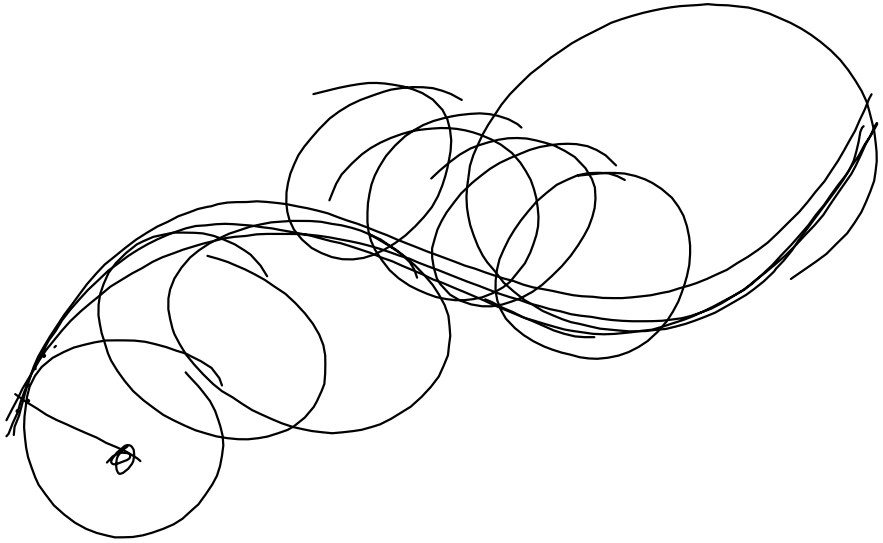
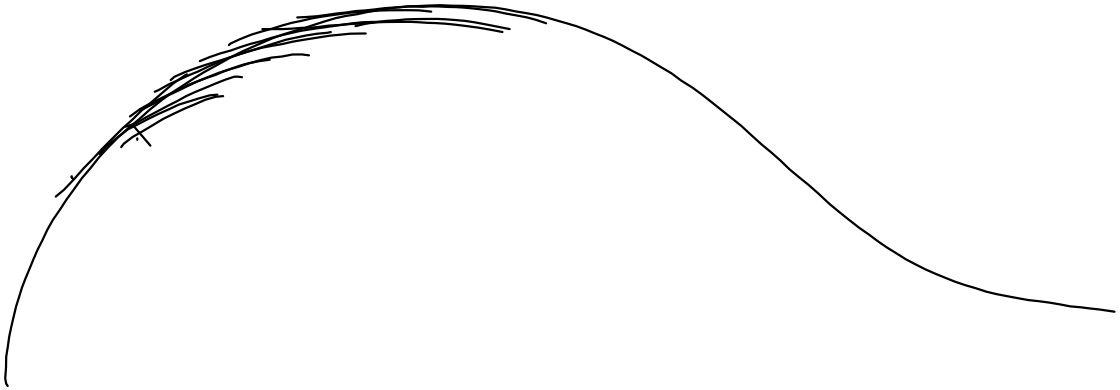
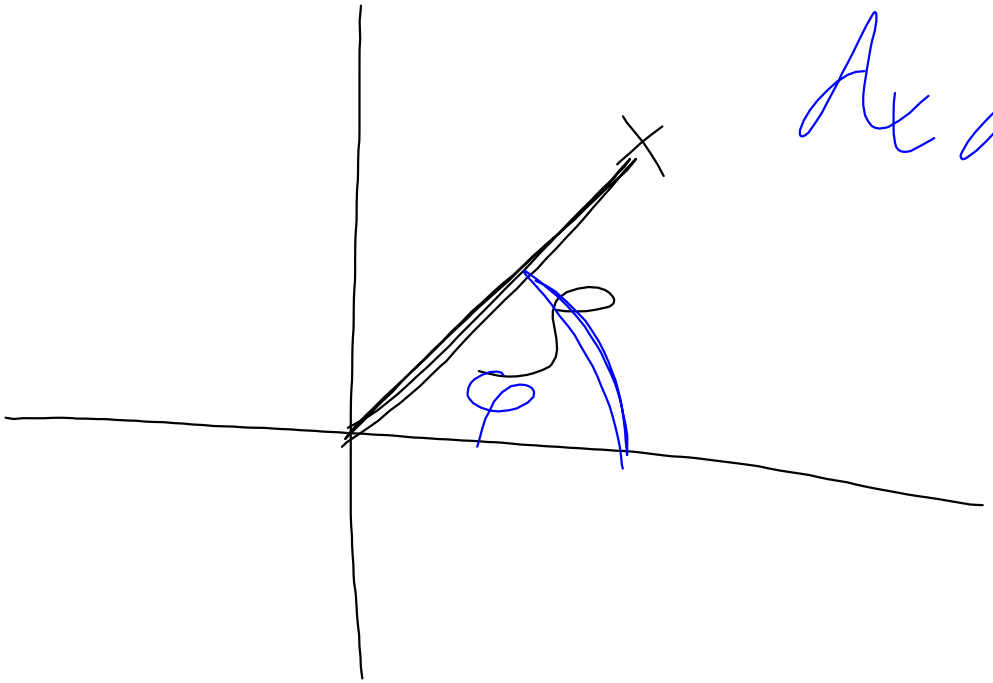
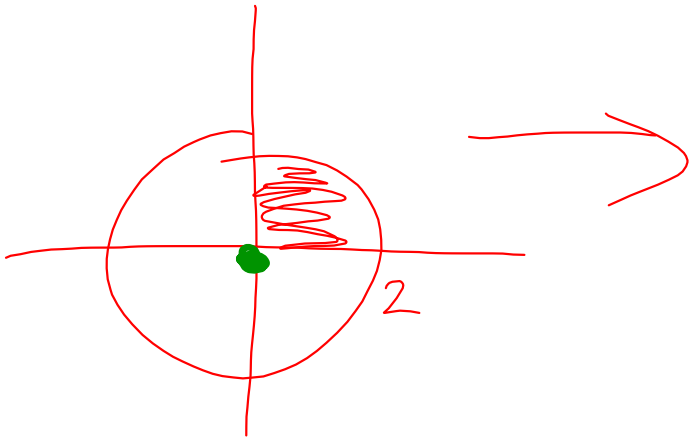


