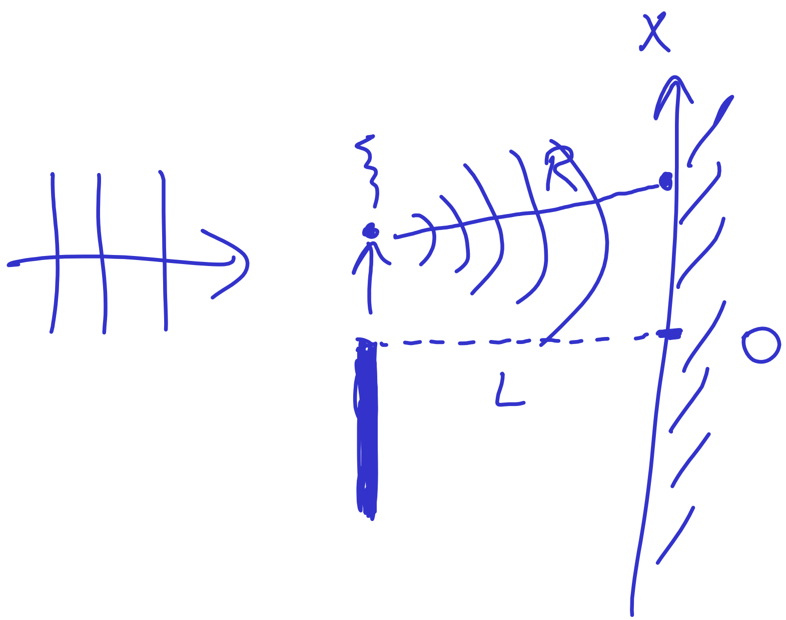
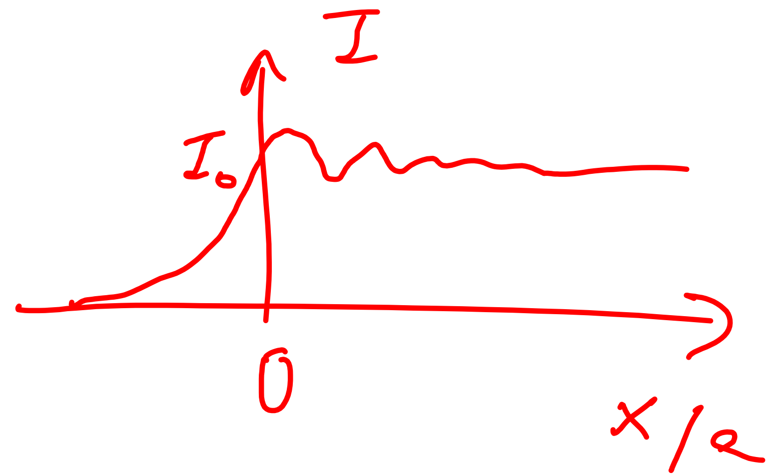
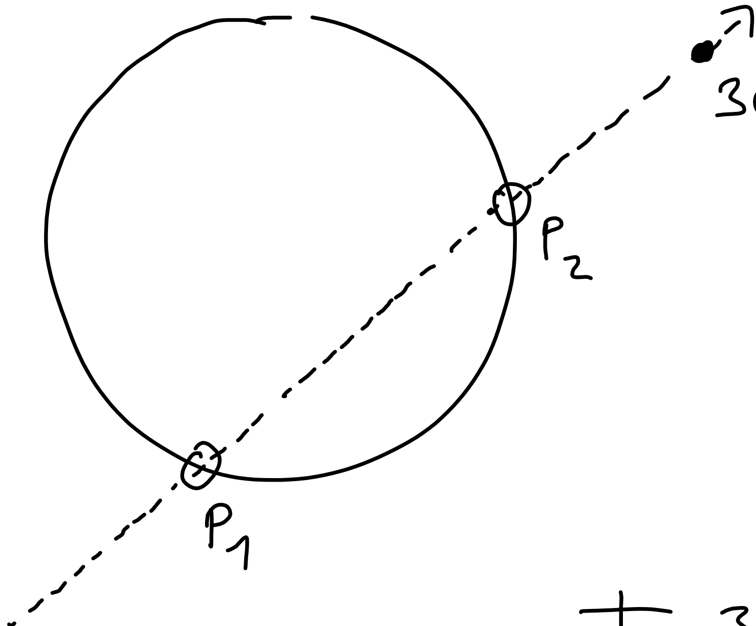
Fresnelova difrakce na hraně

