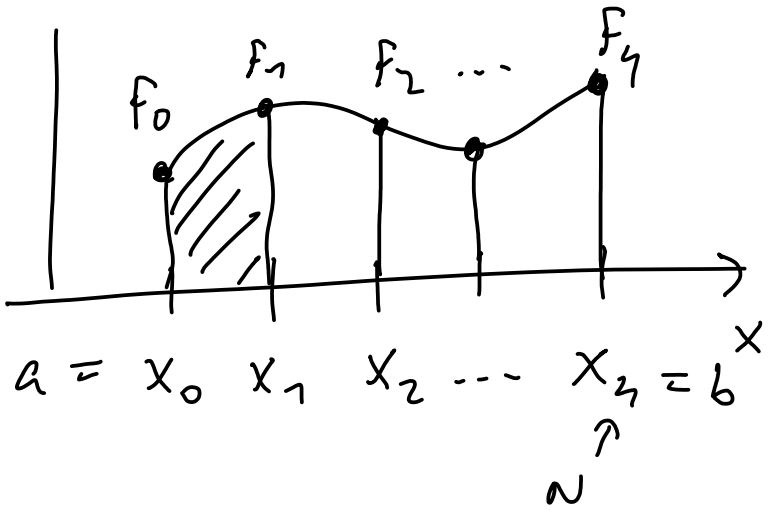
váhy $w_0, w_1, \dots, w_N$

metody
- ekvidistantní uzly (pravidlo obdélníkové, lichoběžníkové, Simpsonovo)
- neekvidist. body (Gaussova)

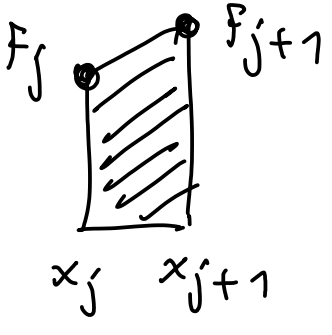
metody
- Fixní mřížka
- adaptivní



klasická integrační pravidla

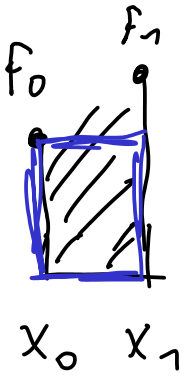


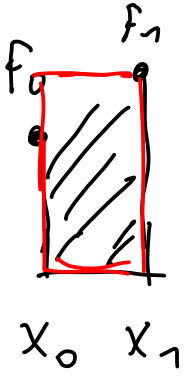
lichoběžníkové



$$(x_{j+1} - x_j)\,\frac{f_j + f_{j+1}}{2}$$

obdélníkové





$$I = \frac{b-a}{N}\left(f_0 + f_1 + f_2 \dots + f_{N-1}\right)$$

$$I = \frac{b-a}{N}\left(f_1 + f_2 + \dots + f_N\right)$$

$$I = \frac{b-a}{N}\left(\frac{1}{2}f_0 + f_1 + f_2 + \dots + f_{N-1} + \frac{1}{2}f_N\right)$$

chyba $\sigma(h^3) \times N$