$$mv^2 \cdot \frac{1}{2}$$



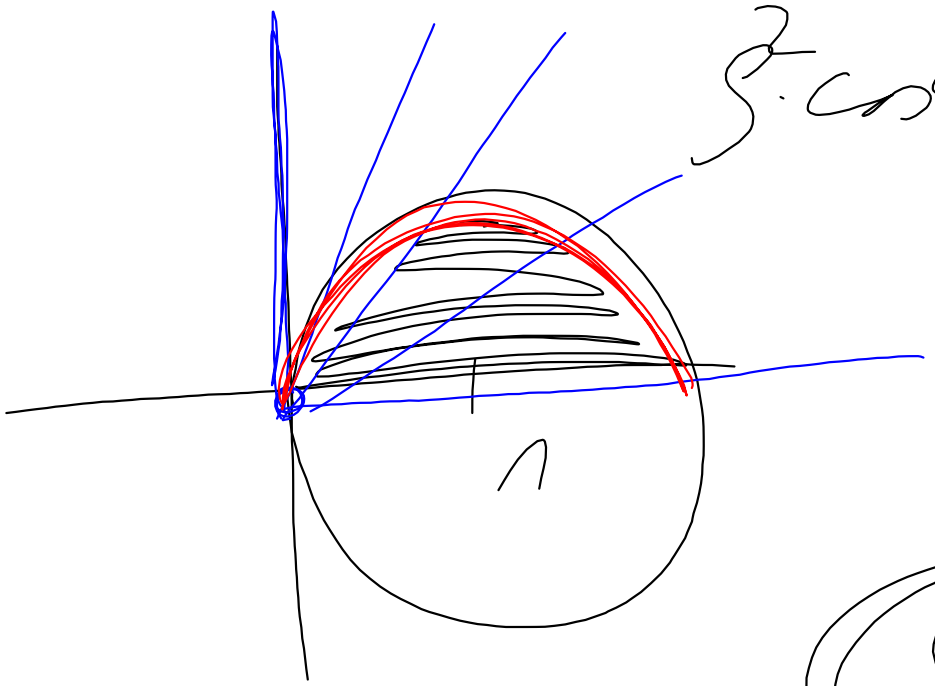


$$dx\,dy = \rho \cdot d\varphi\, d\rho$$



$\rho \in [0, 2]$

$\varphi \in [0, \frac{\pi}{2}]$