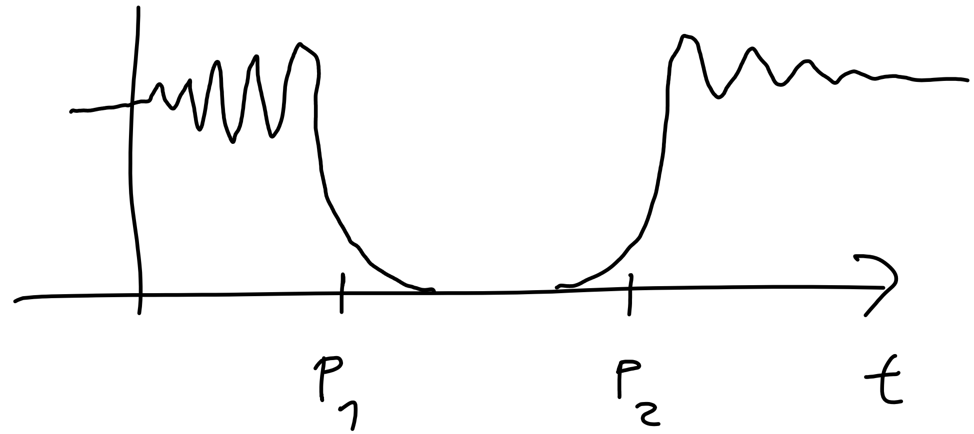


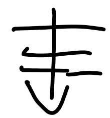


$$I(x) = \frac{I_0}{2}\left\{\left[C\left(\frac{x}{a}\right)+\frac{1}{2}\right]^2 + \left[S\left(\frac{x}{a}\right)+\frac{1}{2}\right]^2\right\} \qquad a = \sqrt{\frac{\pi L}{k}} = \sqrt{\frac{L\lambda}{2}}$$

Fresnelovy integrály $\quad C(w) = \int_0^w \cos\frac{\pi u^2}{2}\,du \qquad S(w) = \int_0^w \sin\frac{\pi u^2}{2}\,du$





 3C273

$$a = \sqrt{\frac{L\lambda}{2}}$$

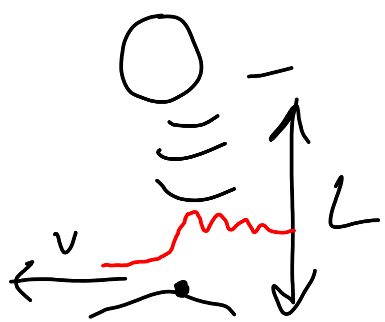
$$v = \frac{2\pi L}{T} \qquad T = 28\,d \qquad v \approx 1\,km/s$$

$$L = 4 \cdot 10^8\,m$$

$$\frac{x}{a} = \frac{vt}{\sqrt{\frac{Lc}{2f}}} \approx 5.0\,\frac{t}{min}$$

$$x = vt$$

$$\lambda = \frac{c}{f} \qquad f = 410\,MHz$$



Obyčejné diferenciální rovnice s počátečními podmínkami

$x_1(t) \ \dots \ x_N(t)$ $\qquad \bar{x}(t_0) = \bar{x}_0$ poč. podmínka

systém ODR

$$\frac{d\bar{x}}{dt} = \bar{F}(t, \bar{x})$$

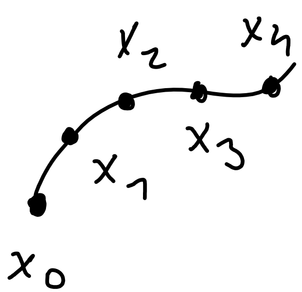
rozepsáno na

$$\frac{dx_1}{dt} = F_1(t, x_1, \dots, x_N)$$

$$\vdots$$

$$\frac{dx_N}{dt} = F_N(t, x_1, \dots, x_N)$$

Eulerova metoda (1768)



