Matrix Analyses

"Populační ekologie živočichů"

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Net reproductive rate (R_0)

- average total number of offspring produced by a female in her lifetime
- equals to finite growth rate

$$R_0 = \sum_{x=0}^{n} l_x m_x$$

Average generation time (T)

- average age of females when they give birth
- not valid for populations with generation overlap

$$T = \frac{\sum_{x=0}^{n} x l_x m_x}{R_0}$$

Expectation of life

- age specific expectation of life – average age that is expected for particular age class
- o .. oldest age

$$e_x = \frac{T_x}{l_x}$$ where $$T_x = \sum_{x}^{o} L_x$$ $$L_x = \frac{l_x + l_{x+1}}{2}$$

Growth rates

- Discrete time/generations

- estimate of λ (finite growth rate) from the life table:

$$\mathbf{A}\tilde{\mathbf{N}}_t = \lambda \tilde{\mathbf{N}}_t$$

where $\tilde{\mathbf{N}}_t$ is vector at stable age distribution

λ is dominant positive eigenvalue of **A**

$$\det(\mathbf{A} - \lambda \mathbf{I}) = 0$$

- or $\lambda \approx \dfrac{R_0}{T}$

- Continuous time

- r can be estimated from λ $\quad r = \ln(\lambda)$

- by approximation

$$r \approx \frac{\ln(R_0)}{T}$$

or by Euler-Lotka method

- valid only for population with SCD

$$1 = \sum_x^{\omega} l_x m_x e^{-rx}$$

Stable Class distribution (SCD)

- relative abundance of different life history age/stage/size categories
  - population approaches stable age distribution:

    $N_0 : N_1 : N_2 : N_3 : ... : N_s$ is stable

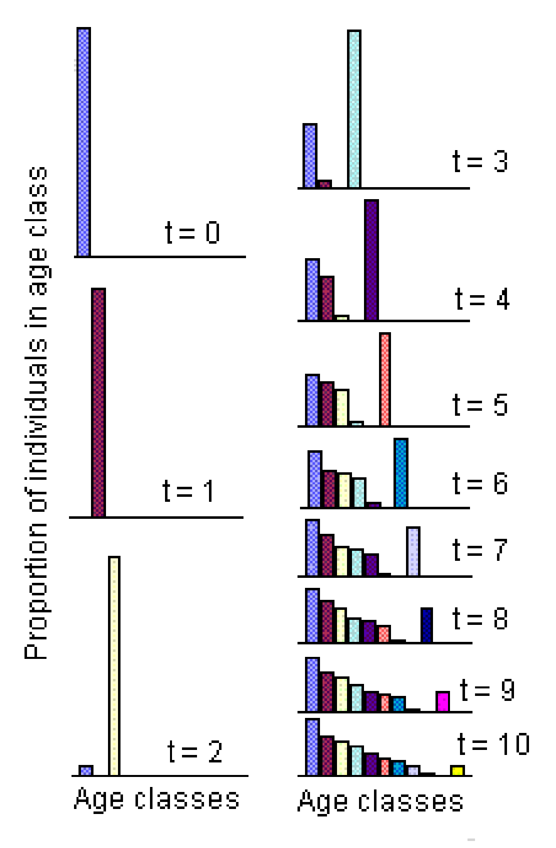
- once population reached SCD it grows exponentially

  - $\mathbf{w}_1$ .. right eigenvector (vector of the dominant eigenvalue)

- provides stable age distribution
- scale $\mathbf{w}_1$ by sum of individuals

$$\mathbf{A}\mathbf{w}_1 = \lambda_1 \mathbf{w}_1$$

$$SCD = \frac{\mathbf{w}_1}{\sum_{i=1}^{S} w_{1i}}$$



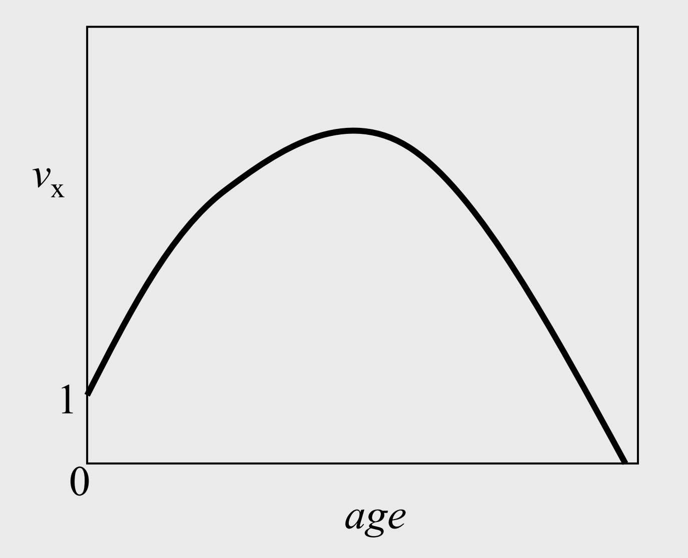
Reproductive value (v_x)

- measures relative reproductive potential and identifies age class that contributes most to the population growth (Fisher 1930)
- such class is under highest selection force
- sum of all expected offspring produced in age x and further
- when population increases then early offspring contribute more to v_x than older ones
- is a function of fertility and survival
- $\mathbf{v}_1$ .. left eigenvector (vector of the dominant eigenvalue of transposed **A**)

$$\mathbf{v}_1 \mathbf{A}' = \lambda_1 \mathbf{v}_1$$

- $\mathbf{v}_1$ is proportional to the reproductive valu and scaled to the first category (class 1 = 1)

$$v_x = \frac{v_{1x}}{v_{11}} \qquad x \neq 1$$



Sensitivity (*s*)

- identifies which process (p, F, G) has largest effect on the population increase (λ_1)
- measures absolute change

- examines change in λ_1 given small change in processes (a_{ij})
- sensitivity is larger for survival of early, and for fertility of older classes
- not used for postreproductive census with class 0

$$s_{ij} = \frac{v_{ij} w'_{ij}}{\langle \mathbf{v}, \mathbf{w} \rangle}$$

← sum of pairwise products

Elasticity (*e*)

- weighted measure of sensitivity

- measures relative contribution to the population increase
- impossible transitions = 0

$$e_{ij} = \frac{a_{ij}}{\lambda_1} s_{ij}$$

Conservation biology (Management)

- to adopt means for population promotion (threatened) or control (pests) or sustainable yield
- in populations with short generation time and higher natality population decline stabilisation will take some delay

Conservation/control procedure

1. Construction of a life table
2. Estimation of the intrinsic rates
3. Sensitivity analysis - helps to decide where conservation /control efforts should be focused - on parameters with high elasticities
4. Development and application of management plan
5. Prediction of future