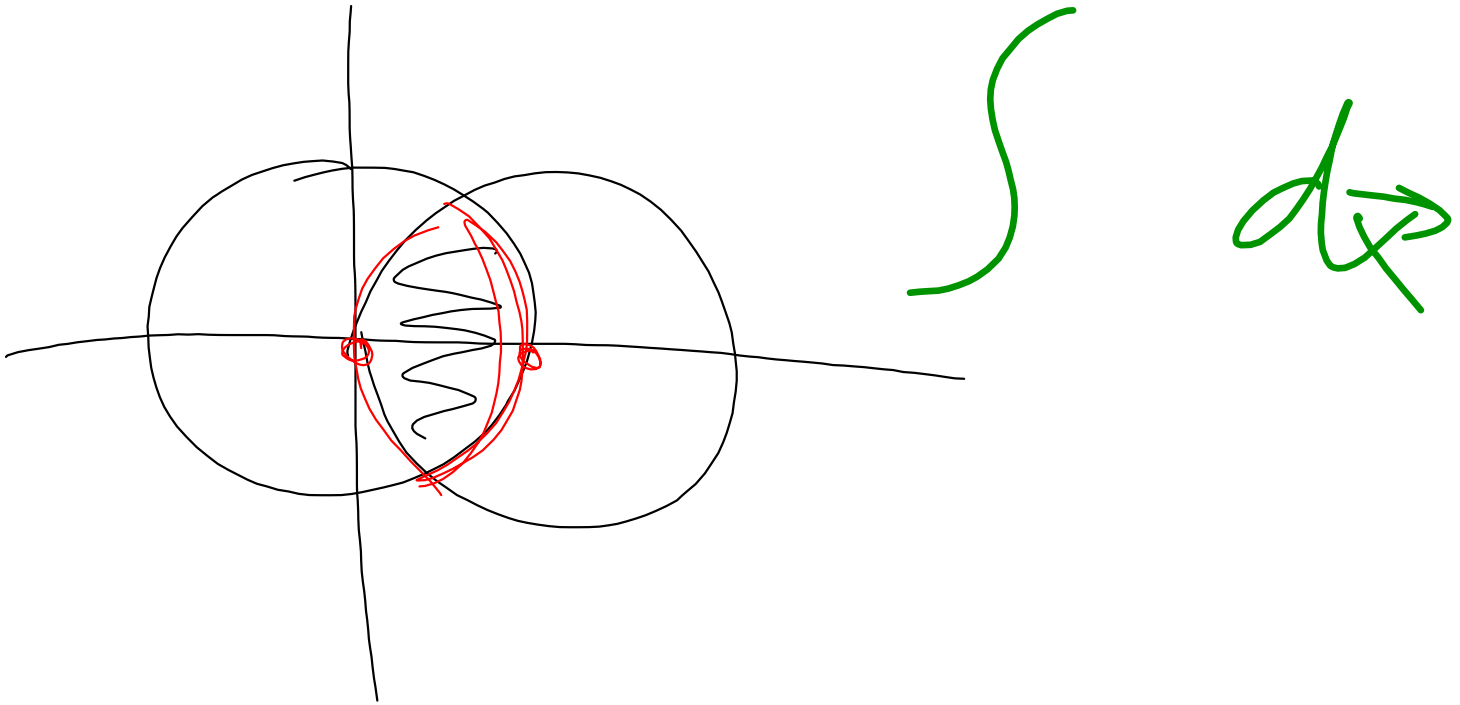


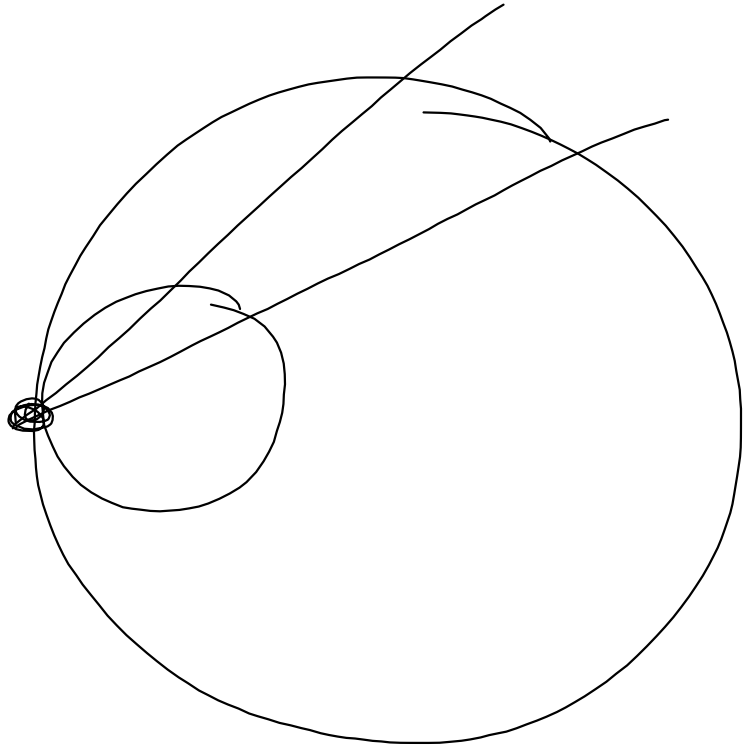
$$\rho^2\cdot\cos^2\varphi+\rho^2\cdot\sin^2\varphi=2\cdot\rho\cdot\cos\varphi$$