$$x_{n+1} = x_n + F(t_n, x_n)\, h$$

$$t_{n+1} = t_n + h$$

Runge-Kutta

Hennova metoda

$$x(t+h) = x(t) + \underbrace{x'(t)}_{f(t,x)} h + \frac{1}{2} \underbrace{x''(t)}_{\frac{d}{dt} x'(t)} h^2 + \sigma(h^3)$$

$$x''(t) = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} x'(t)$$

všechny $\frac{\partial f}{\partial \ldots}$ v t, x

$$f(t+h, x+hf) = f(t,x) + \frac{\partial f}{\partial t} h + \frac{\partial f}{\partial x} hf + \sigma(h^2)$$

$$x''(t) = \frac{f(t+h, x+hf) - f(t,x)}{h} + \sigma(h)$$

$$x_{n+1} = x_n + h f(t_n, x_n) + \frac{1}{2h}\left[f(t_n+h, x_n + h f(t_n, x_n)) - f(t_n, x_n)\right] h^2$$

postup: $k_1 = h f(t_n, x_n)$ $\quad k_2 = h f(t_n+h, x_n + k_1)$ $\quad x_{n+1} = x_n + \frac{1}{2}(k_1 + k_2)$

Runge-Kutta 4. řádu ($O(h^5)$ lokálně)

$$k_1 = h f(t_n, x_n)$$
$$k_2 = h f(t_n + h/2,\ x_n + k_1/2)$$
$$k_3 = h f(t_n + h/2,\ x_n + k_2/2)$$
$$k_4 = h f(t_n + h,\ x_n + k_3)$$

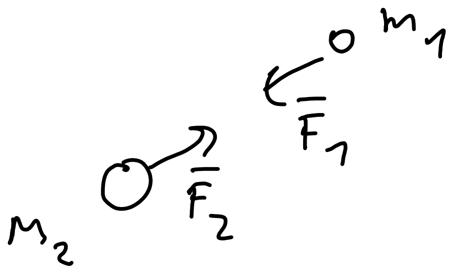
$$t_{n+1} = t_n + h$$
$$x_{n+1} = x_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

- volba kroku

lok. chyba metody řádu $p \sim h^{p+1}$ $\longrightarrow$ odhad glob. chyby $\longrightarrow$ škálování h

$$x_1' = x_1 \qquad x_1(0) = 1 \quad \rightarrow \quad x_1(t) = e^t$$
$$x_2' = -x_2 \qquad x_2(0) = 1 \quad \rightarrow \quad x_2(t) = e^{-t}$$

Př.



m_1: x_1, y_1, v_{x1}, v_{y1}

m_2: x_2, y_2, v_{x2}, v_{y2}

$$F = \frac{G m_1 m_2}{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

r^2

$$\frac{dx_1}{dt} = v_{x1} \qquad \frac{dx_2}{dt} = v_{x2}$$

$$\frac{dy_1}{dt} = v_{y1} \qquad \frac{dy_2}{dt} = v_{y2}$$

$$\frac{dv_{x1}}{dt} = F \, \frac{x_2 - x_1}{r} \, \frac{1}{m_1} \qquad \frac{dv_{x2}}{dt} = F \, \frac{x_1 - x_2}{r} \, \frac{1}{m_2}$$

$$\frac{dv_{y1}}{dt} = F \, \frac{y_2 - y_1}{r} \, \frac{1}{m_1} \qquad \frac{dv_{y2}}{dt} = F \, \frac{y_1 - y_2}{r} \, \frac{1}{m_2}$$

jedno těleso fixní (v počátku), $G = 1$, $m_1 = m_2 = 1$

$$\frac{dx}{dt} = v_x \qquad r = \sqrt{x^2+y^2}$$

$$\frac{dy}{dt} = v_y$$

$$\frac{dv_x}{dt} = \frac{-x}{r^3}$$

$$\frac{dv_y}{dt} = \frac{-y}{r^3}$$

```
def fun(x,t):
    r=np.sqrt(x[0]**2+x[1]**2)
    return [x[2],x[3],-x[0]/r**3,-x[1]/r**3]
```

x y

$$\begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix} \xrightarrow{\text{derivative}} \begin{pmatrix} x[2] \\ x[3] \\ -x[0]/r^3 \\ -x[1]/r^3 \end{pmatrix}$$

$$x0 = \begin{pmatrix} r_0 \\ 0 \\ 0 \\ v_0 \end{pmatrix}$$