$$\rho^2=2\rho\cdot\cos\varphi$$

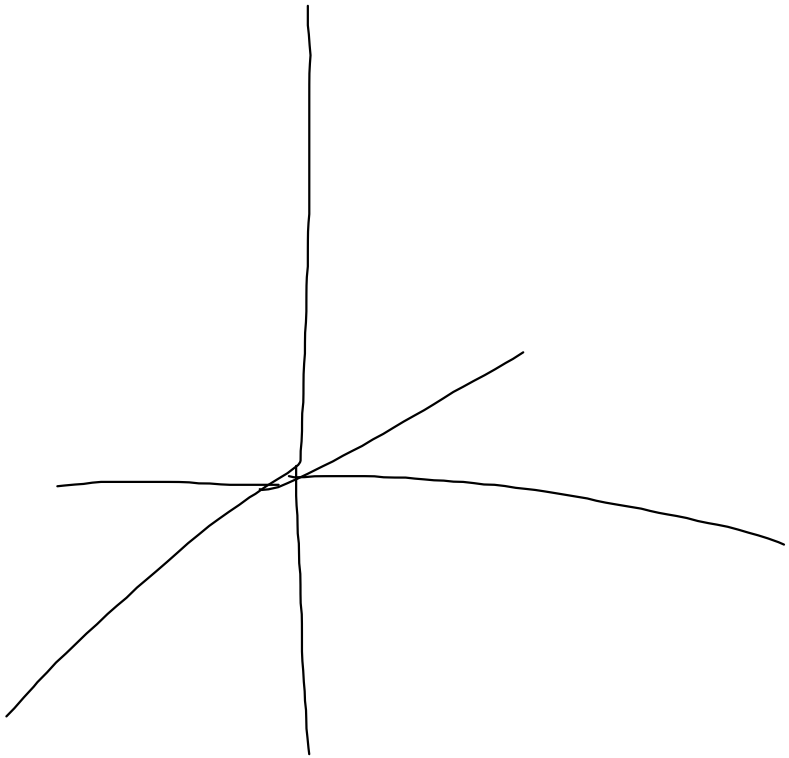
$$\rho=2\cdot\cos\varphi$$

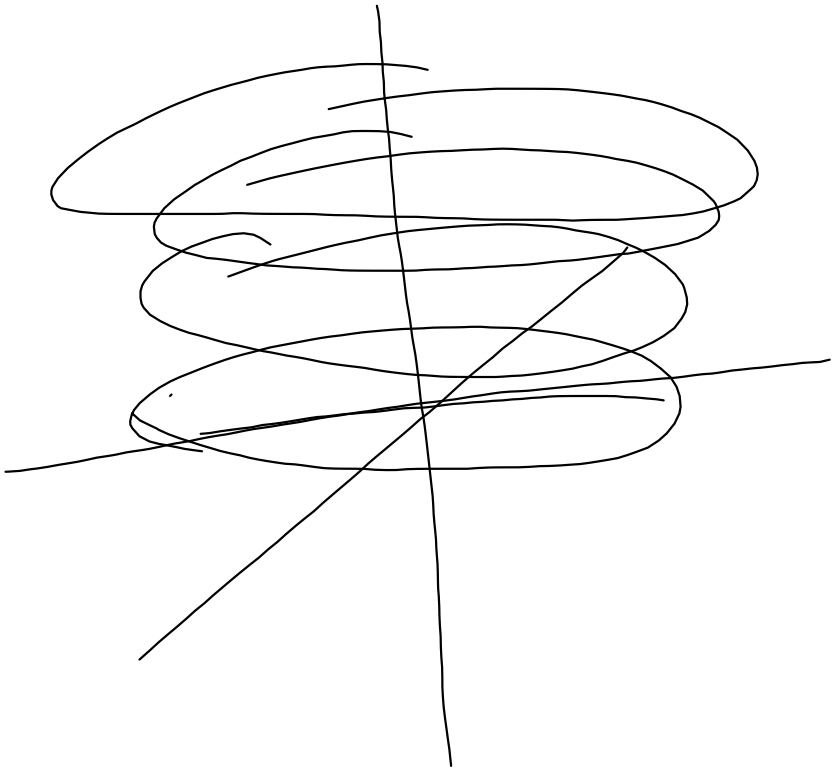
$$\left.\begin{aligned} (x - x_0) &= \rho \cdot \cos\varphi \\ (y - y_0) &= \rho \cdot \sin\varphi \end{aligned}\right| \Rightarrow \begin{aligned} x &= x_0 + \rho \cdot \cos\varphi \\ y &= y_0 + \rho \cdot \sin\varphi \end{aligned}$$

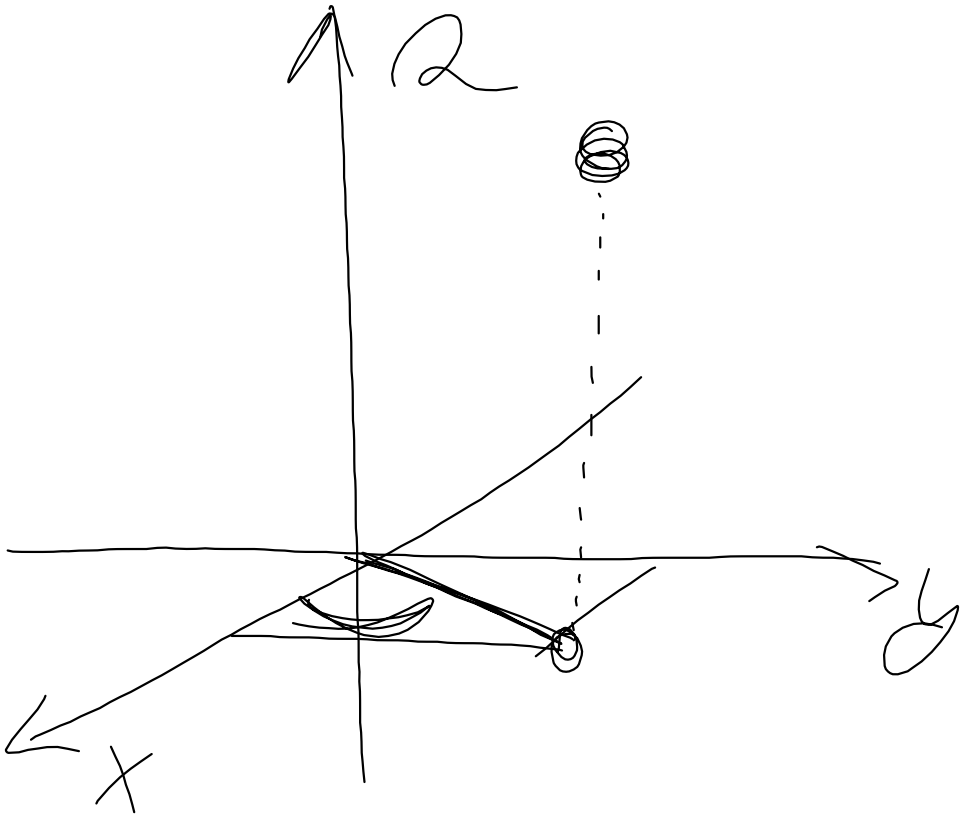


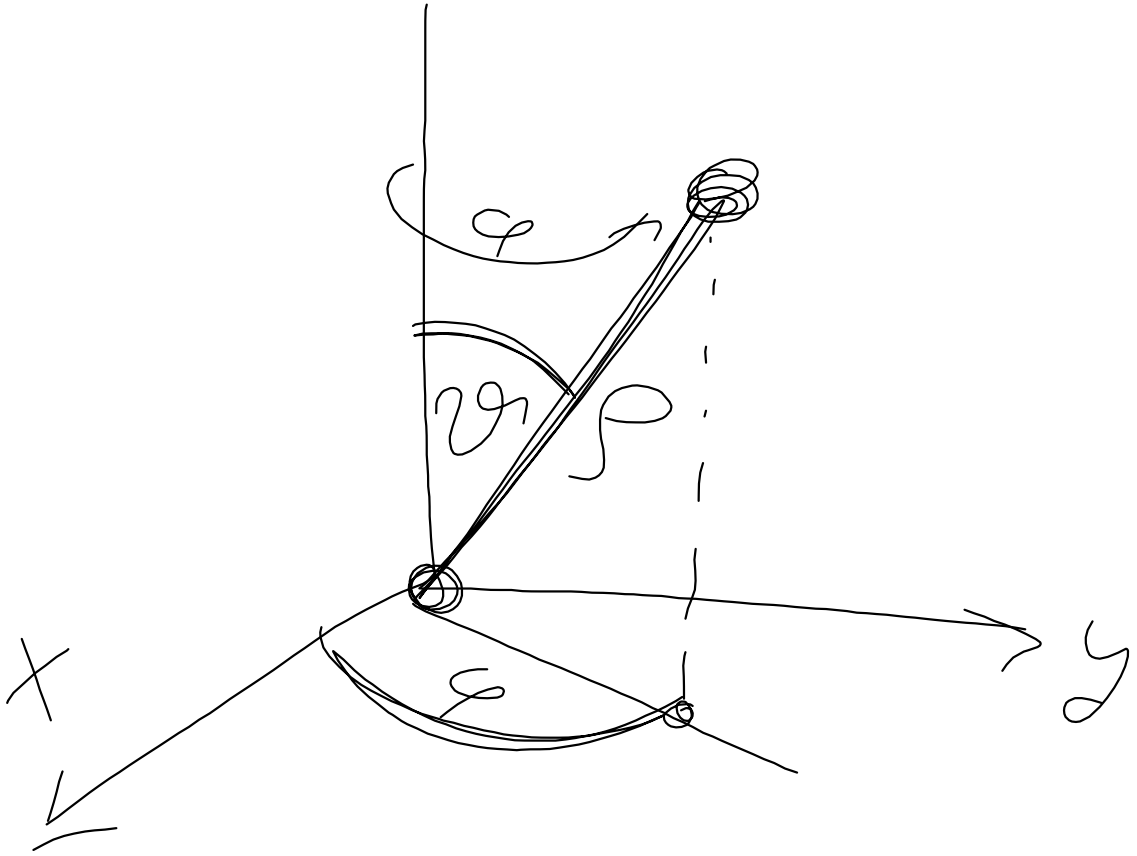


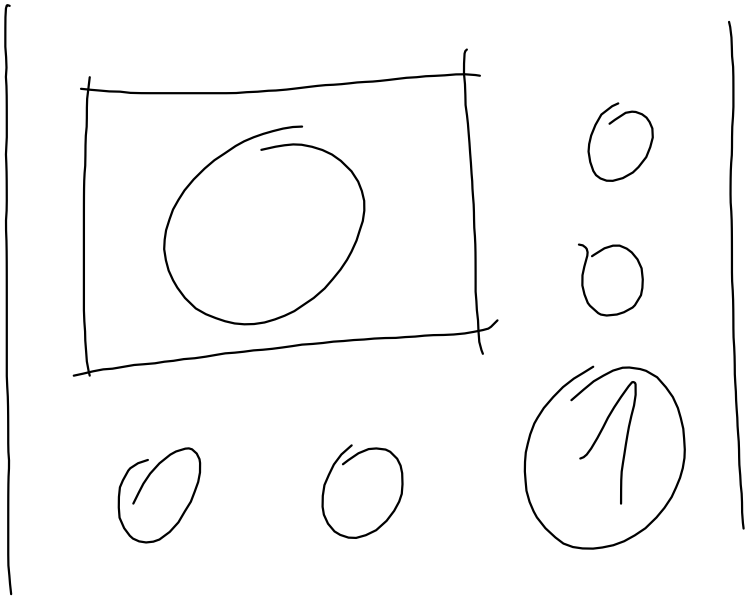
$$\iint_{R} x+y \, dx \, dy$$






z
y
x


φ
ϑ
ρ
x
y


0
0
0
1

$$x^2 + y^2 + z^2 = \rho^2$$

$$L = 0{,}000013\, m v^2$$

$X = m \in [3000, 4000]$ (lb)

$y = v \in [50, 60]$ (mi/h)

$$m(H) = (4000 - 3000) \cdot (60 - 50) = 10^4$$

$$f_{AV} = \frac{1}{m(H)} \cdot \iint_H f(x,y)\, dx\, dy =$$

$$= 10^{-4} \cdot \int_{3000}^{4000} \left(\int_{50}^{60} 0{,}000013\, m v^2\, dv \right) dm =$$

$$= 10^{-4} \int_{3000}^{4000} m \cdot \left[\frac{v^3}{3} \right]_{50}^{60} \cdot 0{,}000013\, dm \doteq 138{,}01667$$

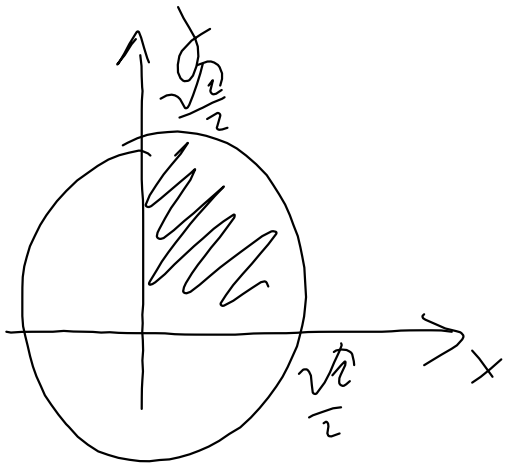
$\cdot 10^{-1}$

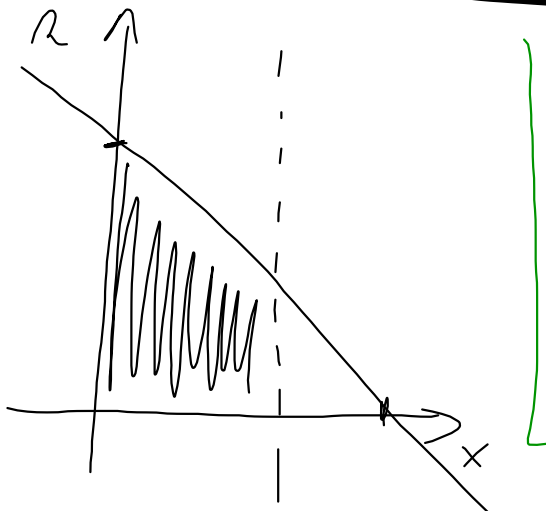
$= 0{,}13$

$$\sqrt{\iint_{\mu} (x^2+y^2)^2 \, \rho(x,y) \, dx \, dy}$$

$$\sqrt{x^2+y^2}$$

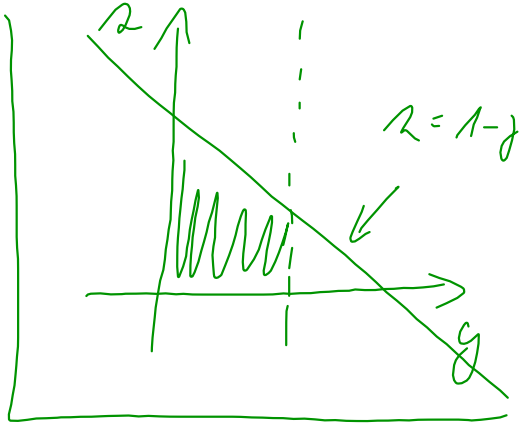
V = ? $x \geq 0, y \geq 0, z \geq 0, x+y+z \leq 1, x^2+y^2 \leq \frac{1}{2}$





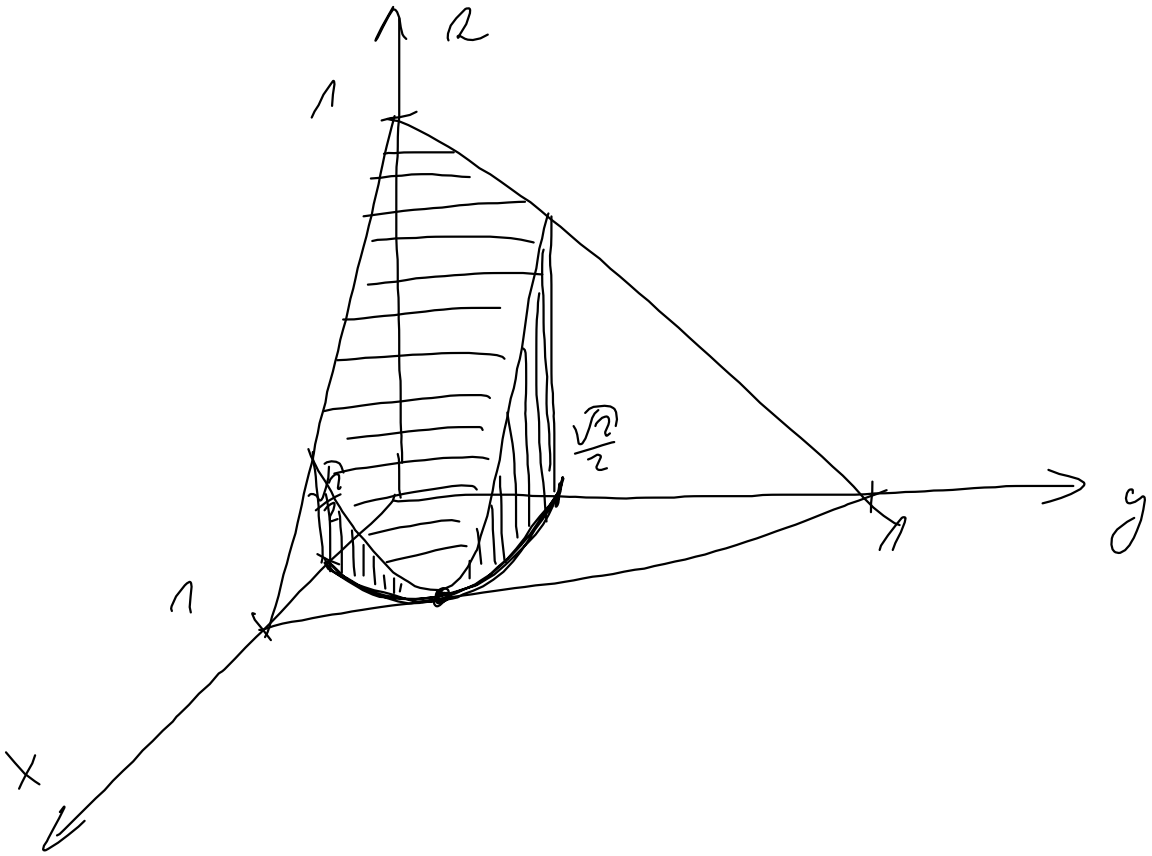
$x + z \leq 1$

$z \leq 1 - x$



$x^2 + \cancel{y^2} \leq \frac{1}{2}$

$x^2 \leq \frac{1}{2} \quad x \leq \frac{\sqrt{2}}{2}$


z
1
$\frac{\sqrt{2}}{2}$
1
1
y
x

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{\sqrt{\frac{1}{2}-x^2}} \int_0^{1-x-y} 1 \, dz \, dy \, dx = \dots$$

$$\dots = \int_0^{\frac{\sqrt{2}}{2}} \sqrt{\frac{1}{2}-x^2} - x \cdot \sqrt{\frac{1}{2}-x^2} - \frac{\frac{1}{2}-x^2}{2} \, dx = \dots$$

$$= \frac{3\pi - 4\sqrt{2}}{24}$$

$x = \rho \cdot \cos\varphi$

$y = \rho \cdot \sin\varphi$

$z = z$

$|J| = \rho$

$\rho \in [0, \frac{\sqrt{2}}{2}]$

$\varphi \in [0, \frac{\pi}{2}]$

$z \in [0, 1 - x - y] = [0, 1 - \rho \cdot \cos\varphi - \rho \cdot \sin\varphi]$

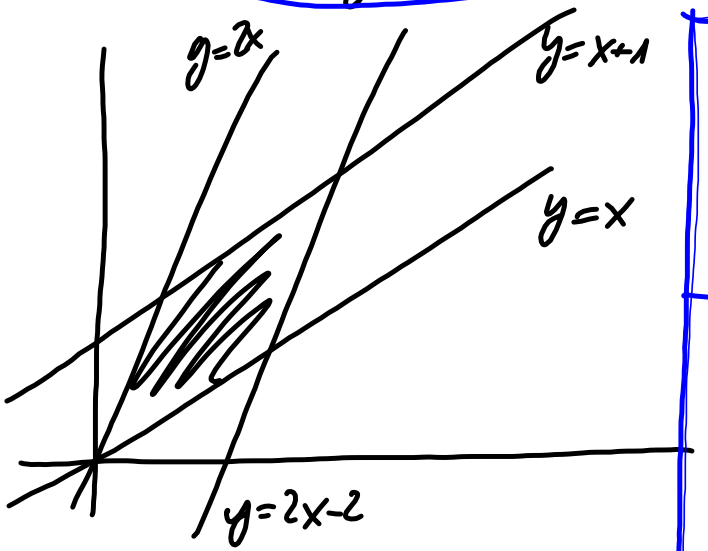
$= [0, 1 - \rho(\cos\varphi + \sin\varphi)]$

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{\frac{\pi}{2}} \int_0^{1-\rho\cdot|\cos\varphi+\sin\varphi|} 1 \cdot \rho \; dz\, d\varphi\, d\rho =$$

$= |J|$

$$= \int_0^{\frac{\sqrt{2}}{2}} \int_0^{\frac{\pi}{2}} \rho\cdot\left(1-\rho\cdot|\cos\varphi+\sin\varphi|\right) d\varphi\, d\rho = \ldots$$

$S = ?$ $\quad x \le y \le x+1, \quad 2x-2 \le y \le 2x$



$0 \le \underbrace{y-x}_{u} \le 1$

$\Rightarrow u = y-x \in [0,1]$

$-2 \le \underbrace{y-2x}_{v} \le 0$

$\Rightarrow v = y-2x \in [-2,0]$

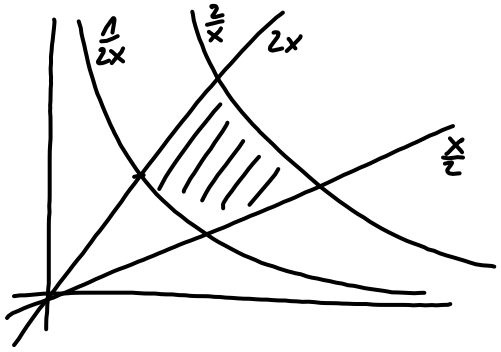
$(*) \Rightarrow x = u-v, \quad y = 2u-v$

$$J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1+2 = 1$$

$$S = \iint_h 1 \, dx\,dy = \int_0^1 \int_{-2}^{0} 1 \cdot 1 \, dv\,du = 2$$

$$\iint_A x^2 y^2 \, dx\,dy = ?$$

A:



$y = \frac{1}{2x}, \; y = \frac{2}{x} \Rightarrow \boxed{y = \frac{u}{x}}$

$u = \left[\frac{1}{2}, 2\right]$

$y = 2x, \; y = \frac{x}{2} \Rightarrow \boxed{y = v \cdot x}, \; v \in \left[\frac{1}{2}, 2\right]$

$u = x \cdot y, \; \boxed{v = \frac{y}{x}}$

$$\sim J = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix} = \frac{y}{x} + \frac{y}{x} = 2 \cdot \frac{y}{x} = J^{-1}$$

$$J = \frac{x}{2y} = \frac{1}{2} \cdot \frac{x}{v \cdot x} = \boxed{\frac{1}{2v}}$$

$x^2 \cdot y^2 = \ldots = \frac{u}{v} \cdot uv = u^2$

$$\Rightarrow \int_{\frac{1}{2}}^{2} \int_{\frac{1}{2}}^{2} u^2 \cdot \frac{1}{2v} \, dv \, du = \ldots = \frac{63}{24} \cdot \ln 2$$